Technical Appendix to "Inflation Rate and Nominal Exchange Rate Volatility brought about by Optimal Monetary Policy under Local Currency Pricing"

OKANO, Eiji

Apr., 2011

A Welfare Criterion for the LCP Model

The second-order Taylor expansion of $U_t = \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\varphi} N_t^{1+\varphi}$ with $U_{CN} = 0$ is given by:

$$\begin{aligned}
\frac{U_t - U}{U_C C} &= c_t + \frac{1-\sigma}{2} c_t^2 + \frac{U_N N}{U_C C}\left(n_t + \frac{1+\varphi}{2} n_t^2\right) + o\left(\|\xi\|^3\right) \\
&= c_t + \frac{1-\sigma}{2} c_t^2 - (1-\Phi)^{-1}\left[c_t + \frac{\eta}{2} s_t + d_t - a_t + \frac{1+\varphi}{2}\left(c_t + \frac{\eta}{2} s_t\right.\right. \\
&\quad \left.\left. + d_t - a_t\right)^2\right] + o\left(\|\xi\|^3\right)
\end{aligned}$$

with $\Phi \equiv 1 - \zeta(1-\tau)$. Because we assume $\Psi = 0$, this equality can be rewritten as:

$$\begin{aligned}
\frac{U_t - U}{U_C C} &= -\frac{1}{2}\left[\eta s_t + 2 d_t + (\sigma+\varphi)\left(y_t^W\right)^2 + (1+\varphi)\eta y_t^W s_t - (1+\varphi) 2 y_t^W a_t\right. \\
&\quad \left. + \frac{(1+\varphi)\eta^2}{4} s_t^2 - (1+\varphi)\eta s_t a_t\right] + \text{t.i.p.} + o\left(\|\xi\|^3\right). \qquad \text{(A.1)}
\end{aligned}$$

Likewise, we have:

$$\begin{aligned}
\frac{U_t^* - U}{U_C C} &= -\frac{1}{2}\left[-\eta s_t + 2 d_t^* + (\sigma+\varphi)\left(y_t^W\right)^2 - (1+\varphi)\eta y_t^W s_t - (1+\varphi) 2 y_t^W a_t^*\right. \\
&\quad \left. + \frac{(1+\varphi)\eta^2}{4} s_t^2 + (1+\varphi)\eta s_t a_t^*\right] + \text{t.i.p.} + o\left(\|\xi\|^3\right). \qquad \text{(A.2)}
\end{aligned}$$

We define $\mathcal{W}_{LCP,t}^W \equiv \mathcal{W}\frac{1}{2}\left(W_{LCP,t} + \mathcal{W}_{LCP,t}^*\right)$ with $\mathcal{W} \equiv \frac{U_t - U}{U_C C}$ and $\mathcal{W}_t^* \equiv \frac{U_t^* - U}{U_C C}$. Plugging Eqs.(**??**) and (A.2) into the definition of $\mathcal{W}_t^W$ yields:

$$\begin{aligned}
\mathcal{W}_t &= -\frac{1}{2}\left[d_t + d_t^* + (\sigma+\varphi)\left(y_t^W - \frac{1+\varphi}{\sigma+\varphi} a_t^W\right)^2 + \frac{(1+\varphi)\eta^2}{4}\left(s_t - \frac{1}{\eta} a_t^R\right)^2\right] \\
&\quad + \text{t.i.p.} + o\left(\|\xi\|^3\right),
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{1}{2}\left[d_t + d_t^* + (\sigma+\varphi)\left(y_t^W - \bar{y}_t^W\right)^2 + \frac{(1+\varphi)\,\eta^2}{4} z_t^2\right] + \text{t.i.p.} \\
&\quad + o\left(\|\xi\|^3\right), \qquad \text{(A.3)}
\end{aligned}
$$

where we use the fact that $a_t^W = \frac{\sigma+\varphi}{1+\varphi}\bar{y}_t^W$ and $a_t^R = \frac{1+\varphi\eta}{\eta(1+\varphi)}\bar{y}_t^R$ to derive the second line.

Combining Eqs.(13) and (14) in the text and integrating yields:

$$
\begin{aligned}
\int_0^1 \frac{Y_t(h)}{Y_t} dh &= \left[\int_0^1 \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon} dh \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\right. \\
&\quad \left. + \int_0^1 \left(\frac{P_t^*(h)}{P_{H,t}^*}\right)^{-\varepsilon} dh \left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta}\right] \left[\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\right. \\
&\quad \left. + \left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta}\right]^{-1}.
\end{aligned}
$$

Log-linearizing this equality, we have:

$$
\begin{aligned}
d_t &= -\frac{\varepsilon}{2} d_{H,t} - \frac{\varepsilon}{2} d_{H,t}^* + \frac{\eta}{4} s_t \\
&= \ln \int_0^1 \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\frac{\varepsilon}{2}} dh + \ln \int_0^1 \left(\frac{P_t^*(h)}{P_{H,t}^*}\right)^{-\frac{\varepsilon}{2}} dh + \frac{\eta}{4} s_t. \qquad \text{(A.4)}
\end{aligned}
$$

