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Understanding the Gains from Wage Flexibility in a Currency Union: The Fiscal Policy Connection *

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Abstract

We investigate two findings in Gali and Monacelli (2016, *American Economic Review*) which are (i) the effectiveness of labor cost adjustments on employment is much smaller in a currency union and (ii) an increase in wage flexibility often reduces welfare, more likely so in an economy that is part of a currency union. First, we introduce a distorted steady state in the small open economy model of GM, in which employment subsidies to make the steady state efficient are not available, and replicate their two findings. Second, an endogenous fiscal policy rule similar to the rule in Bohn (1998, *Quarterly Journal of Economics*) was introduced with a government budget constraint into the model. The results suggest that while the first finding of Gali and Monacelli is still applicable, their second finding is not necessarily applicable. It is, therefore, possible that an increase in wage flexibility reduces welfare loss in an economy that is part of a currency union as long as wage rigidity is high enough. Thus, there is still scope to discuss how wage flexibility is beneficial in a currency union.

Keywords: Sticky Wages, Nominal Rigidities, New Keynesian Models, Monetary and Fiscal Policy, Exchange Rate Policy, Currency Union, Bohn Rule

JEL Classification: E32, E52; E60; F41; F47

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1 Introduction

Gali and Monacelli[22] (GM) find that (i) the effectiveness of labor cost adjustments on employment is much smaller in a currency union and (ii) an increase in wage flexibility often reduces welfare, more likely so in an economy that is part of a currency union. While we support GM's first finding even if there is an endogenous fiscal policy rule such as the Bohn rule advocated by Bohn[7] with a government budget constraint, we cannot necessarily support their second finding as long as there is an endogenous fiscal policy rule such as the Bohn rule with a government budget constraint. At least, an increase in wage flexibility increases welfare in an economy that is part of a currency union as long as wage rigidity is high enough if there is an endogenous fiscal policy rule like the Bohn rule with government budget constraint. There is enough room to discuss how wage flexibility is beneficial in an economy that is part of a currency union.

Gains from wage flexibility have been discussed since Gali[19] examined the role of wages in employment determination in John Maynard Keynes' landmark work *The General Theory of Employment, Interest, and Money* (Keynes[23]) based on New Keynesian models. By interpreting Keynes[23], he shows that employment does not depend on wage adjustments, and the volatility of wage inflation increases when wages become more flexible in New Keynesian models. In addition, he shows that wage flexibility is not always welfare improving. Gali[19] casts doubt on researchers' and policy makers' belief that wage flexibility is desirable. Similar to Gali[19], Bhattarai, Eggertsson and Schoenle[5] show that more flexible wages often reduce welfare under optimal monetary policy.[1] GM, who provide an important research question for this paper, developed a small open economy model based on Gali and Monacelli[20] and compared a small open economy adopting a flexible exchange rate or inflation targeting with a small open economy adopting a fixed exchange rate or that is part of a currency union. Then, they show their two findings mentioned above. They extend their baseline model to a medium-scale dynamic stochastic general equilibrium (DSGE) model and show that their two findings are still applicable. More recently, Billi and Gali[6] show that a zero lower bound (ZLB) constraint on the nominal interest rate generally amplifies the adverse effects of greater wage flexibility on welfare when the central bank follows a conventional Taylor rule. Their finding is consistent with Eggertsson, Ferrero and Raffo[15] who show that structural reforms that increase competition in labor markets, which coincides with reducing wage rigidity, does not support economic activity in the short run as long as the nominal interest rate hits the ZLB.[2]

For now, it seems that the idea that wage flexibility does not necessarily improve welfare not only in an economy that is part of a currency union but also in a closed economy is becoming increasingly accepted. However, those previous studies do not consider the interaction between monetary and fiscal policies. Considering this interaction is not trivial. Leeper and Leith[25] mention that it is always the joint behavior of monetary and fiscal policies that determines inflation (and stabilizes debt). In addition, Mario Draghi, the then President of the European Central Bank (ECB), made the following remark in his speech in December 2018 (Draghi[11]):

> But national budgets will never lose their function as the main stabilization tool during

[1] Their main interest is the effects on price rigidities through their analysis of the effects on wage rigidities.

[2] Although they do not conduct a welfare analysis, Eggertsson and Krugman[14] argue that an increase in wage rigidity may help offset the adverse effects of deflationary shocks in an environment with a binding ZLB. Schmitt-Grohe and Uribe[32] developed a currency union model with downward nominal wage rigidity and showed that optimal capital controls reduce unemployment. However, they did not present explicit findings on how changes in wage rigidity affect welfare.

crises. In the euro area, around 50% of an unemployment shock is absorbed through the automatic stabilizers in national public budgets, significantly more than in the United States. [...] Yet national fiscal policies also need a complement at the European level.

As long as the previous description remains the ECB's policy stance, how fiscal policy, especially national-level fiscal policy which is viewed as another important stabilization tool, affects a currency union along with monetary policy should be discussed. Indeed, certain researchers have been aware of the importance of considering the interaction between monetary and fiscal policies in a currency union, and introducing an endogenous fiscal policy to analyze monetary policy in a currency union is not novel. Gali and Monacelli[21] analyzed optimal monetary and fiscal policy and clarified the importance of fiscal policy in a small open economy that is part of a currency union. Ferrero[17] derived monetary and fiscal policy rules in a currency union that achieved approximate optimal allocation and Okano[30] showed that there are no additional welfare gains from cooperative monetary and fiscal policy in a currency union even if there are nontradable goods. However, they did not introduce wage rigidity in their models and they did not have any policy implications on whether wage flexibility enhances welfare.

Thus, we introduce an endogenous fiscal policy rule similar to the Bohn rule with a government budget constraint, namely, the government financing its deficit by issuing debt and running fiscal surpluses to repay the debt, into the baseline GM model to investigate two findings of GM. First, we develop our baseline model, which is quite similar to the baseline GM model except for the steady-state efficiency. Our steady state is distorted because monopolistically competitive power remains, similar to de Paoli[10] who derived a class of DSGE models assuming a small open economy with a distorted steady state. Using this baseline model (which is referred to as the distorted steady-state model), we compare a small open economy adopting a flexible exchange rate or inflation targeting with a small open economy that is part of a currency union. We replicate GM's two findings using the distorted steady-state model, which means that GM's findings are still applicable even if the steady state is distorted.

Next, we introduce an endogenous fiscal policy rule similar to the Bohn rule with a government budget constraint into the distorted steady-state model. we dub the model, the incorporating government budget constraint (IGBC) model. Analogous to GM, we show dynamic responses to a one percent decrease in the tax rate. Even in the IGBC model, the effect of labor cost adjustments on employment is much smaller in a small open economy that is part of a currency union. That is, GM's first finding is still applicable in the IGBC model. Furthermore, we calculate the welfare loss function of a small open economy that is part of a currency union in that model. GM showed that an increase in wage flexibility often reduced welfare in an economy that is part of a currency union. Our result is that an increase in wage flexibility almost always reduces welfare loss. However, if wage rigidity is high enough, wage flexibility always reduces welfare loss. Wage rigidity is not necessarily favorable in an economy that is part of a currency union. This finding is almost the opposite of GM's second finding.

The reason for our finding is the introduction of an endogenous fiscal policy rule similar to the Bohn rule to close the IGBC model. The Bohn rule implies that the government secures a fiscal surplus to repay the government debt and has the characteristic of procyclicality. In our setting, the tax is effectively levied on output, which is identical to employment taxation as long as productivity remains constant. That is, the Bohn rule is accompanied by distortionary taxation. Moreover, the tax gap (which is the difference between the tax rate and its steady-state value) is the policy instrument, and it varies with changes in the amount of government debt on issue. The tax gap also reacts to employment. The tax gap increases when employment is stagnant to achieve the fiscal surplus necessary to repay the debt (as long as productivity is unchanged). In contrast, the tax gap decreases when employment increases because there is sufficient fiscal surplus to repay the debt (as long as productivity is unchanged). Therefore, the tax gap is negatively related to employment. In other words, stagnant output increases fiscal surplus, and an increase in output decreases fiscal revenue, thus tending to raise government debt. This aspect caused by the Bohn rule with distortionary taxation, led to our finding. Furthermore, when employment increases sufficiently, the tax gap decreases and works to decrease the marginal cost of production, however, there is also upward pressure on marginal cost from an increase in employment. Thus, while the increase in marginal cost puts upward pressure on domestic price inflation, there is also downward pressure on domestic price inflation through the Bohn rule even if employment increases. The pressure to increase domestic price inflation is reduced through the procyclicality of the Bohn rule.

Reducing the pressure on domestic price inflation through a decrease in the tax gap means that introducing the Bohn rule makes domestic price inflation inelastic with respect to changes in the employment gap. Indeed, introducing the Bohn rule makes the slope of the aggregate supply curve, the so-called New Keynesian Phillips curve (NKPC), flatter implicitly. A flatter NKPC implies that domestic price inflation is inelastic with respect to changes in employment. In other words, employment is elastic with respect to changes in domestic price inflation. As wage rigidity increases, the wage markup decreases and workers increase their hours worked in order to offset the stagnation in the nominal wage in response to a demand shock. Combined with a flatter slope of the NKPC associated with the procyclicality of the Bohn rule, which makes employment more elastic with respect to domestic price inflation, the welfare loss associated with the employment gap fluctuation increases drastically as wage rigidity increases. Thus, an increase in wage rigidity has further negative effects on welfare in the IGBC model.

Although the GM model was closely followed in the derivation of this model, some model assumptions and the nature of the monetary regime are slightly different from those in the GM model (Table 1). In contrast to the GM model, this model assumes a distorted steady state, constant returns to scale, and consumer price index (CPI) inflation targeting. The GM model assumes an efficient steady state, decreasing returns to scale, and imperfect substitution between domestic and foreign goods and domestic price (index) inflation targeting. Although we slightly modify GM's New Keynesian small open economy model, we can replicate GM's two findings in our distorted steady-state model. Thus, this modification supports GM's findings. We introduce the distorted steady state to clarify how the introduction of an endogenous fiscal policy rule such as the Bohn rule with a government budget constraint changes GM's two findings. Under the IGBC model, the government levies a tax on firms' sales and this distorts the steady state. Thus, we introduce the assumption of a distorted steady state to clarify that the differences in the results do not depend on the distorted steady state and to examine how the introduction of an endogenous fiscal policy rule such as the Bohn rule with a government budget constraint changes GM's two findings. Note that owing to the distorted steady state, the employment gap on welfare criteria differs from GM. In GM, the employment gap to evaluate welfare is the gap between (actual) employment and natural employment. However, owing to the distorted steady state, the employment gap to evaluate welfare is the gap between (actual) employment and efficient employment, which is the target level of employment in the distorted steady state.

Another difference is that we assume constant returns to scale. If we adopt decreasing returns to, as in the GM model, we cannot obtain welfare criteria that do not have cross terms of the endogenous variables because of the distorted steady state. Instead, welfare should be evaluated using the method developed by Schmitt-Grohe and Uribe[31] and we cannot find the welfare (loss) component which comprises total welfare (loss) and is a decomposition sorted by source of welfare (loss). The GM model clarifies which distortions, such as employment gap, domestic price inflation and wage inflation, generate welfare losses that vary according to the degree of wage rigidity. That is, GM clarify not only how the degree of wage rigidity affects total welfare losses but also how it affects each welfare component, using welfare criteria without the cross terms of the endogenous variables by second-order approximation of the utility function. To examine the welfare components as in the GM model, we must assume constant returns to scale although this modification does not affect our findings because we can replicate GM's two findings in the distorted steady-state model, as mentioned.

Although GM assume the Armingtom form of consumption index, which implies perfect substitution between domestic and foreign goods, Obstfeld and Rogoff[28] argued that the elasticity of substitution between domestic and foreign goods should be 3 to 6; thus, we assume imperfect substitution between domestic and foreign goods. To check the robustness of the results, GM also select 2 as the elasticity of substitution between domestic and foreign goods to calibrate. Therefore, while we assume imperfect substitution between domestic and foreign goods, we choose 2 to calibrate (Thus, we do not violate from GM's set-up because 2 is selected similar to GM.).

Furthermore, in contrast to GM's adoption of domestic price inflation targeting, we adopt CPI inflation targeting. Our setting is more plausible than the GM setting. It is well known that the ECB targets a harmonized index of 2% consumer price inflation rates. We use CPI inflation targeting, which is closer to the ECB's strategy than the inflation targeting in the GM model. By contrast, GM choose domestic inflation targeting [3] Note that although we choose CPI inflation targeting when we analyze the distorted steady-state model, our findings are consistent with GM's two findings. This means that GM's findings are robust even if a more plausible inflation rate is chosen as a target.

Importantly, we can replicate GM's two findings even if we slightly modify their setting, whereas we achieve almost opposite results by introducing an endogenous fiscal policy rule such as the Bohn rule with a government budget constraint in a model of a small open economy that is part of a currency union. As long as we use an endogenous fiscal policy rule such as the Bohn rule with a government budget constraint in an economy that is part of a currency union, we cannot replicate completely GM's second finding. Wage flexibility may reduce welfare loss under several assumptions that are not made by GM, and there is a need to discuss how wage flexibility reduces welfare loss in various settings.

This paper proceeds as follows. Section 2 presents our baseline model, namely the distorted steady-state model. Section 3 presents the welfare criteria and equilibrium in the benchmark model. Section 4 examines the effectiveness of labor cost reduction and discusses wage flexibility and welfare in a currency union in the distorted steady-state model. Section 5 presents the IGBC model. Section 6 discusses the welfare criteria and equilibrium in the IGBC model. Section 7 examines the effectiveness of labor cost reduction and discusses wage flexibility and welfare in the IGBC model. Section 8 conducts a robustness exercise. Section 9 concludes the paper. The

[3] See ECB's website https://www.ecb.europa.eu/mopo/intro/html/index.en.html.

appendix provides details of the derivation of an equality that helps to convince readers of our result.

2 Distorted Steady-state Model: Introducing a Distorted Steady State to GM's New Keynesian Small Open Economy Model

Our baseline model, namely, the distorted steady-state model, is similar to GM's New Keynesian small open economy model, apart from the distorted steady state and constant returns to scale, as we mentioned in Section 1.[4] Another difference relates to the CES aggregator of consumption indexes because we allow imperfect substitution between domestic and foreign goods.[5] The presentation of the model and the notation closely parallels that of the GM model.

2.1 Households

Similar to the GM model, there is a representative household in a small open economy and the household has a continuum of members indexed by $j \in [0,1]$ and is specialized in a differentiated occupation supplying labor services of amount $\mathcal{N}_t(j)$.

The household's utility function is given by:

$$\sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left[U\left(C_t, \{\mathcal{N}_t(j)\}; Z_t\right)\right], \tag{1}$$

where C_t denotes a consumption index, Z_t denotes an exogenous preference shifter, $\beta \equiv \frac{1}{1+\delta} \in (0,1)$ denotes the subjective discount factor and δ denotes the rate of time preference.

Period utility is given by:

$$U\left(C_t, \{\mathcal{N}_t(j)\}; Z_t\right) \equiv \left(\ln C_t - \frac{1}{1+\varphi} \int_0^1 \mathcal{N}_t(j)^{1+\varphi} dj \right) Z_t,$$

with the consumption index:

$$C_t \equiv \left[(1-\upsilon)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \upsilon^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

where $C_{H,t} \equiv \left[\int_0^1 C_{H,t}(i)^{\frac{\epsilon_p - 1}{\epsilon_p}} di \right]^{\frac{\epsilon_p}{\epsilon_p - 1}}$ denotes an index of domestic goods consumption, $C_{F,t}$ denotes quantity consumed of a composite foreign good, ϵ_p denotes the elasticity of substitution across goods produced domestically, $\upsilon \in [0,1]$ denotes a measure of openness, φ denotes the inverse of the elasticity of wage to labor supply and $\eta > 0$ denotes the elasticity of substitution between the domestic and the imported consumption bundle.

The (log) preference shifter is assumed to follow an exogenous AR(1) process as follows:

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z,$$

where lower case letters denote percentage deviation from original variables' steady-state value.

[4] See our website https://www.econ.nagoya-cu.ac.jp/~eiji_okano /papers_e.html for details of the derivation.

[5] The GM model also abandons the assumption of perfect substitution in their medium-scale DSGE model.

The sequence of budget constraints has the following form:

$$\int_0^1 P_{H,t}(i)\, C_{H,t}(i)\, di + P_{F,t}C_{F,t} + \mathrm{E}_t\left(Q_{t,t+1}D^n_{t+1}\right) \leq D^n_t + \int_0^1 W_t(j)\, \mathcal{N}_t(j)\, dj + TR_t + PR_t, \quad (3)$$

where D^n_{t+1} denotes the nominal payoff in period $t+1$ of the portfolio held at the end of period t, $Q_{t,t+1}$ denotes the stochastic discount factor which suffices $\mathrm{E}_t\left(Q_{t,t+1}\right) = \left(1+r_t\right)^{-1}$, r_t denotes the nominal interest rate, $P_{H,t}(i)$ denotes the price of domestic variety i, $P_{F,t}$ denotes the price of imported goods in terms of domestic currency and $W_t(j)$ denotes the nominal wage for type j labor, TR_t denotes the lump-sum transfer and PR_t denotes the nominal profits from the ownership of the firms.

Households maximize Eq. (1) subject to Eq. (3) and the Euler equation is given by:

$$1 = \beta\left(1+r_t\right)\mathrm{E}_t\left(\frac{P_tC_t}{P_{t+1}C_{t+1}}\frac{Z_{t+1}}{Z_t}\right), \quad (4)$$

where

$$P_t \equiv \left[\left(1-\upsilon\right)P^{1-\eta}_{H,t} + \upsilon P^{1-\eta}_{F,t}\right]^{\frac{1}{1-\eta}}, \quad (5)$$

denotes the CPI, $P_{H,t} \equiv \left[\int_0^1 P_{H,t}(i)^{1-\epsilon_p}\, di\right]^{\frac{1}{1-\epsilon_p}}$ denotes the domestic price index and $P_{F,t}$ denotes the price of imported goods in terms of domestic currency.

Under the assumption of complete international financial markets, the equilibrium price (in terms of domestic currency) of a riskless bond denominated in foreign currency is given by $\mathcal{E}_t\left(1+r^*_t\right)^{-1} = \mathrm{E}_t\left(Q_{t,t+1}\mathcal{E}_{t+1}\right)$ where $\mathcal{E}_t$ denotes the nominal exchange rate, which is the price of foreign currency in terms of domestic currency, and r^*_t denotes the nominal world interest rate. The previous pricing equation can be combined with the domestic bond pricing equation, $\left(1+r_t\right)^{-1} = \mathrm{E}_t\left(Q_{t,t+1}\right)$ to obtain a version of the uncovered interest parity condition:

$$\mathrm{E}_t\left\{Q_{t,t+1}\left[\left(1+r_t\right) - \left(1+r^*_t\right)\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right)\right]\right\} = 0.$$

By log-linearizing the previous expression, we have:

$$r_t = r^*_t + \mathrm{E}_t\left(e_{t+1}\right) - e_t, \quad (6)$$

with $e_t \equiv \ln\mathcal{E}_t$.

We assume that the law of one price (LOOP) is applied and there is a relationship between the price of imported goods in terms of domestic currency and the foreign price level as follows:

$$P_{F,t} = \mathcal{E}_tP^*_t,$$

where $P^*_t = 1$ is applied as in the GM model for all t.

Workers specialized in each occupation (or a union representing them) set the corresponding nominal wage subject to an isoelastic demand function for their services. In each period, only a fraction $1-\theta_w$ with $\theta_w \in [0,1]$ of labor types drawn randomly from the corresponding population have their nominal wage reset. This is done in a way consistent with household utility maximization while taking as given the average wage, the price level and other aggregate variables. The remaining fraction θ_w of labor types keep their nominal wage unchanged.

