Technical Appendix to “Inflation Rate and Nominal Exchange Rate Volatility brought about by Optimal Monetary Policy under Local Currency Pricing”

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A Welfare Criterion for the LCP Model

The second-order Taylor expansion of $U_t = \frac{1}{1-\sigma} C^1_t - \frac{1}{1+\varphi} N^{1+\varphi}_t$ with $UCN = 0$ is given by:

$$
\frac{U_t - U}{UCN} = c_t + \frac{1-\sigma}{2} c_t^2 + \frac{UNN}{UCN} \left( n_t + \frac{1+\varphi}{2} n_t^2 \right) + o(\|\xi\|^3)
$$

$$
= c_t + \frac{1-\sigma}{2} c_t^2 - (1-\Phi)^{-1} \left[ c_t + \frac{\eta_2}{2} s_t + d_t - a_t + \frac{1+\varphi}{2} \left( c_t + \frac{\eta}{2} s_t + d_t - a_t \right)^2 \right] + o(\|\xi\|^3)
$$

with $\Phi \equiv 1 - \zeta (1 - \tau)$. Because we assume $Ψ = 0$, this equality can be rewritten as:

$$
\frac{U_t - U}{UCN} = -\frac{1}{2} \left[ \eta_2 s_t + 2 d_t + (\sigma + \varphi) \left( y_t^W \right)^2 + (1 + \varphi) \eta_2 W_s t - (1 + \varphi) 2 y_t^W a_t \right.
$$

$$
\left. + \left( 1 + \varphi \right) \eta_2 s_t - (1 + \varphi) \eta_2 s_t a_t \right] + \text{t.i.p.} + o(\|\xi\|^3). \quad (A.1)
$$

Likewise, we have:

$$
\frac{U_t^* - U}{UCN} = -\frac{1}{2} \left[ - \eta_2 s_t + 2 d_t^* + (\sigma + \varphi) \left( y_t^W \right)^2 - (1 + \varphi) \eta_2 W_s t - (1 + \varphi) 2 y_t^W a_t^* \right.
$$

$$
\left. + \left( 1 + \varphi \right) \eta_2 s_t + (1 + \varphi) \eta_2 s_t a_t^* \right] + \text{t.i.p.} + o(\|\xi\|^3). \quad (A.2)
$$

We define $W_{LCP,t}^W \equiv W_{LCP,t}^W + W^*_{LCP,t}$ with $W \equiv \frac{U_t - U}{UCN}$ and $W_t^* \equiv \frac{U_t^* - U}{UCN}$. Plugging Eqs. (A1) and (A2) into the definition of $W_t^W$ yields:

$$
W_t = -\frac{1}{2} \left[ d_t + d_t^* + (\sigma + \varphi) \left( \frac{y_t^W}{\sigma + \varphi} \right)^2 + (1 + \varphi) \eta_2 \left( s_t - \frac{1}{\eta} a_t^W \right)^2 \right]
$$

$$
+ \text{t.i.p.} + o(\|\xi\|^3),
$$

1
\[
\begin{align*}
\frac{d_t + d_t^*}{t} + (\sigma + \varphi)(y_t^{W} - \bar{y}_t^{W})^2 + \left(\frac{1 + \varphi^2}{4}\right)\xi_t^2 + \text{t.i.p.} + o(\|\xi\|^3),
\end{align*}
\]

where we use the fact that \(d_t^W = \frac{\sigma + \varphi}{1 + \varphi} \bar{y}_t^W\) and \(a_t^R = \frac{1 + \varphi^2}{\eta(1 + \varphi)} \bar{y}_t^R\) to derive the second line.