Note that $D_t = \int_0^1 \frac{Y_t(h)}{Y_t} = \mathrm{E}_h\left(\frac{Y_t(h)}{Y_t}\right)$ and we define $D_{H,t} \equiv \int_0^1 \left(\frac{P_t(h)}{P_{H,t}}\right)$ and $D_{H,t}^* \equiv \left(\frac{P_t^*(h)}{P_{H,t}^*}\right)^{-\varepsilon}$. Similar to Eq.(A.4), we have:

$$
\begin{aligned}
d_t^* &= -\frac{\varepsilon}{2} d_{F,t}^* - \frac{\varepsilon}{2} d_{F,t} - \frac{\eta}{4} s_t \\
&= \ln \int_1^2 \left(\frac{P_t^*(f)}{P_{F,t}^*}\right)^{-\frac{\varepsilon}{2}} df + \ln \int_1^2 \left(\frac{P_t(f)}{P_{F,t}}\right)^{-\frac{\varepsilon}{2}} df - \frac{\eta}{4} s_t. \qquad \text{(A.5)}
\end{aligned}
$$

Combining Eqs.(A.4) and (A.5) yields:

$$
\begin{aligned}
d_t + d_t^* &= \ln \int_0^1 \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\frac{\varepsilon}{2}} dh + \ln \int_0^1 \left(\frac{P_t^*(h)}{P_{H,t}^*}\right)^{-\frac{\varepsilon}{2}} dh + \ln \int_1^2 \left(\frac{P_t^*(f)}{P_{F,t}^*}\right)^{-\frac{\varepsilon}{2}} df \\
&\quad + \ln \int_1^2 \left(\frac{P_t(f)}{P_{F,t}}\right)^{-\frac{\varepsilon}{2}} df \\
&= -\frac{\varepsilon}{2}\left(\mathrm{E}_h\left(\ln P_t(h)\right) - \ln P_{H,t}\right) - \frac{\varepsilon}{2}\left(\mathrm{E}_h\left(\ln P_t^*(h)\right) - \ln P_{H,t}^*\right) \\
&\quad - \frac{\varepsilon}{2}\left(\mathrm{E}_f\left(\ln P_t^*(f)\right) - \ln P_{F,t}^*\right) - \frac{\varepsilon}{2}\left(\mathrm{E}_f\left(\ln P_t(f)\right) - \ln P_{F,t}\right) \\
&= -\varepsilon\left[\frac{1}{2}\left(\mathrm{E}_h\left(\ln P_t(h)\right) + \mathrm{E}_f\left(\ln P_t(f)\right)\right) - \frac{1}{2}\left(\ln P_{H,t} + \ln P_{F,t}\right)\right] \\
&\quad - \varepsilon\left[\frac{1}{2}\left(\mathrm{E}_h\left(\ln P_t^*(h)\right) + \mathrm{E}_f\left(\ln P_t^*(f)\right)\right) - \frac{1}{2}\left(\ln P_{H,t}^* + \ln P_{F,t}^*\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
&= -\varepsilon\left(\mathrm{E}_h\left(\ln P_t^h\right)-\ln P_t\right)-\varepsilon\left(\mathrm{E}_f\left(\ln P_t^{f*}\right)-\ln P_t^*\right)\\
&= \ln\int_0^1\left(\frac{P_t^h}{P_t}\right)^{-\varepsilon}dh+\ln\int_1^2\left(\frac{P_t^{f*}}{P_t^*}\right)^{-\varepsilon}df \qquad \text{(A.6)}
\end{aligned}
$$

with $\ln P_t^h \equiv \frac{1}{2}\left(\ln P_t\left(h\right)+\ln P_t\left(f\right)\right)$ and $\ln P_t^{f*} \equiv \frac{1}{2}\left(\ln P_t^*\left(h\right)+\ln P_t^*\left(f\right)\right)$. Note that $\int_0^1 \ln P_t^h = \mathrm{E}_h\left(\ln P_t^h\right) = \ln P_t$ because Generic households CPI is given analogously to Eq.(4) in the text.

The definition of the price index implies:

$$
\begin{aligned}
1 &= \int_0^1\left(\frac{P_t^h}{P_t}\right)^{1-\varepsilon}dh\\
&= \int_0^1\exp\left(1-\varepsilon\right)\left(p_t^h-p_t\right)dh\\
&= 1+\int_0^1\left(1-\varepsilon\right)\left(p_t^h-p_t\right)dh+\int_0^1\frac{\left(1-\varepsilon\right)^2}{2}\left(p_t^h-p_t\right)^2dh+o\left(\left\|\xi\right\|^3\right).
\end{aligned}
$$