2.2 Firms

2.2.1 Technology, FONC and Steady State

A continuum of firms, indexed by $i \in [0,1]$, are assumed to operate in the home economy. A typical domestic firm produces a differential good using the technology:

$$Y_t(i) = A_t N_t(i),$$

where $Y_t(i)$ is output and $N_t(i) \equiv \left[\int_0^1 N_t(i,j)^{\frac{\epsilon_w - 1}{\epsilon_w}} dj\right]^{\frac{\epsilon_w}{\epsilon_w - 1}}$ is a CES function of the quantities $N_t(i,j)$ of the different types of labor services j and A_t is a stochastic technology parameter. Note that an index for aggregate domestic output is given by $Y_t \equiv \left[\int_0^1 Y_t(i)^{\frac{\epsilon_p - 1}{\epsilon_p}}\right]^{\frac{\epsilon_p}{\epsilon_p - 1}}$. The logarithm of technology follows an exogenous AR(1) process as follows:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a.$$

As mentioned, we assume constant returns to scale, whereas GM assumes decreasing returns to scale.

In each period, a subset of firms of measure $1 - \theta_p$, with $\theta_p \in [0,1]$ being an index of price rigidities drawn randomly from the population, reoptimizes the price of their good, subject to a sequence of isoelastic demand schedules for the latter. The remaining fraction θ_p keep their price unchanged. Prices are set in domestic currency and are the same for both the domestic and export markets and the LOOP is also applied for exports.

A tax is levied on the firm's sales and the (real) marginal cost is given by:

$$MC_t \equiv \frac{W_t}{(1-\tau_t) P_{H,t} A_t}, \tag{7}$$

where MC_t denotes the (real) marginal cost and τ_t denotes the tax rate. Because the firm's marginal cost depends on the number of units of the composite labor input, $N_t(i)$, it employs, whose price is W_t, index j is subtracted from Eq. (7). In Eq. (7), the tax rate appears on the denominator of the right-hand side and the sign is negative. Thus, the higher the tax rate, the higher the marginal cost, and vice versa.

However, the (real) marginal cost in the GM model can be inferred to be:

$$MC_t = \frac{W_t(1+\tau_t)}{P_{H,t} A_t}. \tag{8}$$

In Eq. (8), the tax rate appears in the numerator on the right-hand side. Thus, the higher the tax rate, the higher the marginal cost, and vice versa. Essentially, there is no difference between how changes in tax affect the marginal cost in Eqs. (7) and (8).

The difference between Eqs. (7) and (8) appears in the steady state. Eq. (7) implies that the steady-state wedge between the marginal utility of consumption and the marginal utility of labor is given by:

$$1 - \Phi = \frac{1-\tau}{\frac{\epsilon_p}{\epsilon_p - 1}\frac{\epsilon_w}{\epsilon_w - 1}},$$

where $1 - \Phi \equiv -\frac{U_N}{U_C}$ denotes the steady-state wedge between the marginal utility of consumption and the marginal utility of labor and Φ denotes the parameter measuring efficiency in the steady state. When $\Phi = 0$, monopolistically competitive power disappears in the steady state. Notice that $\frac{\epsilon_p}{\epsilon_p - 1} > 1$ and $\frac{\epsilon_w}{\epsilon_w - 1} > 1$ which are the constant markups on the domestic price and wage. Because $\frac{\epsilon_p}{\epsilon_p - 1} > 1$ and $\frac{\epsilon_w}{\epsilon_w - 1} > 1$ while $\tau \geq 0$, we cannot obtain the efficient steady state and there is monopolistically competitive power because $\Phi > 0$.

Eq. (8), however, implies that the steady-state wedge between the marginal utility of consumption and the marginal utility of labor is given by:

$$1 - \Phi = \frac{1}{\frac{\epsilon_p}{\epsilon_p - 1} \frac{\epsilon_w}{\epsilon_w - 1} (1 + \tau)}.$$

The GM model assumes $\tau < 0$ in the steady state, which should be interpreted as an employment subsidy and implicitly assumes that $\Phi = 0$, which implies that the efficient steady state, where there is no competitive monopolistic power completely disappears by choosing a suitable negative value of τ.

While $\Phi = 0$ in the GM model, $\Phi > 0$ in the distorted steady-state model implies that the steady state is distorted and is not efficient. This difference makes our welfare criteria differ from the welfare criteria in GM's baseline model. However, this difference itself is not important and the reason we introduce a distorted steady state is to clarify how introducing an endogenous fiscal policy rule such as the Bohn rule with a government budget constraint changes GM's findings because the IGBC model incorporates a distorted steady state. Even in the IGBC model, the tax is levied on firm sales and the (real) marginal cost is given by Eq. (7), similar to that in the distorted steady-state model.

2.2.2 International Risk Sharing

Households can access international financial markets and the international risk-sharing condition is applied as follows:

$$C_t = C_t^* \mathcal{Q}_t \left(\frac{Z_t}{Z_t^*} \right), \tag{9}$$

where $\mathcal{Q}_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t}$ denotes the real exchange rate, C_t^* denotes (per capita) world consumption and Z_t^* denotes a discount factor shock in the rest of the world.

2.3 Demand for Exports and Global Shocks

The demand for exports of domestic good i is given by:

$$X_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon_p} X_t,$$

where X_t denotes an aggregate export index. The latter is assumed to be given by:

$$X_t = \upsilon \mathcal{S}_t^{\eta} Y_t^*, \tag{10}$$

where Y_t^* denotes (per capita) world output and $\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}}$ denotes the terms of trade (TOT). In equilibrium, world output equals world consumption, C_t^*, and because of the symmetric steady state, $\mathcal{S} = 1$ and $C = C^* = Y^*$ are applicable. Notice that $X_t = \upsilon \mathcal{S}_t Y_t^*$, implying perfect substitution between home and foreign goods, is applied in the baseline GM model.

There are two global shocks similar to those in the GM model, the export shock and the world interest rate shock. The export shock shifts the export function Eq. (10), leaving the (real) world interest rate unchanged. The world interest rate shock changes the (real) world interest rate while leaving global output unchanged. These variables are endogenous, and GM examines their respective effects on the domestic economy by considering them in isolation. We follow GM and assume that the discount factor shifter for foreign households is given by:

$$Z_t^* = Z_{1,t}^* Z_{2,t}^*,$$

where $Z_{1,t}^*$ and $Z_{2,t}^*$ denote the export and the world interest rate shocks, respectively. These shocks follow exogenous AR(1) processes as follows:

$$\begin{aligned} z_{1,t}^* &= \rho_1^* z_{1,t} + \varepsilon_{1,t}^*, \\ z_{2,t}^* &= \rho_2^* z_{2,t} + \varepsilon_{2,t}^*. \end{aligned}$$

Under the assumption that foreign households have an Euler equation analogous to Eq. (4), that is:

$$1 = \beta \left(1 + r_t^*\right) \mathrm{E}_t \left(\frac{Y_t^*}{Y_{t+1}^*} \frac{Z_{t+1}^*}{Z_t^*} \right), \tag{11}$$

it follows from the assumptions above, and the global market-clearing condition $C_t^* = Y_t^*$ that

$$Y_t^* = Z_{1,t}^*, \tag{12}$$

which implies that Eq. (9) can be rewritten as

$$C_t = \mathcal{Q}_t \frac{Z_t}{Z_{2,t}^*}. \tag{13}$$

The behavior of the world interest rate implied by the assumption above is given by:

$$r_t^* = \delta + \left(1 - \rho_2^*\right) z_{2,t}^*.$$

Thus, the $z_{1,t}^*$ shock affects global output, shifting the demand for home exports Eq. (10). By contrast, $z_{2,t}^*$ alters the world real interest rate and shifts the risk-sharing condition Eq. (13). These shocks only affect aggregate exports through their possible impact on the real exchange rate. This justifies our labeling of these shocks as export and world interest rate shocks, respectively.

2.4 Market-Clearing Condition

The market-clearing condition is given by:

$$Y_t(i) = C_{H,t}(i) + X_t(i). \tag{14}$$

3 Welfare Criteria, Equilibrium, Monetary Regimes and Calibration on the Distorted Steady-state model

While GM show equilibrium in the baseline model before deriving the welfare criteria, we derive the welfare criteria for the distorted steady-state model first. Because the steady state in the distorted steady-state model is distorted literally, the target level of output equals the natural rate of output in GM, whereas it equals the efficient level of output in this paper. [6] The GM model depicts equilibrium using the employment gap (and other gaps such as consumption gap, the TOT gap and real wage gap), which we adopt in our model by obtaining the target level of output using the welfare criteria. Thus, we derive the welfare criteria first.

3.1 Welfare Criteria

Our welfare criteria stem from the second-order approximated utility function, and the linear terms that generate welfare reversal are appropriately eliminated by following Benigno and Woodford[4] and de Paoli[10]. Our welfare criteria in the distorted steady-state model are given by:

$$\mathcal{L} \sim \frac{1}{2}\left[\Lambda_n \mathrm{var}\left(\hat{n}_t\right) + \Lambda_p \mathrm{var}\left(\pi_{H,t}\right) + \Lambda_w \mathrm{var}\left(\pi_t^w\right)\right], \tag{15}$$

where $\hat{n}_t \equiv n_t - n_t^e$ denotes the welfare relevant employment gap, n_t^e denotes the efficient level of employment, $\pi_{H,t}$ denotes (domestic) price inflation, π_t^w denotes wage inflation, $\Lambda_n \equiv \frac{2\Omega_0}{\gamma_v^2}$, $\Lambda_p \equiv \frac{\epsilon_p[(1-\Phi)-\Theta_2(\lambda_p+\varphi)]}{\lambda_p}$ and $\Lambda_w \equiv \frac{\epsilon_w[(1+\epsilon_w\varphi)(1-\Phi)-\Theta_2(1+\varphi)\lambda_w]}{\lambda_w}$ denote the weights on the variances of the welfare relevant employment gap, (domestic) price inflation and wage inflation, respectively, $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}$ and $\lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}$ are slopes of the NKPC and the wage Phillips curve, respectively, and Ω_0 and γ_v are composite blocks of parameters. The definition of the efficient level of output and (domestic) price inflation and wage inflation are shown in Section 3.2.

3.2 Equilibrium in the Distorted Steady-state model

Combined with the market-clearing conditions and after log-linearization around the zero-inflation steady state, optimality conditions can be used to determine the set of conditions characterizing the equilibrium of the small open economy. That equilibrium can be represented by the following system of difference equations. While the welfare criteria are different from those of the GM model, our equilibrium in the distorted steady-state model is almost the same as GM's except that (i) the welfare relevant employment gap replaces the standard employment gap, (ii) the cost push shock which comprises the efficient level of employment, the efficient level of TOT, the demand shock, the export shock and world interest rate shock appear in the NKPC, and (iii) the elasticity of substitution between domestic and imported goods and the parameter measuring returns to scale disappears because of constant returns to scale:

Aggregate Demand Block

$$\begin{aligned}
y_t &= (1-v)\,c_t + \eta v\,(2-v)\,s_t + v z_{1,t}^*, && (16)\\
c_t &= (1-v)\,s_t + z_t - z_{2,t}^*, && (17)\\
c_t &= \mathrm{E}_t\left(c_{t+1}\right) - \left[r_t - \mathrm{E}_t\left(\pi_{t+1}\right)\right] + (1-\rho_z)\,z_t + \delta, && (18)\\
s_t &= e_t - p_{H,t}, && (19)\\
n_t &= y_t - a_t, && (20)
\end{aligned}$$

Aggregate Supply Block

[6] To be precise, the GM model's target level of employment is the natural rate excluding the tax rate.

$$
\begin{aligned}
\pi_{H,t} &= \beta \mathrm{E}_t\left(\pi_{H,t+1}\right)+\lambda_p mc_t, && (21)\\
mc_t &= \hat{\omega}_t+\hat{y}_t+\{\upsilon\left[1-(\eta-1)(2-\upsilon)\right]+1\}\hat{s}_t+\frac{1}{1-\tau}\tau_t+\nu_{p,t}-\frac{1}{1-\tau}\tau, && (22)\\
\pi_{H,t} &\equiv p_{H,t}-p_{H,t-1}, && (23)\\
\pi_t &\equiv p_t-p_{t-1}, && (24)\\
p_t &= p_{H,t}+\upsilon s_t, && (25)\\
\pi_t^w &= \beta \mathrm{E}_t\left(\pi_{t+1}^w\right)-\lambda_w\mu_t^w, && (26)\\
\mu_t^w &= \hat{\omega}_t-\varphi\hat{n}_t-\hat{c}_t, && (27)\\
\pi_t^w &\equiv w_t-w_{t-1}, && (28)\\
\omega_t &\equiv w_t-p_t, && (29)
\end{aligned}
$$

with $s_t \equiv \ln\mathcal{S}_t$ and $\nu_{p,t} \equiv \frac{1+\gamma_\upsilon(1+\varphi)}{\gamma_\upsilon}n_t^e - \left[(\eta-1)\upsilon(2-\upsilon)+1\right]s_t^e + \frac{\upsilon\left[\eta(2-\upsilon)^2-\gamma_\upsilon\right]-\gamma_\upsilon(1-\upsilon)^2}{\gamma_\upsilon(1-\upsilon)}\left(z_t-z_{2,t}^*\right) - \frac{\upsilon(1+\gamma_\upsilon)}{\gamma_\upsilon}z_{1,t}^*$ being the cost push shock where $\hat{\omega}_t \equiv \omega_t-\omega_t^e$, $\hat{y}_t \equiv y_t-y_t^e$, $\hat{s}_t \equiv s_t-s_t^e$ and $\hat{c}_t \equiv c_t-c_t^e$ denote the welfare relevant real (consumption) wage gap, welfare relevant output gap, welfare relevant TOT gap and welfare relevant consumption gap which are deviations from their efficient level, ω_t^e, y_t^e, s_t^e and c_t^e, respectively.

The aggregate demand block consists of Eq. (16) which determines output as a function of aggregate demand, the international risk-sharing condition Eq. (17), Euler equation Eq. (18), the definition of the TOT Eq. (19) and Eq. (20) which determines employment as a function of aggregate output given the technology. The aggregate supply block consists of the NKPC determining inflation dynamics Eq. (21), the wage Phillips curve Eq. (26), the relationship between the CPI and domestic price Eq. (25) and definitions of domestic price inflation $\pi_{H,t}$ Eq. (23), CPI inflation π_t Eq. (24), wage inflation π_t^w Eq. (28) and the real (consumption) wage ω_t Eq. (29). In addition, there are two equalities, Eqs. (22) and (27), which show that marginal cost is a function of the welfare relevant real (consumption) wage gap, the welfare relevant output gap, the welfare relevant TOT gap and the tax rate and varies with changes in the price shock, and show that the wage markup μ_t^w is a function of the welfare relevant real (consumption) wage gap, welfare relevant employment gap and welfare relevant consumption gap, respectively. In contrast to the GM model, we present these two equalities separately from Eqs. (21) and (26) to show how introducing an endogenous fiscal policy rule such as the Bohn rule with a government budget constraint changes GM's two findings.

The efficient levels of employment, output, TOT, consumption and real wage are given by:

$$
\begin{aligned}
n_t^e &\equiv \Omega_1 z_t+\upsilon\Omega_2 z_{1,t}^*+\Omega_3 z_{2,t}^*+\Omega_5 a_t,\\
y_t^e &\equiv n_t^e+a_t,\\
s_t^e &\equiv \frac{1}{\gamma_\upsilon}n_t^e+\frac{1}{\gamma_\upsilon}a_t+\frac{\eta\upsilon(2-\upsilon)-\gamma_\upsilon}{(1-\upsilon)\gamma_\upsilon}z_t-\frac{\upsilon}{\gamma_\upsilon}z_{1,t}^*-\frac{\eta\upsilon(2-\upsilon)-\gamma_\upsilon}{(1-\upsilon)\gamma_\upsilon}z_{2,t}^*,\\
c_t^e &\equiv \frac{1-\upsilon}{\gamma_\upsilon}n_t^e+\frac{1-\upsilon}{\gamma_\upsilon}a_t+\frac{\eta\upsilon(2-\upsilon)}{\gamma_\upsilon}z_t-\frac{\upsilon(1-\upsilon)}{\gamma_\upsilon}z_{1,t}^*-\frac{\eta\upsilon(2-\upsilon)}{\gamma_\upsilon}z_{2,t}^*,\\
\omega_t^e &\equiv \frac{\varphi\gamma_\upsilon+1-\upsilon}{\gamma_\upsilon}n_t^e+\frac{1-\upsilon}{\gamma_\upsilon}a_t+\frac{\eta\upsilon(2-\upsilon)}{\gamma_\upsilon}z_t-\frac{\upsilon(1-\upsilon)}{\gamma_\upsilon}z_{1,t}^*-\frac{\eta\upsilon(2-\upsilon)}{\gamma_\upsilon}z_{2,t}^*,
\end{aligned}
$$

with Ω_1, Ω_2, Ω_3 and Ω_5 being composite blocks of parameters.

3.3 Monetary Regimes

The system consists of the aggregate demand and supply blocks and is closed by the monetary regime equations. Following GM, we analyze two monetary regimes, *inflation targeting* in which a small open economy adopts inflation targeting (or a flexible exchange rate) and *currency union* in which a small open economy is part of a currency union. Under the first regime, namely, *inflation targeting*, the central bank focuses on stabilizing CPI inflation. Formally, we assume:

$$\pi_t = 0, \tag{30}$$

for all t while the exchange rate can fluctuate freely. Note that GM assume $\pi_{H,t} = 0$ for all t, which means that the central bank focuses on stabilizing domestic price inflation, which is different from the current model.

Under the second regime, namely, the *currency union*, the domestic economy is part of a world currency union. Alternatively, it is assumed to peg the exchange rate indefinitely and credibly to the world currency. In either case, we assume:

$$e_t = 0, \tag{31}$$

for all t. Note that the domestic nominal interest rate moves one-for-one with the world interest rate, independently of domestic economic conditions.

3.4 Calibration

Table 2 lists the benchmark parameterization that we use to simulate the distorted steady-state model. Our parameterization is almost consistent with the GM model although there some exceptions. As mentioned, we assume constant returns to scale while GM assume decreasing returns to scale and choose 0.26 as the degree of decreasing returns to scale. In other words, we choose zero as the degree of decreasing returns to scale instead of 0.26. In addition, we set the trade elasticity of substitution η equal to 2, while GM assume an Armington form consumption index that implies the elasticity is unity. However, GM set the elasticity in their medium-scale DSGE model equal to 2 to verify the robustness of their findings.[7] Thus, our choice is not necessarily inconsistent with their parameterization. Rather, our choice is closer to de Paoli[10] who choose 3 and Eaton, Kortum and Neiman[13] who choose 6 following Obstfeld and Rogoff[29].

4 Effectiveness of Labor Cost Reduction, Wage Flexibility, Exchange Rate Policy and Welfare in the Distorted Steady-state model

4.1 Effectiveness of Labor Cost Reduction

We assume that the tax gap in the distorted steady-state model follows:

$$\hat{\tau}_t = \psi_t, \tag{32}$$

$$\psi_t \begin{cases} = \psi_0 & \text{if} \quad t = 0 \\ = 0 & \text{if} \quad t = 1, 2, \cdots, \end{cases} \tag{33}$$

[7] To be exact, GM report robustness tests across $\eta \in \{0.5, 1, 2\}$.

where $\hat{\tau}_t \equiv \tau_t - \tau$ denotes the tax gap which is the percentage deviation of the tax rate from its steady-state value, ψ_t with $\psi_0 < 0$ denotes an exogenous tax shifter that provides pressure to reduce the tax gap (or the tax rate). The tax gap is endogenized following the Bohn rule in the IGBC model, which is derived in Section 5 and has government budget constraints and the Bohn rule to close that model, Eq. (33), which implies that there is no persistency on the tax shifter was introduced into both the distorted steady-state model and the IGBC model while $\psi_t = \rho_\tau^t \tau_0$, which implies that there is persistency of the tax shifter, is implicitly assumed in the GM model.[8] If we used the GM model settings that accompany the persistency of the tax shifter, there would be differences in the dynamic path of the tax gap between the distorted steady-state model where the tax gap is exogenous and the IGBC model where the tax gap is endogenous and follows the Bohn rule. Thus, comparing the dynamic responses to the tax cut in the distorted steady-state model with those in the IGBC model is inappropriate. Therefore, we introduce Eq. (33) to allow a comparison between the dynamic responses in the distorted steady-state model and the IGBC model. Note that Ferrero[17] also chooses no persistency in the exogenous processes, similar to me, to simplify the impulse response functions (IRFs).

