

The Effects of a Money-Financed Fiscal Stimulus without the Irredeemability of Money: The Role of Monetary Accommodation

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Money Finance without Irredeemability

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Abstract

Is money-financed fiscal stimulus powerful because base money is irredeemable, or because the financing regime accommodates? Galí (2020) bundles the two. We separate them by placing base money inside the consolidated government's valuation condition, then fixing the resulting money-growth path in a matched-accommodation counterfactual. The valuation closure removes the permanent net-wealth transfer and direct money financing of government purchases. Nevertheless, a one-percent spending shock raises cumulative output by 1.48, against 0.25 under debt financing and 2.69 in the unbacked benchmark. The remaining effect operates through an interest-payment channel: fluctuations in the government's real interest burden induce an endogenous money-growth path. Imposing the same path on the debt-financed economy reproduces the allocation exactly. Within the maintained model, accommodation—not irredeemability—is the propagation mechanism. The debt-financed analogue of the closure admits no bounded equilibrium under strict inflation targeting. Money financing under immediate backing remains more expansionary than debt financing in a liquidity trap.

Keywords: Money financing; Fiscal backing; Monetary accommodation; Government purchases

JEL Classification: E31, E32, E52, E58, E62, E63

1 Introduction

Can a government stimulate the economy by paying for its spending with newly created money? A prominent strand of the modern debate answers yes, but with a qualification that has hardened into doctrine. Helicopter money works, Bernanke (2003) and Buiter (2014) argue, because fiat base money is *irredeemable*: an asset to the household that is never a liability of the issuer, and therefore pure net wealth when it is handed out. However, permanent unbacked monetization is precisely the feature that modern central-bank arrangements are designed to constrain. Prohibitions on monetary financing, the operational independence of central banks, and the convention of carrying base money as a liability on the central bank's balance sheet all make a permanent, unbacked transfer of money to the private sector institutionally difficult. If the doctrine is right, the most powerful form of fiscal stimulus rests on exactly the ingredient that these institutional arrangements place outside conventional operating frameworks.

Galí (2020) gives the doctrine its sharpest modern formulation. In a standard New Keynesian model, a fiscal expansion financed by money creation (MF) is far more expansionary than the same expansion financed by debt (DF), in normal times and even more so in a liquidity trap. His experiment, however, bundles two distinct ingredients. Because the fiscal rule leaves base money outside the government's solvency condition, the injected money is irredeemable net wealth, a permanent transfer to households. And because the money supply, rather than the interest rate, absorbs the financing need, the policy also delivers a temporary path of monetary accommodation under sticky prices. In Galí's (2020) benchmark, and in much of the subsequent quantitative discussion, these two ingredients remain bundled. Which one carries the effect? The answer decides whether the power of money financing requires the ingredient that institutions exclude, or is available without it.

We distinguish the two mechanisms using two complementary exercises. The first places base money, alongside interest-bearing bonds, inside the consolidated government's intertemporal valuation condition, so that the *entire* consolidated liability must be validated by future fiscal resources; this removes the permanent net-wealth treatment of money, but it also changes the endogenous monetary path. The second, a matched-accommodation counterfactual, re-solves the debt-financing economy along the money-growth path induced by the first, thereby holding that path fixed and isolating whether liability composition has any independent real effect. Following the valuation logic of the fiscal theory of the price level (FTPL) and Cochrane's (2005) "money as stock," the fiscal rule of the first exercise is obtained by iterating the consolidated budget con-

straint forward and imposing a transversality condition (TVC) on bonds and money together. The private-sector block and calibration are otherwise unchanged from Galí (2020). Our central experiment then asks what remains of a money-financed *government-purchases* stimulus once base money is placed inside this valuation condition.

The answer is that the stimulus survives, through a channel Galí’s benchmark leaves implicit. Under the valuation rule, backing is immediate: the direct money-financing of the spending increase is offset by taxation within the period, so the *direct government-purchases channel* of money creation is shut down. Even so, the unweighted cumulative output response is 1.48, against 0.25 under debt financing and 2.69 in Galí’s unbacked benchmark. What sustains it is the *interest-payment channel*: a consolidated government that backs its money still finances part of its real interest burden with money creation, and under sticky prices the resulting temporary monetary accommodation is expansionary. Even without a permanent net-wealth transfer, money financing therefore remains substantially more expansionary than debt financing. Irredeemability is neither necessary nor the propagation mechanism; it nevertheless affects the magnitude indirectly, by changing the endogenous monetary path. The implementation of backing is quantitatively important: the contemporaneous offset embedded in the valuation rule attenuates the response to 1.48, whereas a gradual feedback rule leaves the initial monetary accommodation (and the response, at 2.69) largely intact (Section 4.3). Because these are distinct fiscal closures rather than the same rule at different speeds, that comparison shows that present-value backing alone does not eliminate the stimulus, but does not identify a pure effect of adjustment timing.

The valuation rule does not, by itself, deliver a complete comparison, and the reason is a result in its own right. Applied to debt financing under strict inflation targeting, the same rule leaves government debt with an explosive root equal to the inverse discount factor: debt grows without bound, the TVC fails, and no bounded equilibrium exists, a failure invisible over a standard thirty-two-quarter window but decisive asymptotically. The valuation relation prices liabilities; it does not stabilize them. We therefore introduce an auxiliary Bohn-type rule that taxes the *consolidated* liability gradually and admits a bounded, gradually backed *DF* regime, giving an implementable two-by-two benchmark that isolates the financing margin from the fiscal treatment of money as far as the model allows. It confirms the reading twice over: under strict inflation targeting, lump-sum taxation, and separable real balances, the two *DF* cells coincide in all non-fiscal variables, and under the common gradual rule broad present-value backing leaves the government-purchases stimulus essentially at its unbacked level. A matched-accommodation experiment then isolates the

mechanism: when the money-growth path of a backed MF regime is imposed exogenously on the debt-financing economy, that economy reproduces the MF allocation exactly. Within the maintained model, what propagates the real effects is the induced path of monetary accommodation, not the composition of fiscal liabilities.

A lump-sum rebate serves as a scope check rather than as a second central contribution. Under the valuation rule a rebate is offset within the period and has exactly zero effect, but that neutrality is a property of the contemporaneous offset built into this particular implementation, not a general implication of fiscal backing: implemented gradually, the same present-value backing leaves the rebate effective.

The paper thus makes three contributions. First, it develops a valuation-based fiscal closure that places base money inside the consolidated government's terminal liability condition. Second, it identifies an interest-payment channel through which money-financed government purchases remain expansionary once the direct spending-finance channel is eliminated. Third, it characterizes that closure's implementability and establishes the matched-accommodation result. The auxiliary gradual-backing rule and the rebate exercise serve as robustness and scope checks rather than separate policy experiments, and the central government-purchases findings extend to a liquidity trap.

The remainder of the paper proceeds as follows. Section 2 positions the paper in the literature and Section 3 presents the model, the valuation-based denial of irredeemability, and the policy closures. Sections 4 and 5 report the normal-times and liquidity-trap experiments, Section 6 discusses institutional interpretation and scope, and Section 7 concludes. The appendices collect the equilibrium-existence and determinacy analyses, the intermediate algebra behind the valuation closure, the interpolation between the unbacked and valuation rules, the counterfactual rebate exercise, and additional figures.

2 Related Literature

Bernanke (2003) was an early advocate of the MF fiscal stimulus. Responding to Japanese stagnation, he argued that a fiscal expansion that does not imply higher future taxes is essential for stimulating the economy, and that money financing provides exactly that possibility: unlike a DF stimulus, an MF stimulus is financed by irredeemable money and therefore raises private wealth. Buiter (2014) provides a precise theoretical foundation for this view, deriving a TVC that ensures

the irredeemability of money (IM) and thereby the effectiveness of the *MF* fiscal stimulus. He argues that the IM of base money is required for helicopter money to have wealth effects. Our result is complementary rather than contradictory: money-financed government purchases remain expansionary under immediate backing, but the effect operates through monetary accommodation rather than through a permanent net-wealth transfer (Section 4.2).

Galí (2020) shows, in a standard New Keynesian model, that the *MF* fiscal stimulus is effective both in normal times and in a liquidity trap. He emphasizes the role of sticky prices and the accommodative path of money growth, inflation, and interest rates induced by the financing regime. Although he does not stress the point explicitly, his benchmark fiscal rule is also consistent with the IM, in that the associated TVC leaves base money unbacked (Section 3.2.1). He analyzes both an increase in government purchases and a reduction in lump-sum taxes under the *MF* regime, and that benchmark is our point of departure. Relative to Galí (2020), our primary contribution is to study government purchases under a valuation-based closure that places base money inside the government’s terminal liability condition. This closure removes the direct spending-finance channel of money creation but reveals a distinct interest-payment channel through which money financing remains more expansionary than debt financing, with no permanent net-wealth transfer. We do not dispute Galí’s emphasis on monetary accommodation; we identify the channel through which it operates once irredeemability is denied.

The closest recent challenge to Galí’s (2020) conclusion is van der Kwaak (2026), who shows that monetary financing produces neither high inflation nor large fiscal multipliers once the central bank pays interest on reserves, so that newly created base money is no longer a distinctively cheap liability. The contrast is instructive, because the two papers unbundle Galí’s benchmark along different margins. Van der Kwaak (2026) changes the *remuneration* of money, which removes the interest-cost advantage of money over bonds and thereby weakens the very interest-payment margin we isolate; we hold the non-interest-bearing character of base money fixed and instead change its *fiscal status*, placing it inside the consolidated valuation condition so that it ceases to be permanent net wealth. Taken together, the results suggest that the money-financing premium can survive the removal of irredeemability but depends critically on the return differential between bonds and money.

Okano and Eguchi (2024) extend Galí’s (2020) closed-economy model to the small open economy of Galí and Monacelli (2016) and find money-financed fiscal stimuli more effective there than in the closed economy. As in Galí (2020), they assume the IM, adopting a fiscal policy rule whose

TVC is consistent with irredeemable money.

Our analysis is grounded in the fiscal theory of the price level. Leeper (1991) classifies monetary–fiscal regimes as combinations of “active” and “passive” policies, and Sims (1994), Woodford (1995, 2003), and Cochrane (2005, 2023) show that when fiscal policy is active, so that primary surpluses do not adjust to guarantee solvency, the price level is determined by the valuation of nominal government liabilities, including base money. Cochrane (2005), in particular, treats money as a residual claim on primary surpluses (“money as stock”). We use this machinery in a specific way: rather than invoking the FTPL to determine the price level, we use its valuation logic to construct a fiscal rule that subjects base money to the consolidated valuation condition and thereby denies its irredeemability. Relative to Buiter (2014), whose transversality condition leaves money outside the set of backed liabilities, our rule brings money inside it; cast in Leeper’s terms, Galí’s benchmark is a passive-fiscal rule under which money nonetheless remains unbacked net wealth. Moving from (10) to (14) removes that net-wealth treatment but also changes the endogenous money-growth path; the matched-accommodation counterfactual therefore provides the decisive identification result, showing that, conditional on common money-growth, government-purchases, and preference-shock paths, the composition of fiscal liabilities has no independent effect on the real allocation.

A related literature studies the fiscal backing of the central bank’s balance sheet. Del Negro and Sims (2015) and Hall and Reis (2015) show that a central bank engaged in large-scale balance-sheet operations requires fiscal support to remain solvent without resorting to inflation, and Bassetto and Sargent (2020) review how the consolidated budget of the two authorities disciplines such operations. This literature is the institutional counterpart of our fiscal rule: valuation-backed money finance presumes a form of treasury backing closely related to that formalized in these papers. It also connects to the debate on “helicopter money” as an essentially political rather than purely technical proposition, as emphasized by Turner (2015).

Like Galí (2020), the paper also relates to the New Keynesian literature on monetary policy at the zero lower bound (ZLB) on the nominal interest rate, including Auerbach and Obstfeld (2005), Jung et al. (2005), Eggertsson and Woodford (2003), Werning (2011), Buiter (2001), Svensson (2004), Nakajima (2008), and Fujiwara et al. (2013). Their focus, however, is monetary policy rather than the fiscal treatment of base money.

Several studies examine cases in which the MF fiscal stimulus fails to stabilize the economy. Drawing on Galí (2020), Tsuruga and Wake (2019) analyze how an implementation lag modifies

the effectiveness of the MF fiscal stimulus and find that it can generate a recession even in normal times. The mechanism identified by van der Kwaak (2026), discussed above, belongs to this group as well. English et al. (2017) highlight the role of central-bank credibility, showing that the MF fiscal stimulus is effective if it is communicated successfully and perceived as credible, and ineffective if the public doubts the central bank's commitment to it.

3 The Fiscal and Monetary Policy Framework

Our macroeconomic environment is the same as that in Galí (2020). The only substantive difference concerns fiscal policy, for which we use three closures: the unbacked benchmark rule in Galí (2020), which is consistent with the IM; a valuation rule that follows Cochrane (2005), obtained by iterating the consolidated budget constraint forward under a TVC on bonds and money together, which imposes immediate full backing; and an implementable Bohn-type rule on the consolidated liability, which imposes gradual full backing. Section 3.1 sets out the private-sector block, which is common to every experiment; Section 3.2 introduces the three fiscal closures and explains what each implies for the terminal treatment of base money; and Section 3.3 defines the policy regimes these closures generate, states the identification argument that the paper rests on, and delimits which of the resulting cells admit a bounded equilibrium.

3.1 The Model

Following Galí (2020), we assume a continuum of identical infinitely lived households indexed on $[0, 1]$ that maximize utility. A continuum of monopolistically competitive firms produces differentiated goods using a decreasing-returns technology, and a competitive final-good firm aggregates them with elasticity of substitution ϵ . Individual prices are subject to Calvo rigidity, while wages are flexible. The utility function is given by $[U(C_t, L_t) - V(N_t)] Z_t$, where C_t , $L_t \equiv \frac{M_t}{P_t}$, N_t , Z_t , P_t , and M_t denote consumption, the real money balance, hours worked, the exogenous preference shifter, the price level, and non-interest-bearing money, respectively. The decreasing-returns technology is given by $Y_t = N_t^{1-\alpha}$, where Y_t denotes output. See Galí (2020) for the full statement and derivation of the model.

The analysis below considers equilibrium in a neighborhood of a steady state with zero inflation, no trend growth, and zero government purchases, at which price markups are at their desired level and, real balances being constant, seigniorage is zero. The private-sector steady state and the

log-linear equilibrium conditions are the same as in Galí (2020):

Market-clearing Condition

$$\hat{y}_t = \hat{c}_t + \hat{g}_t, \quad (1)$$

Consumption Euler Equation

$$\hat{\xi}_t = \hat{\xi}_{t+1} + \hat{i}_t - \pi_{t+1} - \hat{\rho}_t, \quad (2)$$

Marginal Utility of Consumption

$$\hat{\xi}_t = -\sigma \hat{c}_t + v \hat{l}_t, \quad (3)$$

First-order Necessary Condition for Firms (New Keynesian Phillips Curve)

$$\pi_t = \beta \pi_{t+1} - \kappa \hat{\mu}_t, \quad (4)$$

Price Markup Gap

$$\hat{\mu}_t = \hat{\xi}_t - \frac{\alpha + \varphi}{1 - \alpha} \hat{y}_t, \quad (5)$$

Money Demand Schedule

$$\hat{l}_t = \hat{c}_t - \eta \hat{i}_t, \quad (6)$$

Logarithmic First Difference of the Real-Balance Definition

$$\hat{l}_{t-1} = \hat{l}_t + \pi_t - \Delta m_t, \quad (7)$$

Government Budget Constraint

$$\hat{b}_t = \hat{g}_t + (1 + \rho) \hat{b}_{t-1} + (1 + \rho) b \left(\hat{i}_{t-1} - \pi_t \right) - \hat{r}_t - \chi \Delta m_t, \quad (8)$$

with $\sigma \equiv -\frac{U_{cc}C}{U_c}$, $\varphi \equiv \frac{V_{nn}N}{V_n}$, $v \equiv \frac{U_{cl}L}{U_c}$, $\eta \equiv \frac{\epsilon_{lc}}{\rho}$, $\epsilon_{lc} \equiv -\frac{1}{h'} \frac{\rho}{1+\rho} V$ and $h\left(\frac{C}{L}\right) \equiv \frac{U_l}{U_c}$ where $\beta \equiv$

$(1 + \rho)^{-1}$ denotes the subjective discount factor, ρ denotes the rate of time preference, which equals the steady-state value of the net nominal interest rate. In addition, $\chi \equiv L/Y$ denotes the steady-state inverse velocity and $b \equiv \mathcal{B}/Y$ the target (steady-state) debt-to-GDP ratio, both of which enter the government budget constraint (8). In (4), $\kappa > 0$ is the slope of the New Keynesian Phillips curve, exactly as in Galí (2020): $\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$, where $\theta \in (0, 1)$ is the Calvo index of price rigidities and $\epsilon > 1$ the elasticity of substitution among goods, not to be confused with the money-demand elasticity ϵ_{lc} defined above. The slope is decreasing in the degree of price stickiness θ , and θ and ϵ enter the equilibrium system only through κ ; their calibrated values are those of Galí (2020) and are reported with the remaining parameters in Section 4. Hats denote either logarithmic deviations (as for $\hat{y}_t, \hat{c}_t, \hat{l}_t$) or steady-state-GDP-normalized level deviations (as for $\hat{g}_t, \hat{b}_t, \hat{tr}_t$), as Table 1 indicates. Here G_t and TR_t denote real government purchases and lump-sum taxes, $SP_t \equiv TR_t - G_t$ denotes the real primary balance, and MC_t denotes real marginal cost; the underlying budget identity and marginal-cost relation are stated below. The presentation and the notation closely parallel Galí (2020).

Table 1: Definitions of Transformed Variables

Variable	Definition	Variable	Definition
$\hat{y}_t$	$\log\left(\frac{Y_t}{Y}\right)$	m_t	$\log M_t$
$\hat{c}_t$	$\log\left(\frac{C_t}{C}\right)$	$\hat{l}_t$	$\log\left(\frac{L_t}{L}\right)$
$\hat{\rho}_t$	$-\log\left(\frac{Z_{t+1}}{Z_t}\right)$	$\hat{sp}_t$	$\log\left(\frac{SP_t}{SP}\right)$
$\hat{g}_t$	$\frac{G_t}{Y}$	$\hat{b}_t$	$\frac{\mathcal{B}_t - \mathcal{B}}{Y}$
$\hat{\xi}_t$	$\log\left(\frac{U_{c,t}}{U_c}\right)$	$\hat{tr}_t$	$\frac{TR_t - TR}{Y}$
π_t	$\log \Pi_t$	μ_t	$-\log MC_t$
$\hat{i}_t$	$\log\left(\frac{1+i_t}{1+\rho}\right)$	$\hat{\mu}_t$	$\mu_t - \mu$

We refer to $\hat{\mu}_t$ as the markup gap.