Hence:

$$
\begin{aligned}
p_t &= \int_0^1 p_t^h dh+\int_0^1\frac{1-\varepsilon}{2}\left(p_t^h-p_t\right)^2dh+o\left(\left\|\xi\right\|^3\right),\\
&= \mathrm{E}_h\left(p_t^h\right)+\int_0^1\frac{1-\varepsilon}{2}\left(p_t^h-p_t\right)^2dh+o\left(\left\|\xi\right\|^3\right). \qquad \text{(A.7)}
\end{aligned}
$$

In addition, we have:

$$
\begin{aligned}
\int_0^1\left(\frac{P_t^h}{P_t}\right)^{-\varepsilon}dh &= \int_0^1\exp\left(-\varepsilon\right)\left(p_t^h-p_t\right)dh\\
&= 1-\varepsilon\int_0^1\left(p_t^h-p_t\right)dh+\frac{\varepsilon^2}{2}\int_0^1\left(p_t^h-p_t\right)^2dh+o\left(\left\|\xi\right\|^3\right)\\
&= 1-\varepsilon\mathrm{E}_h\left(p_t^h\right)+\varepsilon p_t+\frac{\varepsilon^2}{2}\int_0^1\left(p_t^h-p_t\right)^2dh\\
&\quad +o\left(\left\|\xi\right\|^3\right) \qquad \text{(A.8)}
\end{aligned}
$$

Plugging Eq.(A.7) into Eq.(A.8) yields:

$$
\begin{aligned}
\int_0^1\left(\frac{P_t^h}{P_t}\right)^{-\varepsilon}dh &= 1-\varepsilon\left[p_t-\int_0^1\frac{1-\varepsilon}{2}\left(p_t^h-p_t\right)^2dh\right]+\varepsilon p_t\\
&\quad +\frac{\varepsilon^2}{2}\int_0^1\left(p_t^h-p_t\right)^2dh+o\left(\left\|\xi\right\|^3\right)\\
&= 1+\frac{\varepsilon\left(1-\varepsilon\right)}{2}\int_0^1\left(p_t^h-p_t\right)^2dh+\frac{\varepsilon^2}{2}\int_0^1\left(p_t^h-p_t\right)^2dh+o\left(\left\|\xi\right\|^3\right)\\
&= 1+\frac{\varepsilon}{2}\int_0^1\left(p_t^h-p_t\right)^2dh+o\left(\left\|\xi\right\|^3\right)\\
&= 1+\frac{\varepsilon}{2}\mathrm{var}_h\left(p_t^h\right)+o\left(\left\|\xi\right\|^3\right). \qquad \text{(A.9)}
\end{aligned}
$$

As with Eq.(A.9), we have:

$$\int_0^1 \left(\frac{P_t^{f*}}{P_t^*}\right)^{-\varepsilon} df = 1 + \frac{\varepsilon}{2}\text{var}_f \left(p_t^{f*}\right) + o\left(\|\xi\|^3\right) \tag{A.10}$$

Eqs.(A.9) and (A.10) imply as follows:

$$\ln \int_0^1 \left(\frac{P_t^h}{P_t}\right)^{-\varepsilon} dh = \frac{\varepsilon}{2}\text{var}_h \left(p_t^h\right) \ ; \ \ln \int_1^2 \left(\frac{P_t^{f*}}{P_t^*}\right)^{-\varepsilon} df = \frac{\varepsilon}{2}\text{var}_f \left(p_t^{f*}\right) \tag{A.11}$$

Plugging Eq.(A.11) into Eq.(A.6) yields:

$$d_t + d_t^* = \frac{\varepsilon}{2}\text{var}_h \left(p_t^h\right) + \frac{\varepsilon}{2}\text{var}_f \left(p_t^{f*}\right). \tag{A.12}$$

Let $\bar{p}_t^h \equiv \text{E}_h \left(p_t^h\right)$ and $\bar{p}_t^{f*} \equiv \text{E}_f p_t^{f*}$. Hence:

$$\begin{aligned}
\bar{p}_t^h - \bar{p}_{t-1}^h &= \text{E}_h \left(p_t^h - \bar{p}_{t-1}\right) \\
&= \text{E}_h \left\{\theta p_{t-1}^h + (1-\theta)\,\tilde{p}_t - \left[\theta \bar{p}_{t-1} + (1-\theta)\,\bar{p}_{t-1}\right]\right\} \\
&= \theta \text{E}_h \left(p_{t-1}^h - \bar{p}_{t-1}^h\right) + (1-\theta)\left(\tilde{p}_t - \bar{p}_{t-1}^h\right) \\
&= (1-\theta)\left(\tilde{p}_t - \bar{p}_{t-1}^h\right).
\end{aligned}$$