Fig. 1 shows the responses of employment, nominal and real (consumption) interest rates, TOT and tax gap to a one percent decrease in the exogenous tax shifter ψ_0. Similar to the GM model, in Fig. 1, the red lines with diamonds are the responses under *inflation targeting*, while the blue lines with circle are the responses under a *currency union*. While our inflation targeting is not domestic price inflation targeting, but rather CPI inflation targeting, which is different from the GM model, our findings are consistent with GM's first finding: the effectiveness of tax cuts as a means to stimulate employment is much weaker under a *currency union* compared with the case of an autonomous monetary policy focused on price stability, namely, *inflation targeting*. As shown in Panel C, the reduction in the real interest rate in the *currency union* is much smaller than under *inflation targeting*. In addition, as shown in panels D and E, the increase in the TOT and consumption in the *currency union* are much smaller than under *inflation targeting*. These results support GM's first finding.

To understand the previous result, similar to the GM model, we obtain the following two equalities as follows:

$$c_t = -(1-\upsilon)\sum_{k=0}^{\infty} \mathrm{E}_t\left(r_{H,t+k}\right) + \lim_{k\to\infty} \mathrm{E}_t\left(c_{t+k}\right), \tag{34}$$

$$s_t = -\sum_{k=0}^{\infty} \mathrm{E}_t\left(r_{H,t+k}\right) + \lim_{k\to\infty} \mathrm{E}_t\left(s_{t+k}\right), \tag{35}$$

where $r_{H,t} \equiv r_t - \mathrm{E}_t\left(\pi_{H,t+1}\right)$ denotes the real interest rate (and where $\lim_{k\to\infty} \mathrm{E}_t\left(c_{t+k}\right) = 0$ and $\lim_{k\to\infty} \mathrm{E}_t\left(s_{t+k}\right) = 0$ in deviation from steady state). Note that we ignore constants and exogenous shifters in Eqs.(34) and (35). Eq. (34) is derived by iterating Eq. (18), while Eq. (35) is derived by iterating Eq. (6). Eqs. (34) and (35) show that consumption and the TOT are negatively related to the sum of expected future real interest rates and are also derived by the GM model. Eqs. (34) and (35) imply that the smaller the reduction in the real interest rate, the smaller the increase in consumption and the TOT, and vice versa. Thus, GM's first finding is applicable in the distorted steady-state model. Notice that Eqs. (34) and (35) are consistent with the international risk-sharing condition Eq. (17) which shows that consumption is positively

[8] To be exact, the GM model assumes $\hat{\tau}_t = \rho_\tau^t \hat{\tau}_0$.

related to the TOT.

While we can replicate GM's first finding when the distorted steady state is introduced, the following comment regarding this finding is necessary. The strength of the central bank's response to variations in inflation is referred to as the "endogenous policy channel," while the resulting reductions in marginal costs and prices that make domestic firms more competitive are referred to as the "competitiveness channel" in the GM model.[9] GM state that while the "endogenous policy channel" is muted, the employment stimulus only relies on the "competitiveness channel" in the *currency union*. However, this statement is not applicable in the *currency union* in our case. As shown in panels A and C in Fig. 1, employment increases while the real interest rate decreases in the *currency union* (blue line with circles). This phenomenon implies that the "endogenous policy channel" exists. GM show that while the real interest rate increases, employment and TOT only increase slightly. Under the *currency union*, because the nominal interest rate is constant, a decrease in (expected) domestic price inflation causes an increase in the real interest rate. Although GM do not show the dynamic response of domestic price inflation to the tax cut, they admit that domestic price inflation declines. As shown in Panel F in Fig. 1, however, while domestic price inflation declines after a one percent decrease in the tax gap, this decline lasts for only one period and inflation becomes positive after that. Thus, the real interest rate declines after a one percent decrease in the tax gap, which is a different result from the GM model. This result depends on whether the decrease in the tax gap is successive or not. A decrease in the tax gap in the GM model is consecutively and this consecutive decrease in the tax rate causes a consecutive decrease in domestic price inflation. Thus, the real interest rate increases slightly. In our case, a decrease in the tax rate is not consecutive but ends after one period. The real interest rate decreases because of an increase in domestic price inflation after the first period. Thus, the "endogenous policy channel" exists in the current model.

However, it can be said that both the "endogenous policy channel" and "competitiveness channel" have weaker effects on employment fluctuations under the *currency union*. This is clarified by panels D and E in Fig. 1 in which both the responses of TOT and consumption under the *currency union* are weaker than those under *inflation targeting*.

In Section 7.1, we analyze the effectiveness of labor cost reductions on the IGBC model where the tax gap is determined endogenously as in the Bohn rule. Similar to Fig. 1, Fig. 2 shows the dynamic response to a one percent decrease in the tax rate in the IGBC model. As shown in Panel G, the dynamic path of the tax gap is no longer a simple one-period reduction. Instead, the dynamic path of the tax gap is now the same as that in the IGBC model where the tax gap is endogenous and increases slightly after an initial decline to satisfy the Bohn rule which tends to secure a fiscal surplus to repay the government debt. Even if the path of the tax gap is altered, our result is unchanged. As shown in panels A, C, D and E in Fig. 2, the reduction in the real interest rate in the *currency union* is much smaller than under *inflation targeting*, while the increases in employment, TOT and consumption are much smaller in the *currency union* than under *inflation targeting*. Despite the altered dynamic path of the tax gap, GM's first finding remains applicable.

4.2 Wage Flexibility and Welfare in a Currency Union

Fig. 3 shows the average welfare loss for a small open economy in a currency union as a function of the degree of wage stickiness, θ_w, and conditional on each of the four exogenous driving forces

[9]See page 3839 of GM for details on both channels.

introduced above, namely, two domestic shocks (technology and demand) and two external shocks (exports and world interest rate), which follows the GM model. In each case, the welfare losses are expressed as a ratio of those under the benchmark setting $\theta_w = 0.8$, which also follows the GM model.

As Fig. 3 shows, the relationship between the welfare loss and the degree of wage rigidity is nonmonotonic, and this result is independent of the driving forces, as in the GM model. The loss functions are half-moon shaped as in the GM model, although there are some exceptions. Starting from a value of θ_w close to unity, a reduction in that parameter value increases welfare losses. However, if wages are sufficiently flexible to begin with, a further increase in wage flexibility leads to a decline in welfare losses. Although these results are not found for an intermediate degree of wage stickiness following a technology shock, these results are found as long as wage rigidity is sufficiently close to unity or zero. An increase in wage flexibility may raise or lower welfare, depending on the initial degree of wage rigidity, as GM show. Similar to GM, the shape of the welfare loss function varies considerably with the type of shock. The maximum is attained for very different values of wage rigidity in their model and ours. Although the loss function appears to be somewhat monotonic in the case of the technology shock, the loss function is highly monotonic in the case of the export shock in the GM model.

Fig. 4 shows the welfare losses associated with the demand shocks together with the three components of the welfare loss function, each being associated with Eq. (15), as in the GM model. There are three features of the welfare loss function that are consistent with the results of GM. With regard to the first component, associated with the employment gap fluctuations (red line with plus signs), an increase in wage flexibility always reduces the contribution of that component to overall welfare losses, although the size of that reduction is relatively small because of the limited influence of wage adjustments on employment in an economy that belongs to a currency union, as discussed above. Regarding the second component, associated with domestic price inflation (black line with squares), an increase in wage flexibility always raises the volatility of domestic price inflation, and thus the contribution of the latter to welfare losses. With regard to the third component, the wage inflation component of the welfare losses (magenta line with plus signs) displays the same nonmonotonicity as the overall losses (blue line with circles), so its contribution is particularly important to account for the finding in Fig. 3. As GM explain, the reason for the nonmonotonicity is straightforward. While the wage rigidities decrease starting from close to unity, the variance in the wage inflation increases. On the one hand, the weight associated with the wage inflation volatility in the loss function Λ_w rapidly decreases as wages become more flexible because $\frac{\partial \Lambda_w}{\partial \theta_w} = \frac{\epsilon_w(1-\Phi)(1-\beta\theta_w)(2-\theta_w)}{\lambda_w^2} > 0$. Thus, the welfare losses associated with the wage rigidities decrease when the wage rigidities are below a certain level.

Fig. 5 compares the welfare effect of changes in wage flexibility in a small open economy that is part of a currency union with that in a small open economy under inflation targeting. The welfare losses are expressed as a ratio to those under the baseline setting $\theta_w = 0.8$, as in GM. Similar to GM, the differences across the two regimes are clear. Under *inflation targeting*, an increase in wage flexibility is always welfare improving, independent of the initial degree of wage rigidity and source of fluctuations (red line with diamonds).

The differences across the two regimes, both qualitative and quantitative, are clear. In particular, and as Fig. 5 makes clear, under *inflation targeting*, an increase in wage flexibility is always welfare improving, independent of the initial degree of wage rigidity and the source of fluctuations, except for the case of the world interest rate shock. However, except for the intermediate degree of wage rigidity, wage flexibility improves welfare in the case of the world interest rate shock. Thus, our results regarding the welfare loss function under *inflation targeting* are almost the same as those of GM. GM's second finding that an increase in wage flexibility often reduces welfare, more likely so in an economy that is part of a currency union, can be replicated in the distorted steady-state model.

The analysis above has examined the impact of changes in the degree of wage rigidities while keeping the price-rigidity parameter, θ_p, unchanged at its baseline value of 0.8. We follow GM and analyze how welfare changes if both wage and price rigidities change in the same direction. Fig. 6 shows welfare loses under a currency union as a function of the degree of overall nominal rigidities, as captured by variation in a common value for θ_w and θ_p (denoted by θ). Our results are consistent with those obtained by GM. That is, the non-monotonicity that characterized the welfare loss function under a currency union is even shown here when price and wage rigidities are simultaneously varied (at glance, Panel A in Fig. 6 shows that as nominal rigidities increase, welfare losses increase but the welfare losses attain the highest values when both rigidities equal 0.975, so that non-monotonicity remains barely preserved).

5 IGBC Model: Introducing the Government Budget Constraint into the Distorted Steady-state model

In this section, we introduce the government and its budget constraint into the distorted steady-state model. Lump-sum transfers are no longer available and the government issues debt to finance its deficit and to repay the debt issued in the past.

5.1 Households

Instead of Eq. (3), the sequence of budget constraints is:

$$\int_0^1 P_{H,t}(i)\, C_{H,t}(i)\, di + P_{F,t}C_{F,t} + \mathrm{E}_t\left(Q_{t,t+1}D_{t+1}^n\right) \leq D_t^n + \int_0^1 W_t(j)\, \mathcal{N}_t(j)\, dj + PR_t. \qquad (36)$$

Households maximize Eq. (1) subject to Eq. (36) instead of Eq. (3). Different from Eq. (3), lump-sum transfer TR_t disappears in Eq. (36) (Instead of lump-sum transfers, the government levies tax on firms sales). However, this modification does not change the households' optimality conditions and Euler equation Eq. (4) is applicable.

5.2 Government

The government levies the tax on the firms' sales in a small open economy, purchases generic goods and can issue debt to finance its deficits.

5.2.1 Flow Government Budget Constraint

The flow government budget constraint is:

$$B_t^n = B_{t-1}^n\left(1 + r_{t-1}\right) - \int_0^1 P_{H,t}(i)\left(\tau_t Y_t(i) - G_t(i)\right) di,$$

where B_t^n denotes nominal government debt issued by the home country held at the end of period t and maturing in period $t+1$. Note that all households are assumed to be identical, and in equilibrium there is no borrowing or lending among them. Thus, all asset holdings by private agents are in the form of government securities.[10]

The previous flow government budget constraint can be rewritten using the optimal allocation for generic goods as follows:

$$B_t^n = B_{t-1}^n (1 + r_{t-1}) - P_t SP_t, \tag{37}$$

where

$$SP_t \equiv \frac{P_{H,t}}{P_t} (\tau_t Y_t - G_t), \tag{38}$$

denotes the (real) fiscal surplus and $G_t \equiv \left(\int_0^1 G_t(i)^{\frac{\epsilon_p - 1}{\epsilon}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$ denotes the index of government expenditure in a small open economy, analogous to the consumption index. For simplicity, we assume that government expenditure is fully allocated to domestically produced goods as in Gali and Monacelli[21]. Government expenditure follows the exogenous AR(1) process:

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t}.$$

Dividing both sides of Eq. (37) by the CPI yields:

$$B_t = B_{t-1} (1 + r_{t-1}) \left(\frac{P_t}{P_{t-1}} \right)^{-1} - SP_t, \tag{39}$$

where $B_t \equiv \frac{B_t^n}{P_t}$ denotes real government debt.

Iterating Eq. (39) forward and imposing the appropriate transversality condition for government debt $\lim_{k \to \infty} \beta^k \mathrm{E}_t \left(\frac{B_{t+k}^n}{P_{t+k+1}} \right) = 0$, we have

$$1 = \frac{\sum_{k=0}^{\infty} \beta^k \mathrm{E}_t \left(C_{t+k}^{-1} Z_{t+k} SP_{t+k} \right)}{C_t^{-1} Z_t B_{t-1} \left(\frac{P_t}{P_{t-1}} \right)^{-1}}, \tag{40}$$

where we use Eq. (4).

Eq. (40) can be rewritten as a second-order differential equation as follows:

$$(1 + r_{t-1}) B_{t-1} \left(\frac{P_t}{P_{t-1}} \right)^{-1} = SP_t + \beta \mathrm{E}_t \left[\frac{C_t}{C_{t+1}} \frac{Z_{t+1}}{Z_t} \left(\frac{P_{t+1}}{P_t} \right)^{-1} (1 + r_t) B_t \right]. \tag{41}$$

5.3 Market-Clearing Condition

Because of government expenditure, the market-clearing condition in the IGBC model is given by:

$$Y_t(i) = C_{H,t}(i) + X_t(i) + G_t(i),$$

instead of Eq. (14).

[10]That is, $D_t^n = (1 + r_{t-1}) B_{t-1}^n + \mathcal{E}_t \left(1 + r_{t-1}^* \right) B_{t-1}^{n*}$ where B_t^{n*} denotes nominal government debt issued by the foreign government in terms of foreign currency.

6 Welfare Criteria, Equilibrium, Monetary and Fiscal Policy and Calibration of the IGBC Model

6.1 Welfare Criteria

The inclusion of the government budget constraint alters the welfare criteria introduced in Section 3, namely Eq. (15), and the welfare criterion in the IGBC model is given by:

$$\mathcal{L}^F \sim \frac{1}{2}\left[\Lambda_n^F \mathrm{var}\left(\hat{n}_t\right) + \Lambda_p^F \mathrm{var}\left(\pi_{H,t}\right) + \Lambda_w^F \mathrm{var}\left(\pi_t^w\right)\right], \tag{42}$$

where $\Lambda_n^F \equiv \frac{2\Omega_0^F}{\gamma_v^2}$, $\Lambda_p^F \equiv \frac{\epsilon_p\left[(1-\Phi)-\Theta_2^F\sigma_C(\lambda_p+\varphi)\right]}{\lambda_p\sigma_C}$ and $\Lambda_w^F \equiv \frac{\epsilon_w\left[(1+\epsilon_w\varphi)(1-\Phi)-\Theta_2^F\sigma_C(1+\varphi)\lambda_w\right]}{\lambda_w\sigma_C}$ denote the weight of the variances of the welfare relevant output gap, (domestic) price inflation and wage inflation, respectively, Ω_0^F is the composite blocks of parameters and $\sigma_C \equiv \frac{C}{Y}$ denotes the steady-state ratio of consumption to output. Note that Eq. (42) is derived based on the second-order approximated utility function following Benigno and Woodford[3] and Ferrero[17] to eliminate linear terms.

6.2 Equilibrium in the IGBC Model

Using the market-clearing conditions and after log-linearization around the zero-inflation steady state, the optimality conditions, government budget constraint and definition of fiscal surplus can be used to determine the set of conditions characterizing the equilibrium of the small open economy. That equilibrium can be represented using the following system of difference equations similar to Section 3.

Aggregate Demand Block (Modified and Derived Anew Only)

$$y_t = (1-\upsilon)\,\sigma_C c_t + \eta\sigma_C\upsilon\,(2-\upsilon)\,s_t + \upsilon\sigma_C z_{1,t}^* + \sigma_G g_t, \tag{43}$$

$$\begin{aligned} c_t = {} & \mathrm{E}_t\left(c_{t+1}\right) - \left[r_t - \mathrm{E}_t\left(\pi_{t+1}\right)\right] - b_t + \frac{1}{\beta}\left(r_{t-1}-\pi_t\right) + \frac{1}{\beta}b_{t-1} - \frac{1-\beta}{\beta}sp_t \\ & + \left(1-\rho_z\right)z_t + \frac{1-\beta}{\beta}\delta, \end{aligned} \tag{44}$$

$$sp_t = -\upsilon s_t + \frac{\beta}{(1-\beta)\,\sigma_B}\tau_t + \frac{\beta\tau}{(1-\beta)\,\sigma_B}y_t - \frac{\beta\sigma_G}{(1-\beta)\,\sigma_B}g_t - \frac{\beta}{(1-\beta)\,\sigma_B}\tau, \tag{45}$$

$$b_t = \frac{1}{\beta}\left(r_{t-1}-\pi_t\right) + \frac{1}{\beta}b_{t-1} - \frac{1-\beta}{\beta}sp_t - \frac{1}{\beta}\delta, \tag{46}$$

where $\sigma_G \equiv \frac{G}{Y}$ satisfying $\sigma_G = 1 - \sigma_C$ denotes the steady-state ratio of government expenditure to output.

In the aggregate demand block in the IGBC model, Eqs. (43) and (44) replace Eqs. (16) and (18) in the distorted steady-state model and Eqs. (45) and (46), which stem from the definition of the fiscal surplus in Eq. (38) and the government budget constraint in Eq. (39), are newly derived. Other equalities, Eqs. (17), (19) and (20) in the demand block are replaced in the IGBC model. All the equalities in the aggregate supply block as expressed in Eqs. (21) to (29) are replaced in the IGBC model.[11]

[11] The efficient levels of employment, output, TOT, consumption and real wage in the IGBC model are shown in appendices available from our website. See footnote 4 for the URL.

6.3 Monetary and Fiscal Regimes

We now consider whether GM's two findings are still applicable in a domestic economy that is part of a currency union. That is, Eq. (31) is succeeded as a monetary regime. Furthermore, we assume that the government in the domestic economy adopts the Bohn rule. Based on Bohn[7], Mahdavi[27] estimates a key coefficient of the Bohn rule using US government data. The US is often regarded as a currency union because the US consists of 50 states that use a common currency, the "US dollar." Thus, we adopt the fiscal feedback rule from Mahdavi[27] that is a type of Bohn rule.

The fiscal feedback rule in Mahdavi[27] is given by:

$$\frac{SP_t}{Y_t} = \phi_B \frac{B_{t-1}}{Y_t},$$

where ϕ_B takes the value estimated by Mahdavi[27]. This expression implies that the government tends to make the ratio of the fiscal surplus to output larger than the ratio of past government debt to current output depending on the value of ϕ_B. Dividing both sides by the steady-state value of the fiscal surplus, subtracting one from both sides of that equality and taking logarithms, we obtain the logarithmic equality of the fiscal feedback rule in Mahdavi[27] as follows:

$$sp_t = \phi_b b_{t-1} + (\phi_b - 1), \tag{47}$$

where $\phi_b \equiv \frac{\phi_B \beta}{1-\beta}$ denotes the fiscal policy rule coefficient and is dubbed the Bohn rule coefficient hereafter. We assume that the tax gap is an instrument of fiscal policy. Substituting Eq. (45) into the previous expression, we have:

$$\hat{\tau}_t = \frac{\upsilon(1-\beta)\sigma_B}{\beta} s_t - \tau y_t + \frac{(1-\beta)\sigma_B \phi_b}{\beta} b_{t-1} + \sigma_G g_t + \frac{(1-\beta)\sigma_B(\phi_b - 1)}{\beta}, \tag{48}$$

which is the fiscal policy rule based on Mahdavi[27].

Although the equation derived by combining Eqs.(44) and (46) is identical to Eq.(18), which is a log-linearized Euler equation, that is, the Euler equation is the same in the distorted steady state and IGBC models, the IGBC model is still different from the distorted steady-state model. The reason is that the endogenous fiscal-policy rule, Eq.(47), which can be rewritten as Eq.(48), is introduced in the IGBC model.