Combining Eqs.(13) and (14) in the text and integrating yields:

\[
\int_0^1 \frac{Y_t(h)}{Y_t} dh = \left[\int_0^1 \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon} dh \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \right] + \int_0^1 \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon} dh \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[\left(\frac{P_{H,t}}{P_t}\right)^{-\varepsilon}\right].
\]

Log-linearizing this equality, we have:

\[
d_t = -\frac{\varepsilon}{2} d_{H,t} - \frac{\varepsilon}{2} d_{F,t} + \frac{\eta}{4} s_t.
\]

Note that \(D_t = \int_0^1 \frac{Y_t(h)}{Y_t} = E_h \left(\frac{Y_t(h)}{Y_t}\right)\) and we define \(D_{H,t} \equiv \int_0^1 \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon}\) and \(D_{F,t} \equiv \left(\frac{P_t(h)}{P_{F,t}}\right)^{-\varepsilon}\). Similar to Eq.(A.4), we have:

\[
d_t^* = -\frac{\varepsilon}{2} d_{F,t} - \frac{\varepsilon}{2} d_{F,t} - \frac{\eta}{4} s_t.
\]

Combining Eqs.(A.4) and (A.5) yields:

\[
\begin{align*}
d_t + d_t^* &= \ln \int_0^1 \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon} dh + \ln \int_0^1 \left(\frac{P_t(h)}{P_{F,t}}\right)^{-\varepsilon} dh + \ln \int_1^2 \left(\frac{P_t(f)}{P_{F,t}}\right)^{-\varepsilon} df \\
&= -\frac{\varepsilon}{2} \left(\ln P_t(h) \right) - \ln P_{H,t} - \ln P_{F,t} - \frac{\varepsilon}{2} \left(\ln P_t(h) \right) + \ln P_{F,t} + \ln P_{F,t} + \ln P_{F,t} - \ln P_{F,t} - \frac{\varepsilon}{2} \left(\ln P_t(f) \right) \ln P_{F,t} - \ln P_{F,t} - \frac{\varepsilon}{2} \left(\ln P_t(f) \right) \ln P_{F,t} - \ln P_{F,t}
\end{align*}
\]
\[
1 = \int_0^1 \left( \frac{P_h}{P_t} \right)^{1-\varepsilon} \, dh 
= \int_0^1 \exp(1-\varepsilon) (p^h_t - p_t) \, dh 
= 1 + \int_0^1 (1-\varepsilon) (p^h_t - p_t) \, dh + \int_0^1 \frac{(1-\varepsilon)^2}{2} (p^h_t - p_t)^2 \, dh + o\left(\|\xi\|^3\right).
\]

Hence:

\[
p_t = \int_0^1 p^h_t \, dh + \int_0^1 \frac{1-\varepsilon}{2} (p^h_t - p_t)^2 \, dh + o\left(\|\xi\|^3\right),
= E_h(p^h_t) + \int_0^1 \frac{1-\varepsilon}{2} (p^h_t - p_t)^2 \, dh + o\left(\|\xi\|^3\right). \tag{A.7}
\]

In addition, we have:

\[
\int_0^1 \left( \frac{P_h}{P_t} \right)^{-\varepsilon} \, dh = \int_0^1 \exp(-\varepsilon) (p^h_t - p_t) \, dh 
= 1 - \varepsilon \int_0^1 (p^h_t - p_t) \, dh + \frac{\varepsilon^2}{2} \int_0^1 (p^h_t - p_t)^2 \, dh + o\left(\|\xi\|^3\right) 
= 1 - \varepsilon E_h(p^h_t) + \varepsilon p_t + \frac{\varepsilon^2}{2} \int_0^1 (p^h_t - p_t)^2 \, dh 
+ o\left(\|\xi\|^3\right) \tag{A.8}
\]

Plugging Eq.(A.7) into Eq.(A.8) yields:

\[
\int_0^1 \left( \frac{P_h}{P_t} \right)^{-\varepsilon} \, dh = 1 - \varepsilon \left[ p_t - \int_0^1 \frac{1-\varepsilon}{2} (p^h_t - p_t)^2 \, dh \right] + \varepsilon p_t 
+ \frac{\varepsilon^2}{2} \int_0^1 (p^h_t - p_t)^2 \, dh + o\left(\|\xi\|^3\right) 
= 1 + \frac{\varepsilon (1-\varepsilon)}{2} \int_0^1 (p^h_t - p_t)^2 \, dh + \frac{\varepsilon^2}{2} \int_0^1 (p^h_t - p_t)^2 \, dh + o\left(\|\xi\|^3\right) 
= 1 + \frac{\varepsilon}{2} \text{var}_h(p^h_t) + o\left(\|\xi\|^3\right). \tag{A.9}
\]
As with Eq.(A.9), we have:

\[
\int_0^1 \left( \frac{P_i^{f*}}{Pr_i} \right)^{-\varepsilon} df = 1 + \frac{\varepsilon}{2} \text{var}_h \left( p_i^{f*}\right) + o \left( \|\xi\|^3 \right) \tag{A.10}
\]