Similar reasoning, about the dispersion measure $\text{var}_h \left(p_t^h\right)$ yields:

$$\begin{aligned}
\text{var}_h \left(p_t^h\right) &= \text{var}_h \left(p_t^h - \bar{p}_{t-1}^h\right) \\
&= \text{E}_h \left[\left(p_t^h - \bar{p}_{t-1}^h\right)^2\right] - \left[\text{E}_t \left(p_t^h\right) - \bar{p}_{t-1}^h\right]^2 \\
&= 2\text{E}_h \left[\ln \left(\frac{P_t^h}{\bar{P}_{t-1}^h}\right)\right] - \left(\bar{p}_t^h - \bar{p}_{t-1}^h\right)^2 \\
&= 2\text{E}_h \left[\theta p_{t-1}^h + (1-\theta)\,\tilde{p}_t - \bar{p}_{t-1}^h + \theta \bar{p}_{t-1}^h - \theta \bar{p}_{t-1}^h\right] - \left(\bar{p}_t^h - \bar{p}_{t-1}^h\right)^2 \\
&= 2\text{E}_h \left[\theta p_{t-1}^h - \theta \bar{p}_{t-1}^h + (1-\theta)\,\tilde{p}_t - (1-\theta)\,\bar{p}_{t-1}^h\right] - \left(\bar{p}_t^h - \bar{p}_{t-1}^h\right)^2 \\
&= 2\theta \text{E}_h \left(p_{t-1}^h - \bar{p}_{t-1}^h\right) + 2(1-\theta)\left(\tilde{p}_t - \bar{p}_{t-1}^h\right) - \left(\bar{p}_t^h - \bar{p}_{t-1}^h\right)^2 \\
&= \theta \text{E}_h \left[\left(p_{t-1}^h - \bar{p}_{t-1}^h\right)^2\right] + (1-\theta)\left(\tilde{p}_t - \bar{p}_{t-1}^h\right)^2 - \left(\bar{p}_t^h - \bar{p}_{t-1}^h\right)^2 \\
&= \theta \text{E}_h \left[\left(p_{t-1}^h\right)^2 - 2p_{t-1}^h \bar{p}_{t-1}^h - \left(\bar{p}_{t-1}^h\right)^2\right] + (1-\theta)\left[\frac{1}{1-\theta}\left(\bar{p}_t^h - \bar{p}_{t-1}^h\right)\right]^2 \\
&\quad - \left(\bar{p}_t^h - \bar{p}_{t-1}^h\right)^2 \\
&= \theta \text{E}_h \left[\left(p_{t-1}^h\right)^2\right] + (1-\theta)\left[\frac{1}{(1-\theta)^2} - \frac{2}{(1-\theta)^2}\bar{p}_t^h \bar{p}_{t-1}^h\right. \\
&\quad \left. + \frac{1}{(1-\theta)^2}\left(\bar{p}_{t-1}^h\right)^2\right] - \left(\bar{p}_t^h\right)^2 + 2\bar{p}_t^h \bar{p}_{t-1}^h - \left(\bar{p}_{t-1}^h\right)^2 \\
&= \theta \text{var}_h \left(p_{t-1}^h\right) + \frac{1}{1-\theta}\left(\bar{p}_t^h\right) + \frac{1}{1-\theta}\left(\bar{p}_{t-1}^h\right)^2 - \left(\bar{p}_t^h\right)^2 - \left(\bar{p}_{t-1}^h\right)^2 \\
&\quad + 2\bar{p}_t^h \bar{p}_{t-1}^h - \frac{2}{1-\theta}\bar{p}_t^h \bar{p}_{t-1}^h
\end{aligned}$$

$$
\begin{aligned}
&= \theta \text{var}_h \left(p_{t-1}^h\right) + \frac{\theta}{1-\theta}\left[\left(\bar{p}_t^h\right)^2 - 2\bar{p}_t^h \bar{p}_{t-1}^h + \left(\bar{p}_{t-1}^h\right)^2\right] \\
&= \text{var}_h \left(p_{t-1}^h\right) + \frac{\theta}{1-\theta}\left(\bar{p}_t^h - \bar{p}_{t-1}^h\right)^2. \qquad \text{(A.13)}
\end{aligned}
$$

Note that:

$$
\begin{aligned}
\bar{p}_t^h &= \text{E}_h \left(p_t^h\right) \\
&= \ln \int_0^1 \frac{P_t^h}{P_t} dh \\
&= p_t
\end{aligned}
$$

Plugging this into the final line in Eq.(A.13) yields:

$$
\text{var}_h \left(p_t^h\right) = \theta \text{var}_h \left(p_{t-1}^h\right) + \frac{\theta}{1-\theta}\pi_t^2
$$

As shown in Chapter 6, Woodford[1], summing up over time then yields:

$$
\sum_{t=0}^{\infty} \beta^t \text{var}_h \left(p_t^h\right) = \frac{1}{\lambda}\sum_{t=0}^{\infty} \beta^t \pi_t^2. \qquad \text{(A.14)}
$$

Plugging Eq.(A.14) into Summed up Eq.(A.12) over time yields:

$$
\sum_{t=1}^{\infty} \beta^t \left(d_t + d_t^*\right) = \frac{\varepsilon}{2\lambda}\sum_{t=0}^{\infty} \beta_0^t \pi_t^2. \qquad \text{(A.15)}
$$