6.4 Calibration

Table 2 lists the parameterization in the IGBC model that we use for the simulation exercises. Parameterization in the distorted steady-state model is succeeded in the IGBC model. Additional parametrization is necessary because of the introduction of a government that issues government debt and levies tax, as shown in lines 14 to 18 in Table 2. We set $\tau = 0.3$ following Ferrero[17] who analyzed monetary and fiscal policy rules in a currency union. We set $\sigma_B = 4.543$ and $\sigma_G = 0.477$ based on data provided by Eurostat. From 2008 to 2019, the ratio of government debt to GDP in Greece, Italy, Portugal and Spain (GIPS) was 1.1358. The model is based on quarterly data, thus this number must be multiplied by 4. Similarly, the ratio of government expenditure to GDP in the GIPS is 0.477. We set $\phi_b = 6.5$ following the estimation results of Mahdavi[27], who estimated several models with results varying from 0.045 to 0.085, which corresponds to 4.455 to 8.415 in our setting. Thus, we chose 6.5 which is moderate as a benchmark value. [12] We simulate under not only 6.5 but also 4.5 and 8.5, as a robustness check in Section 8. While the GM model does not set the value of ρ_g, which is persistence of the exogenous process of government expenditure because there is no government in their model, we set $\rho_g = 0.9$ following the persistence of the exogenous process of demand and so forth.

7 Effectiveness of Labor Cost Reduction, Wage Flexibility, Exchange Rate Policy and Welfare in the IGBC Model

7.1 Effectiveness of Labor Cost Reduction

We assume that the fiscal policy rule in the IGBC model is as follows:

$$\hat{\tau}_t = \frac{\upsilon (1-\beta)\sigma_B}{\beta} s_t - \tau y_t + \frac{(1-\beta)\sigma_B \phi_b}{\beta} b_{t-1} + \sigma_G g_t + \frac{(1-\beta)\sigma_B(\phi_b - 1)}{\beta} + \psi_t. \tag{49}$$

In Eq.(49), the exogenous tax shifter ψ_t, which follows Eq. (33) is added to Eq. (48), to analyze the effectiveness of a labor cost reduction as in Section 4.1.

Next, we discuss the dynamic responses to a temporary one percent decrease in the tax gap. The tax gap is now endogenous and a one percent decrease in ψ_0 does not reduce the tax gap by one percent because there is bi-directional causality between the tax gap, and TOT and output. That is, a change in the tax gap affects both TOT and output, and changes in both TOT and output affect the tax gap. We find that $\psi_0 \simeq -0.989\%$ results in a one percent decrease in the tax gap, therefore we use this setting.[13]

Similar to Fig. 2, Fig. 7 shows the responses of employment, nominal and real interest rates, TOT, consumption, domestic price inflation, tax gap, government debt and fiscal surplus to a decrease in the exogenous tax shifter which decreases the tax gap by one percent in both the IGBC and distorted steady-state models. The blue lines with circles are responses under the distorted steady-state model and identical to the blue lines with circles in Fig. 2. The magenta lines with plus signs show the responses for the IGBC model. There are notable differences between the responses of employment, TOT and consumption in both models (see panels A, D and E). After the tax reduction, the increase in employment is smaller than in the distorted steady-state model and the responses of TOT and consumption deviate below those in the distorted steady-state model. A decrease in the tax gap decreases the fiscal surplus, while government debt increases, as shown in Eqs. (45) and (46). As a result, consumption decreases more in the IGBC model. In the IGBC model, the government budget constraint Eq. (46) is introduced and Eq. (46) implies that CPI inflation increases in response to a decrease in the fiscal surplus which is used to repay government debt issued in the past. Note that the CPI is a weighted average of domestic prices and imported goods prices which are constant because the domestic economy is part of a currency union and we assume $P_t^* = 1$ for now.[14] Thus, an increase in the domestic price level causes a decrease in the TOT as shown in Panel D.

[12] Although we set ϕ_b to 6.5 following Mahdavi[27], 6.5 is consistent with data on 11 small open economies in the Eurozone (Cyprus, Estonia, Finland, Greece, Latvia, Lithuania, Luxembourg, Malta, Portugal, Slovakia, and Slovenia) from 2007 to 2022. Our empirical analysis following Bohn[7] suggests that ϕ_b is 6.435 or 6.732, which are close to 6.5. See the on-line appendix on https://www.econ.nagoya-cu.ac.jp/~eiji_okano /papers_e.html for details.

[13] To be exact, $\psi_0 = -0.989138307540085\%$.

[14] Eq. (25) can be rewritten as $p_t = (1-\upsilon) p_{H,t} + \upsilon p_{F,t}$.

However, employment in the IGBC model (see Panel A) does not increase substantially after the shock and its response does not differ much from that in the distorted steady-state model. The reason for this is the existence of government expenditure. As shown in Eqs. (16) and (43), the sum of consumption and TOT determines output because of the market-clearing condition, as long as the export shock is ignored. While the steady-state ratio of consumption to output σ_C does not appear in Eq. (16), that parameter appears on Eq. (43) because of government expenditure which affects GDP, along with consumption and TOT, and that parameter is less than unity (our parameterization implies $\sigma_C = 0.523$). Thus, an increase in employment which corresponds to output as long as there are no changes in technology, under the IGBC model, is smaller than the increase in it under the distorted steady-state model, even if consumption and TOT decline more in the IGBC model.

The response of the employment in the IGBC model in the *currency union* is the same as or less than in the distorted steady-state model in the *currency union*. Thus, it can be said that the effectiveness of labor cost adjustments on employment is much smaller in the *currency union*, even if an endogenous fiscal policy rule such as the Bohn rule is introduced. Thus, our analysis supports GM's first finding.

7.2 Wage Flexibility and Welfare in a Currency Union

Similar to Fig. 3, Fig. 8 shows the average welfare loss in a small open economy in a currency union, as a function of the degree of wage stickiness θ_w, and conditional on each of, not four but, five exogenous driving forces, namely the four shocks appearing in the distorted steady-state model and the government expenditure shock. While it is highlighted that the relationship between the welfare loss and the degree of wage rigidity is nonmonotonic in the distorted steady-state model, as shown in Fig. 3, that feature disappears in Fig. 8. Except for the demand and world interest rate shocks, the welfare loss is monotonically increasing, as wage rigidity increases. In addition, as shown in panels A and D which show the response of the welfare loss functions to the demand and world interest rate shocks, respectively, the welfare loss functions are not half-moon shaped but "N" shaped. That is, an increase in wage flexibility reduces the welfare loss if wage rigidity is high enough. Thus, an increase in wage rigidity does not necessarily reduce the welfare loss for all shocks.

Fig. 9 shows the welfare loss associated with the demand shock together with the three components of the welfare loss function, each being associated with Eq. (42), similar to Fig. 4. For the first component, the welfare loss associated with the employment gap fluctuations (red line with diamonds) always exceeds the welfare loss associated with wage inflation fluctuations (magenta line with plus signs) and that loss increases drastically together with an increase in wage rigidity, as wage rigidity exceeds 0.875. In Fig. 4, the welfare loss associated with the employment gap fluctuations is smaller and that loss does not exceed the welfare loss associated with wage inflation fluctuations until wage rigidity reaches 0.75 starting from close to zero and the increase in the welfare loss associated with the employment gap fluctuations is smaller. It is obvious that the nonmonotonic relationship between welfare loss and wage rigidity is eliminated by changes in the welfare loss function associated with employment gap fluctuations, and those changes make the total welfare function (blue line with circles) "N" shaped.

In conclusion, introducing an endogenous fiscal policy rule such as the Bohn rule, namely, introducing Eq. (47), makes the employment gap fluctuate more. The Bohn rule implies that the government achieves a sufficient fiscal surplus to repay the government debt and the rule has procyclicality. To increase the fiscal surplus, an increase in output, which is identical to employment as long as productivity is unchanged as shown in Eq. (20), or an increase in the tax gap is needed if government expenditure and TOT do not change. In our setting, the tax gap is the policy instrument that reacts to government debt issued in the past. The tax gap also reacts to employment. The tax gap increases when employment stagnates to secure the fiscal surplus that is required to repay the government debt (as long as productivity is unchanged). In contrast, the tax gap decreases when employment increases because there is enough fiscal surplus to repay the debt (as long as productivity is unchanged). Thus, the tax gap is negatively related to employment and there is procyclicality in the Bohn rule. The sign of the second term on the right-hand side in Eq. (49) is negative, which implies that the tax gap decreases when employment increases, and vice versa. Furthermore, when employment increases sufficiently, the tax gap decreases and this decreases marginal cost, although the marginal cost will come under pressure to increase from the increase in employment. Thus, while an increase in marginal cost increases domestic price inflation, there is downward pressure on domestic price inflation through the Bohn rule even if employment increases. The pressure to increase domestic price inflation is reduced through the potential procyclicality of the Bohn rule.

Reducing the upward pressure on domestic price inflation through a decrease in the tax gap means that introducing the Bohn rule makes domestic price inflation inelastic to changes in the employment gap. Introducing the Bohn rule makes the slope of the (implied) NKPC flatter in the IGBC model than it is in the distorted steady-state model. By combining Eqs. (21), (22), (27) and (32), we have:

$$\pi_{H,t} = \beta \mathrm{E}_t \left(\pi_{H,t+1} \right) + \lambda_p \left(\sigma_\varsigma + \varphi \right) \hat{n}_t + \lambda_p \mu_t^w, \tag{50}$$

with $\sigma_\varsigma \equiv \left[(\eta - 1) \upsilon (2 - \upsilon) + 1 \right]^{-1}$. Eq. (50) is the (implied) NKPC in the distorted steady-state model. By combining Eqs. (21), (22) and (27) and iterating them and using Eqs. (44), (46) and (49), we have:

$$\begin{aligned} \pi_{H,t} \;\; = \;\; & \beta \mathrm{E}_t \left(\pi_{H,t+1} \right) + \lambda_p \left[\left(\sigma_\varsigma + \varphi \right) - \frac{\beta\tau - \upsilon (1 - \beta) \sigma_B \sigma_\varsigma}{(1 - \tau) \beta} \right] \hat{n}_t \\ & - \frac{\lambda_p}{\beta} \left[\left(\sigma_\varsigma + \varphi \right) - \frac{\beta\tau - \upsilon (1 - \beta) \sigma_B \sigma_\varsigma}{(1 - \tau) \beta} \right] \hat{n}_{t-1} + \lambda_p \mu_t^w - \frac{\lambda_p}{\beta} \mu_{t-1}^w + \frac{\lambda_p}{\beta} mc_{t-1}, \end{aligned} \tag{51}$$

which is the (implied) NKPC in the IGBC model. The exogenous shifters and constants are ignored in Eqs. (50) and (51). Note that the tax gap $\hat{\tau}_t$ is eliminated in both Eqs. (50) and (51) because Eq. (32) is substituted into Eq. (50), while Eq. (49) is substituted into Eq. (51) to show how an endogenous fiscal policy, such as the Bohn rule, affects the slope of the (implied) NKPC.[15] The reason "implied" is highlighted is that Eqs. (50) and (51) are not used in either the distorted steady-state model nor the IGBC model, and are not used to solve those models.

Eqs. (50) and (51) show that domestic price inflation in both models is affected by both the employment gap and the wage markup. In addition, these equalities show that the slope of the NKPC in the IGBC model is $\lambda_p \left[\frac{\beta\tau - \upsilon(1-\beta)\sigma_B \sigma_\varsigma}{(1-\tau)\beta} \right]$, which corresponds to the second term in the squared parentheses in the second term in Eq. (51) multiplied by -1, and is flatter (or smaller) than it is in the distorted steady-state model, with the vertical axis measuring domestic price

[15] See Appendix A for details on the derivation

inflation and the horizontal axis measuring the employment gap. Under the parameterization shown in Table 2, the slope of the (implied) NKPC in the distorted steady-state model is 0.1929, while it is 0.1722 in the IGBC model. A one percent change in the employment gap causes a 0.1929 percent change in domestic price inflation in the distorted steady-state model, while a one percent change in the employment gap causes a 0.1722 percent change in domestic price inflation in the IGBC model, as long as no other variables change.

These slopes imply that a one percent change in domestic price inflation changes the employment gap by $\frac{1}{0.1929}$ percent, which is about 5.18 percent in the distorted steady-state model, while it changes the employment gap by $\frac{1}{0.1722}$ percent, which is 5.81 percent in the IGBC model. That is, the flatter the slope of the (implied) NKPC, the more elastic or sensitive to changes in domestic price inflation is employment. As a result, the red line with diamonds that shows the welfare loss associated with employment gap fluctuations shifts upward and always exceeds the magenta line with plus signs that shows the welfare loss associated with wage inflation fluctuations in Fig. 9, while the red line with diamonds lies at the bottom in Fig. 4. Following an increase in the demand shock, the higher the wage rigidity, the larger the decrease in the wage markup, and vice versa. The reason is that the nominal wage experiences stagnation and employment much increases to offset that stagnation when the wage rigidity is high. Combined with a flatter slope of the (implied) NKPC, which is associated with the procyclicality of the Bohn rule and makes employment more elastic to domestic price inflation, the welfare loss associated with employment gap fluctuations increases substantially as wage rigidity increases. Thus, an increase in wage rigidity reduces welfare further in the IGBC model.

The blue line with circles showing total welfare loss is not half-moon shaped but "N" shaped in Fig. 9, which is different from that line in Fig. 4. As long as wage rigidity is high enough, an increase in wage rigidity reduces welfare loss in the distorted steady-state model. However, this is not applicable in the IGBC model. Even if wage rigidity is high enough, an increase in wage rigidity no longer reduces welfare loss in the IGBC model. Rather, wage rigidity is harmful from the perspective of reducing welfare loss. Note that the red line with diamonds that shows the welfare loss associated with employment gap fluctuations is parallel to the black line with squares that shows welfare loss associated with domestic price inflation fluctuations in Fig. 9 until wage rigidity reaches a value of 0.825 starting from close to zero because the employment gap is elastic or more sensitive to changes in domestic price inflation, as mentioned.

Fig. 10 shows the IRFs of employment and domestic price inflation to the demand shock in both the distorted steady-state and IGBC models. The red line with diamonds, blue line with circles and magenta line with plus signs are the IRFs under $\theta_w = 0.625$, 0.8 and 0.975, respectively. As panels A and C show, there are no notable differences in the IRFs even if wage rigidity θ_w varies in the distorted steady-state model. Panel D shows that domestic price inflation is more stable as wage rigidity increases in the IGBC model. However, as shown in Panel B, the magenta line with plus signs, which is the IRF of employment, becomes flatter implying that the stabilization is postponed for $\theta_w = 0.975$ in the IGBC model. This implies that employment becomes elastic or more sensitive to changes in domestic inflation as wage rigidity increases.

Fig. 11 compares the welfare effect of changes in wage flexibility in the IGBC model with that in the distorted steady-state model. The welfare losses are expressed as a ratio to those under the baseline setting $\theta_w = 0.8$, as in Fig. 5. In Fig. 11, the blue line with circles is identical to that in Fig. 5, while the magenta line with plus signs is identical to that in Fig. 8. As we have already seen, the differences between the two models are clear. Under the IGBC model, an increase in wage flexibility is almost welfare improving, independent of the initial degree of rigidity and the source of the fluctuations, except for the response to demand shocks. In addition, as long as wage rigidity is high enough, an increase in wage flexibility reduces the welfare loss even under demand shocks.

GM's second finding is that an increase in wage flexibility often reduces welfare, more likely so in an economy that is part of a currency union. However, as shown in panels B, C and D in Figs. 8 and 11, the welfare loss increases as wage rigidity increases. In addition, as long as the degree of wage rigidity is high enough, an increase in wage flexibility reduces the welfare loss as shown in Panel A in Figs. 8 and 11. GM's second finding is not necessarily applicable in the IGBC model in which there is an endogenous fiscal policy rule such as the Bohn rule with a government budget constraint. Druante et al.[12] and Knell[24] show that the frequency of changes in wages is once in 4.967 quarters (once in 14.9 months) in Europa, based on survey data. Based on Calvo pricing, this result implies $\theta_w = 0.8$, which is identical to our benchmark parameterization.[16] $\theta_w = 0.8$ is not necessary extremely high; however, as shown in Panels B, C, and E in Figure 8, a decrease in nominal wage rigidity at $\theta_w = 0.8$ improves welfare losses to technology, export, and government-expenditure shocks. Indeed, welfare losses might be mitigated through a decrease in wage rigidity.

Finally, we show results regarding how welfare changes if both wage and price rigidities change in the same direction, and we analyze them following GM. Similar to Fig. 6, Fig. 12 shows welfare losses under a currency union as a function of the degree of overall nominal rigidities, as captured by variation in a common value for θ_w and θ_p. Our results are consistent with those obtained by GM as well as with our results in Subsection 4.2. The non-monotonicity that characterizes the welfare loss function under a currency union is even shown here when both price and wage rigidities simultaneously vary.

7.3 Replicating GM's Second Finding

As mentioned, an endogenous fiscal policy rule such as the Bohn rule makes the slope of the (implied) NKPC flatter and makes employment more sensitive or elastic to changes in domestic price inflation. The slope of the (implied) NKPC in the IGBC model is $\lambda_p \left[\frac{\beta\tau - \upsilon(1-\beta)\sigma_B\sigma_\varsigma}{(1-\tau)\beta}\right]$, smaller than that in the distorted steady-state model. Two important fiscal parameters, which are the steady-state tax rate τ and the steady-state ratio of government debt to output σ_B, are related to the difference in the slope of the (implied) NKPC. we can make the slope of the (implied) NKPC in the IGBC model the same as the slope in the distorted steady-state model by choosing these important fiscal parameters as follows:

$$\tau = \frac{\upsilon\left(1-\beta\right)\sigma_B\sigma_\varsigma}{\beta} \quad \text{or} \quad \sigma_B = \frac{\beta\tau}{\upsilon\left(1-\beta\right)\sigma_\varsigma}.$$

[16] Recent studies estimating New Keynesian wage Phillips curve (NKWPC) report that the absolute value of slope of the NKWPC is 0.308 to 0.739 (Schryder, Peersman and Wauters[33] and Gabriel[18]). These authors used lagged unemployment rather than wage markups to estimate. Viegi and Dadam[34] suggest that the slope of the NKWPC is $-\frac{\lambda_w\varphi}{1-\beta}$ rather than $-\lambda_w$ if the explanatory variable is unemployment rather than wage markup. By using that relationship, we can obtain suggested θ_w and those estimation results implies that θ_w is 0.58 to 0.95, premising our benchmark parameterization. Our benchmark is within the suggested value for θ_w.

Substituting any equalities in the previous expression into Eq. (51) yields:

$$\pi_{H,t} = \beta \mathrm{E}_t \left(\pi_{H,t+1}\right) + \lambda_p \left(\sigma_\varsigma + \varphi\right) \hat{n}_t - \frac{\lambda_p \left(\sigma_\varsigma + \varphi\right)}{\beta} \hat{n}_{t-1} + \lambda_p \mu_t^w - \frac{\lambda_p}{\beta} \mu_{t-1}^w + \frac{\lambda_p}{\beta} mc_{t-1},$$

in which the slope is the same as the slope of the (implied) NKPC in the distorted steady-state model in Eq. (50) $\lambda_p \left(\sigma_\varsigma + \varphi\right)$ and that expression corresponds to Eq. (50) except for the presence of lagged variables.