Eqs.(A.9) and (A.10) imply as follows:

\[
\ln \int_0^1 \left( \frac{P_{ih}}{Pr_i} \right)^{-\varepsilon} dh = \frac{\varepsilon}{2} \text{var}_h \left( p_{ih}\right) ; \ln \int_1^2 \left( \frac{P_i^{f*}}{Pr_i} \right)^{-\varepsilon} df = \frac{\varepsilon}{2} \text{var}_f \left( p_i^{f*}\right) \tag{A.11}
\]

Plugging Eq.(A.11) into Eq.(A.6) yields:

\[
d_t + d_t' = \frac{\varepsilon}{2} \text{var}_h \left( p_{ih}\right) + \frac{\varepsilon}{2} \text{var}_f \left( p_i^{f*}\right) \tag{A.12}
\]

Let \(p_i^{h} \equiv E_h \left( p_i^{h}\right) \) and \(p_i^{f*} \equiv E_f p_i^{f*} \). Hence:

\[
\begin{align*}
p_i^{h} - p_{ih-1} & = E_h \left( p_i^{h} - \bar{p}_{i-1}\right) \\
& = E_h \left\{ \theta p_i^{h-1} + (1 - \theta) \bar{p}_t - \left[ \theta \bar{p}_{i-1} + (1 - \theta) \bar{p}_{i-1} \right] \right\} \\
& = \theta E_h \left( p_i^{h-1} - p_{ih-1}\right) + (1 - \theta) \left( \bar{p}_t - p_{ih-1}\right) \\
& = (1 - \theta) \left( \bar{p}_t - p_{ih-1}\right) .
\end{align*}
\]

Similar reasoning, about the dispersion measure \( \text{var}_h \left( p_{ih}\right) \) yields:

\[
\begin{align*}
\text{var}_h \left( p_i^{h}\right) &= \text{var}_h \left( p_i^{h} - p_{ih-1}\right) \\
&= E_h \left[ \left( p_i^{h} - p_{ih-1}\right)^2 \right] - \left[ E_t \left( p_i^{h}\right) - p_{ih-1}\right]^2 \\
&= 2E_h \left[ \ln \left( \frac{P_i^{h}}{P_{ih-1}} \right) \right] - \left( p_i^{h} - p_{ih-1}\right)^2 \\
&= 2E_h \left[ \theta p_i^{h-1} + (1 - \theta) \bar{p}_t - p_{ih-1} - \theta p_{ih-1} - \theta p_{ih-1} - (p_i^{h} - p_{ih-1})^2 \right] \\
&= 2E_h \left[ \theta p_i^{h-1} + (1 - \theta) \bar{p}_t - (1 - \theta) p_{ih-1} \right] - \left( p_i^{h} - p_{ih-1}\right)^2 \\
&= 2\theta E_h \left( p_i^{h-1} - p_{ih-1}\right) + 2 (1 - \theta) \left( \bar{p}_t - p_{ih-1}\right) - \left( p_i^{h} - p_{ih-1}\right)^2 \\
&= \theta E_h \left[ \left( p_i^{h-1} - p_{ih-1}\right)^2 \right] + (1 - \theta) \left( \bar{p}_t - p_{ih-1}\right)^2 - \left( p_i^{h} - p_{ih-1}\right)^2 \\
&= \theta E_h \left[ \left( p_i^{h-1}\right)^2 - 2p_i^{h-1} p_{ih-1} - (p_{ih-1})^2 \right] + (1 - \theta) \left[ \frac{1}{1 - \theta} \left( p_i^{h} - p_{ih-1}\right) \right]^2 \\
&= \theta E_h \left[ \left( p_i^{h-1}\right)^2 \right] + (1 - \theta) \left[ \frac{1}{1 - \theta} \right] - \frac{2}{(1 - \theta)^2} p_i^{h} p_{ih-1} \\
&+ \frac{1}{(1 - \theta)^2} \left( p_i^{h-1}\right)^2 - \left( p_i^{h} \right)^2 + 2p_i^{h} p_{ih-1} - \left( p_{ih-1}\right)^2 \\
&= \theta \text{var}_h \left( p_{ih-1}\right) + \frac{1}{1 - \theta} (p_i^{h}) + \frac{1}{1 - \theta} \left( p_i^{h-1}\right)^2 - (p_i^{h})^2 - (p_{ih-1})^2 \\
&+ 2p_i^{h} p_{ih-1} - \frac{2}{1 - \theta} p_i^{h} p_{ih-1}
\end{align*}
\]
\[ \var_h (p^h_t) = \theta \var_h (p^h_{t-1}) + \frac{\theta}{1-\theta} \left[ \left( \bar{p}^h_t \right)^2 - 2 \bar{p}^h_t \bar{p}^h_{t-1} + (\bar{p}^h_{t-1})^2 \right] \]