Summing up over time Eq.(A.3) yields:

$$
\begin{aligned}
\mathcal{W}_{LCP}^W &= -\frac{1}{2}\sum_{t=0}^{\infty} \beta^t \text{E}_0 \left[d_t + d_t^* + (\sigma+\varphi)\left(x_t^W\right)^2 + \frac{(1+\varphi)\,\eta^2}{4} z_t^2\right] + \text{t.i.p.} \\
&\quad + o\left(\|\xi\|^3\right). \qquad \text{(A.16)}
\end{aligned}
$$

Plugging Eq.(A.15) and its counterpart in country F into this equality yields:

$$
\begin{aligned}
\mathcal{W}_{LCP}^W &= -\sum_{t=0}^{\infty} \beta^t \text{E}_0 \left[\frac{\varepsilon}{4\lambda}\pi_t^2 + \frac{\varepsilon}{4\lambda}\left(\pi_t^*\right)^2 + \frac{\sigma+\varphi}{2}\left(x_t^W\right)^2 + \frac{(1+\varphi)\,\eta^2}{8} z_t^2\right] + \text{t.i.p.} + o\left(\|\xi\|^3\right) \\
&= -\mathcal{L}_{LCP}^W,
\end{aligned}
$$

which is consistent with Eq.(48) in the text.

B Gains from Policy Cooperation

In this section, we calculate gains from policy cooperation following Monacelli[2].

B.1 Gains from Policy Cooperation under PCP

Combining Eq.(47) in the text and its counterpart in country F yields:

$$
\begin{aligned}
\pi_t^W &= \beta \text{E}_t \left(\pi_{t+1}^W\right) + \kappa_\alpha x_t^W \qquad \text{(B.1)} \\
\pi_{P,t}^R &= \beta \text{E}_t \left(\pi_{P,t+1}^R\right) + \frac{\lambda\,(1+\eta\varphi)}{\eta} x_t^R \qquad \text{(B.2)}
\end{aligned}
$$

Plugging Eqs.(28), (41) and their counterparts in country F into the definition of the deviation of the TOT from its efficient level z_t yields:

$$x_t^R = \eta z_t - \frac{\Gamma_0}{\eta^2 (1+\varphi)} \bar{y}_t^R. \tag{B.3}$$

Combining the equality in line 2 in page 25 in the text and its counterpart in country F is given by:

$$\bar{y}_t^R = \frac{\omega_2 2\eta (\sigma+\varphi)}{\omega_3} \xi_t^R. \tag{B.4}$$

Combining Eqs.(B.3) and (B.4) yields:

$$x_t^R = \eta z_t - \frac{4(\sigma+\varphi)\Gamma_0}{\omega_3} \xi_t^R. \tag{B.5}$$

Plugging Eq.(B.5) into Eq.(B.2) yields:

$$\pi_{P,t}^R = \beta \mathrm{E}_t \left(\pi_{P,t+1}^R\right) + \lambda (1+\eta\varphi) z_t - \frac{4\lambda (1+\eta\varphi)(\sigma+\varphi)\Gamma_0}{\eta\omega_3} \xi_t^R. \tag{B.6}$$

Under the cooperative setting, Lagrangean is given by:

$$\begin{aligned} \pounds \quad = \quad & \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t L_{PCP,t}^W + \mu_{3,t} \left[\pi_t^W - \beta \mathrm{E}_t \left(\pi_{P,t+1}^W\right) - \kappa_\alpha x_t^W\right] \right. \\ & + \mu_{4,t} \left[\pi_{P,t}^R - \beta \mathrm{E}_t \left(\pi_{P,t+1}^R\right) - \lambda (1+\eta\varphi) z_t \right. \\ & \left.\left. + \frac{4(\sigma+\varphi)\Gamma_0}{\omega_3} \xi_t^R\right]\right\} \end{aligned}$$

FONCs are given by:

$$\begin{aligned} \frac{\varepsilon}{2\lambda}\pi_{P,t} + \frac{1}{2}\mu_{3,t} + \mu_{4,t} &= 0, \\ \frac{\varepsilon}{2\lambda}\pi_{P,t}^{*} + \frac{1}{2}\mu_{3,t} - \mu_{4,t} &= 0, \\ (\sigma+\varphi) x_t^W - \kappa_\alpha \mu_{3,t} &= 0, \\ \frac{(1+\varphi)\eta^2}{4} z_t - \lambda (1+\eta\varphi) \mu_{4,t} &= 0. \end{aligned} \tag{B.7}$$

Combining the first and the second equality in Eq.(B.7) yields:

$$\frac{\varepsilon}{2\lambda}\pi_{P,t}^R + 2\mu_{4,t} = 0 \tag{B.8}$$

$$\frac{\varepsilon}{2\lambda}\pi_t^W + \mu_{3,t} = 0 \tag{B.9}$$

Plugging Eq.(B.9) into the third equality in Eq.(B.7), we have:

$$\pi_t^W = -\frac{1}{\varepsilon} x_t^W \tag{B.10}$$

Plugging Eq.(B.10) into Eq.(B.1) yields:

$$x_t^W = \frac{\beta}{1+\kappa_\alpha} \mathrm{E}_t \left(x_{t+1}^W\right). \tag{B.11}$$