The reason GM's second finding is not necessarily applicable in the IGBC model is that we introduce the Bohn rule which has procyclicality latently and make the slope of the (implied) NKPC flatter than that in the distorted steady-state model. By choosing either $\tau = \frac{\upsilon(1-\beta)\sigma_B\sigma_\varsigma}{\beta}$ or $\sigma_B = \frac{\beta\tau}{\upsilon(1-\beta)\sigma_\varsigma}$ and calibrating the model, we can verify if our explanation is appropriate. While there are two options, we choose $\tau = \frac{\upsilon(1-\beta)\sigma_B\sigma_\varsigma}{\beta}$ which implies that the steady-state tax rate is 0.0091 under our parameterization shown in Table 2. There is also the option to choose $\sigma_B = \frac{\beta\tau}{\upsilon(1-\beta)\sigma_\varsigma}$ which implies that the balance of government debt is 37.3725 times higher than output under our parameterization.[17] From 2008 to 2019, the minimum of the (annual) ratio of government debt to output was 0.79 in 2008 and it reached a maximum of 1.25 in 2014 in the GIPS and 37.3725 is too inconsistent with the data. While setting the steady-state tax rate to 0.0091 is not plausible, this setting might be better than setting $\sigma_B = \frac{\beta\tau}{\upsilon(1-\beta)\sigma_\varsigma}$.[18]

Fig. 13 shows the IRFs of employment and domestic price inflation to a demand shock under various degrees of wage rigidity $\theta_w \in \{0.625, 0.8, 0.975\}$ and how setting $\tau = \frac{\upsilon(1-\beta)\sigma_B\sigma_\varsigma}{\beta}$, which makes the slope of the (implied) NKPC in the IGBC the same as in the distorted steady state, contributes to stabilizing employment even if the degree of wage rigidity is very high. In Fig. 13, the red line with diamonds, the blue line with circles and the magenta line with plus signs show the IRFs to a demand shock under $\theta_w = 0.625$, 0.8 and 0.975, respectively. In Panel A, which shows the IRFs under $\tau = 0.3$, the IRF of employment with $\theta_w = 0.975$ (the magenta line with plus signs) deviates from the other lines which are the IRFs under lower wage rigidity and implies that stabilization in the employment gap takes a substantial amount of time if wage rigidity is high. However, in Panel B, which shows the IRFs under $\tau = \frac{\upsilon(1-\beta)\sigma_B\sigma_\varsigma}{\beta}$, the IRF of employment under $\theta_w = 0.975$ (magenta line with plus signs) is close to the other lines and implies that even if wage rigidity is high, the time taken to stabilize employment is not so different from the time under lower wage rigidity. These results imply that a flatter slope of the (implied) NKPC in the IGBC model makes the employment gap more volatile.

Fig. 14 compares the welfare effect of changes in wage flexibility in the IGBC model and shows how the steady-state tax rate $\tau = \frac{\upsilon(1-\beta)\sigma_B\sigma_\varsigma}{\beta}$ replicates GM's second finding. In Fig. 14, the magenta line with plus signs shows the welfare loss for $\tau = 0.3$ which is identical to the magenta line with plus signs in Fig. 11 and the black line with squares shows the welfare loss for $\tau = \frac{\upsilon(1-\beta)\sigma_B\sigma_\varsigma}{\beta}$. Different from the welfare loss in $\tau = 0.3$, $\tau = \frac{\upsilon(1-\beta)\sigma_B\sigma_\varsigma}{\beta}$ is not necessarily a monotonically increasing function of wage rigidity. In particular, for a demand shock, the welfare loss under $\tau = \frac{\upsilon(1-\beta)\sigma_B\sigma_\varsigma}{\beta}$ is not "N" shaped, but rather half-moon shaped. As long as $\tau = \frac{\upsilon(1-\beta)\sigma_B\sigma_\varsigma}{\beta}$, wage rigidity often contributes to reduce the welfare loss and GM's second

[17] $\sigma_B = 149.49$ is applied under the benchmark parameterization. Because the timing of the model is quarterly, we have to divide this by four.

[18] According to the European Commission[16], in the GIPS, the reduced rate of VAT ranges from 5% (Italy) to 10% (Spain) and 4% is adopted as the super-reduced rate (Spain). Although regarding the steady-state tax rate in the IGBC model as the VAT rate is not necessarily appropriate, our choice, which implies a 0.91% VAT rate, could be worse.

finding is applicable and cannot be denied even if there is an endogenous fiscal policy rule such as the Bohn rule with a government budget constraint. In other words, an endogenous fiscal policy rule such as the Bohn rule with a government budget constraint eliminates nonmonotonicity (or half-moon shape) of the welfare loss functions and hinders the realization of GM's second finding.

8 Robustness Exercise on the Bohn Rule

8.1 Robustness Exercise on the Bohn-Rule Coefficient

This section conducts robustness exercises across three different values of the Bohn rule coefficient ϕ_b in Eq. (47) and shows that variation in the Bohn rule coefficient does not necessarily result in nontrivial findings as long as the Bohn Rule coefficient ranges within the values estimated by Mahdavi[27].

Fig. 15 shows how changes in the Bohn rule coefficient ϕ_b affect the dynamic responses to a one percent decrease in the tax gap. Based on Mahdavi[27], ϕ_b ranges from 4.455 to 8.415, thus, we analyze not only the case of $\phi_b = 6.5$, which is our benchmark, but also the case of $\phi_b = 4.5$ and 8.5.[19] [20]

In Fig. 15, the blue line with circles which is identical to the magenta line with plus signs in Fig. 7 which shows the dynamic response to a one percent decrease in the tax gap with benchmark parameterization, that is, the Bohn rule coefficient ϕ_b is set to 6.5. The red line with diamonds and magenta line with plus signs show the dynamic responses to a one percent decrease in the tax gap with $\phi_b = 4.5$ and 8.5, respectively. As shown in panels I and J, the increase in the fiscal surplus after the shock increases as ϕ_b increases. This result is consistent with the Bohn rule Eq. (47) implying that the bigger the value of ϕ_b, the more secure the fiscal surplus with which to repay debt, and vice versa. As a result, as shown in Panel E, the decrease in consumption increases as ϕ_b increases. Because of the international risk-sharing condition Eq. (17), TOT is positively related to consumption. Thus, similar to the consumption response, as shown in Panel D, the decrease in TOT increases as ϕ_b increases.

To understand the consumption and TOT responses, we derive two equalities similar to Eqs. (34) and (35). By iterating Eq. (44) with Eqs. (45), (46) and (48), we obtain:

$$c_t = -\frac{(1-\beta)(\phi_b - 1)}{\beta} \sum_{k=0}^{\infty} \mathrm{E}_t (b_{t+k}) + b_t - \lim_{k\to\infty} \mathrm{E}_t (b_{t+k}) + \lim_{k\to\infty} \mathrm{E}_t (c_{t+k}), \quad (52)$$

while substituting Eq. (17) into Eq. (52) yields

$$s_t = -\frac{(1-\beta)(\phi_b - 1)}{\beta(1-\upsilon)} \sum_{k=0}^{\infty} \mathrm{E}_t (b_{t+k}) + \frac{1}{1-\upsilon} b_t - \lim_{k\to\infty} \frac{1}{1-\upsilon} \mathrm{E}_t (b_{t+k}) + \lim_{k\to\infty} \mathrm{E}_t (c_{t+k}), \quad (53)$$

while ignoring the constants and exogenous shifters where $\lim_{k\to\infty} \mathrm{E}_t (c_{t+k}) = 0$ and $\lim_{k\to\infty} \mathrm{E}_t (b_{t+k}) = 0$ (in terms of deviations from the steady state) in the case of a tax cut. Eqs. (52) and (53) imply that consumption and TOT are negatively related to the sum of expected future government debt and that the larger the value of ϕ_b, the stronger the relationship between both consumption and

[19] Mahdavi[27] estimates $\phi_B = \frac{\phi_b(1-\beta)}{\beta}$ and the results for ϕ_B range from 0.045 to 0.085.

[20] Leeper, Traum, and Walker[26] analyze the case of ϕ near to zero. In our model, under benchmark parameterization, $\phi \geq 1$ (to be exact, 0.999) is necessary to verify rank conditions, and thus we analyze neither the case of $\phi_b = 0$ nor that of ϕ_b near to zero.

TOT and the sum of government debt, and vice versa. Thus, it can be said that the faster the repayment of debt, the faster the increase in consumption and TOT, and vice versa. In fact, as shown in Panel H, the lines shift downward as ϕ_b increases. This means that the higher the value of ϕ_b, the faster the repayment of government debt. As a result, the higher the value of ϕ_b, the faster consumption is stabilized. Similarly, the higher the value of ϕ_b, the faster TOT is stabilized. While panels D and E do not show clearly how a larger value of ϕ_b makes consumption and TOT stabilize faster, the percentage deviation of consumption from its steady state (vertical axis of Panel E) under $\phi_b = 4.5$, 6.5 and 8.5 are –0.0105, –0.0084 and –0.0064, respectively in period 30. Indeed, the higher the value of ϕ_b, the faster consumption is stabilized. To summarize these results, it can be said that the higher the value of ϕ_b, the larger the decrease in consumption and TOT, the larger the value of ϕ_b, and the faster consumption and TOT are stabilized.

Because of Eq. (43), which implies that employment is the weighted average of both consumption and TOT (through the technology function), the above-mentioned applies to employment. That is, the larger the value of ϕ_b, the larger the decrease in employment and the faster employment is stabilized (Panel A in Fig. 15).

This feature of the relationship between government debt and employment affects the relationship between wage rigidities and welfare. Fig. 16 shows wage rigidities and welfare under $\phi_b \in \{4.5, 6.5, 8.5\}$. In each panel, there are no notable differences among the three lines; the red line with diamonds, the blue line with circles and the magenta line with plus signs show the welfare losses under $\phi_b = 4.5$, 6.5 and 8.5, respectively. The reason for this result is obvious. The higher the value of ϕ_b, the larger the decrease in employment and the faster employment is stabilized. Thus, changes in ϕ_b neither improve welfare nor worsen welfare, as long as its value ranges within the estimated values.

8.2 Robustness Exercise on the Bohn Rule with Tax Smoothing

According to the standard tax-smoothing theory (Barro[1] and [2]), when the level of output is temporarily low due to a recession, the timely increase in tax rates necessary to guarantee a balanced budget would cause unnecessary economic distortions by influencing agents' choices of optimal time paths for labor, production, and consumption. Under these circumstances, to avoid welfare-decreasing distortions, the government finds it optimal to let the primary surplus-GDP ratio decrease and, consequently, the debt-GDP ratio increase. In other words, the primary surplus rule Eq.(47) should include a countercyclical component depending on the output gap. Indeed, the primary surplus rule in Bohn[7] and [8] and Mahdavi[27] includes the output gap.

To test for the robustness of our results, we examine the Bohn rule including the output gap. Following Mahdavi[27], we set a fiscal-policy rule:

$$\frac{SP_t}{Y_t} = \phi_B \frac{B_{t-1}}{Y_t} + \phi_X \left(\frac{Y_t - Y_t^n}{Y_t} \right),$$

where ϕ_X is the tax-smoothing coefficient and Y_t^n denotes the natural rate of output. Dividing both sides by the steady-state value of the fiscal surplus, subtracting one from both sides of that equality, and taking a first-order approximation, we obtain the logarithmic equality of the fiscal-feedback rule:

$$sp_t = \phi_b b_{t-1} + \phi_x x_t, \tag{54}$$

with $\phi_x \equiv \frac{\beta\phi_X}{(1-\beta)\sigma_B}$, $y_t^n \equiv \frac{(1+\varphi)(1+\upsilon\Psi_1)}{\Psi_2}a_t - \frac{\upsilon(\upsilon+\Psi_1)}{\Psi_2}z_t + \frac{\upsilon(1+\upsilon)}{\Psi_2}z_{1,t}^* + \frac{\upsilon(\upsilon+\Psi_1)}{\Psi_2}z_{2,t}^* - \frac{1+\upsilon\Psi_1}{\Psi_2(1-\tau)}\hat{\tau}_t$, $\Psi_1 \equiv (\eta-1)(2-\upsilon)$ and $\Psi_2 \equiv (1+\varphi) + \upsilon(1+\varphi\Psi_1)$ where $x_t \equiv y_t - y_t^n$ denotes the output gap.

Substituting Eq.(45) into Eq.(54), we have the following:

$$\hat{\tau}_t = \frac{\upsilon(1-\beta)\sigma_B}{\beta}s_t - \left[\tau - \frac{(1-\beta)\sigma_B\phi_x}{\beta}\right]x_t - \tau y_t^n + \frac{\phi_b(1-\beta)\sigma_B}{\beta}b_{t-1} + \sigma_G g_t - \tau, \tag{55}$$

which is the fiscal-policy rule with tax smoothing. By comparing Eq.(48) with Eq.(55), it can be understood that the tax-smoothing parameter, ϕ_x, mitigates procyclicality. Eq.(47) implies that the government achieves a sufficient fiscal surplus to repay its debt and also ensures that the rule has procyclicality. Considering the definition of $x_t \equiv y_t - y_t^n$, we understand that the coefficient of the output gap on the tax gap is $-\tau$ in Eq.(47), whereas in Eq.(55), the coefficient is $-\left[\tau - \frac{(1-\beta)\sigma_B\phi_x}{\beta}\right]$. This suggests that procyclicality, which contradicts the second finding of GM[22] in the IGBC model, is mitigated as the tax-smoothing parameter, ϕ_X, increases.

Next, we choose the value of the tax-smoothing parameter to calibrate the model. Mahdavi[27] reports a value of almost zero and Bohn[8] reports 0.088 based on US data. Our estimated ϕ_X based on data on 11 small open economies in the Eurozone (Cyprus, Estonia, Finland, Greece, Latvia, Lithuania, Luxembourg, Malta, Portugal, Slovakia, and Slovenia) from 2007 to 2022 is 0.027[21]. Among the results reported by the authors above, 0.088, which corresponds to $\phi_x = 1.92$, is the highest, and we set $\phi_x = 1.92$ to calibrate the model.

Fig. 17 shows how the tax-smoothing parameter, ϕ_x, changes the dynamic responses to a one percent decrease in the tax gap. In Fig. 17, the blue line with circles, which is identical to the magenta line with plus signs in Fig. 7, shows the dynamic response to a one percent decrease in the tax gap with benchmark parameterization with no tax smoothing ($\phi_x = 0$). The magenta line with plus signs shows the dynamic responses to a one percent decrease in the tax gap under tax smoothing ($\phi_x = 1.92$). As shown in each panel, there is no noticeable difference between the blue line with circles and the magenta line with plus signs. This result means that the tax smoothing does not mitigate the procyclicality brought about by the Bohn rule.

Fig. 18 shows wage rigidities and welfare under $\phi_x \in \{0, 1.92\}$. In each panel, there are no noticeable differences between the blue line with circles showing the welfare losses without tax smoothing (benchmark, $\phi_x = 0$) and the magenta line with plus signs showing the welfare losses with tax smoothing ($\phi_x = 1.92$). The result suggests that GM's second finding does not necessarily hold even if tax smoothing is incorporated into the Bohn rule. The fiscal-policy rule, Eq.(55), implies that tax smoothing mitigates procyclicality; however, the actual tax-smoothing policy implied by empirical results does not change our result. Indeed, even if the tax smoothing parameter, ϕ_x, is set to 1.92, the absolute value of the coefficient of the ratio of the output gap to tax gap, $-\left[\tau - \frac{(1-\beta)\sigma_B\phi_x}{\beta}\right]$, is 0.212. Although this value is less than 0.3, which is the absolute value of the coefficient of the output gap to tax gap in the benchmark fiscal policy rule, Eq. (48), the sign of the coefficient remains negative, and the actual tax-smoothing policy is not sufficient to mitigate procyclicality. Thus, the tax smoothing does not change our result and GM's second finding does not necessarily hold. Wage flexibility may contribute to reducing welfare losses, even if tax smoothing is introduced.

[21] See the on-line appendix on https://www.econ.nagoya-cu.ac.jp/~eiji_okano /papers_e.html for details

9 Conclusion

In this paper, we investigated GM's two findings using a small open economy model. As long as an endogenous fiscal policy such as the Bohn rule is adopted in an economy that is part of a currency union, GM's second finding does not necessarily hold. Owing to the Bohn rule with distortionary taxation, an increase in output decreases fiscal revenue, hence tending to raise government debt. This aspect, caused by the Bohn rule with distortionary taxation, led to our findings. Wage flexibility may contribute to reducing welfare losses and how wage flexibility reduces welfare loss in various settings should be examined.

For example, the Stability and Growth Pact (SGP) is worthy of consideration. Under the SGP, a member of a currency union faces two constraints. That is, the ratio of fiscal deficit to GDP must be below 3% and the ratio of government debt to GDP must be below 60%. The analysis would be difficult if those two constraints were introduced into our model, and the results of the analysis would be more beneficial to the European Monetary Union. Another example is introducing heterogeneous agents. As in Debortoli and Gali[9], the existence of heterogeneous agents increases the real interest rate elasticity of output. In a currency union, GM's first finding that the effectiveness of labor cost adjustments on employment is much smaller is supported by our findings. Because of the high interest rate elasticity of output in a New Keynesian economy with heterogeneous agents, the effectiveness of labor cost adjustments on employment might be larger and this phenomenon might change my result.

Finally, the fact that the analysis does not regard the cost on behavioral change to participate in a currency union should be noted. Inflation targeting was simply compared to a currency union, irrespective of that cost. Further discussion regarding this cost is necessary.

Appendix

A Derivation of Eq. (51)

Eq. (22) can be rewritten as:

$$mc_t = \frac{1}{1-\tau}\tau_t + \mu_t^w + (\sigma_\varsigma + \varphi)\,\hat{n}_t,$$

where the exogenous shocks and constant are excluded. Substituting Eq. (48) into the previous expression yields:

$$mc_t = \left[(\sigma_\varsigma + \varphi) - \frac{\beta\tau - \upsilon(1-\beta)\sigma_B\sigma_\varsigma}{(1-\tau)\beta}\right]\hat{n}_t + \frac{(1-\beta)\sigma_B\phi_b}{(1-\tau)\beta}b_{t-1} + \mu_t^w.$$

Iterating the previous expression, we obtain:

$$\begin{aligned} mc_{t+k} \;=\; & \left[(\sigma_\varsigma + \varphi) - \frac{\beta\tau - \upsilon(1-\beta)\sigma_B\sigma_\varsigma}{(1-\tau)\beta}\right]\hat{n}_{t+k} + \frac{(1-\beta)\sigma_B\phi_b}{(1-\tau)\beta}\sum_{h=0}^{k-1}\beta^{-(k-h)}\mathrm{E}_t\left(r_{t+h-1} - \pi_{t+h}\right) \\ & - \frac{(1-\beta)^2\sigma_B\phi_b}{(1-\tau)\beta}\sum_{h=0}^{k-1}\beta^{-(k-h)}\mathrm{E}_t\left(sp_{t+h}\right) + \mu_{t+k}^w. \end{aligned}$$

Multiplying by β^k on both sides of the previous expression yields:

$$\begin{aligned}\beta^k mc_{t+k} &= \beta^k\left[(\sigma_\varsigma+\varphi)-\frac{\beta\tau-\upsilon(1-\beta)\sigma_B\sigma_\varsigma}{(1-\tau)\beta}\right]\hat{n}_{t+k}+\frac{(1-\beta)\sigma_B\phi_b}{(1-\tau)\beta}\sum_{h=0}^{k-1}\beta^h\mathrm{E}_t(r_{t+h-1}-\pi_{t+h})\\ &\quad -\frac{(1-\beta)^2\sigma_B\phi_b}{(1-\tau)\beta}\sum_{h=0}^{k-1}\beta^h\mathrm{E}_t(sp_{t+h})+\beta^k\mu^w_{t+k}. \qquad (A.1)\end{aligned}$$

Combining Eqs. (44) and (46) yields:

$$r_{t-1}-\pi_t=c_t-c_{t-1},$$

where we ignore the exogenous shocks and constant. Substituting the previous expression into Eq. (A.1) yields:

$$\begin{aligned}\beta^k mc_{t+k} &= \beta^k\left[(\sigma_\varsigma+\varphi)-\frac{\beta\tau-\upsilon(1-\beta)\sigma_B\sigma_\varsigma}{(1-\tau)\beta}\right]\hat{n}_{t+k}+\frac{(1-\beta)\sigma_B\phi_b}{(1-\tau)\beta}\mathrm{E}_t\left[\beta^{k-1}(c_{t+k-1}-c_{t+k-2})\right.\\ &\quad \left.+\beta^{k-2}(c_{t+k-2}-c_{t+k-3})+\cdots+\beta(c_{t+1}-c_t)+(c_t-c_{t-1})\right]\\ &\quad -\frac{(1-\beta)^2\sigma_B\phi_b}{(1-\tau)\beta}\sum_{h=0}^{k-1}\beta^h\mathrm{E}_t(sp_{t+h})+\beta^k\mu^w_{t+k}\\ &= \beta^k\left[(\sigma_\varsigma+\varphi)-\frac{\beta\tau-\upsilon(1-\beta)\sigma_B\sigma_\varsigma}{(1-\tau)\beta}\right]\hat{n}_{t+k}\\ &\quad +\frac{(1-\beta)\sigma_B\phi_b}{(1-\tau)\beta}\sum_{h=0}^{k-1}\beta^h\mathrm{E}_t(c_{t+h}-c_{t+h-1})-\frac{(1-\beta)^2\sigma_B\phi_b}{(1-\tau)\beta}\sum_{h=0}^{k-1}\beta^h\mathrm{E}_t(sp_{t+h})\\ &\quad +\beta^k\mu^w_{t+k}. \qquad (A.2)\end{aligned}$$