Any variable without a time subscript corresponds to its steady-state value. Δ is the difference operator, so Δm_t denotes money growth. The disturbance $\hat{\rho}_t$ is the preference (“Z”) shock defined in Table 1; it should not be confused with the time-preference rate ρ .

Equations (5)–(7) are, respectively, the log-linearized real marginal cost $MC_t = \frac{V_{n,t}}{U_{c,t}} \frac{N_t^\alpha}{1-\alpha}$, the household optimality condition yielding a money demand that depends on consumption and the nominal interest rate, and the definition of the real money balance. Equation (8) log-linearizes the

consolidated government budget constraint,

$$G_t + \mathcal{B}_{t-1}\mathcal{R}_{t-1} = TR_t + \mathcal{B}_t + \frac{\Delta M_t}{P_t}, \quad (9)$$

where G_t , i_t , $\Pi_t \equiv \frac{P_t}{P_{t-1}}$, $\mathcal{B}_t \equiv \frac{B_t}{P_t}$, B_t , TR_t , and $\mathcal{R}_t \equiv (1 + i_t)\Pi_{t+1}^{-1}$ denote the (real) government purchases index, the net nominal interest rate, the (gross) inflation, the real domestic government bond outstanding, the nominal riskless one-period domestic government bond, the lump-sum tax, and the (*ex-post*) gross real interest rate. As defined above, $SP_t \equiv TR_t - G_t$ is the real primary balance, while MC_t denotes real marginal cost. We assume that the government, consisting of the fiscal and monetary authorities acting in a coordinated manner, finances its expenditures through lump-sum taxes, a riskless nominal one-period bond, and non-interest-bearing money.

3.2 Fiscal Backing Rules

3.2.1 The Unbacked-Money Bohn Rule

We first introduce a simple tax rule, adopted in Galí (2020), which is a Bohn-type rule in the sense of Bohn (1998), as follows:

Unbacked-Money Bohn Rule

$$\hat{tr}_t = \psi_b \hat{b}_{t-1}. \quad (10)$$

Equation (10) shows that tax variations depend on $\psi_b \hat{b}_{t-1}$, which is endogenous and varies in response to deviations in the debt ratio from its long-run target, where ψ_b is a tax adjustment parameter. In the absence of further disturbances, the debt deviation evolves with root $1 + \rho - \psi_b$. Local stability therefore requires $\rho < \psi_b < 2 + \rho$; the benchmark value $\psi_b = 0.02$ lies comfortably inside this interval, and the debt ratio converges to its long-run target. Accordingly, the government's TVC:

$$\lim_{k \rightarrow \infty} \Lambda_{t,t+k} \mathcal{B}_{t+k} = 0, \quad (11)$$

is satisfied for bounded equilibrium price and interest-rate paths, provided the discount factor $\Lambda_{t,t+k} \equiv \prod_{j=0}^{k-1} \mathcal{R}_{t+j}^{-1}$, the cumulative discount between t and $t+k$ implied by the gross real interest rate $\mathcal{R}_t$ defined below (9), converges to zero as $k \rightarrow \infty$. Because the terminal condition in (11) applies only to interest-bearing bonds and excludes base money, fiat base money is treated as an

asset (wealth) to its holders but not as a terminal liability of the issuing central bank, itself part of the consolidated government, in Buiter’s (2014) sense. The IM is thus assumed as long as (10) is in force: Galí (2020) adopts (10) as the fiscal policy rule, with (11) following endogenously as the associated transversality condition, so that, although he does not state this explicitly, he analyzes the effectiveness of an *MF* fiscal stimulus under the IM. In the taxonomy of Leeper (1991), (10) is a passive-fiscal rule: taxes respond to inherited debt strongly enough ($\rho < \psi_b < 2 + \rho$) to stabilize it. The backing it provides, however, extends only to interest-bearing debt.

3.2.2 The Immediate-Backing Valuation Closure

We then derive a fiscal policy rule denying the IM. Throughout the paper, “denial of irredeemability” is used in a fiscal-valuation sense: base money is included in the consolidated government’s terminal liability condition and backed by future surpluses. It does not mean that currency holders receive a legal conversion or redemption claim against the central bank. By rewriting (9) in terms of consolidated liabilities (equivalently, by adding the imputed interest saving on beginning-of-period money balances), then iterating forward, using the Euler equation $U_{c,t}Z_t = \beta \mathcal{R}_t U_{c,t+1}Z_{t+1}$, and imposing the following transversality condition,

$$\lim_{k \rightarrow \infty} \Lambda_{t,t+k} (\mathcal{B}_{t+k} + L_{t+k}) = 0, \quad (12)$$

we obtain the valuation relation:

$$\begin{aligned} U_{c,t}Z_t\mathcal{R}_{t-1}(\mathcal{B}_{t-1} + L_{t-1}) &= \sum_{j=0}^{\infty} \beta^j U_{c,t+j}Z_{t+j}SP_{t+j} \\ &\quad + \sum_{j=0}^{\infty} \beta^{j-1} U_{c,t+j-1}Z_{t+j-1} \left(\frac{i_{t+j-1}}{1+i_{t+j-1}} \right) L_{t+j-1}, \end{aligned} \quad (13)$$

where SP_t is the primary balance defined above and $\Lambda_{t,t+k}$ is the discount factor introduced with (11).

$\left(\frac{i_t}{1+i_t} \right) L_t$ is the opportunity cost borne by households from holding real money balances, so (13) shows that the consolidated government’s liability, measured in units of the marginal utility of consumption, equals the sum of the present value of the primary balance in those units and the present value of the opportunity cost of holding real money balances.

The TVC (12) requires that the discounted value of consolidated liabilities vanish asymptotically; convergence of $\Lambda_{t,t+k}$ to zero is necessary but not sufficient, since liabilities must not grow

faster than the discount factor shrinks. Under the bounded MF equilibria studied below, $\hat{b}_t = 0$ and real balances remain bounded, so the consolidated TVC is satisfied. This is a property of those equilibria, not an automatic consequence of holding $\hat{b}_t$ constant. Under debt financing, by contrast, endogenous debt may activate an explosive liability root, and the restriction becomes decisive (Section 3.3 and Appendix A). Buiter (2014) instead imposes a terminal condition only on interest-bearing bonds, $\lim_{k \rightarrow \infty} \Lambda_{t,t+k} \mathcal{B}_{t+k} = 0$: bonds must ultimately be redeemed or “inflated away,” whereas base money remains outside the issuer’s terminal liability set. Equation (12) changes this by including both bonds and real balances, and is the consolidated-liability TVC underlying *both* full-backing implementations: the valuation rule (14) is its immediate representation, while the feedback rule (16) of Section 3.2.4 satisfies it gradually along a bounded path.

Equation (13) has the structure of the FTPL valuation equation of Cochrane (2005), in which nominal government debt, including the monetary base, is a residual claim on the present value of primary surpluses,

$$\frac{\text{Nominal Government Debt}}{\text{Price Level}} = \text{Expected Present Value of Primary Surpluses},$$

so that (13) inherits the character of Cochrane’s (2005) FTPL. Rewriting (13) so that the gross nominal interest factor $1 + i_{t-1}$ appears on the left-hand side, and then log-linearizing the transformed expression, isolates the role of seigniorage in the narrow sense (an imputed *liquidity-service return*, or equivalently the interest saving from issuing non-interest-bearing money rather than bonds), as distinct from the broader seigniorage term $\frac{\Delta M_t}{P_t}$ of Galí (2020). A key step is that this liquidity-service return lowers the coefficient on the inherited nominal interest rate from $\frac{b}{\chi} + 1$ to $\frac{b}{\chi}$, so that the interest saving mitigates the burden of financing interest payments; this feature is what makes the MF fiscal stimulus operative under the immediate-backing valuation rule. We call this margin the *interest-payment channel*, as distinct from the *government-purchases channel* through which money creation finances the spending increase itself; Section 4.2 defines both formally from the equilibrium money-growth relations and measures their relative contributions. The intermediate algebra is collected in Appendix B. Plugging the logarithmic definition of the primary balance $\hat{p}_t = \frac{\beta}{b(1-\beta)} \hat{tr}_t - \frac{\beta}{b(1-\beta)} \hat{g}_t$ into the resulting relation, we obtain:

Immediate-Backing Valuation Closure

$$\hat{tr}_t = \hat{b}_{t-1} + b(\hat{i}_{t-1} - \pi_t) + \frac{b(1-\beta)^2 + \chi\beta^2}{\beta} \hat{l}_{t-1} + \hat{g}_t - \beta \hat{b}_t - \beta \chi \hat{l}_t - \chi \beta \pi_t, \quad (14)$$

which is our immediate valuation closure. The terms involving the nominal interest rate and inflation are combined into the second term on the right-hand side, which is expressed in terms of the real interest rate, while the last term retains the possibility of ‘inflating away’ liabilities, so that (14) allocates the resources required to service inherited consolidated liabilities among primary surpluses, the issuance or rollover of liabilities, and changes in their real value through inflation. Equation (14) is not an independently estimated fiscal reaction function; it is the tax relation implied jointly by the period budget constraint, the consolidated valuation condition, its terminal condition, and the selected financing closure. We therefore call it a valuation closure; “rule” is used below as shorthand for its implementation through the tax sequence.

Although (8) and (14) ultimately stem from the same consolidated government’s budget constraint (9), they are not log-linearizations of the same object in the same form: (8) approximates the period-by-period flow budget constraint directly, in terms of conventional seigniorage $\frac{\Delta M_t}{P_t}$, whereas (14) is obtained only after transforming (9) into the intertemporal valuation relation (13) under the stronger TVC (12) and rewriting that relation so that $1 + i_{t-1}$ appears on the left-hand side (Appendix B). Equation (14) is therefore a different reduced-form representation of the same underlying budget identity under a different fiscal backing, not an alternative direct linearization of (9).

In the polar active-fiscal regime of Leeper (1991) and Cochrane (2005, 2023), primary surpluses are set exogenously and the price level adjusts so that the real value of nominal liabilities, interest-bearing debt and base money alike, equals the present value of those surpluses. Equation (14) borrows the valuation content of that regime but embeds it in a rule under which taxes respond to the full stock of consolidated liabilities: it backs the bond component, as (10) does, and, unlike (10), also backs the money component by including previously issued real balances in the liability to which the implied tax sequence responds. Base money is thereby backed rather than irredeemable, and the residual “inflating-away” term is the vestige of the active-fiscal valuation channel, operating only insofar as surpluses and newly issued liabilities fall short of servicing inherited liabilities. Our contribution is therefore not a new determinacy result within the FTPL, but the use of its valuation logic to remove the permanent net-wealth treatment of money; the matched-accommodation counterfactual then identifies whether liability composition has any independent effect once the monetary path is held fixed. Because (14) is a *local difference representation* of the intertemporal valuation relation, it must be supplemented by the terminal condition and does not by itself exclude explosive liability paths. Whether a particular monetary–fiscal closure delivers

such a bounded path is a separate question: under money financing the rule is closed by fixing $\hat{b}_t = 0$, whereas under debt financing, where debt is endogenous, it need not deliver a bounded equilibrium at all (Section 3.3).

3.2.3 A Reduced-Form Interpolation between the Fiscal Rules

The two fiscal rules introduced above are polar cases: (10) leaves base money entirely unbacked (full irredeemability), while (14) backs it entirely. The distinction need not generate a discontinuous response, but because the intermediate cases are not central to our argument we relegate them to Appendix C, which introduces a reduced-form interpolation between the two rules and maps the response across it. The main text works with the two polar rules and the implementable backed rule introduced next.

3.2.4 An Implementable Fully Backed Rule

The valuation-based rule (14) is derived from the exact intertemporal valuation relation but is itself a forward-looking local log-linear representation, and, as we show in Section 3.3, it need not deliver a bounded path under every monetary closure. It is therefore useful to pair it with an *implementable* feedback rule that shares its economic content, backing both bonds and money by future surpluses, while stabilizing the consolidated liability directly. Let

$$q_t \equiv \hat{b}_t + \chi \hat{l}_t \quad (15)$$

denote the consolidated real liability of the government (interest-bearing debt plus real balances, in steady-state-GDP units), and consider the Bohn-type rule

$$\hat{tr}_t = \psi_q q_{t-1} + \hat{tr}_t^*, \quad (16)$$

where $\psi_q > 0$ is a feedback coefficient and $\hat{tr}_t^* \leq 0$ is an exogenous lump-sum rebate. The admissible set is closure-specific. Under debt financing, convergence to the liability target requires $\rho < \psi_q < 2 + \rho$, obtained analytically below. Under money financing, the connected determinate region containing the benchmark must be located numerically (Appendix A.3). Unlike (10), which stabilizes only $\hat{b}_t$ and leaves base money outside the solvency condition, (16) makes taxes respond to the *entire* consolidated liability. It therefore (i) backs both bonds and money by current and future surpluses, so that no permanent net-wealth transfer arises; (ii) stabilizes q_t for a suitable ψ_q ,

so that the transversality condition (12) holds along a bounded path; and (iii) is compatible with strict inflation targeting, so that a well-defined gradually backed *DF* regime exists. The backing it imposes, moreover, is *gradual* rather than contemporaneous: a rebate $\hat{tr}_t^* < 0$ is serviced by higher taxes spread over future periods in proportion to ψ_q , rather than offset one-for-one within the period. The rule thus shows that contemporaneous offset, rather than present-value backing itself, is responsible for rebate neutrality under (14). Equation (16) serves throughout as an auxiliary benchmark that completes the regime comparison, the paper's central object remaining the valuation-based implementation (14) and the channels that operate under it.

To characterize the benchmark-relevant regions under the two closures, we begin with the law of motion of the consolidated liability. Substitute the definition of money growth, (7), into the government budget constraint, (8), and add $\chi \hat{l}_t$ to both sides. Using $(1 + \rho) \hat{b}_{t-1} + \chi \hat{l}_{t-1} = (1 + \rho) q_{t-1} - \rho \chi \hat{l}_{t-1}$ and the rule (16), the consolidated liability obeys

$$q_t = (1 + \rho - \psi_q) q_{t-1} - \rho \chi \hat{l}_{t-1} + (1 + \rho) b \left(\hat{i}_{t-1} - \pi_t \right) - \chi \pi_t + \hat{g}_t - \hat{tr}_t^*. \quad (17)$$

Under debt financing, once the disturbances have vanished and all endogenous variables other than the consolidated-liability deviation have returned to steady state, real balances are back at their steady-state level, so that $q_t = \hat{b}_t$ and (17) collapses to the autonomous difference equation $q_t = (1 + \rho - \psi_q) q_{t-1}$, whose root lies strictly inside the unit circle if and only if $\rho < \psi_q < 2 + \rho$. The interval is open because the root equals +1 at $\psi_q = \rho$ and -1 at $\psi_q = 2 + \rho$; at either endpoint the liability remains bounded but fails to converge to its target. Equation (17) thus establishes $\psi_q > \rho$ as a *necessary* condition, obtained analytically, for stabilizing the consolidated liability *under debt financing*; it is the exact consolidated-liability counterpart of the condition $\rho < \psi_b < 2 + \rho$ that (10) imposes on interest-bearing debt alone. The economics is the interest cost of carrying the liability: a coefficient below the steady-state real rate ρ does not retire enough of the liability each period to offset the interest accruing on it, so the consolidated position grows without bound and (12) fails. Setting $\psi_q = 0$ recovers $q_t = (1 + \rho) q_{t-1} = \beta^{-1} q_{t-1}$, the same asymptotic root β^{-1} as the valuation-rule *DF* experiment of Appendix A, although the two fiscal closures are distinct; the common root reflects the absence of sufficiently strong fiscal feedback in both cases.

Under money financing the same scalar reduction is unavailable. Since $\hat{b}_t = 0$ at every date, $q_t = \chi \hat{l}_t$, so the consolidated-liability deviation is proportional to real balances rather than an

independent liability state, and the term $-\rho\chi\hat{l}_{t-1}$ in (17) is itself $-\rho q_{t-1}$. Substituting this restriction into (17) combines the first two lagged terms into $(1 - \psi_q)q_{t-1}$, in place of $1 + \rho - \psi_q$, but q_t remains jointly determined with the private-sector variables, so no autonomous scalar recursion in q_t alone survives. Hence the debt-financing lower bound $\psi_q > \rho$ does not apply under *MF*: as $\psi_q \rightarrow 0$ the rule approaches the unbacked *MF* benchmark of (10), which is itself a bounded equilibrium. The connected *MF* determinacy region containing the benchmark must therefore be located numerically, using the reduced-system procedure described in Appendix A, rather than read off (17). The root counts reported in Appendix A.3 establish that the benchmark $\psi_q = 0.02$ lies well inside the connected determinate region containing it, whose upper endpoint is $\bar{\psi}_q \simeq 0.1434$; that appendix also records the full range of ψ_q examined and the mechanism behind the endpoint. Since $\psi_q = 0.02$ is admissible under both closures, we use it throughout the implementable two-by-two comparison.

Equation (16) equalizes the fiscal feedback *function* across financing schemes, not the realized tax sequence or the present value of taxes: because consolidated liabilities evolve differently under money and debt financing (money-financed inflation erodes the real liability base), an identical feedback coefficient generally generates different tax paths. The rule is therefore used purely as an implementability and identification benchmark; it does *not* study money financing under the same future tax burden as debt financing, a distinct experiment outside the scope of this paper.

3.3 Policy Regimes, Identification, and Equilibrium Existence

Below, we analyze two stylized fiscal interventions in the same New Keynesian environment used by Galí (2020): an exogenous lump-sum rebate and an exogenous increase in government purchases. These are two *separate* experiments, not a joint intervention. The intervention is announced in period zero and implemented from that period onward. For concreteness, in the rebate experiment we assume

$$\hat{tr}_t^* = -\delta^t < 0, \quad \hat{g}_t = 0,$$

while in the government-purchases experiment we assume

$$\hat{g}_t = \delta^t > 0, \quad \hat{tr}_t^* = 0,$$

for $t = 0, 1, 2, \dots$, where $\delta \in [0, 1)$ measures the persistence of the fiscal stimulus. In both experiments, the size of the intervention is normalized to 1% of steady-state output in period zero. Because taxation is lump sum throughout, we refer to the tax intervention as a lump-sum rebate (a transfer) rather than a tax cut, reserving the latter for reductions in *distortionary* tax rates.