Iterating this yields:

$$x_t^W = 0 \tag{B.12}$$

Plugging Eq.(B.12) into Eq.(B.10), we have:

$$\pi_t^W = 0. \tag{B.13}$$

The fourth equality in Eq.(B.7) can be rewritten as:

$$\mu_{4,t} = \frac{(1+\varphi)\,\eta^2}{\lambda\,(1+\eta\varphi)\,4} z_t. \tag{B.14}$$

Pluggging Eq.(B.14) into Eq.(B.8) yields:

$$z_t = -\frac{\varepsilon\,(1+\eta\varphi)}{(1+\varphi)\,\eta^2}\pi_{P,t}^R. \tag{B.15}$$

Plugging Eq.(B.15) into Eq.(B.6) yields:

$$\pi_{P,t}^R = \frac{(1+\varphi)\,\eta^2\beta}{\Gamma_1}\mathrm{E}_t\left(\pi_{P,t+1}^R\right) - \frac{4\lambda\,(1+\eta\varphi)\,(\sigma+\varphi)\,\Gamma_0\,(1+\varphi)\,\eta}{\Gamma_1\omega_3}\xi_t^R.$$

Iterating this forward, we have:

$$\pi_{P,t}^R = -\frac{4\lambda\,(1+\eta\varphi)\,(\sigma+\varphi)\,\Gamma_0\,(1+\varphi)\,\eta}{\Gamma_1\omega_3}\xi_t^R. \tag{B.16}$$

Combinig Eqs.(B.13) and (B.16) yields:

$$\pi_{P,t} = -\frac{2\lambda\,(1+\eta\varphi)\,(\sigma+\varphi)\,\Gamma_0\,(1+\varphi)\,\eta}{\Gamma_1\omega_3}\xi_t^R \tag{B.17}$$

$$\pi_{P,t}^* = \frac{2\lambda\,(1+\eta\varphi)\,(\sigma+\varphi)\,\Gamma_0\,(1+\varphi)\,\eta}{\Gamma_1\omega_3}\xi_t^R \tag{B.18}$$

Plugging Eq.(B.16) into Eq.(B.15), we have:

$$z_t = \frac{4\lambda\varepsilon\,(1+\eta\varphi)^2\,(\sigma+\varphi)\,\Gamma_0}{\Gamma_1\omega_3\eta}\xi_t^R \tag{B.19}$$

Plugging Eqs.(B.13), (B.17), (B.18) and (B.19) into Eq.(50) in the text yields:

$$L_{PCP,t}^W = \frac{2\lambda\varepsilon\,(1+\eta\varphi)^2\,(\sigma+\varphi)^2\,\Gamma_0^2\,(1+\varphi)}{\Gamma_1\omega_3^2}\left[\xi_t^2 + (\xi_t^*)^2\right] \tag{B.20}$$

Under the non-cooperative setting, Lagrangean is given by:

$$\begin{aligned} \pounds &= \mathrm{E}_0\Bigg\{\sum_{t=0}^{\infty} L_{PCP,t}^{NC} + \mu_{3,t}\left[\pi_t^W - \beta\mathrm{E}_t\left(\pi_{P,t+1}^W\right) - \kappa_\alpha x_t^W\right] \\ &\quad + \mu_{4,t}\left[\pi_{P,t}^R - \beta\mathrm{E}_t\left(\pi_{P,t+1}^R\right) - \frac{\lambda\,(1+\eta\varphi)}{\eta}x_t^R\right]\Bigg\} \end{aligned}$$

FONCs are given by:

$$\begin{aligned}\frac{\varepsilon}{\lambda}\pi_{P,t}+\frac{1}{2}\mu_{3,t}+\mu_{4,t} &= 0,\\ \frac{1+\varphi}{2}x_t-\frac{\kappa_\alpha}{2}\mu_{3,t}-\frac{\lambda(1+\eta\varphi)}{\eta}\mu_{4,t} &= 0.\end{aligned}$$

Alternative Lagrangean is given by:

$$\begin{aligned}\pounds &= \mathrm{E}_0\left\{\sum_{t=0}^{\infty}L_{PCP,t}^{NC*}+\mu_{3,t}\left[\pi_t^W-\beta\mathrm{E}_t\left(\pi_{P,t+1}^W\right)-\kappa_\alpha x_t^W\right]\right.\\ &\left.+\mu_{4,t}\left[\pi_{P,t}^R-\beta\mathrm{E}_t\left(\pi_{P,t+1}^R\right)-\frac{\lambda(1+\eta\varphi)}{\eta}x_t^R\right]\right\}\end{aligned}$$