By bringing Eq. (A.2) backward one period, we obtain:

$$\begin{aligned}\beta^{k-1}mc_{t+k-1} &= \beta^{k-1}\left[(\sigma_\varsigma+\varphi)-\frac{\beta\tau-\upsilon(1-\beta)\sigma_B\sigma_\varsigma}{(1-\tau)\beta}\right]\hat{n}_{t+k-1}\\ &\quad +\frac{(1-\beta)\sigma_B\phi_b}{(1-\tau)\beta}\sum_{h=0}^{k-1}\beta^{h-1}\mathrm{E}_t(c_{t+h-1}-c_{t+h-2})\\ &\quad -\frac{(1-\beta)^2\sigma_B\phi_b}{(1-\tau)\beta}\sum_{h=0}^{k-1}\beta^{h-1}\mathrm{E}_t(sp_{t+h-1})+\beta^{k-1}\mu^w_{t+k-1},\end{aligned}$$

where both sides are multiplied by β^{-1}. Substituting the previous expression into Eq. (A.2) yields:

$$\begin{aligned}\beta^k mc_{t+k} &= \beta^k\left[(\sigma_\varsigma+\varphi)-\frac{\beta\tau-\upsilon(1-\beta)\sigma_B\sigma_\varsigma}{(1-\tau)\beta}\right]\hat{n}_{t+k}+\beta^{k-1}\mathrm{E}_t(mc_{t+k-1})+\beta^k\mathrm{E}_t\left(\mu^w_{t+k}\right)\\ &\quad -\frac{(1-\beta)\sigma_B\phi_b}{(1-\tau)\beta}\beta^{-1}\left[c_{t-1}-c_{t-2}-(1-\beta)sp_{t-1}\right]\\ &\quad +\beta^{k-1}\left[(\sigma_\varsigma+\varphi)-\frac{\beta\tau-\upsilon(1-\beta)\sigma_B\sigma_\varsigma}{(1-\tau)\beta}\right]\hat{n}_{t+k-1}-\beta^{k-1}\mathrm{E}_t\left(\mu^w_{t+k-1}\right)\\ &\quad +\frac{(1-\beta)\sigma_B\phi_b}{(1-\tau)\beta}\beta^{k-1}\left[c_{t+k-1}-c_{t+k-2}-(1-\beta)sp_{t+k-1}\right].\end{aligned}$$

Substituting $k=0$ into the previous expression yields:

$$\begin{aligned}mc_t &= \left[(\sigma_\varsigma+\varphi)-\frac{\beta\tau-\upsilon(1-\beta)\sigma_B\sigma_\varsigma}{(1-\tau)\beta}\right]\hat{n}_t-\frac{1}{\beta}\left[(\sigma_\varsigma+\varphi)-\frac{\beta\tau-\upsilon(1-\beta)\sigma_B\sigma_\varsigma}{(1-\tau)\beta}\right]\hat{n}_{t-1}+\mu^w_t\\ &\quad -\frac{1}{\beta}\mu^w_{t-1}+\frac{1}{\beta}mc_{t-1}.\end{aligned}$$

Substituting the previous expression into Eq. (21) yields:

$$\begin{aligned}\pi_{H,t} &= \beta \mathrm{E}_t\left(\pi_{H,t+1}\right)+\lambda_p\left[\left(\sigma_\varsigma+\varphi\right)-\frac{\beta\tau-\upsilon\left(1-\beta\right)\sigma_B\sigma_\varsigma}{\left(1-\tau\right)\beta}\right]\hat{n}_t \\ &\quad -\frac{\lambda_p}{\beta}\left[\left(\sigma_\varsigma+\varphi\right)-\frac{\beta\tau-\upsilon\left(1-\beta\right)\sigma_B\sigma_\varsigma}{\left(1-\tau\right)\beta}\right]\hat{n}_{t-1}+\lambda_p\mu_t^w-\frac{\lambda_p}{\beta}\mu_{t-1}^w+\frac{\lambda_p}{\beta}mc_{t-1},\end{aligned}$$

which is Eq. (51) in the text.

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Table 1: Differences on the Set-up

	GM	I
Steady State	Efficient	Distorted
Economies of Scale	Decreasing Returns	Constant Returns
Substitutability between Domestic and Foreign Goods	Perfect	Imperfect
Employment Gap on Welfare Criteria	Gap bet. Employment and Natural Employment	Gap bet. Employment and Efficient Employment
Targetted Inflation	Domestic Price	CPI

Table 2: Parameterization

Model	Parameter	Description	Value	Source
Distorted Steady State Model	φ	Curvature of Labor Disutility	2.2	GM
	η	Trade Elasticity of Substitution	2	GM
	ϵ_w	Elasticity of Substitution (Labor)	4.3	GM
	ϵ_p	Elasticity of Substitution (Goods)	3.8	GM
	θ_p	Calvo Index of Price Rigidities	0.8	GM
	θ_w	Calvo Index of Wage Rigidities	0.8	GM
	υ	Openness	0.3	GM
	β	Discount Factor	0.99	GM
	ρ_a	Persistence of Exo. Process	0.9	GM
	ρ_z		0.9	GM
	ρ_1^*		0.9	GM
	ρ_2^*		0.9	GM
IGBC Model	τ	S.S. Tax Rate	0.3	Ferrero[17]
	σ_B	S.S. Share of Gov. Debt to GDP	4.543	Average in GIPS 2008–2019
	σ_G	S.S. Share of Gov. Exp. to GDP	0.477	
	ϕ_b	Bohn Rule Coefficient	6.5	Mahdavi[27]
	ρ_g	Persistence of Exo. Process	0.9	(Unless Specified Otherwise)

Figure 1: Dynamic Response to One Percent Decrease in the Tax Rate in the Distorted S.S. Model

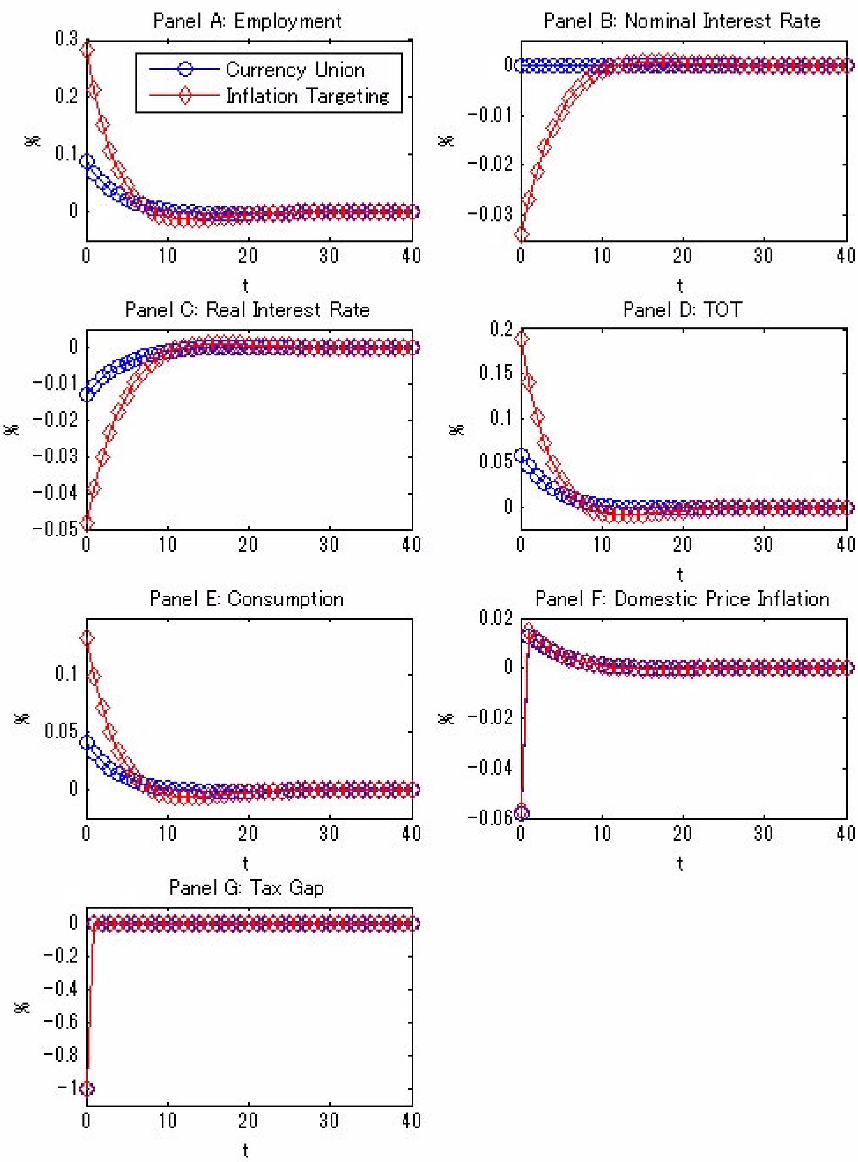

Figure 2: Dynamic Response to One Percent Decrease in the Tax Rate in the Distorted S.S. Model (Under the Dynamic Path of Tax Gap which is Identical to it under IGBC Model with Bohn Rule)

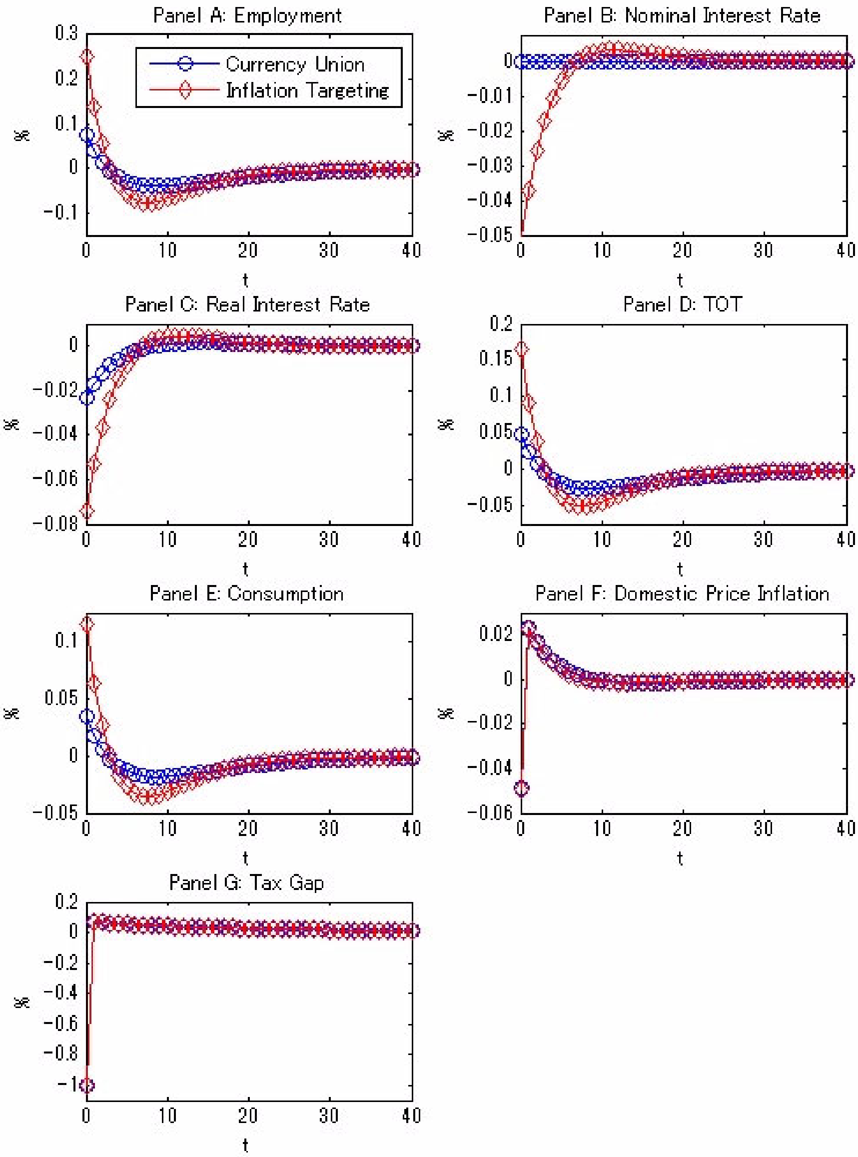

Figure 3: Wage Rigidities and Welfare in a Currency Union

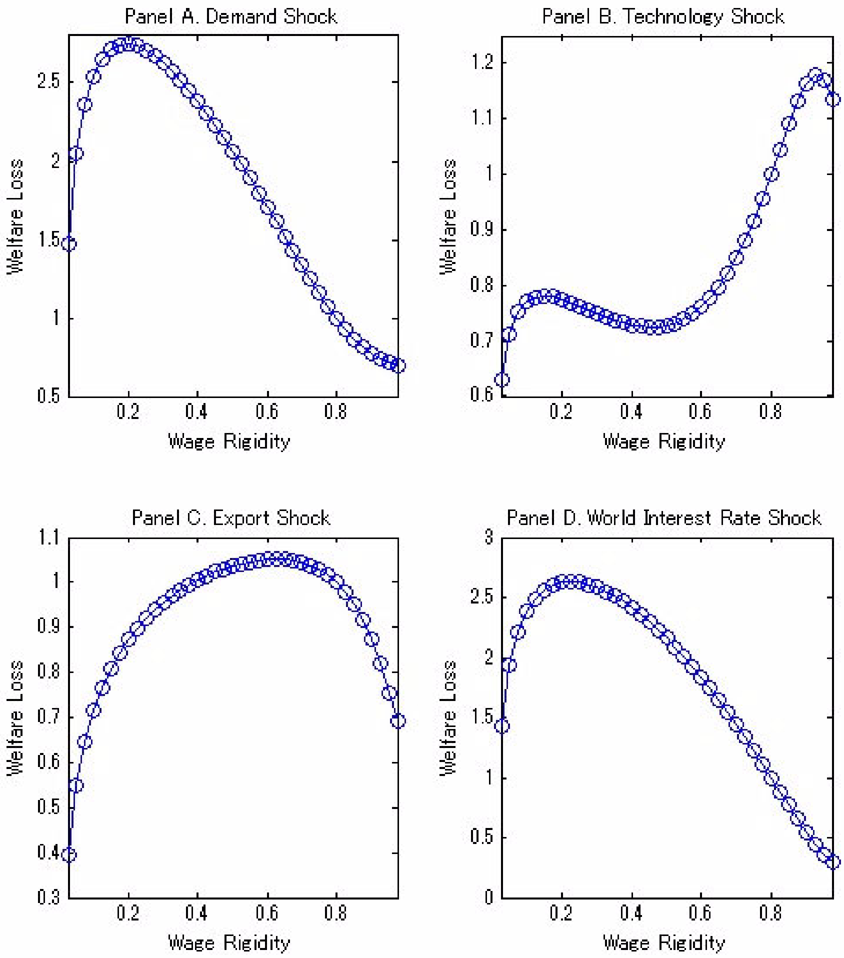

Figure 4: Wage Rigidities in a Currency Union: Welfare Components

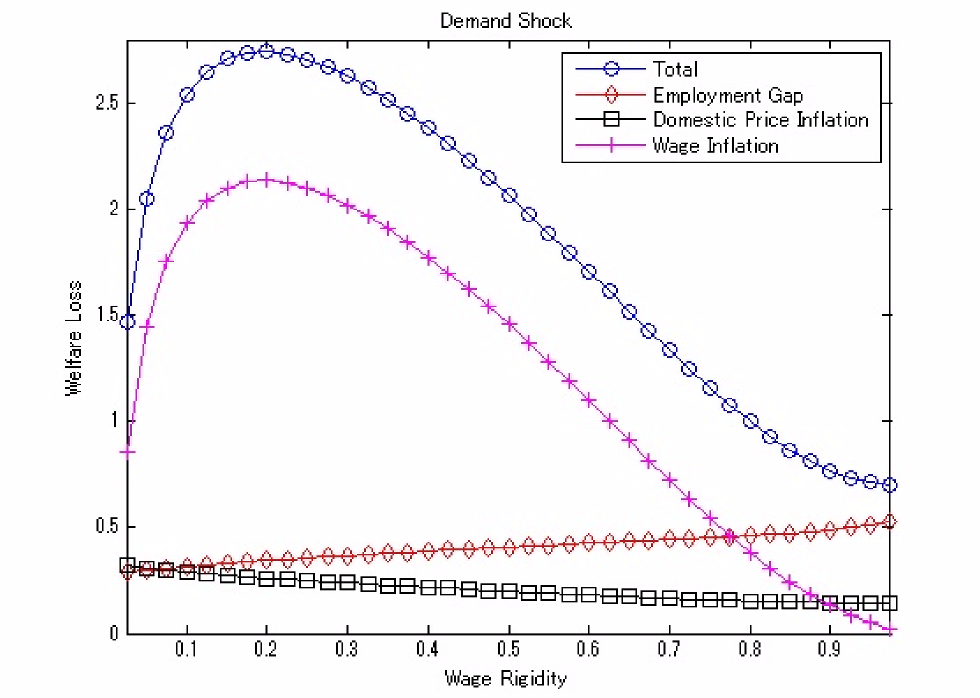

Figure 5: Wage Rigidities and Welfare: Currency Union vs Inflation Targeting

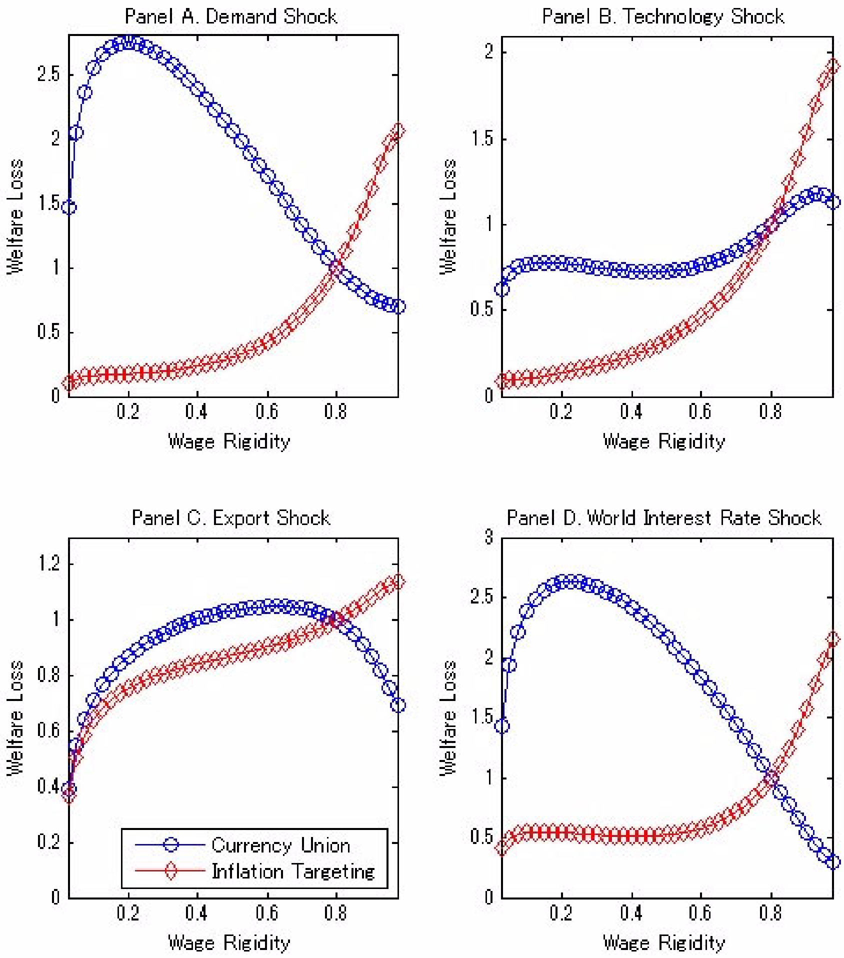

Figure 6: Nominal Rigidities and Welfare in a Currency Union ($\theta_w = \theta_p = \theta$)