\[ \var_h (p^h_t) = \var_h (p^h_{t-1}) + \frac{\theta}{1-\theta} (\bar{p}^h_t - \bar{p}^h_{t-1})^2. \quad (A.13) \]

Note that:

\[ \bar{p}^h_t = \text{E}_h (p^h_t) = \ln \int_0^1 \frac{p^h_t}{p_t} \, dh = p_t \]

Plugging this into the final line in Eq.(A.13) yields:

\[ \var_h (p^h_t) = \theta \var_h (p^h_{t-1}) + \frac{\theta}{1-\theta} \pi^2_t. \]

As shown in Chapter 6, Woodford[1], summing up over time then yields:

\[ \sum_{t=0}^{\infty} \beta^t \var_h (p^h_t) = \frac{1}{\lambda} \sum_{t=0}^{\infty} \beta^t \pi^2_t. \quad (A.14) \]

Plugging Eq.(A.14) into Summed up Eq.(A.12) over time yields:

\[ \sum_{t=1}^{\infty} \beta^t (d_t + d^*_t) = \frac{\varepsilon}{2\lambda} \sum_{t=0}^{\infty} \beta^t \pi^2_t. \quad (A.15) \]

Summing up over time Eq.(A.3) yields:

\[ W_{LCP}^W = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t E_0 \left[ d_t + d^*_t + (\sigma + \varphi) (x^W_t)^2 + \frac{(1+\varphi)\eta^2}{8} z^2_t \right] + \text{t.i.p.} \]

\[ + o \left( \| \xi \|^3 \right) . \quad (A.16) \]

Plugging Eq.(A.15) and its counterpart in country $F$ into this equality yields:

\[ W_{LCP}^W = -\sum_{t=0}^{\infty} \beta^t E_0 \left[ \frac{\varepsilon}{4\lambda} \pi^2_t + \frac{\varepsilon}{4\lambda} (\pi^2_t)^2 + \frac{\sigma + \varphi}{2} (x^W_t)^2 + \frac{(1+\varphi)\eta^2}{8} z^2_t \right] + \text{t.i.p.} + o \left( \| \xi \|^3 \right) \]

which is consistent with Eq.(48) in the text.

B Gains from Policy Cooperation

In this section, we calculate gains from policy cooperation following Monacelli[2].