FONCs are given by:

$$\begin{aligned}\frac{\varepsilon}{\lambda}\pi_{P,t}^*+\frac{1}{2}\mu_{3,t}-\mu_{4,t} &= 0,\\ \frac{1+\varphi}{2}x_t^*-\frac{\kappa_\alpha}{2}\mu_{3,t}+\frac{\lambda(1+\eta\varphi)}{\eta}\mu_{4,t} &= 0.\end{aligned}$$

Combining those FONCs mutually, yields:

$$\begin{aligned}\pi_{P,t}^W &= -\frac{1+\varphi}{\varepsilon+\varphi}x_t^W,\\ \pi_{P,t}^R &= -\frac{(1+\varphi)\eta}{(1+\eta\varphi)\varepsilon 2}x_t^R.\end{aligned}\tag{B.21}$$

Plugging the first equality in Eq.(B.21) into Eq.(B.1), we have:

$$x_t^W=\frac{\beta(1+\varphi)}{1+\varphi+\varepsilon\lambda(\sigma+\varphi)^2}\mathrm{E}_t\left(x_{t+1}^W\right).$$

iterating this forward yields:

$$x_t^W=0\tag{B.22}$$

Plugging the second equality in Eq.(B.21) into Eq.(B.2), we have:

$$x_t^R=\frac{(1+\varphi)\beta\eta^2}{(1+\varphi)\eta^2+\lambda\varepsilon 2(1+\eta\varphi)^2}\mathrm{E}_t\left(x_{t+1}^R\right)$$

iterating this forward yields:

$$x_t^R=0\tag{B.23}$$

Plugging Eqs.(B.22) and (B.23) into Eq.(B.21), we have:

$$\pi_{P,t}^W=\pi_{P,t}^R=0\tag{B.24}$$

Eqs.(B.22), (B.23) and (B.24) imply as follows:

$$\begin{aligned}\pi_{P,t}=0 \quad &; \quad \pi_{P,t}^*=0\\ x_t=0 \quad &; \quad x_t^*=0\end{aligned}\tag{B.25}$$

Plugging the third and the fourth equalities in Eq.(B.25) into Eq.(B.5), we have:

$$z_t = \frac{4\Gamma_0(\sigma+\varphi)}{\omega_3\eta}\xi_t^R \tag{B.26}$$

Plugging Eqs.(B.25) and (B.26) into Eq.(50) in the text, we have:

$$L_{PCP,t}^{NCW} = \frac{2(1+\varphi)\Gamma_0^2(\sigma+\varphi)^2}{\omega_3^2}\left[\xi_t^2 + (\xi_t^*)^2\right] \tag{B.27}$$

Plugging Eqs.(B.20) and (B.27) into $\mathcal{L}_{PCP}^{NCW} - \mathcal{L}_{PCP}^{W}$ yields:

$$\mathcal{L}_{PCP}^{NCW} - \mathcal{L}_{PCP}^{W} = \frac{2(1+\varphi)\Gamma_0^2(\sigma+\varphi)^2}{(1-\beta)\omega_3^2}\left[1 - \frac{\lambda\varepsilon(1+\eta\varphi)^2}{\Gamma_1}\right]\left[\text{var}(\xi_t) + \text{var}(\xi_t^*)\right],$$

which is consistent with the equality in line 4 in page 37 in the text.

B.2 Gains from Policy Cooperation under LCP

Combining Eq.(35) and its counterpart in country F is given by

$$\pi_t^W = \beta \mathrm{E}_t\left(\pi_{t+1}^W\right) + \kappa_\alpha x_t^W \tag{B.28}$$
$$\pi_t^R = \beta \mathrm{E}_t\left(\pi_{t+1}^R\right) \tag{B.29}$$

Under the cooperative setting, Lagrangean is given by:

$$\begin{aligned} \pounds &= \mathrm{E}_0\left\{\sum_{t=0}^{\infty}\beta^t L_{LCP,t}^W + \mu_{3,t}\left[\pi_t^W - \beta\mathrm{E}_t\left(\pi_{t+1}^W\right) - \kappa_\alpha x_t^W\right]\right. \\ &\quad \left. + \mu_{4,t}\left[\pi_t^R - \beta\mathrm{E}_t\left(\pi_{t+1}^R\right)\right]\right\} \end{aligned}$$

FONCs are given by:

$$\begin{aligned} \frac{\varepsilon}{2\lambda}\pi_t + \frac{1}{2}\mu_{3,t} + \mu_{4,t} &= 0 \\ \frac{\varepsilon}{2\lambda}\pi_t^* + \frac{1}{2}\mu_{3,t} - \mu_{4,t} &= 0 \\ x_t^W - \lambda\mu_{3,t} &= 0 \\ z_t &= 0 \end{aligned} \tag{B.30}$$

Combinig the first to the third equalities in Eq.(B.30) yields:

$$\pi_t^W = -\frac{1}{\varepsilon}x_t^W. \tag{B.31}$$

Plugging Eq.(B.30) into Eq.(B.28) yields:

$$x_t^W = \frac{\beta}{1+\varepsilon\kappa_\alpha}\mathrm{E}_t\left(x_{t+1}^W\right). \tag{B.32}$$