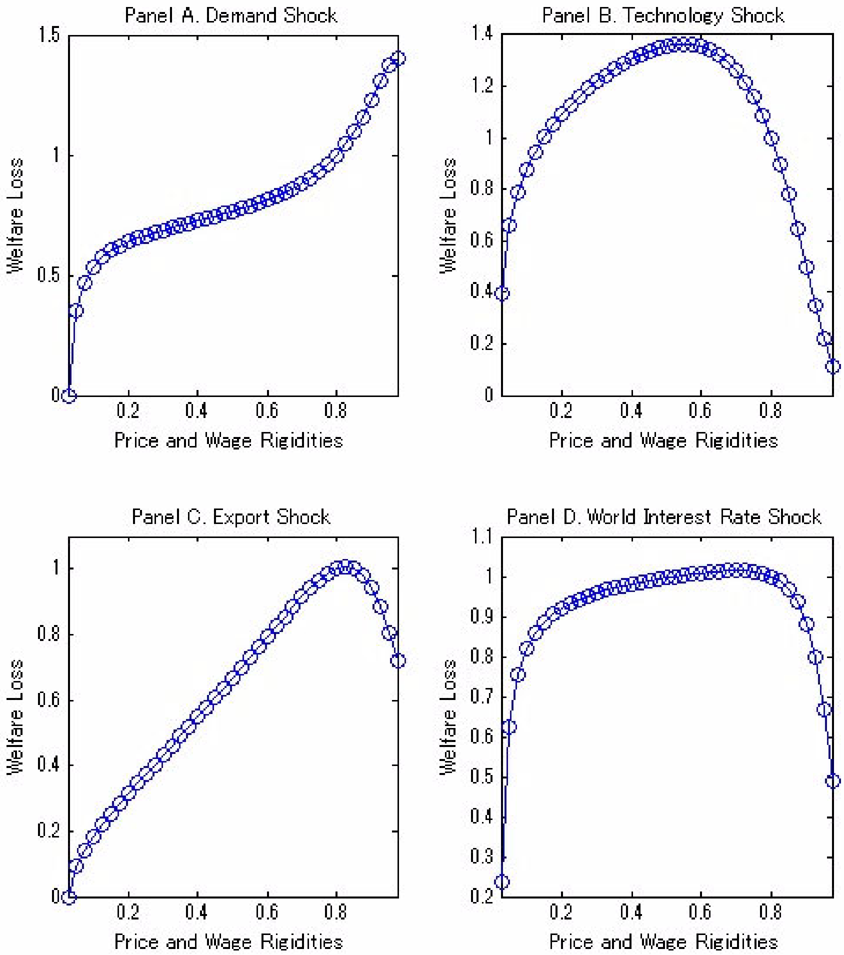

Figure 7: Dynamic Response to One Percent Decrease in the Tax Rate in the IGBC model

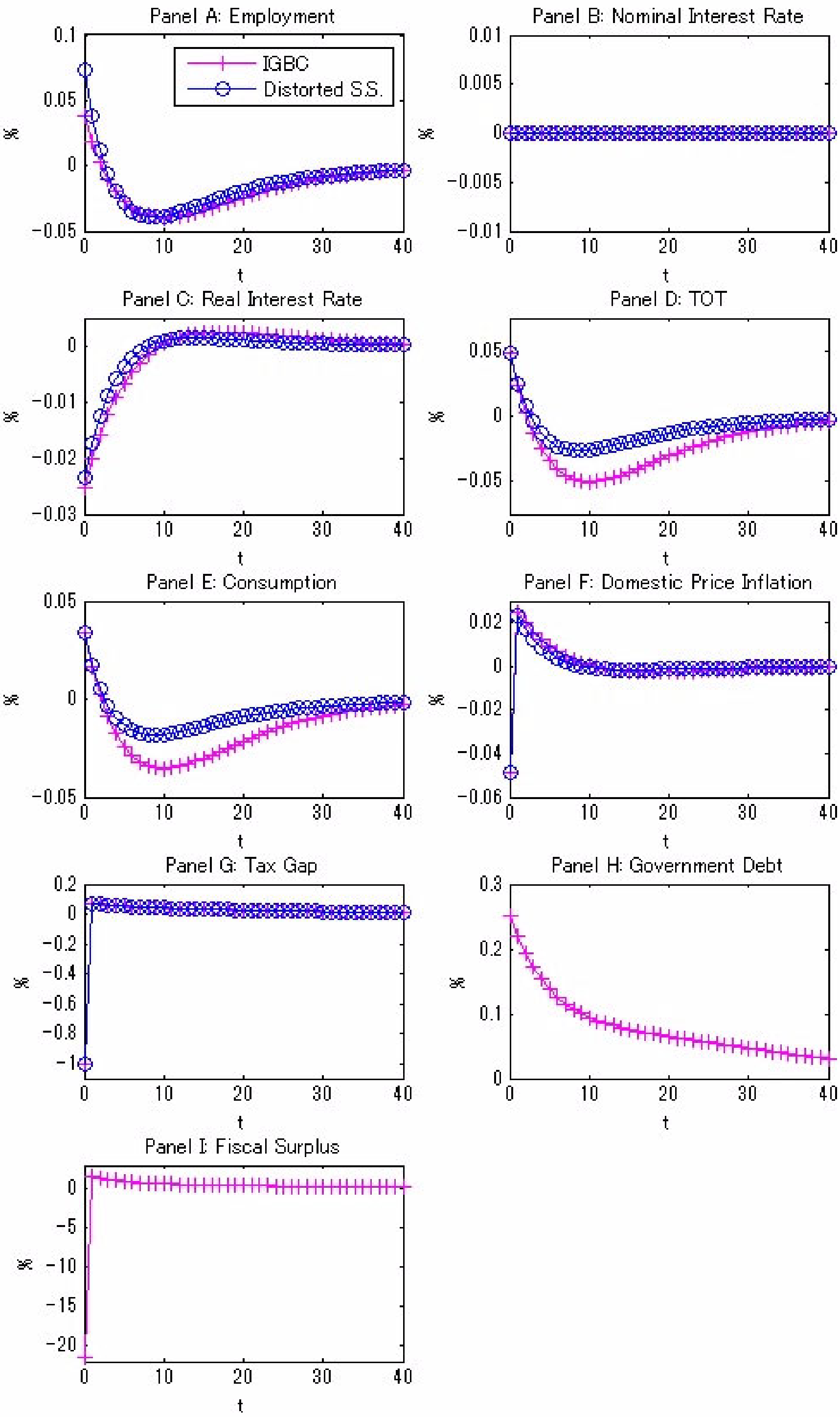

Figure 8: Wage Rigidities and Welfare in a Currency Union: IGBC Model

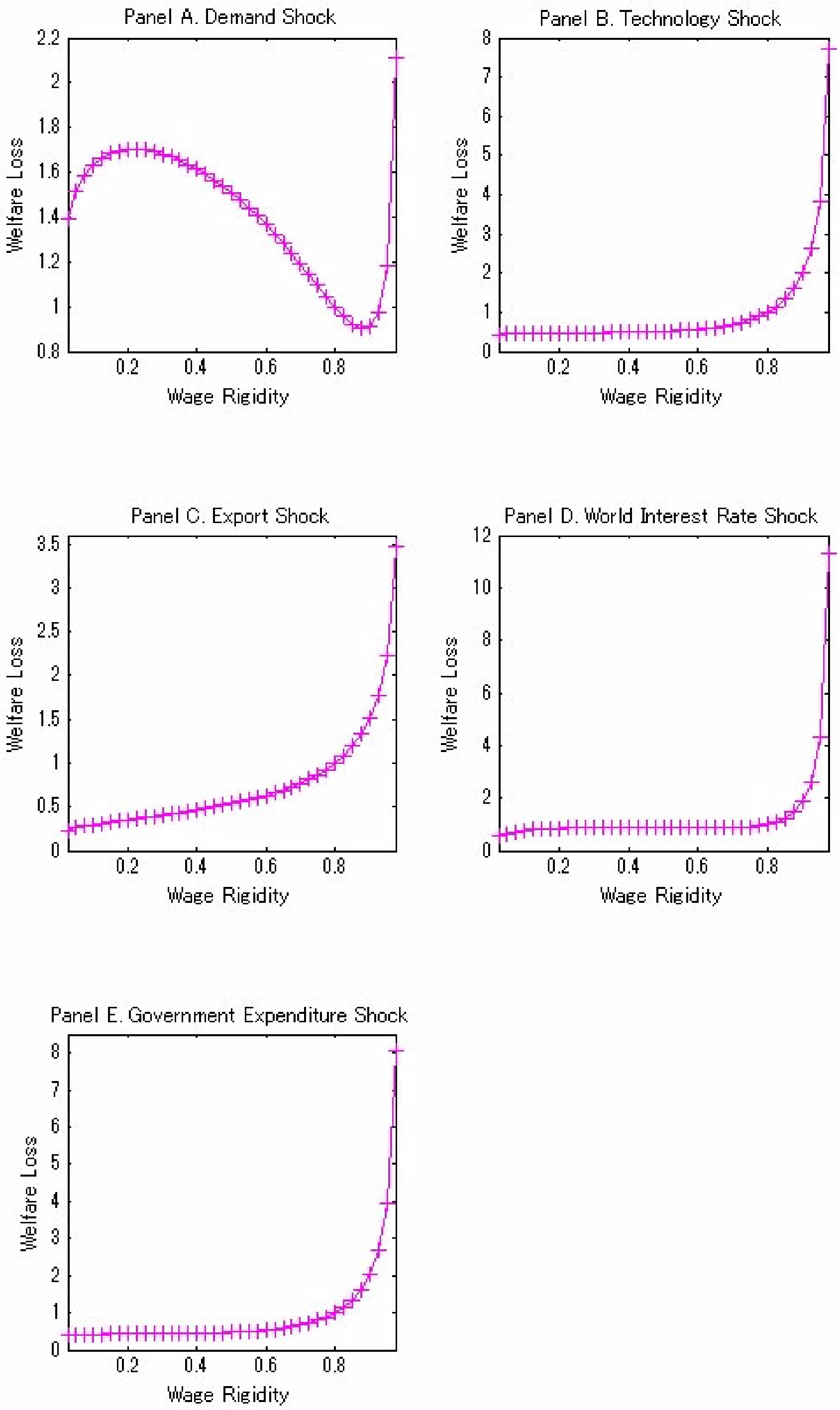

Figure 9: Wage Rigidities in a Currency Union in the IGBC model: Welfare Components

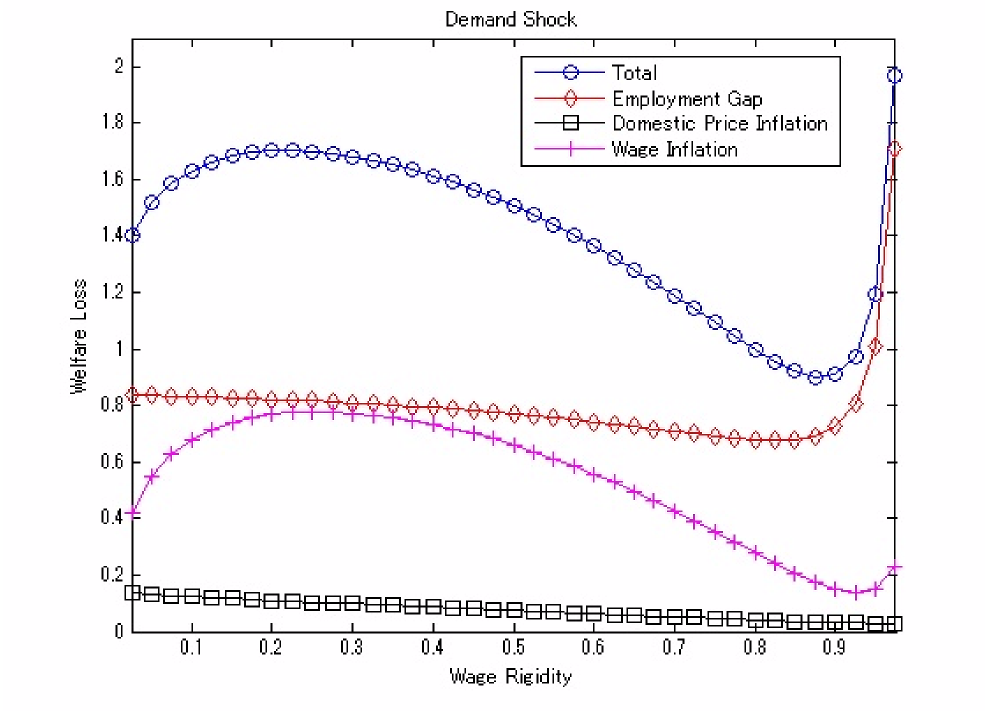

Figure 10: IRFs to Demand Shock in the Distorted Steady State and the IGBC model

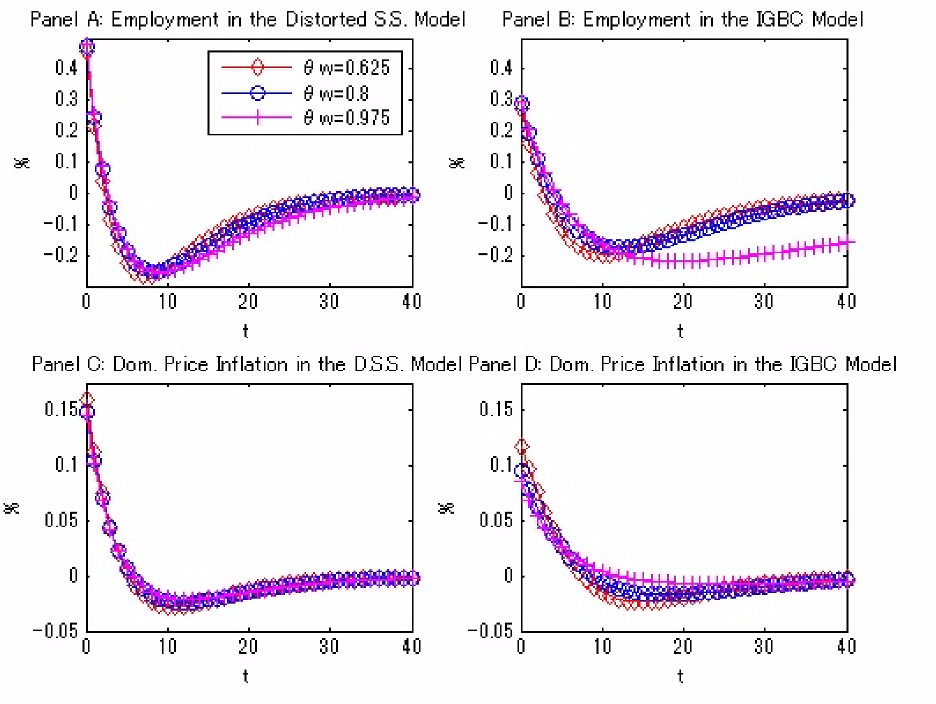

Figure 11: Wage Rigidities and Welfare in a Currency Union: the IGBC Model vs Distorted Steady-state model

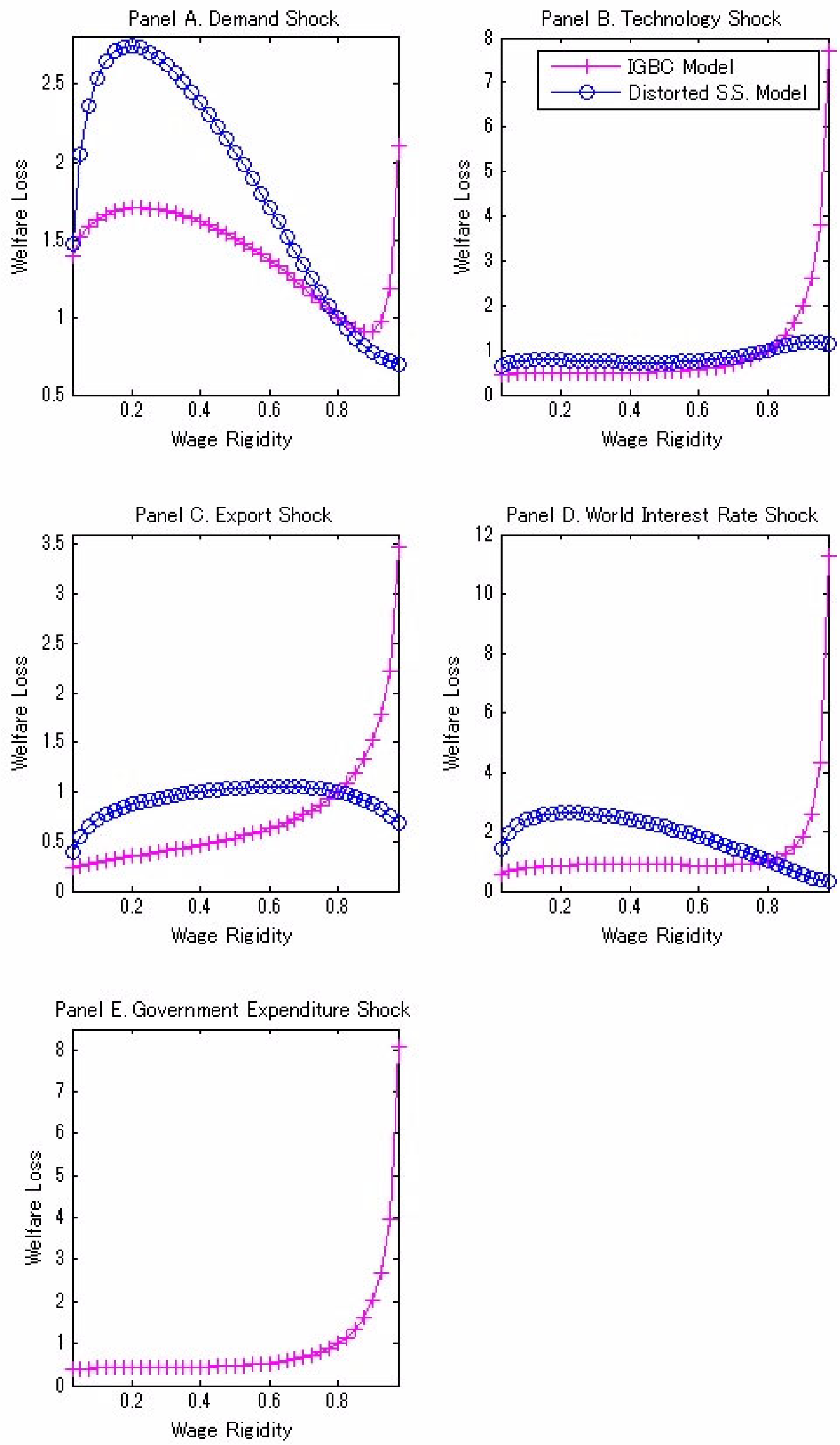

Figure 12: Nominal Rigidities and Welfare in a Currency Union in the IGBC model ($\theta_w = \theta_p = \theta$)