Under the *MF* scheme, the real stock of government debt is kept constant, as in Galí (2020). By substituting $\hat{b}_t = 0$ into (8), we obtain

Monetary Policy Rule under the MF Fiscal Stimulus

$$\Delta m_t = \frac{1}{\chi} \left[\hat{g}_t - \hat{r}_t + (1 + \rho) b \left(\hat{i}_{t-1} - \pi_t \right) \right], \quad (18)$$

for $t = 0, 1, 2, \dots$. Under the *MF* scheme, therefore, the monetary authority gives up control of the nominal interest rate and instead adjusts the money supply to meet the government's financing needs.

Under the *DF* scheme, the fiscal authority issues debt to finance the fiscal stimulus, eventually adjusting the tax path to attain the long-run debt target $\mathcal{B}$, as in Galí (2020), while the monetary authority pursues an independent price stability mandate. For concreteness, we assume that, if feasible, it conducts a policy such that

Monetary Policy Rule under the DF Fiscal Stimulus

$$\pi_t = 0, \quad (19)$$

for all t . Equation (19) represents strict inflation targeting and applies to the *DF* regimes. The money supply then adjusts endogenously to support the interest rate required to stabilize prices, as in the regime generally assumed in the New Keynesian fiscal-policy literature. Galí's conventional *DF* benchmark combines (10) with strict inflation targeting, (19). We additionally combine (19) with the gradual-backing rule (16), and use its combination with the valuation rule (14) only as an equilibrium-existence diagnostic (Section 3.3.1).

Table 2 provides a compact summary of the policy regimes considered in the analysis.

Table 2: Policy Regimes: Fiscal Rules, Backing, and Equilibrium Status

Regime	Fiscal rule	Backing	Tax closure	Debt	Monetary closure	Equilibrium
<i>MF</i> , unbacked	(10)	bonds only	none (purchases); exogenous (rebate)	fixed	money-growth closure	bounded
<i>DF</i> , unbacked	(10)	bonds only	gradual debt feedback	endogenous	$\pi_t = 0$	bounded
<i>MF</i> , gradual backing	(16)	bonds + money	gradual liability feedback	fixed	money-growth closure	bounded
<i>DF</i> , gradual backing	(16)	bonds + money	gradual liability feedback	endogenous	$\pi_t = 0$	bounded
<i>MF</i> , immediate backing	(14)	bonds + money	contemporaneous offset	fixed	money-growth closure	bounded
<i>DF</i> , immediate backing	(14)	bonds + money	n/a (no bounded eq.)	endogenous	$\pi_t = 0$	no bounded eq.

Our definitions of the *MF* and *DF* fiscal stimuli are the same as those in Galí (2020): under *MF* the real stock of government bonds is held at its target level under every fiscal rule, and under *DF* inflation is fixed at zero, with taxes and money growth determined endogenously in each case (Table 2). In the government-purchases experiment, taxes remain constant under the unbacked *MF* stimulus, whereas they rise with government purchases within the period under the immediate-backing rule and gradually at speed ψ_q under (16) in either financing scheme. For brevity we refer to the three money-financing regimes by short names throughout: the *unbacked MF* stimulus (money financing under the unbacked Bohn rule (10), Galí’s benchmark), the *immediate-backing MF* stimulus (money financing under the immediate-backing valuation rule (14)), and the *gradually backed MF* stimulus (money financing under the gradual-backing rule (16)). On the debt-financing side we likewise distinguish the *gradually backed DF* regime (debt financing under the gradual-backing rule (16), which admits a bounded equilibrium) from the *valuation-rule DF* experiment (debt financing under the valuation rule (14), which admits no bounded equilibrium; Appendix A).

3.3.1 The Immediate-Backing Valuation Benchmark and Equilibrium Existence

The valuation rule (14) is the paper’s central benchmark: it is our immediate valuation-based implementation of money as a fiscally backed liability. Under money financing it delivers a bounded equilibrium, studied in Section 4. Under debt financing the analogous experiment is special. At the level of the *real* allocation it is redundant: with lump-sum taxes and strict inflation targeting, the non-fiscal block (1)–(5) determines output, consumption, and the real interest rate independently of the fiscal rule, and, with $v = 0$, money demand (6) is a residual that supports the required interest-rate path. At the level of the *complete* equilibrium, however, this regime (the last row of Table 2) is not well defined, because no fiscal path supports that allocation while satisfying the consolidated TVC. Combining (14) with strict inflation targeting, (19), and endogenous debt makes government debt inherit the explosive root $\beta^{-1} = 1 + \rho > 1$, so any non-zero inherited debt

position diverges and violates the transversality condition (12) (Appendix A). The reason is that (14) is a forward-looking valuation relation rather than a debt-stabilizing feedback of the Bohn type: under money financing it is closed by holding $\hat{b}_t = 0$, but under debt financing nothing pins debt down. Hence the valuation-rule *DF* experiment admits no bounded rational-expectations equilibrium under this closure (short-run series can be simulated, but they do not constitute an equilibrium). This nonexistence is distinct from the *indeterminacy* that arises under the ψ_q rule over the intermediate region immediately above $\bar{\psi}_q$: there, too many stable roots leave multiple bounded equilibria; here, an explosive root on predetermined debt leaves none.

This has two implications. First, the missing regime is a nonexistence result rather than a numerical failure, and it prevents (14) alone from identifying a pure financing effect, which motivates the implementable comparison based on (16). Appendix A verifies it numerically: simulated debt grows at exactly the gross rate $\beta^{-1} = 1.005$ per quarter, and the discounted consolidated liability $Q_t \equiv \beta^t(\hat{b}_t + \chi \hat{l}_t)$, the first-order counterpart of the level transversality condition (12), converges to $2.197 \neq 0$ instead of to the zero that condition requires. Second, a distinct and *well-behaved* gradually backed *DF* regime does exist once the forward-looking valuation rule is replaced by the implementable feedback rule (16): its real allocation coincides with the conventional *DF* allocation to machine precision, its debt path is stable, and its discounted consolidated liability converges to zero (Figure 1).

3.3.2 The Matched-Accommodation Counterfactual

A matched-accommodation counterfactual pins down the source of the observed *MF–DF* regime contrast. We re-solve the debt-financing economy while imposing, as an exogenous sequence, the equilibrium money-growth path generated by the immediate-backing *MF* regime.¹ The resulting real allocation coincides with that of *MF* under immediate backing to machine precision (the largest discrepancy across output, consumption, inflation, and the interest rate is of order 10^{-16}), while the fiscal variables differ: debt rises and taxes barely move on impact. Within the maintained model, and conditional on common exogenous and money-growth paths, what distinguishes money financing from debt financing for real outcomes is therefore the path of monetary accommodation it induces, not the composition of fiscal liabilities. This illustrates the following analytical equivalence, which formalizes the mechanism that Galí (2020) emphasizes: the superiority of *MF* arises because the regime commits the central bank to temporary accommodation under sticky prices.

Proposition 1 (Matched-accommodation equivalence). *In the maintained representative-agent*

economy with lump-sum taxation, no segmented asset markets, and common initial conditions, fix common bounded paths for money growth, government purchases, and the preference disturbance. For any feasible fiscal closure that implements the money-growth path, admits a bounded equilibrium, and levies lump-sum taxes, the equilibrium paths of output, consumption, inflation, and the nominal interest rate are invariant to the composition of fiscal liabilities and to the fiscal rule.

Proof. Conditional on the fixed paths of government purchases and the preference disturbance, the private-sector block, (1)–(7), depends on the financing regime only through the money-growth path. Given those common exogenous paths and a bounded $\{\Delta m_t\}$, the private-sector block determines the non-fiscal variables uniquely under the standard Blanchard–Kahn conditions of the underlying model; the budget constraint and the fiscal rule then determine taxes and debt residually. Because the money-growth path is imposed exogenously here, the private-sector block is self-contained, so, unlike Proposition 2, the equivalence does not rely on separable real balances ($v = 0$). ■

The proposition holds for *any* common path: we verify it numerically both for the immediate-backing path (largest real discrepancy of order 10^{-16}) and for the gradual-backing path (of the same order, with cumulative output 2.69).

3.3.3 The Implementable Two-by-Two Design

To verify that our conclusions do not hinge on the immediacy of the valuation-rule implementation, we also cross the two financing schemes with the two *implementable* fiscal rules, the unbacked Bohn rule (10) and the consolidated-liability rule with gradual full backing (16), setting $\psi_q = \psi_b = 0.02$ so that the benchmark feedback coefficient is held fixed while only the object of the tax response (bonds alone or the consolidated liability) changes. The design thus separates financing from the fiscal treatment of money as closely as the model permits. Within the debt-financing column the fiscal-rule margin is inert, as we now record.

Proposition 2 (Invariance across the implementable fiscal rules under debt financing). *Under strict inflation targeting, lump-sum taxation, and separable real balances ($v = 0$), the equilibrium paths of output, consumption, inflation, and the nominal interest rate under debt financing are identical under (10) and (16); the two rules affect only the paths of taxes and debt.*

Proof. With $\pi_t = 0$ imposed by (19), the NKPC, (4), pins the markup gap at zero; the markup condition, (5), the marginal-utility relation, (3), which reduces to $\hat{\xi}_t = -\sigma \hat{c}_t$ under $v = 0$, and market clearing, (1), then jointly determine output, consumption, and the marginal utility of

consumption from the path of government purchases alone; the Euler equation, (2), determines the nominal rate; and money demand, (6), determines identical real balances, so (7) implies identical money growth. The fiscal rule enters only the government budget constraint, which determines taxes and debt residually. ■

Section 4 reports the resulting cumulative output effects.

4 The Effects of the Fiscal Stimulus in Normal Times

In this section, we study the effects of fiscal stimulus in normal times using the basic New Keynesian model given by (1)–(8), which is identical to the model in Galí (2020) and is closed by the relevant fiscal and monetary rules.² We examine both a lump-sum rebate and an increase in government purchases in normal times, that is, when the ZLB does not bind. Our calibration, reported in Table 3, is the same as in Galí (2020) and is maintained throughout the paper. Unless otherwise stated, the cumulative output effect is the unweighted sum $\sum_{t=0}^{\infty} \hat{y}_t$ of percent output deviations, reported without a percent sign; for comparisons across shock persistence we follow Galí (2020) and report the normalized multiplier $(1 - \delta) \sum_{t=0}^{\infty} \hat{y}_t$, which at $\delta = 0.5$ is half the former and hence not directly comparable to it.

Table 3: Parameterization

Parameter	Description	Value
σ	Relative Risk Aversion	1
β	Discount Factor	0.995
φ	Curvature of Labor Disutility	5
α	Index of Decreasing Returns to Labor	0.25
ϵ	Elasticity of Substitution among Goods	9
θ	Calvo Index of Price Rigidities	0.75
χ	Steady-State Inverse Velocity	$\frac{1}{3}$
η	Semi-elasticity of Money Demand	7
v	Separability of Real Balances	0
ψ_b	Tax Adjustment (Bohn Rule)	0.02
ψ_q	Tax Adjustment (Consolidated-Liability Rule)	0.02
b	Target Debt Ratio	2.4
δ	Persistence	0.5

4.1 *DF* Fiscal Stimulus

Under the *DF* fiscal stimulus, monetary policy is inflation targeting, (19), so that the inflation target disciplines the joint behavior of output, consumption, and the nominal interest rate. Because debt is not held fixed exogenously, the fiscal expansion is initially financed through debt issuance, with taxes adjusting only gradually through the fiscal rule.

For an increase in government purchases, the dynamics are standard and coincide with those in Galí (2020): upward pressure on inflation is absorbed by a higher nominal interest rate, so the real interest rate rises and consumption is crowded out. Quantitatively, cumulative consumption falls by 1.75, whereas cumulative output rises by only 0.25; although higher government purchases raise output mechanically through market clearing, the overall expansion under debt financing remains limited.

For a lump-sum rebate, the fiscal rule becomes

$$\hat{tr}_t = \psi_b \hat{b}_{t-1} + \hat{tr}_t^*, \quad (20)$$

instead of (10), simply adding the exogenous rebate component to the standard debt rule. As Galí (2020) notes, a debt-financed lump-sum rebate has no effect on any variable other than debt and taxes, which is a straightforward implication of Ricardian equivalence under lump-sum taxation.

4.2 Government Purchases under the Immediate-Backing Valuation Rule

We now turn to the paper's central experiment. Under the immediate-backing valuation rule, base money is part of the consolidated liability that future fiscal resources must validate, so the direct financing of the spending increase by money creation is offset within the period. The question is what, if anything, remains of the money-financing advantage once that direct channel is closed. Figure 2 reports the full set of impulse responses underlying the normal-times comparison. We first summarize the headline dynamics and then use (21)–(23), together with the money-growth decomposition in Appendix E, Figure E.1, to identify the source of the remaining monetary accommodation.

4.2.1 Headline Dynamic Responses

Figure 2 shows the dynamic effects of an increase in government purchases under the *MF* fiscal stimulus in normal times, the solid line with circles and the dashed line with crosses reporting the unbacked rule and the immediate-backing valuation rule, respectively; all panel references in this subsection are to that figure. Output rises under both regimes, although the increase is smaller under immediate backing (Panel 1). The difference is driven by consumption, which falls under the immediate-backing valuation rule but rises under the unbacked rule (Panel 2), and, consistently, the real interest rate declines under the unbacked rule but rises under immediate backing (Panel 3): by the Euler equation, a lower real interest rate stimulates consumption through a decline in the marginal utility of consumption $\hat{\xi}_t$, whereas a higher one has the opposite effect. This difference in real interest rates is in turn explained by inflation, which rises in both cases because higher government purchases raise demand and narrow the markup gap, but by much more under the unbacked rule (Panel 4).

4.2.2 The Interest-Payment Channel

The difference in inflation reflects the different responses of lump-sum taxes. Under the unbacked *MF* stimulus, the right-hand side of (10) is identically zero, so $\hat{tr}_t = 0$ (Panel 10) and money injection is essential:

$$\Delta m_t = \frac{1}{\chi} \hat{g}_t + \frac{1}{\chi} (1 + \rho) b (\hat{i}_{t-1} - \pi_t), \quad (21)$$

which is obtained by substituting $\hat{b}_t = 0$ and (10) into (18). Because the lump-sum tax is fixed at zero, the term multiplied by $1/\chi$ in (21), namely $\hat{g}_t + (1 + \rho) b (\hat{i}_{t-1} - \pi_t)$, is the fiscal deficit: the negative of the fiscal surplus, where the fiscal surplus is defined as the primary balance minus the real interest payment. Division by χ converts that financing need into the rate at which base money must grow. An increase in government purchases is therefore financed by money injection, and money growth rises (Panel 7); we call this source of money-growth variation the “government-purchases channel.”

Equation (21) also implies that a decline in the real interest payment, brought about by higher inflation, improves the fiscal surplus and thereby dampens money growth; we call this mechanism the “interest-payment channel.” Following an increase in government purchases the real interest rate declines (Panel 3), so the two channels pull in opposite directions, but the government-

purchases channel dominates: the fiscal surplus turns negative and money is injected (Panels 7 and 11).³ This money injection raises consumption through the decline in the real interest rate caused by higher inflation (Panels 2 and 3). As a result, output increases. The cumulative increase in output is 2.69 under the unbacked *MF* stimulus.

Under the immediate-backing *MF* stimulus, tax revenue finances the increase in government purchases, so the direct purchases-finance term disappears. Positive money injection after impact arises overwhelmingly through the interest-payment channel. The inherited-real-balance component is zero on impact and makes only a small positive contribution thereafter, because real balances lie below their steady-state level. The fiscal rule is then:

$$\hat{tr}_t = b \left(\hat{i}_{t-1} - \pi_t \right) + \frac{b(1-\beta)^2 + \chi\beta^2}{\beta} \hat{l}_{t-1} - \beta\chi\hat{l}_t - \chi\beta\pi_t + \hat{g}_t, \quad (22)$$

obtained by substituting $\hat{b}_t = 0$ into (14). Substituting (22) into (18) yields:

$$\begin{aligned} \Delta m_t &= \frac{1}{\chi} \left\{ \hat{g}_t - \left[b \left(\hat{i}_{t-1} - \pi_t \right) + \frac{b(1-\beta)^2 + \chi\beta^2}{\beta} \hat{l}_{t-1} - \beta\chi\hat{l}_t - \chi\beta\pi_t + \hat{g}_t \right] \right\} \\ &\quad + \frac{(1+\rho)b}{\chi} \left(\hat{i}_{t-1} - \pi_t \right), \\ &= -\frac{b(1-\beta)}{\chi\beta} \hat{l}_{t-1} + \frac{\rho b}{\chi(1-\beta)} \left(\hat{i}_{t-1} - \pi_t \right). \end{aligned} \quad (23)$$

The expression in square brackets on the first line of (23) is the right-hand side of (22), the driving force of lump-sum tax revenue, which includes government purchases $\hat{g}_t$.⁴ The last term in that line is the real interest-payment term scaled by $1/\chi$, so the first line expresses money growth as the fiscal deficit divided by χ . At the level of the period budget identity, then, both *MF* regimes finance the residual deficit through money creation; what differs is the *composition* of that deficit. Using the real-balance identity (7) to combine current real balances and inflation into money growth, the explicit government-purchases and inflation terms cancel, leaving the inherited-balance and real-interest-payment components in the second line of (23). The first captures the effect of inherited real balances, whose sign depends on whether real balances lie above or below their steady-state level. In the normal-times experiment, real balances fall below steady state after impact, so this term adds slightly to money growth. The direct spending-finance channel is nevertheless eliminated, although government purchases still affect money growth indirectly through their equilibrium effect on real interest payments.