Iterating this forward yields:

$$x_t^W = 0. \tag{B.33}$$

Plugging Eq.(B.33) into Eq.(B.31) yields:

$$\pi_t^W = 0. \tag{B.34}$$

Iterating Eq.(B.29) forward, we have:

$$\pi_t^R = 0. \tag{B.35}$$

Combinig Eqs.(B.34) and (B.35), we obtain:

$$\pi_t = 0 \ ; \ \pi_t^* = 0 \tag{B.36}$$

Plugging the last equality in Eqs.(B.30), (B.33) and (B.36) into Eq.(49) in the text, we have:

$$L_{LCP}^W = 0. \tag{B.37}$$

Under the non-cooperative setting, Lagrangean is given by:

$$\begin{aligned} \pounds &= \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t L_{LCP,t}^{NC} + \mu_{3,t} \left[\pi_t^W - \beta \mathrm{E}_t \left(\pi_{t+1}^W \right) - \kappa_\alpha x_t^W \right] \right. \\ &\quad \left. + \ \mu_{4,t} \left[\pi_t^R - \beta \mathrm{E}_t \left(\pi_{t+1}^R \right) \right] \right\}. \end{aligned}$$

FONCs are given by:

$$\begin{aligned} \frac{\varepsilon}{\lambda} \pi_t + \frac{1}{2} \mu_{3,t} + \mu_{4,t} &= 0, \\ (1+\varphi) x_t - \frac{\kappa_\alpha}{2} \mu_{3,t} &= 0. \end{aligned} \tag{B.38}$$

Alternative Lagrangean is given by:

$$\begin{aligned} \pounds &= \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t L_{LCP,t}^{NC*} + \mu_{3,t} \left[\pi_t^W - \beta \mathrm{E}_t \left(\pi_{t+1}^W \right) - \kappa_\alpha x_t^W \right] \right. \\ &\quad \left. + \ \mu_{4,t} \left[\pi_t^R - \beta \mathrm{E}_t \left(\pi_{t+1}^R \right) \right] \right\}. \end{aligned}$$

FONCs are given by:

$$\begin{aligned} \frac{\varepsilon}{\lambda} \pi_t^* + \frac{1}{2} \mu_{3,t} - \mu_{4,t} &= 0, \\ (1+\varphi) x_t^* + \frac{\kappa_\alpha}{2} \mu_{3,t} &= 0. \end{aligned} \tag{B.39}$$

Combining Eqs.(B.39) and (B.40), we have:

$$\pi_t^W = \frac{1+\varphi}{\varepsilon(\sigma+\varphi)} x_t^W, \tag{B.40}$$

$$x_t^R = 0. \tag{B.41}$$

Plugging Eq.(B.40) into Eq.(B.28) yields:

$$x_t^W = \frac{\varepsilon(\sigma+\varphi)}{1+\varphi+\varepsilon\lambda(\sigma+\varphi)^2} \mathrm{E}_t \left(x_{t+1}^W \right). \tag{B.42}$$

Iterating this forward yields:

$$x_t^W = 0 \tag{B.43}$$

Plugging Eq.(B.43) into Eq.(B.41) yields:

$$\pi_t^W = 0. \tag{B.44}$$

Iterating Eq.(B.29) yields:

$$\pi_t^R = 0. \tag{B.45}$$

Combining Eqs.(B.44) and (B.45) yields:

$$\pi_t = 0 \ ; \ \pi_t^* = 0. \tag{B.46}$$

Plugging Eq.(B.41) into Eq.(B.5) yields:

$$z_t = \frac{4\,(\sigma + \varphi)\,\Gamma_0}{\eta \omega_3} \xi_t^R \tag{B.47}$$

Plugging Eqs.(B.43), (B.46) and (B.47) into Eq.(49) in the text, we have:

$$L_{LCP,t}^{NCW} = \frac{2\,(1+\varphi)\,(\sigma+\varphi)^2\,\Gamma_0^2}{\omega_3^2} \left[\xi_t^2 + (\xi_t^*)^2\right] \tag{B.48}$$

Plugging Eqs.(B.38) and (B.48) into $\mathcal{L}_{LCP}^{NCW} - \mathcal{L}_{LCP}^{W}$ yields:

$$\mathcal{L}_{LCP}^{NCW} - \mathcal{L}_{LCP}^{W} = \frac{2\,(1+\varphi)\,(\sigma+\varphi)^2\,\Gamma_0^2}{(1-\beta)\,\omega_3^2} \left[\mathrm{var}\,(\xi_t) + \mathrm{var}\,(\xi_t^*)\right], \tag{B.49}$$

which is consistent with the equality in line 13 in page 37 in the text.

References

[1] Woodford, Michael (2003), "Interest and Prices," *Princeton University Press*, Princeton, NJ.

[2] Monacelli, Tommaso (2004), "Principles of Optimal Monetary Policy," Lecture Notes provided at the URL: http://www.igier.unibocconi.it/whos.php?vedi=2647&tbn=albero&id_doc=177.