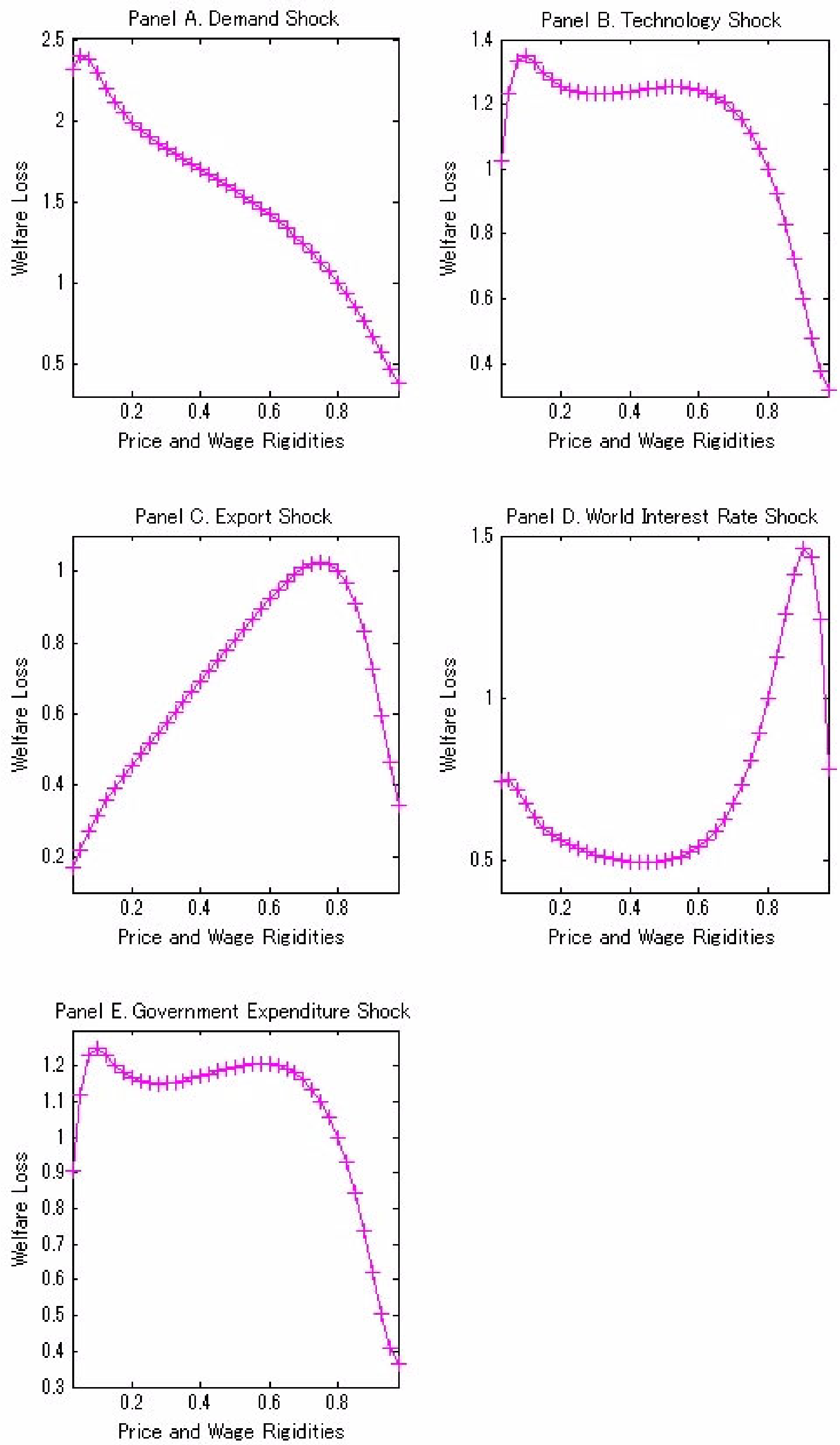

Figure 13: IRFs to Demand Shock in the IGBC model: Variation in the Steady-state Tax Rate

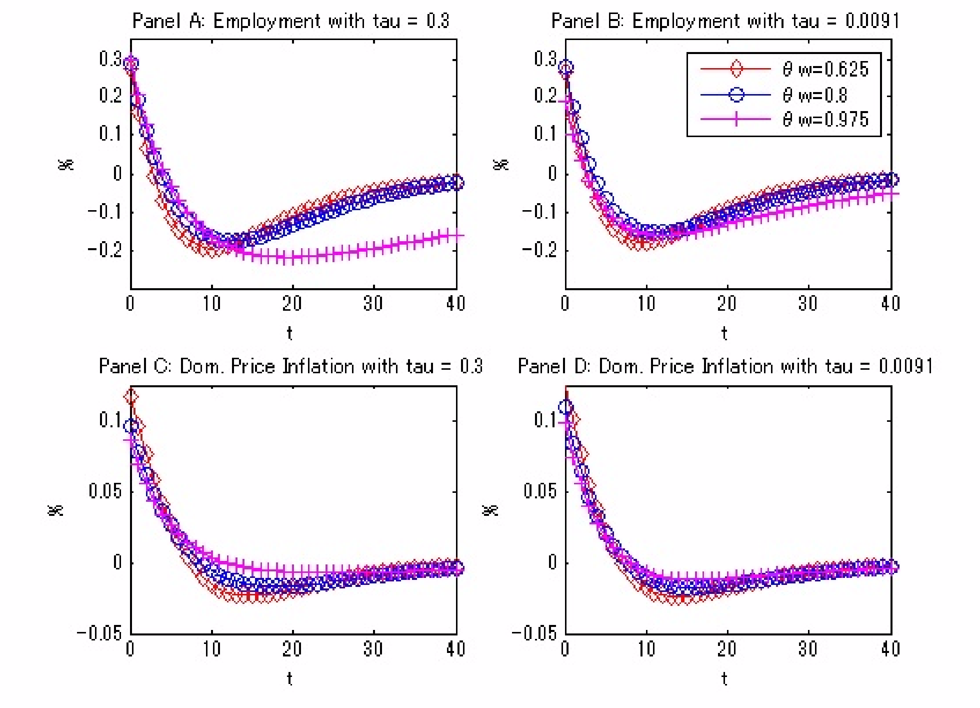

Figure 14: Wage Rigidities and Welfare in a Currency Union: Variation in the Steady-state Tax Rate

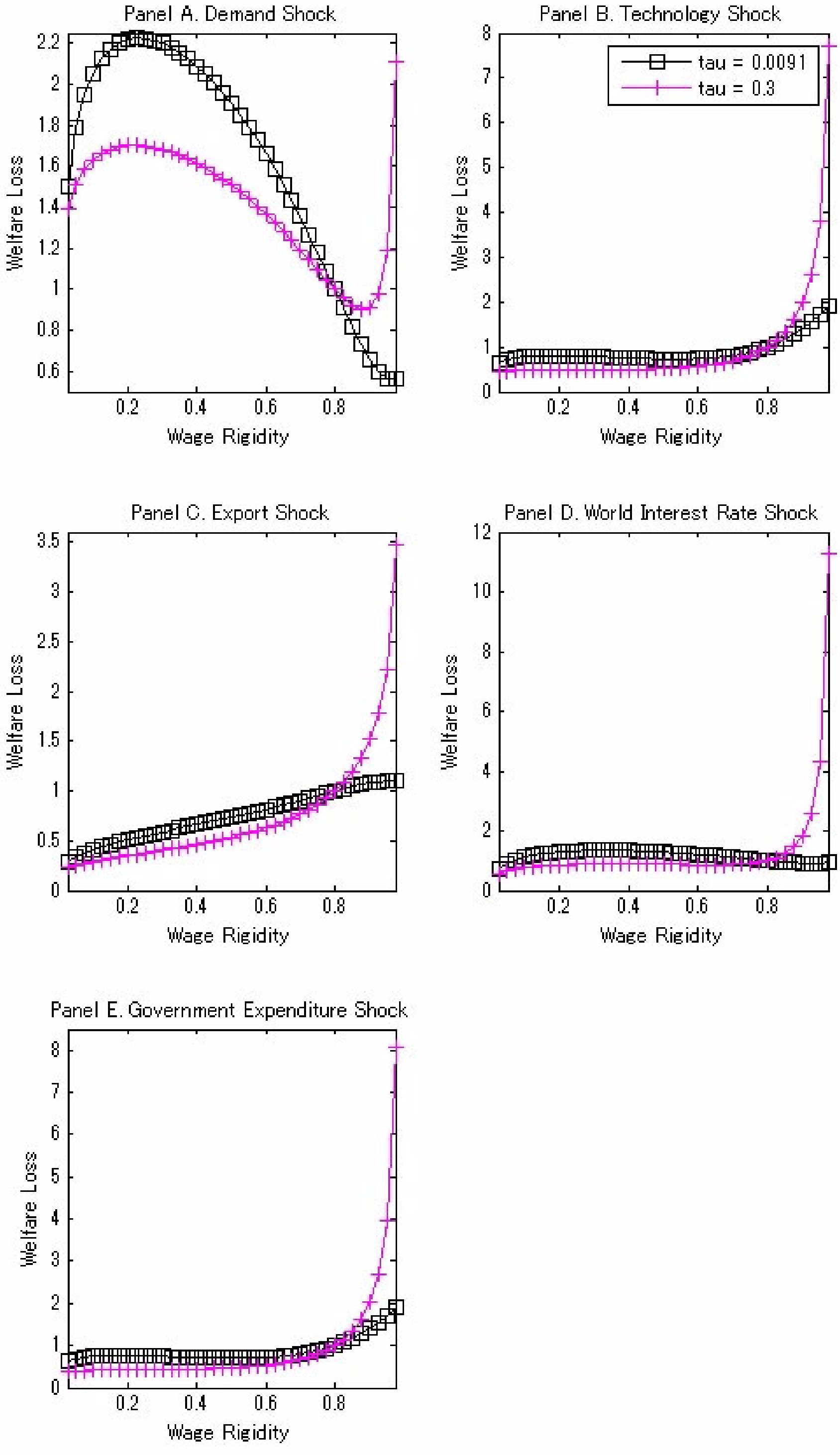

Figure 15: Dynamic Response to One Percent Decrease in the Tax Rate in the IGBC model: Role of Bohn Rule Coefficient

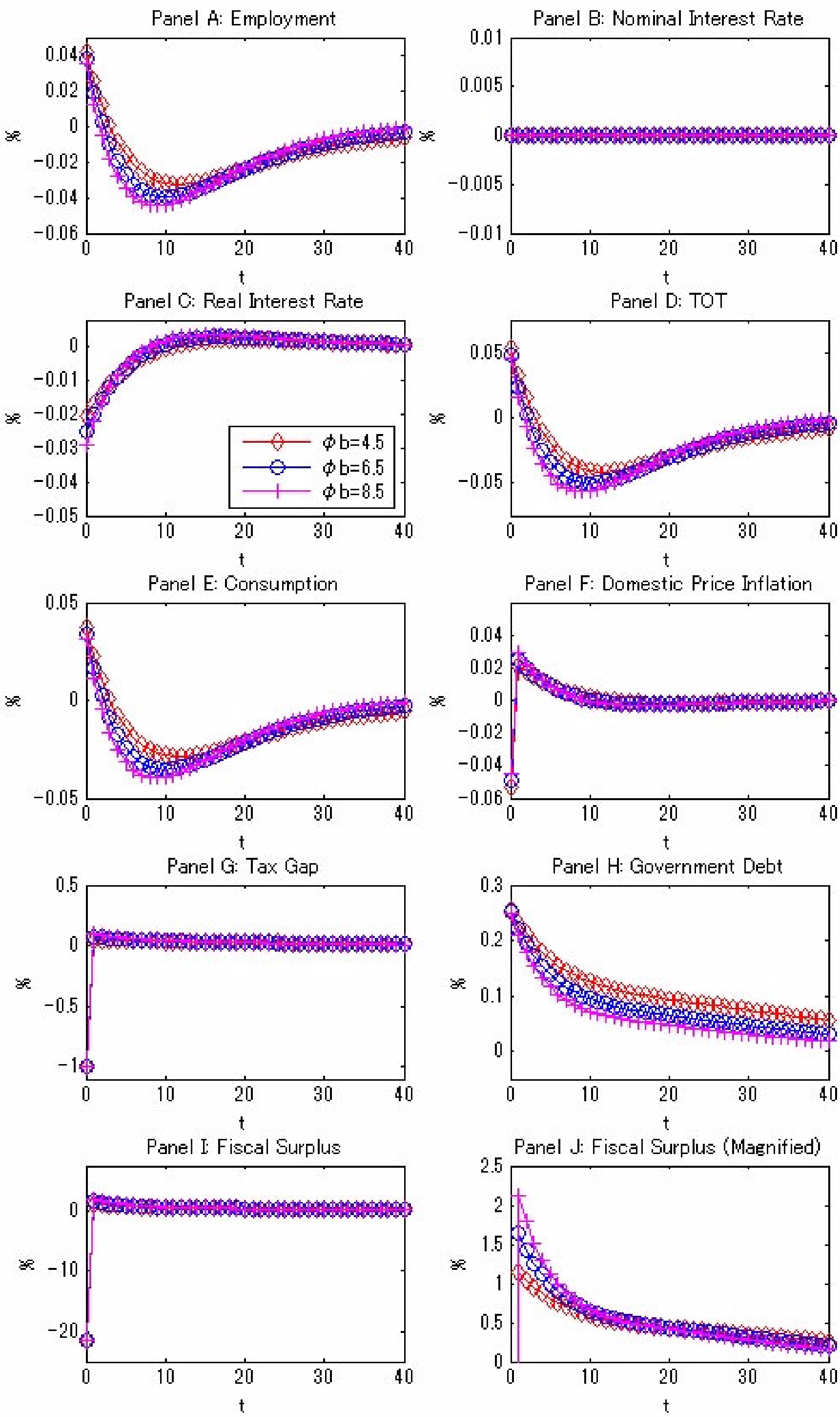

Figure 16: Wage Rigidities and Welfare in a Currency Union: Variation in the Coefficient in the Bohn Rule

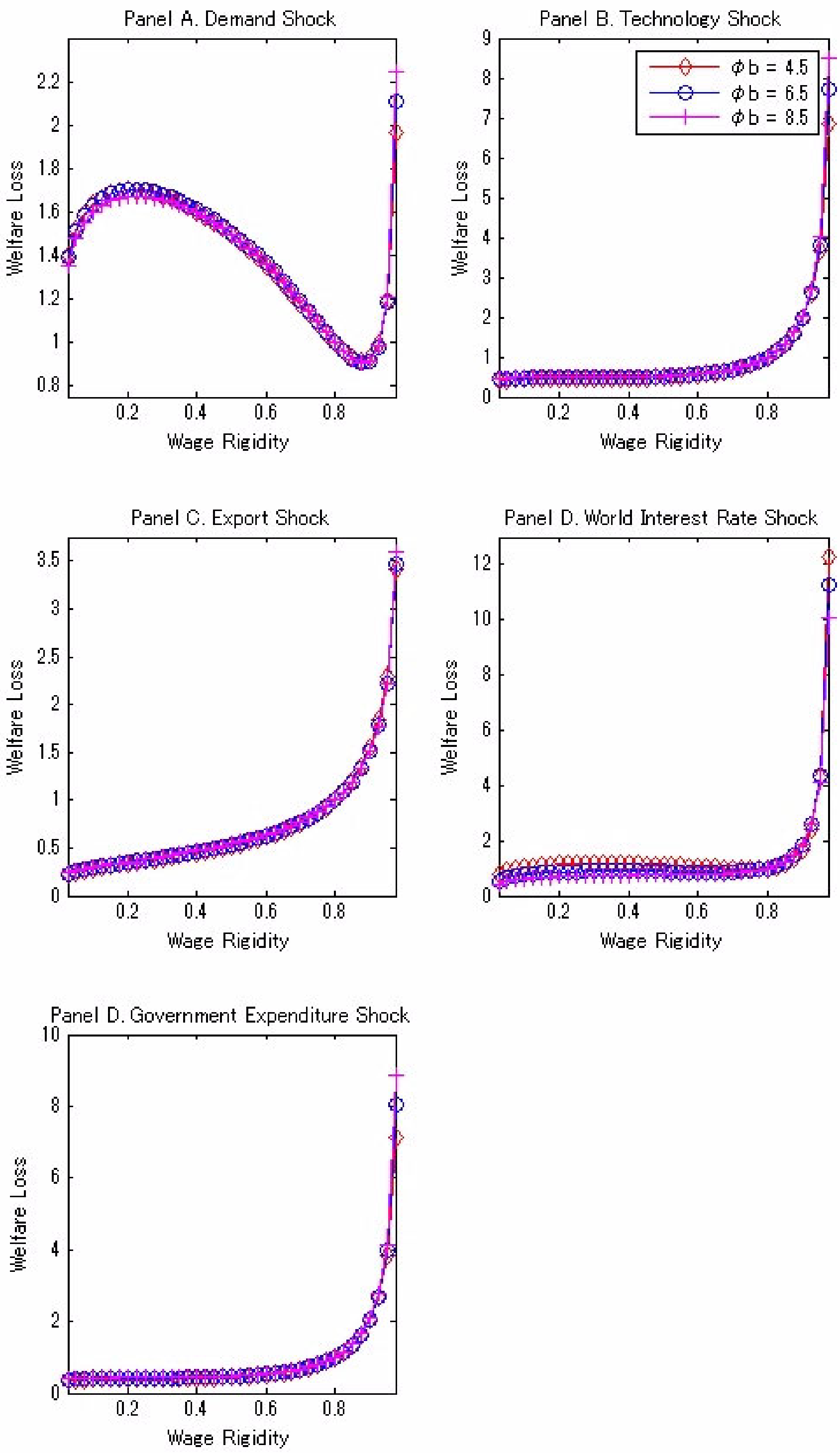

Figure 17: Dynamic Response to One Percent Decrease in the Tax Rate in the IGBC model: Role of Tax Smoothing

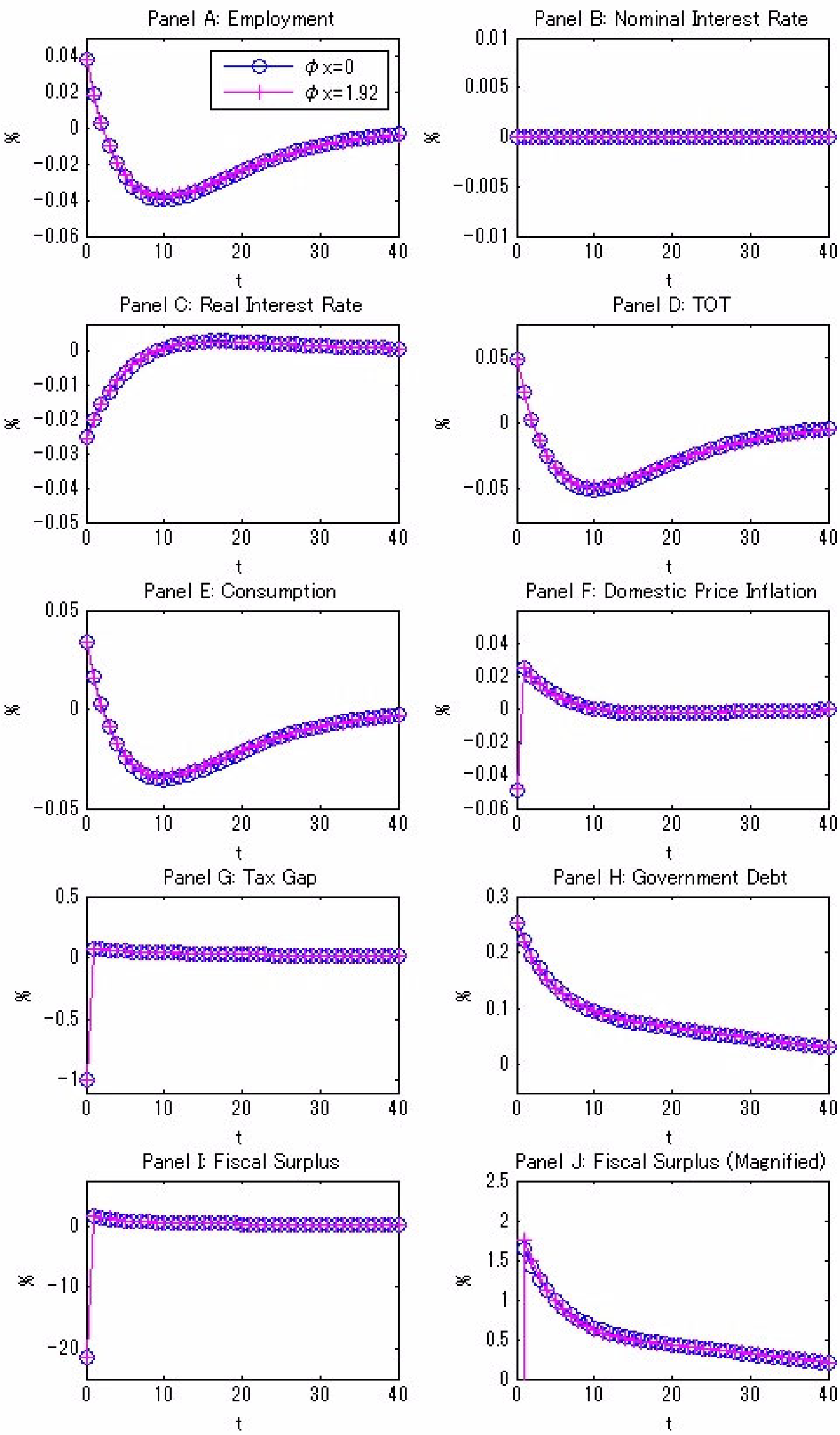

Figure 18: Wage Rigidities and Welfare in a Currency Union: Role of Tax Smoothing