B.1 Gains from Policy Cooperation under PCP

Combining Eq.(47) in the text and its counterpart in country $F$ yields:

\[ \pi^W_t = \beta E_t (\pi^W_{t+1}) + \kappa_\alpha x^W_t \quad (B.1) \]

\[ \pi^R_{p,t} = \beta E_t (\pi^R_{p,t+1}) + \frac{\lambda (1+\varphi)}{\eta} x^R_t \quad (B.2) \]
Plugging Eqs. (28), (41) and their counterparts in country $F$ into the definition of the deviation of the TOT from its efficient level $z_t$ yields:

$$x_t^R = \eta z_t - \frac{\Gamma_0}{\eta^2 (1 + \varphi)} \bar{s}_t^R.$$  

(B.3)

Combining the equality in line 2 in page 25 in the text and its counterpart in country $F$ is given by:

$$\bar{s}_t^R = \frac{\omega_2 \eta (\sigma + \varphi) \bar{\epsilon}_t^R}{\omega_3}.$$  

(B.4)

Combining Eqs. (B.3) and (B.4) yields:

$$x_t^R = \eta z_t - \frac{4 (\sigma + \varphi) \Gamma_0 \bar{s}_t^R}{\eta \omega_3}.$$  

(B.5)

Plugging Eq. (B.5) into Eq. (B.2) yields:

$$\pi_{R,t} = \beta \bar{E}_t \left( x_{R,t+1} \right) + \lambda (1 + \eta \varphi) z_t - \frac{4 \lambda (1 + \eta \varphi) (\sigma + \varphi) \Gamma_0}{\eta \omega_3} \bar{s}_t^R.$$  

(B.6)

Under the cooperative setting, Lagrangean is given by:

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t L_{PCP,t} + \mu_{3,t} \left[ \pi_t^W - \beta \bar{E}_t \left( \pi_{P,t+1}^W \right) - \kappa \alpha x_t^W \right] 
+ \mu_{4,t} \left[ \pi_t^W - \beta \bar{E}_t \left( \pi_{P,t+1}^R \right) - \lambda (1 + \eta \varphi) z_t 
+ \frac{4 (\sigma + \varphi) \Gamma_0}{\omega_3} \bar{s}_t^R \right] \right\}.$$  

FONCs are given by:

$$\frac{\varepsilon}{2 \lambda} \pi_{P,t} + \frac{1}{2} \mu_{3,t} + \mu_{4,t} = 0, \quad \frac{\varepsilon}{2 \lambda} \pi_{P,t} + \frac{1}{2} \mu_{3,t} - \mu_{4,t} = 0, \quad (\sigma + \varphi) x_t^W - \kappa \alpha \mu_{3,t} = 0, \quad \frac{(1 + \varphi) \eta^2}{4} z_t - \lambda (1 + \eta \varphi) \mu_{4,t} = 0.$$  

(B.7)

Combining the first and the second equality in Eq. (B.7) yields:

$$\frac{\varepsilon}{2 \lambda} \pi_{P,t} + 2 \mu_{4,t} = 0 \quad \text{(B.8)}$$

$$\frac{\varepsilon}{2 \lambda} \pi_{t}^W + \mu_{3,t} = 0 \quad \text{(B.9)}$$

Plugging Eq. (B.9) into the third equality in Eq. (B.7), we have:

$$\pi_t^W = -\frac{1}{\varepsilon} x_t^W$$  

(B.10)

Plugging Eq. (B.10) into Eq. (B.1) yields:

$$x_t^W = \frac{\beta}{1 + \kappa \alpha} \bar{E}_t \left( x_{t+1}^W \right).$$  

(B.11)
Iterating this yields:

\[ x_t^W = 0 \quad \text{(B.12)} \]

Plugging Eq.(B.12) into Eq.(B.10), we have:

\[ \pi_t^W = 0. \quad \text{(B.13)} \]

The fourth equality in Eq.(B.7) can be rewritten as:

\[ \mu_{4,t} = \frac{(1 + \varphi) \eta^2}{\lambda (1 + \eta \varphi)} z_t. \quad \text{(B.14)} \]

Plugging Eq.(B.14) into Eq.(B.8) yields:

\[ z_t = \frac{\varepsilon (1 + \eta \varphi)}{(1 + \varphi) \eta^2 \pi_{P,t}^R}. \quad \text{(B.15)} \]