Even under immediate backing, however, the *MF* fiscal stimulus still raises output and inflation

(Panels 1 and 4), because the interest-payment channel survives the elimination of the direct government-purchases channel, as the second term on the right-hand side of the second line of (23) shows. Real interest payments are part of consolidated government expenditure, but, unlike government purchases, they are only partially covered by lump-sum taxation: the consolidated issuer earns an imputed liquidity-service return on non-interest-bearing money, so that issuing the zero-interest liability economizes on its interest burden, and fluctuations in that burden induce money creation even when the spending increase itself is tax financed. Decomposing the equilibrium money growth of (23) into its two terms confirms this quantitatively (Figure E.1): the interest-payment component accounts for essentially all variation in money growth, including virtually all of the positive money injection after impact, whereas the inherited-real-balance component is an order of magnitude smaller throughout. The residual monetary accommodation that sustains the 1.48 response therefore works almost entirely through the interest-payment term.

The gap between the gross real interest payment in (8) and the mitigated burden in (14), namely $\rho b(\hat{i}_{t-1} - \pi_t)$, reflects the mitigation provided by the imputed liquidity-service return on non-interest-bearing money; it is absorbed jointly by inherited real balances and current money creation (Appendix B).

On impact, higher inflation reduces the real servicing burden on inherited nominal government bonds, so the fiscal surplus improves (Panel 11); additional money injection is then unnecessary, and both money growth and real money balances decline (Panels 7 and 8). By the money-demand schedule in (6), the fall in real balances raises the nominal and hence the real interest rate (Panels 3 and 5).

This initial increase in the real interest rate crowds out consumption, which falls here although it rises under the unbacked *MF* stimulus (Panel 2). The crowding-out is nevertheless not fatal, because after impact the higher real interest rate raises real interest payments and reverses the impact-period improvement in the fiscal surplus (Panel 11). The interest-payment channel operates with the same sign in both regimes, since the coefficient on $\hat{i}_{t-1} - \pi_t$ in (21) and (23) is positive. The real interest payment, however, moves in opposite directions: it falls under the unbacked rule, where the real interest rate declines, damping money growth, whereas it rises under the immediate-backing valuation rule, so that money growth increases and the increase persists for a while (Panel 7), which pushes up inflation and brings the real interest rate back down (Panels 3 and 4). Consumption then recovers, with a cumulative response of -0.52 , higher than under the *DF* fiscal stimulus, and cumulative output rises by 1.48, smaller than under the unbacked *MF*

stimulus but larger than under the DF fiscal stimulus. Hence, even without a permanent net-wealth transfer to households through money injection, the MF fiscal stimulus remains effective, in contrast to the immediate-offset rebate case of Section 4.4, in which the within-period reversal of the money injection eliminates the effect.

4.3 An Implementability Check: Completing the Two-by-Two Design

This exercise is not the paper’s central policy experiment; its purpose is narrower, to establish that the government-purchases result under (14) is not an artifact of the missing debt-financed cell, whose absence leaves the comparison in Section 4.2 varying financing and backing at once. Because the valuation rule admits no bounded DF equilibrium (Section 3.3.1), we build the benchmark from the auxiliary implementable rule (16), with $\psi_q = \psi_b = 0.02$, as described in Section 3.3.3 and reported in Table 4.

The financing-regime contrast is sharp: money financing delivers cumulative output of 2.69 against 0.25 under debt financing, a gap of 2.44, and moving from the unbacked Bohn rule to the backed rule changes essentially nothing within either column (2.69 versus 2.69 under MF ; 0.25 versus 0.25 under DF), so broad present-value backing does not overturn the result. The central valuation-rule benchmark of Section 4.2, at 1.48, sits below the unbacked MF value: within the valuation closure, contemporaneous offset eliminates the direct spending-finance term, whereas the gradual feedback closure leaves the initial financing need largely unoffset. This difference generates sharply different monetary paths. Proposition 2 establishes the invariance of the DF column.

Table 4: An Implementable Two-by-Two Benchmark: Cumulative Output Effects of a Government-Purchases Stimulus

Fiscal rule	MF	DF
Unbacked money (Bohn rule, (10))	2.6883	0.2500
Consolidated backing (ψ_q rule, (16), $\psi_q = 0.02$)	2.6884	0.2500

MF – DF financing-regime contrast at a common fiscal rule: $2.69 - 0.25 = 2.44$. Fiscal-rule contrast within each financing column, between (10) and (16), ≈ 0 . Analytical benchmark of *immediate* backing (valuation rule, (14), MF): 1.48. The DF column is invariant under strict inflation targeting, lump-sum taxation, and $v = 0$; the valuation rule under DF admits no bounded equilibrium (Appendix A).

Figure 3 displays the four implementable cells together with the immediate-backing benchmark.

Three features stand out. First, as Proposition 2 predicts, the two DF paths coincide in all non-fiscal variables but differ in taxes and debt. Second, the unbacked and gradually backed MF paths are nearly indistinguishable, whereas the immediate-backing MF path differs substantially. Third, by the matched-accommodation result of Section 3.3.2, the gap between any MF regime and DF would vanish entirely if the same money-growth path were imposed on DF . The gradual rule tracks the unbacked one because, under (16), taxes respond only to *inherited* liabilities: the spending increase is not offset on impact, and much of the fiscal adjustment occurs after the initial sticky-price effects of the monetary accommodation have weakened. The near-equivalence holds at every value examined within the connected determinate region containing the benchmark.⁵ The upshot for the paper is that broad present-value backing does not eliminate the effectiveness of money-financed government purchases; what the immediate valuation rule removes is the direct spending-finance channel, leaving the interest-payment channel of Section 4.2 to carry the effect.

4.4 A Lump-Sum Rebate as a Scope Check

The lump-sum rebate serves as a scope check, not a second central contribution. Throughout, $\widehat{tr}_t$ denotes total net taxes in the government's budget constraint (8). Let $\widehat{tr}_t^* < 0$ be the exogenous rebate and $\widehat{\tau}_t^o$ the endogenous offsetting tax component, both steady-state-GDP-normalized level deviations as in Table 1, so that $\widehat{tr}_t = \widehat{\tau}_t^o + \widehat{tr}_t^*$. Under the MF implementations, neither (10) nor (16) offsets the current rebate one for one within the period, so net taxes fall on impact. Under (16) the feedback component subsequently raises taxes through the dynamics of the consolidated liability; under unbacked MF with (10) the debt-feedback component remains zero, because debt is held fixed. The immediate-backing valuation closure instead pins down *net* taxes themselves, since (14) is the tax relation implied by the valuation condition:

$$\widehat{\tau}_t^o + \widehat{tr}_t^* = F_t \equiv \widehat{b}_{t-1} + b(\widehat{i}_{t-1} - \pi_t) + \frac{b(1-\beta)^2 + \chi\beta^2}{\beta}\widehat{l}_{t-1} + \widehat{g}_t - \beta\widehat{b}_t - \beta\chi\widehat{l}_t - \chi\beta\pi_t,$$

in which the rebate does not appear on the right-hand side. Hence $\widehat{\tau}_t^o = F_t - \widehat{tr}_t^*$: the endogenous component offsets the rebate one for one, *rising* by $-\widehat{tr}_t^* > 0$ within the period, so total net taxes remain $\widehat{tr}_t = F_t$, with no effect on output, consumption, inflation, or money growth.

This neutrality is a property of the *contemporaneous* offset built into (14), not a general implication of fiscal backing. The transversality condition (12) requires discounted consolidated liabilities to vanish asymptotically; neither it nor *present-value* backing requires the rebate to be offset

within the same period. Under the gradual-backing rule (16) the rebate is again effective, with a cumulative output effect close to the unbacked benchmark and no permanent net-wealth transfer.⁶

4.5 Sensitivity Analysis

Hereafter we return to the increase in government purchases. The sensitivity exercises in this subsection assess the government-purchases results across the bounded regimes (unbacked, gradually backed, and immediate-backing *MF*, and *DF*), focusing on price stickiness θ , shock persistence δ , and the balance-sheet ratio b/χ ; the gradual-backing rebate is analyzed separately in Section 4.4. Following Galí (2020), we report the $(1 - \delta)$ -weighted multiplier defined at the start of Section 4, plotted against θ in Panel (A) and against δ in Panel (B) of Figure 4, whose note describes the plotted lines and axes. The rightmost computed value is 0.975 in both panels.⁷

Two observations emphasized by Galí (2020) carry over to all three *MF* closures (Figure 4, Panel (A)). First, the multiplier is invariant to stickiness under the *DF* fiscal stimulus but increases with it under money financing, because the liquidity effect strengthens as prices become stickier.⁸ Second, the *MF* multiplier remains above the *DF* multiplier and converges to it only as prices become fully flexible. The increase is much weaker under immediate backing, however: at $\theta = 0.975$ the multiplier is 0.997 under immediate backing against 1.938 under the unbacked rule. The reason is that an increase in government purchases is then financed by lump-sum taxation, so that both money growth and the upward pressure on inflation are weaker, even though the real interest rate remains higher than under the unbacked *MF* regime throughout the interior of the plotted range. The gradually backed regime, by contrast, tracks the unbacked rule closely across the entire range of stickiness and persistence (both panels of Figure 4): at $\theta = 0.975$ its multiplier is 1.901 against 1.938 unbacked, and the two lines are visually indistinguishable elsewhere. The near-equivalence of Section 4.3 is thus not specific to the benchmark calibration.

The *DF* multiplier is likewise independent of persistence, as Galí (2020) also reports, whereas the three *MF* multipliers eventually decline as δ approaches one within the plotted stationary range (Figure 4, Panel (B)).⁹ The immediate-backing multipliers lie below the unbacked ones throughout, for the reason just given. They decline in persistence even though the government-purchases channel is shut down, because more persistent purchases sustain the upward pressure on inflation for longer, which lowers real money balances and raises the real interest rate through the nominal interest rate.

It is also natural to ask how sensitive the results are to the two parameters governing the

consolidated balance sheet: the target debt ratio b and the steady-state inverse velocity χ . The comparative statics can be read directly from (23): the strength of the interest-payment channel is governed by the coefficient $\frac{\rho b}{\chi(1-\beta)}$ on the real interest payment, and the contribution of inherited real balances by $\frac{b(1-\beta)}{\chi\beta}$. Both are strictly increasing in b , strictly decreasing in χ , and depend on the two parameters only through the ratio b/χ . Intuitively, a larger debt stock enlarges the interest-payment base that money injection accommodates, while a smaller monetary base translates a given financing need into a larger movement in money growth through (18). Because both terms scale with b/χ , their net effect cannot be signed from the coefficients alone, and the decomposition of Figure E.1 is computed at the benchmark rather than cell by cell. The numerical mapping below therefore reports the equilibrium outcome rather than a term-by-term comparison: the net effect remains expansionary relative to DF throughout the connected bounded-equilibrium region computed here, not as a global feature of the model.

Figure 5 maps the (b, χ) plane numerically, computing the cumulative output effect of the government-purchases stimulus under MF with immediate backing over $b \in [0.4, 4.8]$ and $\chi \in [0.1, 1]$; under DF the corresponding value is 0.25 throughout, the real allocation being independent of b and χ . The map delivers three results. First, as the coefficient analysis predicts, the cumulative output effect depends on the two parameters only through b/χ : cells with equal b/χ deliver identical values to the reported precision. Second, it is increasing in b/χ , from about 1.09 at low ratios to about 1.72 near the upper end of that region, and exceeds the DF value of 0.25 at *every* computed cell in the connected bounded region containing the benchmark, while displaying pronounced diminishing increases: moving from the lowest ratio to the benchmark 7.2 raises the interest-payment coefficient $\frac{\rho b}{\chi(1-\beta)}$ of (23) by more than an order of magnitude but raises cumulative output only from 1.09 to 1.48. The ranking of MF under immediate backing over DF is thus not a knife-edge feature of the calibration ($b = 2.4$, $\chi = 1/3$, ratio 7.2, comfortably interior). Third, the region itself has a boundary: as b/χ rises past approximately 15, a further eigenvalue crosses outside the unit circle and the immediate valuation-rule MF closure ceases to admit a bounded local equilibrium, a property of that particular closure, not of gradually backed money financing in general. That closure does admit bounded solutions again on a second, disconnected branch at $b/\chi \gtrsim 28.5$, but there the cumulative output effect is negative and hence below the DF value; the two results reported above are therefore stated for the connected region containing the benchmark, and Figure 5 marks the two excluded cases separately.¹⁰

The interpolation between the unbacked and valuation rules is reported in Appendix C. It

confirms that the *MF* cumulative output response remains above the *DF* response throughout the computed rule-weight grid, and that the response approaches the unbacked benchmark as the weight on the valuation rule falls.

5 The Effects of the Fiscal Stimulus in a Liquidity Trap

The interest-payment channel operates in normal times because movements in the government's real interest burden induce an endogenous money-growth path that, under sticky prices, affects inflation, real interest rates, and consumption. A liquidity trap is the natural place to press the question further: it is where Galí (2020) finds the case for money financing strongest, and it is also where the conventional policy rate can no longer deliver accommodation on its own, so that whatever accommodation the financing regime supplies must come from the money-growth path itself. If the interest-payment channel is what carries the effect, it should carry it here too. Following Galí (2020), this section therefore compares the effectiveness of the *MF* and *DF* fiscal stimuli in response to a temporary adverse demand shock. We assume that the adverse demand shock is large enough to prevent the central bank from fully stabilizing output and inflation because of the ZLB on the nominal interest rate. As in Galí (2020), the ZLB constraint takes the form $\hat{i}_t \geq \log \beta$, and the experiment assumes that $\hat{\rho}_t = -\gamma < \log \beta$ for $t = 0, 1, 2, \dots, T$ and $\hat{\rho}_t = 0$ for $t = T + 1, T + 2, \dots$, where $\gamma > 0$ is the size of the disturbance and T the last quarter in which it is active; both are calibrated below. Note that $\hat{\rho}_t$ appears in the consumption Euler equation, (2), so a decline in $\hat{\rho}_t$ puts downward pressure on consumption through an increase in the marginal utility of consumption. This process describes a temporary adverse demand shock that drives the natural interest rate into negative territory up to period T , after which it disappears. The shock is fully unanticipated; once realized, however, the path of $\{\hat{\rho}_t\}$ and the corresponding policy responses are known with certainty. We also consider a different adverse demand disturbance, namely a decline in the natural interest rate itself. To distinguish that case from the baseline experiment, we refer to $\hat{\rho}_t$ as the Z shock, since it is defined as $-\log \left(\frac{Z_{t+1}}{Z_t} \right)$ in Table 1 and is labeled that way in Figures 6 to 8 of Galí (2020); in what follows, an adverse demand shock means a decline in the Z shock.¹¹

The ZLB constraint is incorporated into the equilibrium conditions through the real money demand schedule: the nominal interest rate is bounded below, and the bound and the money-

demand inequality hold with complementary slackness,

$$\hat{i}_t \geq \log \beta, \quad \left(\hat{i}_t - \log \beta \right) \left(\hat{l}_t - \hat{c}_t + \eta \hat{l}_t \right) = 0,$$

for all t , where

$$\hat{l}_t \geq \hat{c}_t - \eta \hat{l}_t, \tag{24}$$

represents the demand for real money balances.¹² The inequality in (24) reflects an assumption we make explicit here: as in Galí (2020), liquidity services are locally satiated at the zero nominal rate, so real balances may exceed the level implied by the interior money-demand condition when the lower bound binds, without further reducing the nominal rate or altering the marginal valuation of money.

In addition to the previous changes, under the *DF* fiscal stimulus and *no response* benchmark, (19) must be replaced by

$$\left(\hat{i}_t - \log \beta \right) \pi_t = 0, \tag{25}$$

for all t , together with (19) for $t = T + 1, T + 2, \dots$. Equation (25) should be read as a *piecewise monetary policy rule* rather than as an equilibrium complementarity condition: the central bank sets $\pi_t = 0$ whenever the shadow nominal rate implied by strict inflation targeting lies above the lower bound, and sets $\hat{i}_t = \log \beta$ when the bound binds, inflation then being determined endogenously by the remaining equilibrium conditions. Which branch applies in each period during the shock is therefore endogenous; in the benchmark *DF* government-purchases experiment the bound binds for five of the six shock quarters, because the anticipated withdrawal of government purchases raises the shadow rate above the bound in the final quarter. Under the *MF* fiscal stimulus, by contrast, (18) determines the money supply for all t , and (24) holds with equality while the nominal rate is positive and with inequality once the ZLB is reached and real balances overshoot their satiation level. We assume $\gamma = 0.01$ and $T = 5$; with $\beta = 0.995$ this lowers the implied real rate from approximately 2 to approximately -2 percent annually for six quarters.

The government-purchases scenario is a 1% increase in the steady-state ratio to output, lasting for the duration of the decline in the Z shock ($\hat{g}_t = 0.01$, for $t = 0, 1, \dots, 5$), under both the *MF* and the *DF* fiscal stimulus.

5.1 *No Response*

We start with the case of *no response*, in which there is no fiscal stimulus in response to a decline in the Z shock ($\hat{g}_t = 0$ for all t). Monetary policy is described by (19) and (25) and fiscal policy by (10), and the responses are identical to those in the *no response* case in Galí (2020). The decline in the Z shock puts downward pressure on inflation, which raises the real interest payment on government bonds and so generates a revenue shortfall (a loss of inflation-tax revenue) that is financed by issuing bonds. Combined with the limited decline in the nominal interest rate imposed by the ZLB, the lower inflation raises the real interest rate. Inflation in period zero is -2.97% , and cumulative output and cumulative consumption are both -17.66 .

5.2 *DF Fiscal Stimulus*

The responses under the *DF* fiscal stimulus are similar both to those in Galí (2020) and to those in the *no response* case, although cumulative output improves because government purchases increase. In period zero, inflation is -1.82% ; cumulative consumption is -16.10 ; and cumulative output is -10.10 .

5.3 *MF Fiscal Stimulus*

Figure E.5 (Appendix E) shows the dynamic effects of an increase in government purchases under the *MF* fiscal stimulus in a liquidity trap, with the unbacked regime drawn as a solid line with circles and the immediate-backing regime as a dashed line with crosses; the variable-by-variable responses are described in the note to that figure, and we confine the text to the mechanism.

A decline in the Z shock reduces inflation through the consumption Euler equation, (2), the NKPC, (4), and the price-markup gap, (5). The lower inflation raises the real interest payment on government bonds and can therefore be read as a loss of inflation-tax revenue. Under the unbacked *MF* stimulus, the increase in government purchases enters the first term on the right-hand side of (21) and the higher real interest burden enters the second; both financing needs are accommodated through money creation. Under the immediate-backing *MF* stimulus, government purchases are themselves tax financed, so the direct government-purchases channel disappears. The higher real interest burden still induces positive money creation through the interest-payment component of (23). In this liquidity-trap experiment, real balances remain above steady state, so the inherited-real-balance component offsets part of that increase. Money growth therefore rises under both

rules, even though consumption is crowded out by more under immediate backing.

Both rules consequently mitigate the decline in inflation, though through different channels: the unbacked rule operates through the government-purchases and interest-payment channels together, whereas immediate backing removes the former and leaves the latter as the source of the positive money injection. Inflation in period zero is -0.58% under the unbacked and -1.40% under the immediate-backing *MF* stimulus, in both cases smaller in absolute value than under *no response* and the *DF* fiscal stimulus. Under both rules the nominal interest rate reaches the ZLB while the real interest rate turns negative; under immediate backing it does so during the remaining shock window after impact, having been positive in period zero, so the crowding-out of consumption is mitigated over most of the trap. Cumulative consumption is -13.58 under immediate backing, higher than under the *DF* fiscal stimulus, and output, after falling initially, recovers. Cumulative output is -2.69 under the unbacked rule and -7.58 under immediate backing: more negative under immediate backing than under the unbacked rule, but less negative than under *no response* and the *DF* fiscal stimulus.

5.4 Comparing the Effects of the *MF* Fiscal Stimulus with the *DF* Fiscal Stimulus in a Liquidity Trap

The top three panels of Figure 6 compare the effectiveness of the *MF* fiscal stimulus with that of the *DF* fiscal stimulus in a liquidity trap, using the line styles of the rest of the paper as described in the figure note. Under both *MF* regimes shown in Figure 6, the fiscal stimulus stabilizes output and inflation more effectively than under *DF*, and although immediate backing is weaker than the unbacked rule, it remains more effective than the *DF* fiscal stimulus.

The two-by-two design of Section 3.3 extends to the trap. Solving the gradually backed *DF* regime under the implementable rule (16) in the same experiment delivers a bounded equilibrium whose real allocation again coincides with the conventional *DF* allocation exactly: cumulative output is -10.10 , debt returns to its steady state, and the discounted consolidated liability converges to zero.¹³ The fiscal-rule contrast under debt financing is therefore zero in the liquidity trap as well. Proposition 2 is stated for strict inflation targeting, but its logic extends: conditional on the common ZLB monetary closure, the fiscal rule enters only the fiscal block. Solving money financing under the same ψ_q rule completes the trap two-by-two design, and the pattern of Section 4.3 recurs, with the financing-regime contrast again much larger than the within-column fiscal-rule contrast: at the common backed rule it is 7.18 percentage points in the Z-shock trap (cumulative

output -2.92 under the gradually backed MF stimulus against -10.10 under gradually backed DF), while the fiscal-rule contrast within each financing column remains small. Table 5 collects the cumulative output responses (Panel A) and the stabilization gains relative to *no response* (Panel B); the central ranking and the interest-payment mechanism carry over to the trap, with the immediate-backing benchmark preserving roughly one-third to three-fifths of the unbacked $MF-DF$ advantage depending on the disturbance.

Table 5: Cumulative Output across Experiments: Responses and Stabilization Gains

<i>Panel A: Unweighted cumulative output responses</i>					
Scenario	<i>DF</i> under (10) or (16)	<i>MF</i> unbacked	<i>MF</i> gradual	<i>MF</i> immediate	Share retained
Normal times	0.25	2.69	2.69	1.48	$\approx 1/2$
Z-shock trap	-10.10	-2.69	-2.92	-7.58	$\approx 1/3$
Natural-rate trap	-16.91	-4.22	-4.58	-9.36	$\approx 3/5$
<i>Panel B: Gain relative to no response (no-response cumulative output: -17.66 in both traps)</i>					
Z-shock trap	+7.56	+14.97	+14.74	+10.08	
Natural-rate trap	+0.75	+13.44	+13.08	+8.30	
"Share retained" is the fraction of the unbacked <i>MF</i> - <i>DF</i> advantage preserved by immediate backing.					

5.5 Another Scenario: A Decrease in the Natural Interest Rate

5.5.1 Derivation of the Natural Interest Rate

Galí (2020) interprets the Z -shock experiment as a decline in the natural interest rate. Because the model-implied natural rate also depends on the anticipated path of government purchases, we report an alternative experiment that fixes the natural-rate disturbance directly.¹⁴ According to Galí (2015), the natural interest rate is the real interest rate that would prevail in the flexible-price equilibrium.¹⁵ We therefore derive the natural interest rate directly from the model, following Galí and Monacelli (2005), as follows:

$$\bar{r}r_t = \hat{\rho}_t - \frac{\varphi + \alpha}{1 + \varphi} (\hat{g}_{t+1} - \hat{g}_t), \quad (26)$$

where $\bar{r}r_t$ denotes the natural interest rate.¹⁶ Under (26), the New Keynesian IS curve can be written as $\hat{x}_t = \hat{x}_{t+1} - \hat{i}_t + \pi_{t+1} + \bar{r}r_t$, which implies that the flexible-price allocation is attained when the real interest rate is aligned with the natural interest rate. Here $\hat{x}_t \equiv \hat{y}_t - \bar{y}_t$, where $\hat{x}_t$ and $\bar{y}_t$ denote the output gap and the logarithmic natural level of output, respectively. In this sense, (26) gives the natural interest rate relevant for our model.

Although the Z shock $\hat{\rho}_t$ is closely related to the natural interest rate $\bar{r}r_t$, identifying the two

mechanically understates how contractionary a decline in the natural rate defined by (26) would be when government purchases increase. From periods zero through four, purchases remain constant at one percent of steady-state output, so $\hat{g}_{t+1} - \hat{g}_t = 0$ and $\bar{r}\bar{r}_t = \hat{\rho}_t$. In period five, their anticipated withdrawal implies $\hat{g}_{t+1} - \hat{g}_t = -0.01$, so $\bar{r}\bar{r}_t = \hat{\rho}_t + \frac{\varphi+\alpha}{1+\varphi}\hat{g}_t$: the model-implied natural rate is then less negative than the Z shock, and the baseline experiment does not generate a full one-percent decline in it.

We therefore report a robustness experiment that holds the model-implied natural rate at $-\gamma$ throughout the trap, assuming $\bar{r}\bar{r}_t = -\gamma < \log\beta$ for $t = 0, 1, \dots, T$ and $\bar{r}\bar{r}_t = 0$ thereafter. The consumption Euler equation, (2), is then replaced by:

$$\hat{c}_t = \hat{c}_{t+1} - \hat{i}_t + \pi_{t+1} + \left[\frac{\varphi + \alpha}{1 + \varphi} (\hat{g}_{t+1} - \hat{g}_t) + \bar{r}\bar{r}_t \right], \quad (27)$$

which is obtained by plugging (26) into (2) (also using (3) and $v = 0$, so that $\hat{c}_t$ replaces $\hat{\xi}_t$). The expression in square brackets is the effective demand disturbance in the Euler equation, and because $\hat{g}_{t+1} - \hat{g}_t = -0.01$ at $t = 5$, the downward pressure on consumption is stronger under the natural-rate disturbance than under the Z shock.

5.5.2 Responses to a Decrease in the Natural Interest Rate

The bottom three panels of Figure 6 compare the immediate-backing and unbacked *MF* stimuli with the *DF* fiscal stimulus in the alternative liquidity-trap scenario described above. Holding the model-implied natural rate at $-\gamma$ (a per-quarter deviation of 0.01, about four percentage points at an annual rate) is more contractionary than the same-sized decline in the Z shock, so the economy remains more depressed under every financing scheme. Because the underlying shock differs, output levels compare the depth of the recession, not the effectiveness of the stimulus; the proper metric is the stabilization gain relative to the no-response benchmark, whose cumulative output is -17.66 under *both* disturbances (absent a fiscal response the two shocks coincide). Read this way (Table 5, Panel B), debt financing loses almost all of its stabilizing power under the natural-rate disturbance (its gain falls from $+7.56$ in the Z-shock trap to $+0.75$), whereas money financing loses comparatively little, its gain slipping only modestly across the two disturbances. The immediate-backing regime accordingly preserves a larger share of the unbacked *MF* advantage over *DF* than in the baseline: about three-fifths (7.55 of 12.69 percentage points of cumulative output) under the natural-rate disturbance, against about one-third (2.52 of 7.41) under the Z

shock.

The immediate-backing MF stimulus is relatively more effective under the natural-rate disturbance because the DF fiscal stimulus becomes markedly less effective there. The mechanism can be read off the variable-by-variable responses in Appendix E, Figures E.6–E.8, which report the unbacked MF , immediate-backing MF , and DF regimes, respectively, with the solid lines with squares denoting the Z-shock experiment and the dashed lines with triangles the natural-rate experiment. The natural-rate disturbance depresses consumption more strongly in period five, as (27) shows, so consumption and inflation fall more in all three regimes (Panels 2 and 4 of Figures E.6–E.8; the inflation comparison is summarized in Panels 3 and 6 of Figure 6). Under DF the larger decline in inflation sharply raises the real interest rate and deepens the crowding-out of consumption and output, whereas under MF stronger money growth cushions that increase (Panels 3 and 7 of Figures E.6–E.8). Output therefore deteriorates much more under DF (Panel 1 of those figures), which is why immediate backing preserves a larger share of the MF advantage here; as in the baseline, it does so without a permanent net-wealth transfer, so such a transfer is not essential under a natural-rate disturbance either.

Repeating the common-rule experiment under (16) yields a cumulative output response of -4.58 under MF and -16.91 under DF . Both gradually backed regimes remain bounded, and the DF real allocation again coincides with its conventional counterpart, so the trap version of the two-by-two design of Section 4.3 carries over to the natural-rate experiment as well.

6 Institutional Interpretation and Scope

Before concluding, we draw out the institutional content of the analysis, address the plausibility of its central assumption, and state its scope conditions.

6.1 Policy Regimes, the FTPL, and Institutional Realism

It is useful to place the policy regimes in the active/passive language of Leeper (1991); three points suffice. First, the conventional DF benchmark is the canonical active-monetary/passive-fiscal configuration: monetary policy is active on inflation, (19), and fiscal policy is passive through the unbacked Bohn rule, (10). Second, under the MF fiscal stimulus money growth passively meets the consolidated financing need, (18); this is *monetary accommodation* rather than “active money” in Leeper’s sense, since the money supply is set to satisfy the government’s budget rather

than to determine the price level. The three *MF* variants share that closure and differ only in their treatment of consolidated liabilities. Third, (14) is a valuation closure rather than a conventional feedback rule and does not map one-for-one into the binary taxonomy: under money financing it validates the *entire* consolidated liability, base money included, through the implied tax sequence, whereas under debt financing with endogenous debt it admits no bounded equilibrium. We therefore treat it as a separate case, with Galí’s benchmark as the polar alternative in which base money is excluded from the solvency condition and remains unbacked net wealth.

Two qualifications about realism attach to this framing. Treating money as fully backed by future surpluses imposes fiscal credibility by assumption: it presumes that the treasury will run the surpluses needed to service the consolidated balance sheet. Whether that is institutionally plausible is the question studied in the literature on the fiscal backing of central banks (Del Negro and Sims (2015), Hall and Reis (2015), and Bassetto and Sargent (2020)), in which the treasury stands behind the central bank’s balance sheet through remittances and indemnities. Credibility, moreover, is rarely perfect. As a reduced-form illustration (Appendix C), a lower weight on the immediate-backing rule shifts outcomes toward Galí’s benchmark without reversing the ranking of *MF* over *DF* for government purchases; within that rule family, however, the response is highly nonlinear near the full-backing endpoint, so even a modest weight on the unbacked rule moves it back toward Galí’s benchmark (Figure E.9). If the public also doubts the central bank’s commitment to the financing path itself, the effectiveness of money financing can erode more sharply, as English et al. (2017) emphasize. Our results therefore characterize the polar, fully credible arrangement; the appendix exercise is an illustration, not a structural prediction.

6.2 What Valuation-Backed Money Finance Means in Practice

One possible institutional interpretation of valuation-backed money finance is a coordinated fiscal–monetary arrangement in which the financing closure generates an endogenous path of base-money adjustment. Under the immediate closure, that path is state dependent. In the normal-times government-purchases experiment, money contracts on impact and expands subsequently as the interest-payment channel operates. In the liquidity-trap experiment, the larger real interest burden instead induces positive money growth already on impact. In both cases, the treasury credibly backs the consolidated balance sheet through future surpluses. Money growth rather than the policy rate absorbs the financing need over the episode, before returning to its steady-state rate, so that the expansion is a temporary provision of liquidity rather than a permanent transfer of net

wealth. The model does not specify the legal instrument, the asset side of the central-bank balance sheet, or the division of responsibilities between the two authorities. Nor is the base-money path an instrument: it is the endogenous outcome of the financing closure, so the arrangement described here is an institutional reading of that path rather than a policy rule. The arrangement differs from conventional quantitative easing based on asset purchases and reserve creation, and it is deliberately distinct from “helicopter money” in the sense of a permanent, unbacked monetization that the private sector perceives as net wealth, which is the arrangement that prohibitions on monetary financing are typically intended to limit. The central message is that a substantial share of the expansionary effect of money financing for government purchases survives without permanent unbacked monetization, because it comes from temporary monetary accommodation under sticky prices. That share is roughly half of its advantage over debt financing in normal times, and between one-third and three-fifths in the liquidity-trap experiments (Table 5). Helicopter-money proposals that instead rely on the wealth-transfer mechanism ask for exactly that permanently unbacked ingredient.

This reading needs two clarifications. First, *valuation-backed money finance* names only the operational arrangement just described; it is not a claim about the legality of any particular operation in any particular jurisdiction, which turns on statutes and institutions (central-bank capital, reserve remuneration, government guarantees) that the model does not contain. Second, the money in our model is non-interest-bearing base money, and the interest-payment channel rests on the spread between the return on bonds and the zero return on money. Where reserves are remunerated at close to market rates, that spread, and with it the channel, shrinks, as van der Kwaak (2026) emphasizes. Our policy reading therefore applies most directly to cash or non-interest-bearing reserves; extending it to remunerated-reserve regimes, by introducing a money interest rate so that the opportunity cost of holding money is the spread, is an important next step.

6.3 Scope and External Validity

Several simplifications delimit the scope of our conclusions, and each points to a direction for further work. Taxes are lump sum, which keeps the conventional debt-financing allocation Ricardian and the fiscal decomposition transparent; with distortionary taxation the interest-payment channel may remain operative, but whether its expansionary effects continue to outweigh the additional distortions created by taxation is an open quantitative question. The analysis also abstracts

from distributional effects, though the contrast is instructive rather than merely a limitation, because a wealth transfer through irredeemable money and a tax-backed money injection have very different incidence. Finally, we take the fiscal backing as politically feasible, whereas in practice the willingness to run the requisite surpluses is itself a political variable, which is the sense in which, as Turner (2015) stresses, monetary finance is ultimately a political question. The calibration follows Galí (2020), and Section 4 shows that the ranking of immediate-backing MF over DF is preserved throughout the connected bounded-equilibrium region considered for the debt ratio and the inverse velocity, the two parameters most directly tied to the consolidated balance sheet.

7 Conclusion

Does money financing require irredeemable money? For government purchases, no. Once base money is included in the consolidated government's intertemporal valuation condition, the direct spending-finance term disappears, but the *interest-payment channel* continues to generate temporary monetary accommodation. The unweighted cumulative output response is 1.48, compared with 0.25 under debt financing and 2.69 in Galí's unbacked benchmark, and the advantage survives with no permanent net-wealth transfer. It is monetary accommodation, not the treatment of money as net wealth, that carries the effect: irredeemability is neither necessary nor the propagation mechanism, though its fiscal treatment still shapes the magnitude of the response by changing the endogenous monetary path. Because the immediate and gradual cases employ distinct fiscal closures, their contrast establishes that present-value backing itself does not eliminate the stimulus; it does not identify a pure effect of adjustment timing.

The valuation rule also reveals an implementability constraint that is a result in its own right. Its debt-financed analogue admits no bounded equilibrium: with strict inflation targeting and endogenous debt, government debt inherits an explosive root equal to the inverse discount factor, so the valuation equation is a pricing relation, not a stabilizing fiscal rule, a divergence invisible over a standard impulse-response window but decisive asymptotically. A matched-accommodation result then shows that, within the maintained model and wherever a bounded implementation exists, the real effects are mediated entirely by the induced money-growth path rather than by the composition of fiscal liabilities. Accordingly, under the maintained assumptions of strict inflation targeting, lump-sum taxation, and separable real balances, switching from the debt-only rule (10)

to the consolidated-liability rule (16) within the implementable two-by-two benchmark changes no non-fiscal variable under debt financing.

An auxiliary consolidated-liability feedback rule confirms that broader, gradually imposed present-value backing does not overturn the government-purchases result, and the central findings carry over to a liquidity trap. The rebate experiment delimits rather than extends this reading: neutrality under the valuation rule is specific to its contemporaneous offset and should not be read as a general consequence of backing.

The assumptions that mark the way forward are those set out in Section 6.3, together with two additional features of the model and the valuation closure: the money in the model is non-interest-bearing base money, so the interest-payment channel lives on the spread between bonds and money and shrinks where reserves are remunerated at market rates, and the immediate-backing valuation closure ceases to admit a bounded local equilibrium over an interval of high debt-to-base ratios, above which bounded solutions reappear only on a branch disconnected from the benchmark, along which money financing is contractionary. Quantifying the channel under remunerated reserves, endogenizing the fiscal backing as a function of credibility, and conducting a full welfare and policy-design analysis of alternative future-tax paths, for which (16) serves here only as an implementability and scope benchmark, are natural next steps. Much of the stimulus that helicopter money promises does not require the helicopter’s cargo to be a gift; a substantial part of it comes from the flight—the monetary accommodation that the financing regime induces.

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Competing interests

The authors declare none.

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Appendix

A Equilibrium Existence and Determinacy under the Fiscal Closures

This appendix collects the equilibrium-existence and determinacy results used in the main text. Subsection A.1 establishes nonexistence under the immediate-backing valuation rule (14) with debt financing. Subsection A.2 locates the balance-sheet boundaries of the same rule under money financing, which delimit the map of Figure 5. Subsection A.3 characterizes numerically the determinacy region of the gradual-backing rule (16) under money financing.