Plugging Eq.(B.15) into Eq.(B.6) yields:

\[ \pi_{P,t} = \frac{4 \lambda (1 + \eta \varphi) (\sigma + \varphi) \Gamma_0 (1 + \varphi) \eta \xi R}{(1 + \eta \xi R) \xi_t}. \quad \text{(B.16)} \]

Iterating this forward, we have:

\[ \pi_{P,t} = \frac{4 \lambda (1 + \eta \varphi) (\sigma + \varphi) \Gamma_0 (1 + \varphi) \eta \xi R}{(1 + \eta \xi R) \xi_t}. \quad \text{(B.17)} \]

\[ \pi_{P,t} = \frac{2 \lambda (1 + \eta \varphi) (\sigma + \varphi) \Gamma_0 (1 + \varphi) \eta \xi R}{(1 + \eta \xi R) \xi_t}. \quad \text{(B.18)} \]

Plugging Eq.(B.16) into Eq.(B.15), we have:

\[ z_t = \frac{4 \lambda (1 + \eta \varphi)^2 (\sigma + \varphi) \Gamma_0 \xi R}{(1 + \eta \xi R) \xi_t}. \quad \text{(B.19)} \]

Plugging Eqs.(B.13), (B.17), (B.18) and (B.19) into Eq.(50) in the text yields:

\[ L_{P} = \frac{2 \lambda \varepsilon (1 + \eta \varphi)^2 (\sigma + \varphi)^2 \Gamma_2^3 (1 + \varphi) \xi_t^2}{(1 + \eta \xi R) \xi_t^2} \left[ \xi_t^2 + (\xi_t^*)^2 \right]. \quad \text{(B.20)} \]

Under the non-cooperative setting, Lagrangean is given by:

\[
\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} L_{P_2}^{NC} + \mu_{3,t} \left[ \pi_t^W - \beta E_t (\pi_{P,t+1}^W) - \kappa_0 x_t^W \right] \\
+ \mu_{4,t} \left[ \pi_{P,t}^R - \beta E_t (\pi_{P,t+1}^R) - \frac{\lambda (1 + \eta \varphi)}{\eta} x_t^R \right] \right\}
\]
FONCs are given by:

\[
\frac{\varepsilon}{\lambda} \pi_{P,t} + \frac{1}{2} \mu_{3,t} + \mu_{4,t} = 0,
\]

\[
\frac{1 + \varphi}{2} x_t - \frac{\kappa}{2} \mu_{3,t} - \frac{\lambda (1 + \eta \varphi)}{\eta} \mu_{4,t} = 0.
\]

Alternative Lagrange is given by:

\[
\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} I_{\text{NC}*}^P \pi_{P,t} + \mu_{3,t} \left[ \pi_{P,t}^W - \beta E_t \left( \pi_{P,t+1}^W \right) - \kappa_0 x_t^W \right] + \mu_{4,t} \left[ \pi_{P,t}^R - \beta E_t \left( \pi_{P,t+1}^R \right) - \frac{\lambda (1 + \eta \varphi)}{\eta} x_t^R \right] \right\}
\]

FONCs are given by:

\[
\frac{\varepsilon}{\lambda} \pi_{P,t}^* + \frac{1}{2} \mu_{3,t} - \mu_{4,t} = 0,
\]

\[
\frac{1 + \varphi}{2} x_t^* - \frac{\kappa}{2} \mu_{3,t} + \frac{\lambda (1 + \eta \varphi)}{\eta} \mu_{4,t} = 0.
\]

Combining those FONCs mutually, yields:

\[
\pi_{W,t} = \frac{1 + \varphi}{\varepsilon + \varphi} x_t^W,
\]

\[
\pi_{R,t} = -\frac{(1 + \varphi) \eta}{(1 + \eta \varphi) \varepsilon 2 x_t^R}. \tag{B.21}
\]

Plugging the first equality in Eq.(B.21) into Eq.(B.1), we have:

\[
x_t^W = \frac{\beta (1 + \varphi)}{1 + \varphi + \varepsilon \lambda (\sigma + \varphi)^2} E_t \left( x_{t+1}^W \right).
\]

iterating this forward yields:

\[
x_t^W = 0 \quad \tag{B.22}
\]