A.1 Nonexistence under Debt Financing

This subsection substantiates the claim in Section 3.3.1 that the valuation-rule DF cell is not a well-defined bounded equilibrium under the maintained closure. Consider the immediate-backing valuation closure, (14), combined with the debt-financing monetary closure of strict inflation targeting, (19), and with government debt left endogenous (that is, without imposing $\hat{b}_t = 0$). Substituting (14) into the flow budget constraint (8) and using the definition of real balances (7) yields a difference equation for the interest-bearing government debt component. Once the fiscal and non-fiscal disturbances have vanished, so that $\hat{i}_t = \hat{l}_t = \pi_t = 0$, that difference equation reduces to

$$\hat{b}_t = \beta^{-1} \hat{b}_{t-1}. \quad (\text{A.1})$$

Because $\beta = (1 + \rho)^{-1} < 1$, the root $\beta^{-1} = 1 + \rho > 1$ is explosive. Any non-zero inherited debt deviation $\hat{b}_{t-1}$ therefore diverges, so the discounted liability $\Lambda_{t,t+k} (\mathcal{B}_{t+k} + L_{t+k})$ does not converge to zero and the transversality condition (12) is violated, unless the post-stimulus debt position is exactly zero, a nongeneric knife-edge case that is not generated by the fiscal experiment considered here. Hence, under strict inflation targeting with endogenous debt, the valuation rule (14) does not support a bounded equilibrium: the valuation-rule DF regime is not well defined under this closure.

The economic content of (A.1) is that (14) is a forward-looking valuation relation rather than a debt-stabilizing feedback rule. Under money financing it is closed by fixing $\hat{b}_t = 0$, so the explosive root never operates; under debt financing debt is endogenous and unpinned, so it does.

We verify the unstable law of motion numerically by forward-simulating the linearized system for 2,100 quarters. This exercise is a diagnostic of the explosive root, not a bounded perfect-foresight equilibrium, and the long horizon should be read as a diagnostic on the linearized law of motion rather than as an economic prediction. The discounted-liability statistic $Q_t \equiv \beta^t(\hat{b}_t + \chi \hat{l}_t)$ is the first-order counterpart of the level TVC, obtained by discounting the linearized consolidated liability at the steady-state real rate. Under the valuation rule with strict inflation targeting, the short-run paths are computable and the real allocation coincides with the conventional DF allocation exactly, but debt grows without bound: it reaches 2.54 by quarter 30, 26.8 by quarter 500, and 8.1×10^4 by quarter 2,100, with a measured gross growth rate of 1.005025 per quarter, equal to β^{-1} to six decimal places. Correspondingly, the discounted consolidated liability $Q_t \equiv \beta^t(\hat{b}_t + \chi \hat{l}_t)$ converges not to zero but to 2.197, so the transversality condition (12) fails (Figure 1). The generalized-eigenvalue calculation confirms an explosive root of modulus 1.005 attached to a predetermined state, and the Blanchard–Kahn rank condition is not satisfied.¹⁷ A truncated impulse-response window is misleading here: over the first 32 quarters debt rises by only about 17% in total, so the path can appear stable even though it diverges asymptotically at rate β^{-1} .

A bounded and *implementable* gradually backed DF regime is obtained by replacing (14) with the consolidated-liability feedback rule (16), $\hat{r}_t = \psi_q q_{t-1} + \hat{r}_t^*$ with $q_t \equiv \hat{b}_t + \chi \hat{l}_t$, which stabilizes the consolidated liability directly while still backing base money by future surpluses. With the benchmark value $\psi_q = 0.02$, the consolidated-liability feedback rule stabilizes the DF regime and satisfies the Blanchard–Kahn and transversality conditions: debt peaks and returns to steady state, $Q_t \rightarrow 0$, and the real allocation again coincides with the conventional DF allocation to machine precision. This regime completes the two-by-two design of Table 4 and underlies the decomposition reported in Section 4.3.

A.2 Balance-Sheet Boundaries under Money Financing

The valuation closure also has existence boundaries under *money* financing, in the balance-sheet ratio b/χ , and these bound the map of Figure 5. Under money financing the real debt stock is held constant, $\hat{b}_t = 0$, and (18) is precisely that restriction imposed on (8). Solved with $\hat{b}_t$ nonetheless retained as a variable, the system carries the debt block (A.1) as a decoupled scalar equation

that does not feed back into any other equation, so it contributes the explosive root β^{-1} to the eigenvalue count without affecting the allocation. We therefore impose $\hat{b}_t = \hat{b}_{t-1} = 0$ directly and conduct the eigenvalue analysis on the reduced system that determines the allocation. The moduli of the remaining roots are unchanged by this reduction, but the counts become directly comparable with the number of non-predetermined variables, which is two: the marginal utility of consumption $\hat{\xi}_t$ and the inflation rate π_t , the only two variables entering (1)–(8) with a $t + 1$ dating.

At the benchmark calibration exactly two eigenvalues of the reduced system lie outside the unit circle, so the Blanchard–Kahn rank condition is satisfied and the *MF* equilibrium is locally determinate. Repeating the count cell by cell over the grid that generates Figure 5, it is invariant at two throughout the connected region containing the benchmark. The borderline root is real and negative, and its modulus rises from 0.956 at $b/\chi = 14.667$ to 0.998 at 14.96 and 1.001 at 14.98; a third explosive root (one more than the number of non-predetermined variables) therefore appears between $b/\chi = 14.974$ and 14.976, so no bounded local equilibrium exists over the intermediate interval until the root count returns to two. The grid underlying Figure 5 brackets the crossing between its last admissible cells, at $b/\chi = 14.667$, and its first inadmissible ones, at 16.

The count returns to two, with the rank condition again verified, between $b/\chi = 28.52$ and 28.55, where a second real root re-enters the unit circle: those cells do admit a bounded solution, but on a branch disconnected from the benchmark along which the cumulative output effect is negative, at -0.76 for $b/\chi = 32$ and -0.11 for $b/\chi = 48$, hence below the *DF* value throughout the computed disconnected branch. The claims in Section 4 are restricted to the connected region accordingly, and Figure 5 distinguishes the two excluded cases.

A.3 Determinacy of the Gradual-Backing Rule under Money Financing

Applying the reduced-system procedure of Subsection A.2 to the consolidated-liability rule (16), the number of unstable roots is two at $\psi_q = 0.1434$ and one at $\psi_q = 0.1435$. Because the reduced *MF* system contains two non-predetermined variables, the connected determinate region containing the benchmark ends at $\bar{\psi}_q \in (0.1434, 0.1435)$, which we summarize as $\bar{\psi}_q \simeq 0.1434$. The count is invariant at two at every value examined in $0 < \psi_q \leq 0.1434$, including the benchmark $\psi_q = 0.02$, and equals one at every value examined over $0.1435 \leq \psi_q \leq 1.076$.

This upper endpoint is of a different nature from the debt-financing bounds of Section 3.2.4, which follow analytically from (17). Table A.1 reports the *DF* liability root together with the

equilibrium status of each closure at four values of ψ_q , under the benchmark $\beta = 0.995$ of Table 3, i.e., $\rho \approx 0.0050$. The root $1 + \rho - \psi_q$ remains comfortably inside the unit circle at $\psi_q = 0.150$, so the loss of *MF* determinacy there is *not* generated by the consolidated-liability recursion. It arises instead from a different eigenvalue of the full system, which under money financing crosses from outside to inside the unit circle as ψ_q rises past $\bar{\psi}_q$: because base money is endogenous under *MF*, tightening the tax response to the consolidated liability feeds back on the money-growth path through (7) and (8) rather than on debt alone. Just above $\bar{\psi}_q$ the number of unstable roots is one smaller than the number of forward-looking variables, so the model is indeterminate and admits a continuum of bounded equilibria. Under debt financing, by contrast, strict inflation targeting (19) fixes the real allocation independently of the fiscal rule under the maintained calibration $v = 0$ and lump-sum taxation. Over the benchmark-relevant range shown in Table A.1, the analytical condition $\rho < \psi_q < 2 + \rho$ is satisfied throughout, so no analogous *DF* boundary is encountered near the first *MF* threshold.

Table A.1: Consolidated-Liability Root and Equilibrium Status Under (16)

ψ_q	<i>DF</i> liability root $1 + \rho - \psi_q$	<i>DF</i> root status	<i>MF</i> equilibrium status	<i>DF</i> equilibrium status
0.004	1.0010	explosive	determinate	no bounded equilibrium
0.020	0.9850	stable	determinate (benchmark)	determinate
0.140	0.8650	stable	determinate	determinate
0.150	0.8550	stable	indeterminate	determinate

The second and third columns report the *DF* liability root $1 + \rho - \psi_q$ and its stability status, respectively. This scalar reduction of (17), $q_t = (1 + \rho - \psi_q) q_{t-1}$, applies under *DF* only. Under *MF* the consolidated liability satisfies $q_t = \chi \hat{l}_t$, so no autonomous recursion in q_t alone exists; the *MF* status is obtained from numerical root counts using the reduced-system procedure of Subsection A.2.

Extending the sweep to much larger feedback coefficients, the count returns to two: a second determinate region appears between $\psi_q = 1.076$ and 1.078, and determinacy persists, with the rank condition again verified, at every larger value examined up to 100. The mechanism parallels the upper endpoint of the *DF* interval in Section 3.2.4: the eigenvalue that enters the unit circle at $\bar{\psi}_q$ becomes negative as ψ_q rises further and exits again through -1 , restoring the number of unstable roots required by the Blanchard–Kahn condition. Over the range examined the determinate set is therefore not connected, and we do not claim otherwise. This second region begins at a feedback coefficient roughly seven and a half times larger than $\bar{\psi}_q$ and lies far outside the values used anywhere in the paper; as with the disconnected branch of Subsection A.2, the claims in Section 4 are restricted to the connected region $0 < \psi_q < \bar{\psi}_q$ accordingly, and the benchmark $\psi_q = 0.02$

lies well inside it. We do not claim an analytic characterization of $\bar{\psi}_q$; it is a numerical property of the benchmark calibration, established over the range of ψ_q examined here, as is the boundary of the connected bounded-equilibrium region in the b - χ map of Figure 5.

B Derivation of the Immediate-Backing Valuation Closure

This appendix collects the intermediate algebra that takes the intertemporal valuation relation (13) to the immediate-backing valuation closure (14) of Section 3.2.2.

Equation (13) can be rewritten as:

$$\begin{aligned} \frac{U_{c,t} Z_t (1 + i_{t-1}) (B_{t-1} + M_{t-1})}{P_t} &= \sum_{j=0}^{\infty} \beta^j U_{c,t+j} Z_{t+j} S P_{t+j} \\ &+ \sum_{j=0}^{\infty} \beta^{j-1} U_{c,t+j-1} Z_{t+j-1} \left(\frac{i_{t+j-1}}{1 + i_{t+j-1}} \right) L_{t+j-1}. \end{aligned} \quad (\text{B.1})$$

According to Cochrane (2005), the FTPL treats nominal debt, including the monetary base, as a residual claim on government primary surpluses. If the present value of primary surpluses falls short of the outstanding nominal liabilities, valuation balance is restored through an adjustment in the price level rather than being guaranteed by assumption. In the standard FTPL interpretation, the price level is determined by the valuation equation for nominal government liabilities:

$$\frac{\text{Nominal Government Debt}}{\text{Price Level}} = \text{Expected Present Value of Primary Surpluses.}$$

Equation (B.1) is analogous to this valuation equation; accordingly, (13) inherits the valuation structure of Cochrane's (2005) FTPL.

Equation (13) can be rewritten as the following second-order difference equation:

$$\begin{aligned} U_{c,t} Z_t S P_t + \beta^{-1} U_{c,t-1} Z_{t-1} \frac{i_{t-1}}{1 + i_{t-1}} L_{t-1} &= U_{c,t} Z_t (1 + i_{t-1}) (B_{t-1} + L_{t-1}) \Pi_t^{-1} \\ &- \beta U_{c,t+1} Z_{t+1} (1 + i_t) (B_t + L_t) \Pi_{t+1}^{-1}. \end{aligned}$$

In the previous expression, the left-hand side is the consolidated government's revenue in units of the marginal utility of consumption. The first term on the right-hand side represents the real servicing burden on inherited consolidated liabilities. The second represents the rollover or new issuance of consolidated liabilities.

Using the Euler equation $\beta \frac{U_{c,t+1} Z_{t+1}}{U_{c,t} Z_t} = \frac{1}{1+i_t} \Pi_{t+1}$, the previous expression can be simplified as:

$$1 + i_{t-1} = -\Pi_t \frac{SP_t}{L_{t-1}} + (1 + i_{t-1}) \left(\frac{\mathcal{B}_{t-1}}{L_{t-1}} + 1 \right) - \Pi_t \left(\frac{\mathcal{B}_t}{L_{t-1}} + 1 \right) + 1, \quad (\text{B.2})$$

where the primary balance is moved to the right-hand side. Equation (B.2) isolates the imputed gross nominal return to liquidity services, that is, the return generated by supplying one unit of real money balances. This return can offset a primary deficit, for instance one created by a rebate, or reduce the resources needed to service inherited consolidated liabilities, including the monetary base. Any remaining shortfall is met through new liability issuance or through an adjustment in the price level, as in Cochrane (2005). We exploit this property of the gross nominal return on liquidity services to derive our immediate valuation closure. In other words, we derive the fiscal policy rule by focusing on seigniorage in a narrow sense, namely the liquidity-service return from the provision of liquidity services. We distinguish that narrow notion from the seigniorage term in Galí (2020), $\frac{\Delta M_t}{P_t}$, which is the broader concept commonly used in macroeconomics. In this paper, unless otherwise noted, “seigniorage” refers to the latter concept.

The previous expression can be log-linearized as:

$$\hat{i}_{t-1} + \frac{b(1-\beta)}{\chi\beta} \hat{sp}_t = \frac{b+\chi}{\chi} \hat{i}_{t-1} + \frac{1}{\chi} \hat{b}_{t-1} + \frac{b(1-\beta)^2 + \chi\beta^2}{\chi\beta} \hat{l}_{t-1} - \frac{\beta}{\chi} \hat{b}_t - \beta \hat{l}_t - \frac{b+\chi\beta}{\chi} \pi_t,$$

where $\chi \equiv \frac{L}{Y}$ and $b \equiv \frac{B}{Y}$ are the steady-state inverse velocity and the target debt ratio, respectively. In the previous expression, (the percentage deviation of) the primary balance is moved to the left-hand side. Therefore, the left-hand side is the consolidated government’s (logarithmic) revenue, which consists of the liquidity-service return from providing liquidity services and the primary balance. The first three terms on the right-hand side represent the resources required to service inherited government bonds and real balances. The fourth and fifth terms represent the rollover or new issuance of government bonds and real balances. The sixth term captures the effect of inflation on the real value of outstanding liabilities. Positive inflation lowers the resources required on the right-hand side; equivalently, when moved to the revenue side, it represents the inflation-tax contribution. In that sense, the term can be interpreted as the remaining trace of the possibility of ‘inflating away’ liabilities in Cochrane (2005).

The previous expression can be rewritten as:

$$\frac{b(1-\beta)}{\chi\beta} \hat{sp}_t = \frac{b}{\chi} \hat{i}_{t-1} + \frac{1}{\chi} \hat{b}_{t-1} + \frac{b(1-\beta)^2 + \chi\beta^2}{\chi\beta} \hat{l}_{t-1} - \frac{\beta}{\chi} \hat{b}_t - \beta \hat{l}_t - \frac{b+\chi\beta}{\chi} \pi_t, \quad (\text{B.3})$$

in which the imputed liquidity-service return is moved from the revenue side, and the coefficient on the first term, the nominal interest rate, decreases from $\frac{b}{\chi} + 1$ to $\frac{b}{\chi}$. This suggests that this liquidity-service return mitigates the burden of financing interest payments: the imputed liquidity-service return offsets part, but not all, of the interest burden on government bonds. This feature is important for making the *MF* fiscal stimulus operative under the immediate-backing valuation rule.

Plugging the logarithmic definition of the primary balance $\hat{p}_t = \frac{\beta}{b(1-\beta)}\hat{tr}_t - \frac{\beta}{b(1-\beta)}\hat{g}_t$ into the left-hand side in (B.3) yields the immediate-backing valuation closure, (14) of the main text.

Finally, the financing of the mitigated interest payment used in Section 4.2 follows from the same relations. Subtracting the valuation closure (14) from the flow budget constraint (8) yields

$$\rho b(\hat{i}_{t-1} - \pi_t) = (1 - \beta)\hat{b}_t - \rho\hat{b}_{t-1} + \frac{b(1-\beta)^2}{\beta}\hat{l}_{t-1} + \chi(1 - \beta)\Delta m_t.$$

Imposing $\hat{b}_t = 0$ leaves $\rho b(\hat{i}_{t-1} - \pi_t) = \frac{b(1-\beta)^2}{\beta}\hat{l}_{t-1} + \chi(1 - \beta)\Delta m_t$, so the residual interest burden $\rho b(\hat{i}_{t-1} - \pi_t)$ is absorbed jointly by inherited real balances and current money creation, which rearranges to the second line of (23).

C A Reduced-Form Interpolation between the Two Fiscal Rules

The two polar fiscal rules of the main text, the unbacked rule (10) (full irredeemability) and the valuation rule (14) (no irredeemability), bracket a one-parameter family of intermediate rules. To illustrate whether equilibrium responses must change discontinuously between the two rule endpoints, we map a reduced-form interpolation between them and show that the government-purchases ranking is preserved along it. Consider the one-parameter family of fiscal rules

$$\hat{tr}_t(\lambda) = \lambda \hat{tr}_t^{\text{noIM}} + (1 - \lambda) \hat{tr}_t^{\text{IM}}, \quad \lambda \in [0, 1], \quad (\text{C.1})$$

where $\hat{tr}_t^{\text{noIM}}$ is the right-hand side of (14) and $\hat{tr}_t^{\text{IM}} = \psi_b \hat{b}_{t-1}$ is the right-hand side of (10). The parameter λ is a reduced-form rule weight that interpolates between the two polar fiscal rules. It should not be interpreted as a literal share of liabilities that is fiscally backed: we do not derive it from a partial-backing transversality condition, and the model contains no default probability

or belief structure. At $\lambda = 1$ the valuation rule applies and the IM is denied; at $\lambda = 0$ we recover Galí's benchmark and the IM holds. Because (C.1) enters the linear equilibrium system through coefficients that depend continuously on λ , the equilibrium responses vary continuously in λ wherever the equilibrium is determinate. Continuity alone does not imply that the responses lie between the two polar outcomes; whether they do is a numerical question.

Figure E.9 computes the cumulative output effect of the government-purchases stimulus on a numerical grid spanning the entire interval. The resulting profile is continuous but sharply nonlinear, and it lies between the two polar outcomes, 2.69 and 1.48, at every value of λ evaluated. The cumulative output effect is essentially flat near Galí's benchmark (at $\lambda = 0.5$, with equal weights on the two polar rules, cumulative output is 2.68, barely below the benchmark 2.69) and it remains at 2.63 even at $\lambda = 0.9$. Only near the full-backing endpoint does the response converge sharply toward the valuation-rule outcome: cumulative output is 2.28 at $\lambda = 0.99$, 1.68 at $\lambda = 0.999$, and 1.48 at $\lambda = 1$. In this reduced-form rule family, even a small weight on the unbacked rule generates a highly persistent component in the simulated money path, and the cumulative effect therefore declines steeply only near the full-backing endpoint. Two lessons follow. The response is highly nonlinear in the rule weight: even a ten-percent weight on the unbacked rule moves the response close to Galí's benchmark, so the transition to the valuation-rule outcome is concentrated at the full-backing boundary, which is why we model that outcome as an explicit fiscal rule rather than as an approximation. More important for the paper, a lower weight on the valuation rule moves the response toward the unbacked benchmark without overturning the ranking over DF : cumulative output exceeds the debt-financed value of 0.25 throughout the numerical grid. This interpolation changes the magnitude of the stimulus, not the direction of the comparison.

D A Counterfactual Rebate Exercise

This appendix retains the original counterfactual rebate experiment as a diagnostic of the valuation closure (14). It is not a separate policy regime, and it is not used to establish a general result about fiscal-adjustment timing. Its purpose is narrower: to determine whether the rebate neutrality of Section 4.4 follows from present-value backing itself or from the contemporaneous offset embedded in (14).

Using Galí's (2020) rebate-shock profile, we insert an exogenous rebate component u_t^A into the right-hand side of (14), so that, unlike in Section 4.4, the rebate is *added to* the net taxes implied

by the valuation closure rather than absorbed by the endogenous component:

$$\hat{tr}_t = \hat{b}i_{t-1} + \hat{b}_{t-1} + \frac{b(1-\beta)^2 + \chi\beta^2}{\beta}\hat{l}_{t-1} - \beta\hat{b}_t - \beta\chi\hat{l}_t - (b + \chi\beta)\pi_t + \hat{g}_t + \omega u_t^A, \quad (\text{D.1})$$

where $\omega > 0$ scales the exogenous rebate component and $u_t^A = -\delta^t$. We calibrate $\omega = 0.005$ numerically, so that the realized impact decline in net taxes equals one percent of steady-state output.¹⁸ Figure E.4 reports the responses under the inserted-rebate valuation rule (dashed line with crosses) and the unbacked rule (solid line with circles): the two are nearly identical, and in particular the paths of output, real money balances, and money growth under the inserted-rebate case almost coincide with the unbacked benchmark (Panels 1, 7, and 8, Figure E.4). The rebate thus remains highly expansionary, in contrast to the exact neutrality under the immediate-backing valuation rule itself (Panel 1, Figure E.3). Present-value backing is nonetheless preserved: with debt fixed at $\hat{b}_t = 0$ under money financing, the discounted consolidated liability $Q_t \equiv \beta^t(\hat{b}_t + \chi\hat{l}_t)$ falls monotonically from 0.44 at impact to below 10^{-5} within twenty quarters, so no discounted residual consolidated liability remains in the limit, in contrast to the DF valuation-rule case of Appendix A, where Q_t converges to 2.197.

Removing the contemporaneous offset therefore restores a temporary monetary path resembling Galí's unbacked benchmark, even though the consolidated transversality condition (12) remains satisfied. The exercise diagnoses the source of neutrality under (14): what eliminates the rebate effect is the within-period reversal of the money injection, not the absence of a permanent net-wealth transfer. It is an ad hoc, less-structured counterpart of the gradual-backing rule (16) of Section 4.3, not a reintroduction of irredeemability; as Section 4.4 shows, an implementable rule with gradual but complete backing sustains a similar temporary monetary expansion without a permanent net-wealth transfer.

E Additional Dynamic Results and Robustness Figures

This appendix collects the complete impulse responses and diagnostic figures that support the main-text results. The figures are referenced in the relevant sections of the paper, while the main figures retain only the headline comparisons needed to follow the central argument. They are grouped by topic below, numbered E.1–E.10, and gathered after the main figures at the end of the paper.

The Interest-Payment Channel

Figure E.1 decomposes the equilibrium money growth under immediate backing into its interest-payment and inherited-real-balance components (Section 4.2).

The Gradual-Backing Benchmark

Figure E.2 reports the fiscal-adjustment speed and cumulative output under gradual full backing (Section 4.3).

Rebate Experiments

Figure E.3 reports the lump-sum rebate impulse responses under the *MF* fiscal stimulus in normal times (Section 4.4), and Figure E.4 the counterfactual inserted-rebate exercise of Appendix D.

Liquidity-Trap Dynamics

Figure E.5 reports the government-purchases responses under the unbacked and immediate-backing *MF* regimes in the liquidity trap, and Figures E.6–E.8 the complete variable-by-variable responses under the unbacked *MF*, immediate-backing *MF*, and *DF* regimes for the Z-shock and natural-rate experiments (Section 5).

Reduced-Form Rule Interpolation

Figure E.9 maps the cumulative output response of the government-purchases stimulus across the reduced-form interpolation between the two fiscal rules (Appendix C).

Fiscal Diagnostics

Figure E.10 reports the tax and debt paths underlying the implementable two-by-two comparison in Figure 3 (Section 4.3).

Notes

¹The inflation-targeting equation is dropped and replaced by the exogenous money-growth path; the Bohn rule, the flow budget constraint, and the money-demand and private-sector blocks are retained, so debt and taxes are endogenous and the nominal rate is read off money demand. This is a hybrid regime that issues debt while following the *MF* money path rather than the *DF* monetary closure. The equivalence is analytic rather than accidental: given identical money-growth and exogenous paths, the private-sector block determines the same allocation regardless of the composition of fiscal liabilities, and the fiscal block adjusts residually; the reported budget-constraint residuals are of the order of machine precision.

²The closures are: *MF* unbacked, (10) and (18); *MF* gradual backing, (16) and (18); *DF* unbacked, (10) and (19); *DF* gradual backing, (16) and (19); and *MF* immediate backing, (14) and (18). For the unbacked rebate experiment under either financing scheme, (20) replaces (10) to introduce the exogenous rebate component. Under *MF*, $\hat{b}_t = 0$ would otherwise make the Bohn-rule component identically zero.

³Panel 11 reports the fiscal surplus, measured as the steady-state-GDP ratio $\hat{tr}_t - [\hat{g}_t + (1 + \rho)b(\hat{i}_{t-1} - \pi_t)]$, that is, the primary balance minus the real interest payment on government debt; the same convention applies wherever Panel 11 appears in the impulse-response figures.

⁴Equation (23) is a reduced-form equilibrium relation derived from a system in which the transversality condition on $B_t + L_t$ has already been imposed. Under the *MF* regime, where $\hat{b}_t = 0$, that condition reduces to the requirement that discounted real money balances vanish asymptotically. Hence, Equation (23) should be understood as being consistent with the transversality condition, not as constituting that condition by itself.

⁵Cumulative output is 2.688–2.689 at every $\psi_q \in \{0.0075, 0.02, 0.05, 0.10, 0.14\}$; see Figure E.2.

⁶Under the unbacked rule a money-financed rebate reproduces Galí (2020) (cumulative output 1.25); under (16) it yields 1.24 at $\psi_q = 0.0075$, declining to 1.11 at $\psi_q = 0.14$, near the first determinacy boundary ($\psi_q \simeq 0.1434$). Figure E.3 in Appendix E reports the rebate responses under the two main rules. Appendix D describes the counterfactual shown in Figure E.4, which removes the contemporaneous offset while retaining the consolidated transversality condition, confirming that what the offset eliminates is the temporary monetary expansion, not a permanent net-wealth transfer.

⁷When stickiness is 1, the model has no solution, so the multiplier cannot be calculated under either the *DF* or the *MF* fiscal stimulus.

⁸With stickier prices, a given nominal money injection translates more strongly into real balances and less immediately into prices, strengthening the liquidity effect through the money-demand relation (6); the fiscal multipliers therefore increase with price stickiness.

⁹ $\delta = 1$ lies outside the stationary impulse-response experiment, since cumulative government purchases diverge; the plotted range is restricted to $\delta < 1$, and the limit at $\delta = 1$ should not be read as an economic multiplier.

¹⁰The boundary is a change in the root count, not a numerical artifact. Counting eigenvalues in the reduced system that determines the allocation, the number lying outside the unit circle equals the number of non-predetermined variables, two, throughout the connected region, so the Blanchard–Kahn rank condition holds there; it rises to three just below $b/\chi = 14.98$ and returns to two just above 28.5. Appendix A details both crossings and the disconnected branch.

¹¹At first glance, the figures in Galí (2020) may appear to display a different path for the Z shock. In those figures, however, the Z shock is expressed as a deviation from its steady state, where $\rho = i$, and is reported at an annual rate. Once those conventions are taken into account, the dynamics of the Z shock are the same in Galí (2020) and in our analysis.

¹²The real money demand schedule is given by (6).

¹³The ZLB is implemented as in the main experiments, with the binding window verified against the lower-bound and money-demand complementarity conditions together with the piecewise monetary policy rule (25). Under the valuation rule (14), by contrast, no bounded *DF* equilibrium exists in the trap either. Once the ZLB episode ends and the non-fiscal disturbances have vanished, debt again obeys (A.1), so a temporary ZLB episode does not remove the asymptotic explosive root or restore the consolidated transversality condition.

¹⁴The baseline liquidity-trap experiment in Galí (2020) is the same as ours: $\hat{\rho}_t = -\gamma$ with $\gamma = 0.01$. He interprets that experiment as an unanticipated drop in the natural interest rate. See the first paragraph on page 15 of Galí (2020).

¹⁵See page 63, Chap. 3 of Galí (2015).

¹⁶Equation (26) is written for the calibration $\sigma = 1$; for general σ the coefficient on $\hat{g}_{t+1} - \hat{g}_t$ is $\frac{\sigma(\varphi+\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$, which reduces to $\frac{\varphi+\alpha}{1+\varphi}$ at $\sigma = 1$.

¹⁷The Blanchard–Kahn condition requires the number of explosive eigenvalues of the linearized system to equal the number of non-predetermined variables; here the explosive root is attached to predetermined debt, so no bounded rational-expectations solution exists.

¹⁸The diagnostic shock ωu_t^A of this appendix is normalized on the *realized* impact fall in net taxes rather than on the inserted component itself: because $\hat{tr}_t$ in (D.1) also responds endogenously through $\hat{l}_t$ and π_t , an inserted component of $\omega u_0^A = -0.005$ delivers $\hat{tr}_0 = -1$, the same impact magnitude as the exogenous rebate $\hat{tr}_0^* = -1$ of Section 4.4 and as the one-percent government-purchases stimulus, so that the experiments are directly comparable.

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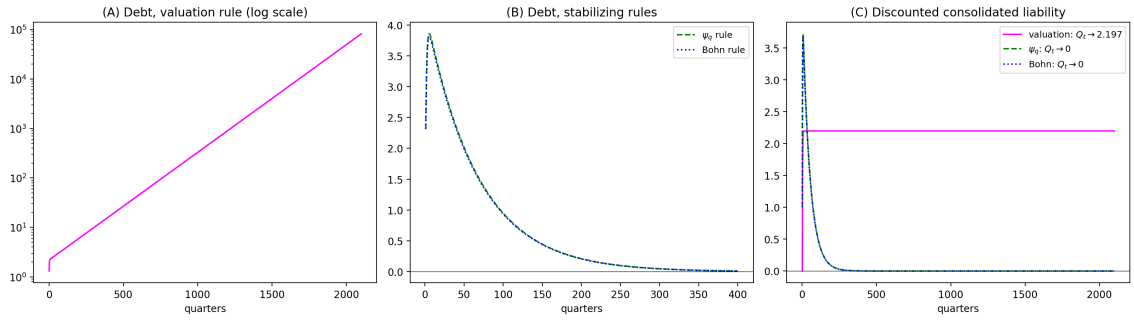
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Figures

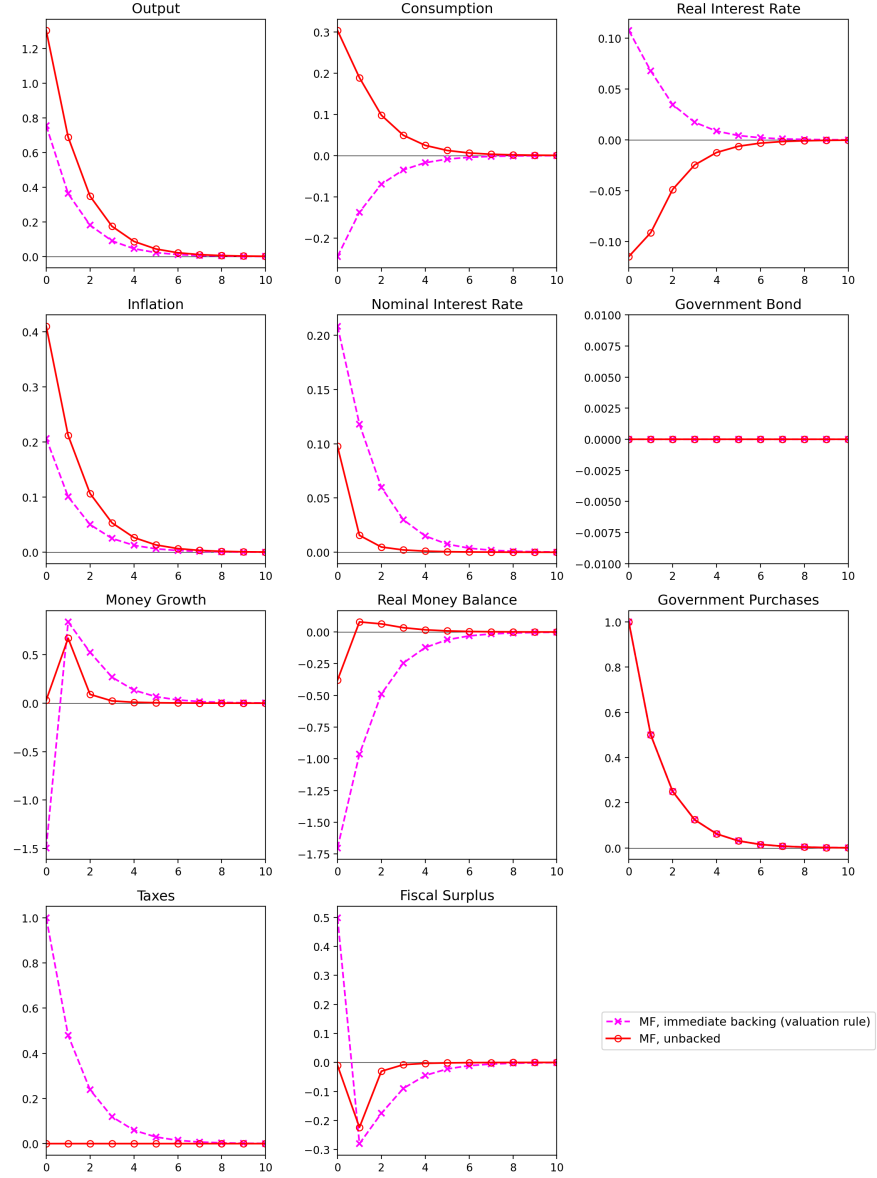
All figures are designed to remain legible in grayscale: the regimes are distinguished by line style as well as by color, and each regime carries a distinct plotting marker throughout the paper. The unbacked *MF* regime, (10) with (18), is drawn as a solid line with circles; the immediate-backing *MF* regime, (14) with (18), as a dashed line with crosses; the gradually backed *MF* regime, (16) with (18), as a dash-dotted line with diamonds; and the *DF* regimes, which use (19), as a dotted line with pluses. Where the two *DF* rules must be separated, the unbacked rule keeps the dotted line with pluses and the gradual-backing rule is drawn as a dash-dot-dotted line with squares. The same assignment is used in every figure, so that a reader who identifies a regime in one figure can identify it in all of them without reference to color. In the few figures that compare two *disturbances* rather than two regimes, the *Z*-shock path is drawn as a solid line with squares and the natural-rate path as a dashed line with triangles. Each figure carries a descriptive note explaining what is plotted.

Figure 1: Debt Dynamics under the Valuation-Rule *DF* Experiment and Stabilizing Alternatives



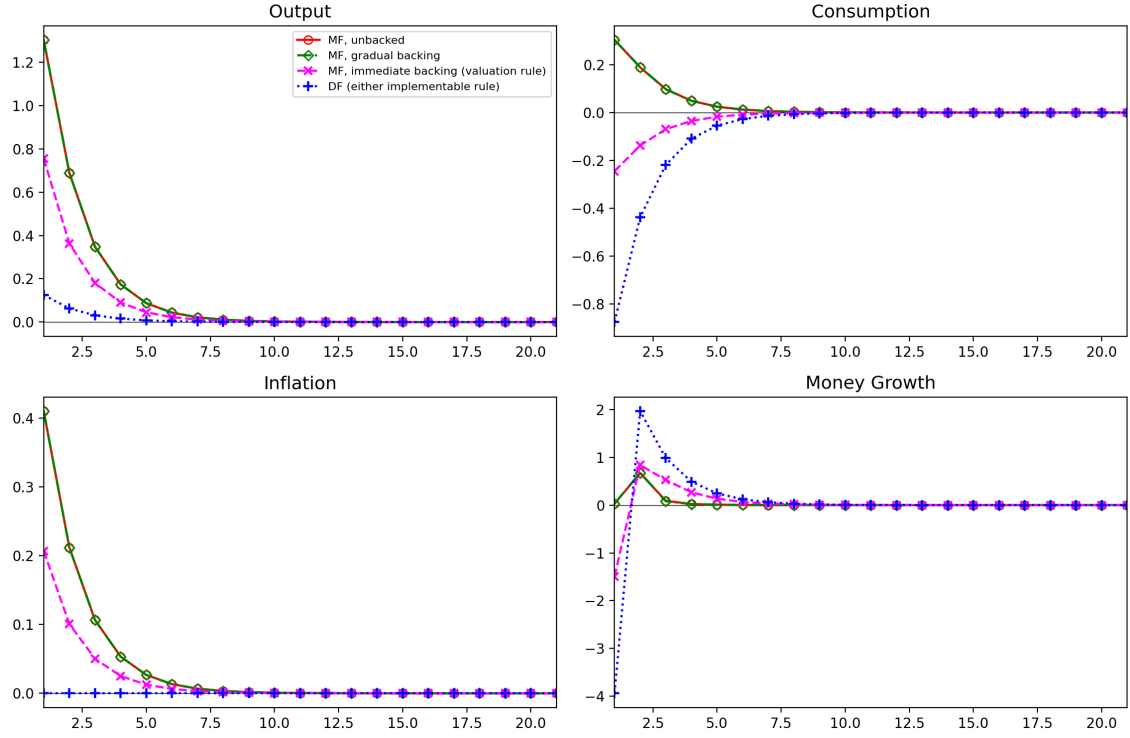
Note: Panel (A) plots debt under the valuation rule (14) on a log scale: growth at gross rate $\beta^{-1} = 1.005$ per quarter, invisible over a standard impulse-response window. Panel (B) plots debt under the two stabilizing rules on a linear scale. Panel (C) plots $Q_t = \beta^t(\hat{b}_t + \chi\hat{l}_t)$: it converges to $2.197 \neq 0$ under the valuation rule, violating the TVC (12), and to zero under the stabilizing rules.