Plugging the second equality in Eq.(B.21) into Eq.(B.2), we have:

\[
x_t^R = \frac{(1 + \varphi) \beta \eta^2}{(1 + \varphi) \eta^2 + \lambda \varepsilon 2 (1 + \eta \varphi)^2} E_t \left( x_{t+1}^R \right)
\]

iterating this forward yields:

\[
x_t^R = 0 \quad \tag{B.23}
\]

Plugging Eqs.(B.22) and (B.23) into Eq.(B.21), we have:

\[
\pi_{W,t}^* = \pi_{R,t}^* = 0 \quad \tag{B.24}
\]

Eqs.(B.22), (B.23) and (B.24) imply as follows:

\[
\pi_{P,t} = 0 \quad ; \quad \pi_{P,t}^* = 0
\]

\[
x_t = 0 \quad ; \quad x_t^* = 0 \quad \tag{B.25}
\]
Plugging the third and the fourth equalities in Eq. (B.25) into Eq. (B.5), we have:

\[ z_t = 4\Gamma_0 (\sigma + \varphi) \zeta_t \]  

(B.26)

Plugging Eqs. (B.25) and (B.26) into Eq. (50) in the text, we have:

\[ L_{NCW}^{PCP, t} = 2(1 + \varphi) \Gamma_0^2 (\sigma + \varphi)^2 \left[ \xi_t^2 + (\xi_t^*)^2 \right] \]  

(B.27)

Plugging Eqs. (B.20) and (B.27) into \( L_{NCW}^{PCP} - L_{W}^{PCP} \), yields:

\[ L_{NCW}^{PCP} - L_{W}^{PCP} = \frac{2 (1 + \varphi) \Gamma_0^2 (\sigma + \varphi)^2}{(1 - \beta) \omega_3^2} \left[ 1 - \frac{\lambda \varepsilon (1 + \eta \varphi)}{\Gamma_1} \right] \left[ \text{var} (\xi_t) + \text{var} (\xi_t^*) \right], \]

which is consistent with the equality in line 4 in page 37 in the text.

B.2 Gains from Policy Cooperation under LCP

Combining Eq. (35) and its counterpart in country \( F \) is given by:

\[ \begin{align*}
\pi^W_t &= \beta E_t \left( \pi^W_{t+1} \right) + \kappa \alpha x^W_t \\
\pi^R_t &= \beta E_t \left( \pi^R_{t+1} \right)
\end{align*} \]  

(B.28)

(B.29)

Under the cooperative setting, Lagrangean is given by:

\[ \mathcal{L} = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t L_{PCP, t} \right\} + \mu_3, t \left[ \pi^W_t - \beta E_t \left( \pi^W_{t+1} \right) - \kappa \alpha x^W_t \right] + \mu_4, t \left[ \pi^R_t - \beta E_t \left( \pi^R_{t+1} \right) \right] \]

FONCs are given by:

\[ \begin{align*}
\frac{\varepsilon}{2\lambda} \pi_t + \frac{1}{2} \mu_{3, t} + \mu_{4, t} &= 0 \\
\frac{\varepsilon}{2\lambda} \pi^*_t + \frac{1}{2} \mu_{3, t} - \mu_{4, t} &= 0 \\
x^W_t - \lambda \mu_{3, t} &= 0 \\
z_t &= 0
\end{align*} \]  

(B.30)

Combining the first to the third equalities in Eq. (B.30) yields:

\[ \pi^W_t = -\frac{1}{\varepsilon} x^W_t. \]  

(B.31)

Plugging Eq. (B.30) into Eq. (B.28) yields:

\[ x^W_t = \frac{\beta}{1 + \varepsilon \kappa} E_t \left( x^W_{t+1} \right). \]  

(B.32)

Iterating this forward yields:

\[ x^W_t = 0. \]  

(B.33)
Plugging Eq.(B.33) into Eq.(B.31) yields:

\[ \pi^W_t = 0. \]  
(B.34)

Iterating Eq.(B.29) forward, we have:

\[ \pi^R_t = 0. \]  
(B.35)

Combining Eqs.(B.34) and (B.35), we obtain:

\[ \pi_t = 0 ; \pi^*_t = 0 \]  
(B.36)

Plugging the last equality in Eqs.(B.30), (B.33) and (B.36) into Eq.(49) in the text, we have:

\[ L^W_{LCP} = 0. \]  
(B.37)

Under the non-cooperative setting, Lagrangean is given by:

\[
\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t L^NC_{LCP,t} + \mu_{3,t} \left[ \pi^W_t - \beta E_t \left( \pi^W_{t+1} \right) - \kappa_\alpha x^W_t \right] \right. \\
+ \left. \mu_{4,t} \left[ \pi^R_t - \beta E_t \left( \pi^R_{t+1} \right) \right] \right\}.
\]

FONCs are given by:

\[
\frac{\varepsilon}{\lambda} \pi^* + \frac{1}{2} \mu_{3,t} + \mu_{4,t} = 0, \\
(1 + \varphi) x_t - \frac{\kappa_\alpha}{2} \mu_{3,t} = 0.
\]  
(B.38)

Alternative Lagrangean is given by:

\[
\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t L^{NC}_{LCP,t} + \mu_{3,t} \left[ \pi^W_t - \beta E_t \left( \pi^W_{t+1} \right) - \kappa_\alpha x^W_t \right] \right. \\
+ \left. \mu_{4,t} \left[ \pi^R_t - \beta E_t \left( \pi^R_{t+1} \right) \right] \right\}.
\]

FONCs are given by:

\[
\frac{\varepsilon^*}{\lambda} \pi^*_t + \frac{1}{2} \mu_{3,t} - \mu_{4,t} = 0, \\
(1 + \varphi) x^*_t + \frac{\kappa_\alpha}{2} \mu_{3,t} = 0.
\]  
(B.39)

Combining Eqs.(B.39) and (B.40), we have:

\[ \pi^W_t = \frac{1 + \varphi}{\varepsilon (\sigma + \varphi)} x^W_t, \]  
(B.40)

\[ x^R_t = 0. \]  
(B.41)

Plugging Eq.(B.40) into Eq.(B.28) yields:

\[ x^W_t = \frac{\varepsilon (\sigma + \varphi)}{1 + \varphi + \varepsilon \lambda (\sigma + \varphi)^2} E_t \left( x^W_{t+1} \right). \]  
(B.42)
Iterating this forward yields:

\[ x_t^W = 0 \]  \hspace{1cm} (B.43)

Plugging Eq.(B.43) into Eq.(B.41) yields:

\[ \pi_t^W = 0. \]  \hspace{1cm} (B.44)

Iterating Eq.(B.29) yields:

\[ \pi_t^R = 0. \]  \hspace{1cm} (B.45)

Combining Eqs.(B.44) and (B.45) yields:

\[ \pi_t = 0 ; \pi_t^* = 0. \]  \hspace{1cm} (B.46)

Plugging Eq.(B.41) into Eq.(B.5) yields:

\[ z_t = \frac{4(\sigma + \varphi)\Gamma_0\xi_t}{\eta\omega^3}. \]  \hspace{1cm} (B.47)

Plugging Eqs.(B.43), (B.46) and (B.47) into Eq.(49) in the text, we have:

\[ L_{NCW,LCP} = 2(1 + \varphi)(\sigma + \varphi)^2 \frac{\Gamma_0^2}{\omega^3} \left[ \xi_t^2 + (\xi_t^*)^2 \right] \]  \hspace{1cm} (B.48)

Plugging Eqs.(B.38) and (B.48) into \( L_{NCW,LCP} - L_{W,LCP} \) yields:

\[ L_{NCW,LCP} - L_{W,LCP} = \frac{2(1 + \varphi)(\sigma + \varphi)^2 \Gamma_0^2}{(1 - \beta)\omega^3} \left[ \text{var}(\xi_t) + \text{var}(\xi_t^*) \right], \]  \hspace{1cm} (B.49)

which is consistent with the equality in line 13 in page 37 in the text.

References