Figure 2: Dynamic Effects of an Increase in Government Purchases under the *MF* Fiscal Stimulus in Normal Times



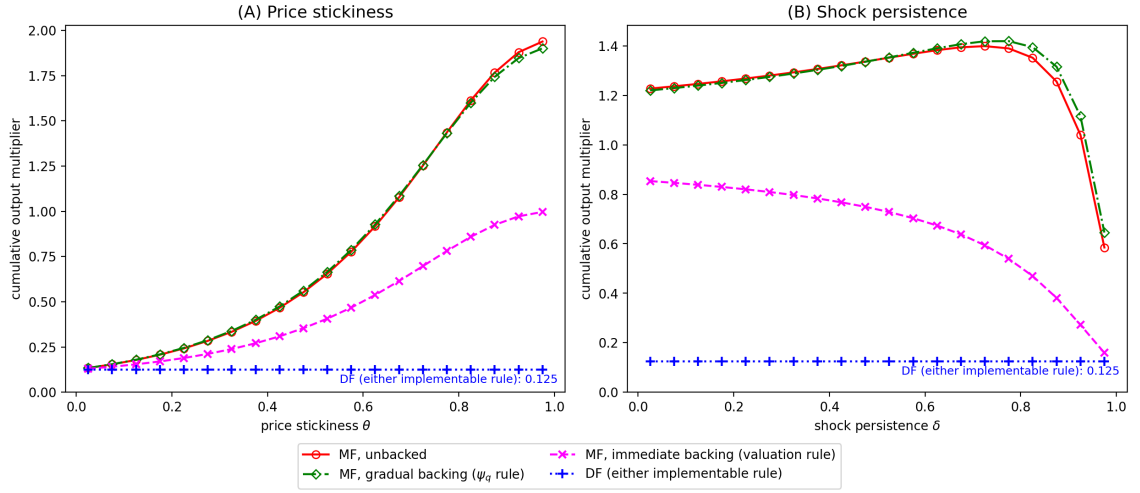
Note: Eleven panels plot the impulse responses of output, consumption, the real interest rate, inflation, the nominal interest rate, government bonds, money growth, real money balances, government purchases, taxes, and the fiscal surplus to a one-percent increase in government purchases in normal times, over eleven quarters on the horizontal axis. Two *MF* regimes are compared: the unbacked regime, (10) with (18), drawn as a solid line with circles, and the immediate-backing valuation regime, (14) with (18), drawn as a dashed line with crosses. The unbacked responses of output, consumption, and inflation are larger on impact; money growth turns negative on impact under the immediate-backing regime and positive thereafter.

Figure 3: Dynamic Effects of an Increase in Government Purchases: The Implementable Two-by-Two Benchmark and the Immediate-Backing Valuation Benchmark (Normal Times)



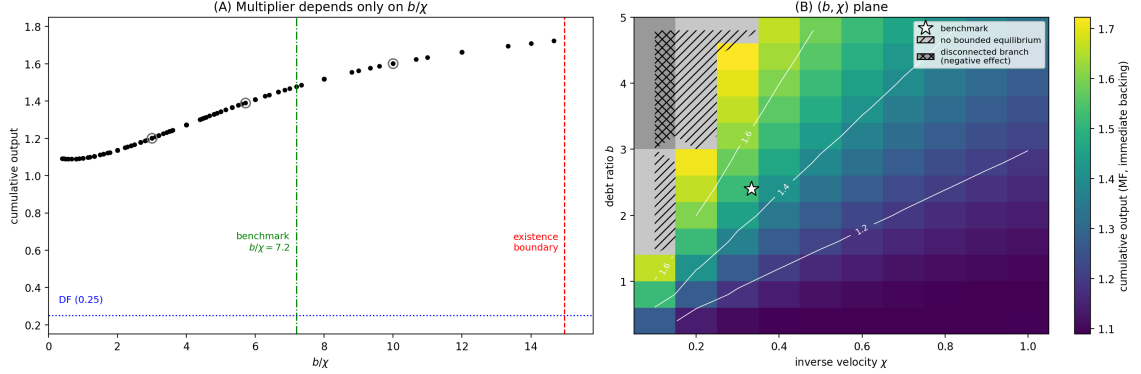
Note: Four panels plot the responses of output, consumption, inflation, and money growth to a one-percent increase in government purchases over twenty-one quarters. Five policy regimes underlie the figure, but only four distinct macroeconomic paths are visible, because the two implementable DF regimes coincide exactly in these variables. The paths shown are unbacked MF (solid line, circles), gradually backed MF (dash-dotted line, diamonds), immediate-backing MF (dashed line, crosses), and DF (dotted line, pluses). The single DF line applies exactly under either implementable fiscal rule, (10) and (16) (Proposition 2). The unbacked and gradually backed MF lines lie on top of each other throughout; the immediate-backing line lies visibly below them for output, consumption, and inflation. The corresponding fiscal diagnostics, in which the two DF rules do differ, are reported separately in Figure E.10.

Figure 4: Cumulative Output Multipliers, Price Stickiness, and Shock Persistence



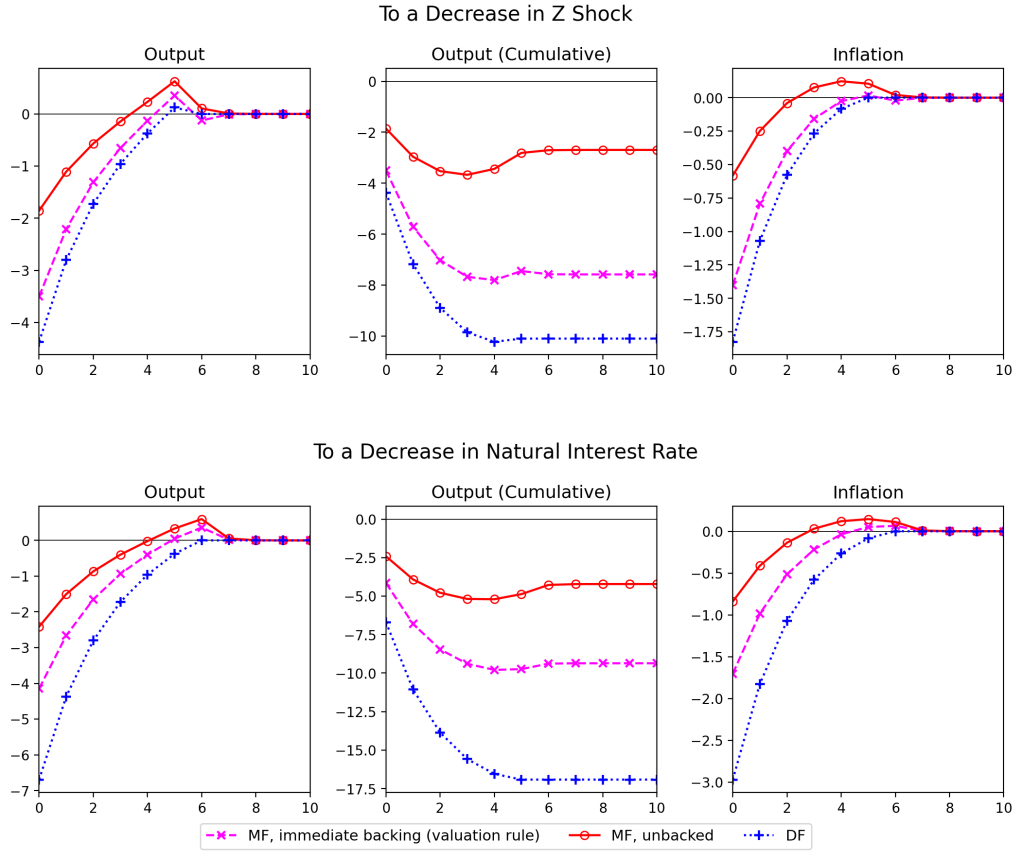
Note: Panel (A) plots the $(1 - \delta)$ -weighted cumulative output multiplier against the degree of price stickiness θ ; Panel (B) plots it against shock persistence δ . In both panels the three MF regimes are drawn as curves: unbacked (solid line, circles), gradually backed (dash-dotted line, diamonds), and immediate-backing (dashed line, crosses). The DF multiplier, which is invariant to both parameters and identical under either implementable fiscal rule, is drawn as a horizontal dotted reference line with plus markers at 0.125. The unbacked and gradually backed curves nearly coincide throughout; the immediate-backing curve lies below them but above the DF line at every computed point.

Figure 5: Robustness in the Debt Ratio and the Inverse Velocity: Cumulative Output under MF with the Immediate-Backing Valuation Rule



Note: Panel (A) plots the cumulative output effect for every computed (b, χ) cell of the connected bounded region against the ratio b/χ : all points lie on a single increasing curve, so the two parameters matter only through their ratio. The corresponding DF value is 0.25 everywhere. Panel (B) shows the underlying (b, χ) plane; the white iso-cumulative-output contours (labelled at 1.2, 1.4, and 1.6) are the level sets of the curve in Panel (A) and run as near-straight rays through the origin, confirming the b/χ dependence. The cells excluded from Panel (A) are classified in Panel (B) by the number of eigenvalues outside the unit circle: the diagonally hatched light grey cells in the upper-left corner are those at which one further explosive root has appeared and no bounded equilibrium exists ($14.97 \lesssim b/\chi \lesssim 28.5$), while the cross-hatched darker cells along the left edge ($b/\chi \gtrsim 28.5$) lie on a branch disconnected from the benchmark that does admit a bounded solution but delivers a negative cumulative output effect, below the DF value. The star marks the benchmark calibration, $b = 2.4$ and $\chi = 1/3$.

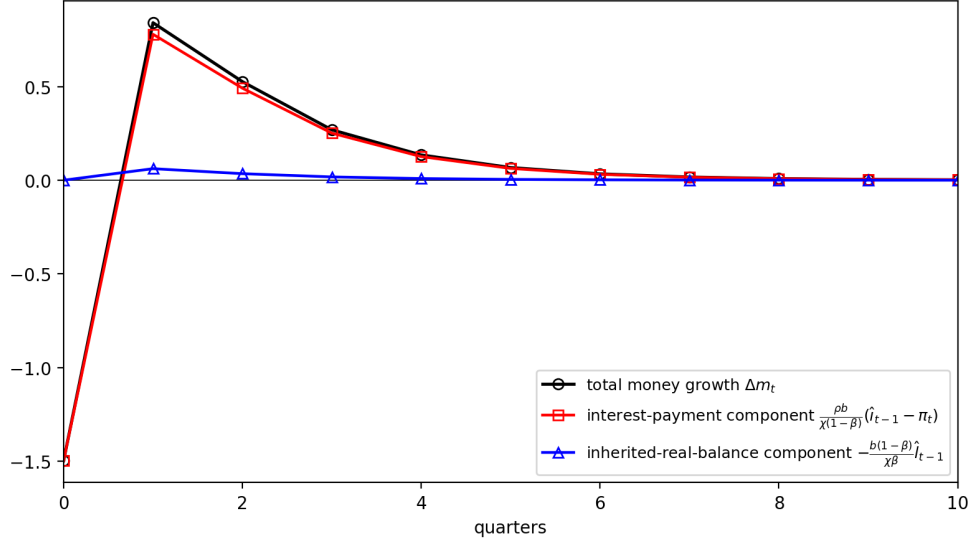
Figure 6: Dynamic Effects of an Increase in Government Purchases in a Liquidity Trap: Comparison of the MF and DF Schemes



Note: The upper row plots the responses to a decrease in the Z shock and the lower row those to a decrease in the natural interest rate; within each row the three panels are output, cumulative output, and inflation, over eleven quarters. Three regimes are compared: unbacked MF (solid line, circles), immediate-backing MF (dashed line, crosses), and DF (dotted line, pluses). In both rows the ranking of cumulative output is the same, with the unbacked MF path least negative and the DF path most negative. The gradually backed MF path is omitted for readability because it closely tracks the unbacked MF path; its cumulative effects are reported in Table 5.

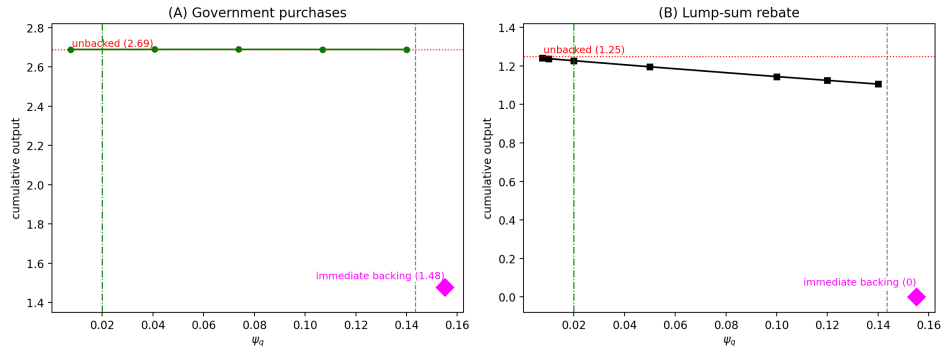
Figures for Appendix E

Figure E.1: Decomposition of Equilibrium Money Growth under Immediate Backing (Government Purchases, Normal Times)



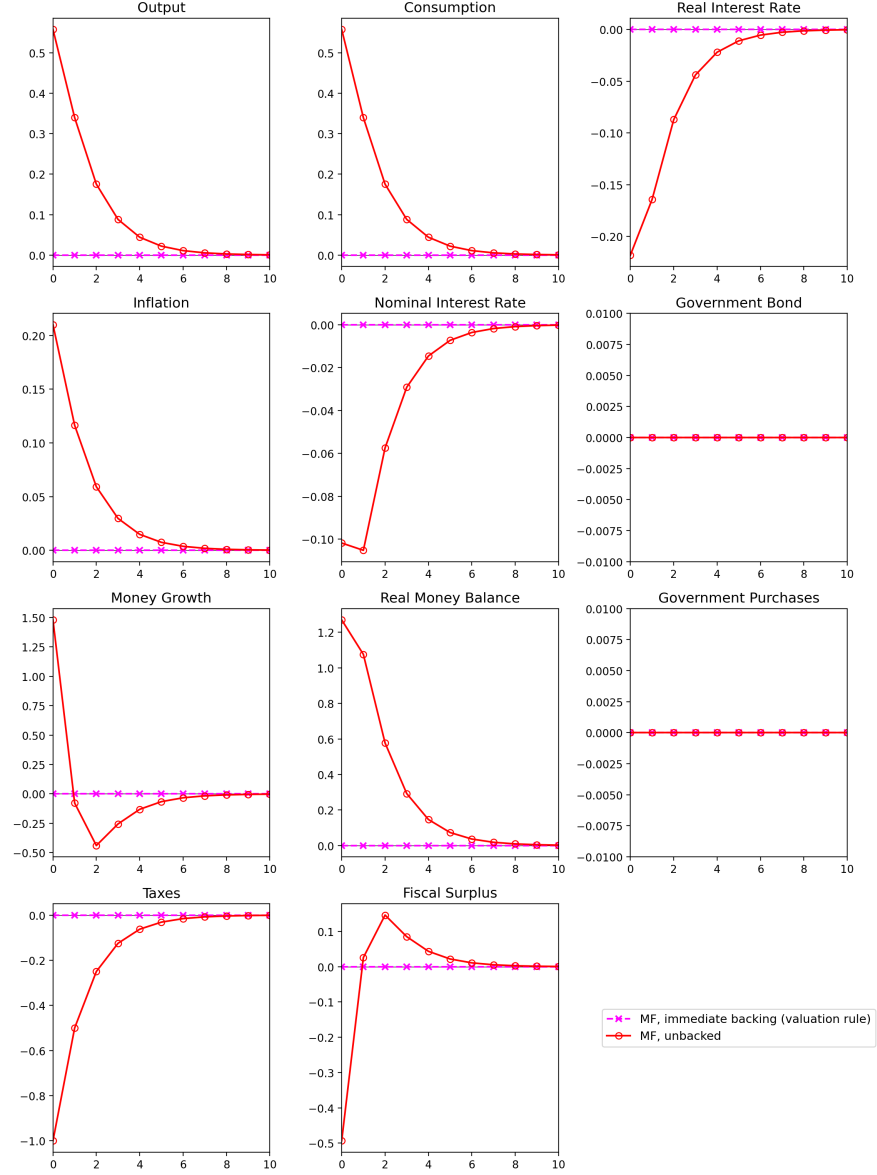
Note: The figure decomposes the equilibrium money growth Δm_t of the immediate-backing *MF* regime, (23), into the inherited-real-balance component $-\frac{b(1-\beta)}{\chi\beta}\hat{l}_{t-1}$ and the interest-payment component $\frac{\rho b}{\chi(1-\beta)}(\hat{l}_{t-1} - \pi_t)$. The two components sum to total money growth to machine precision. The interest-payment component accounts for essentially all variation in money growth (including virtually all of the positive money injection after impact), confirming that the monetary accommodation sustaining the government-purchases response operates through the interest-payment channel.

Figure E.2: Fiscal-Adjustment Speed and Cumulative Output under Gradual Full Backing



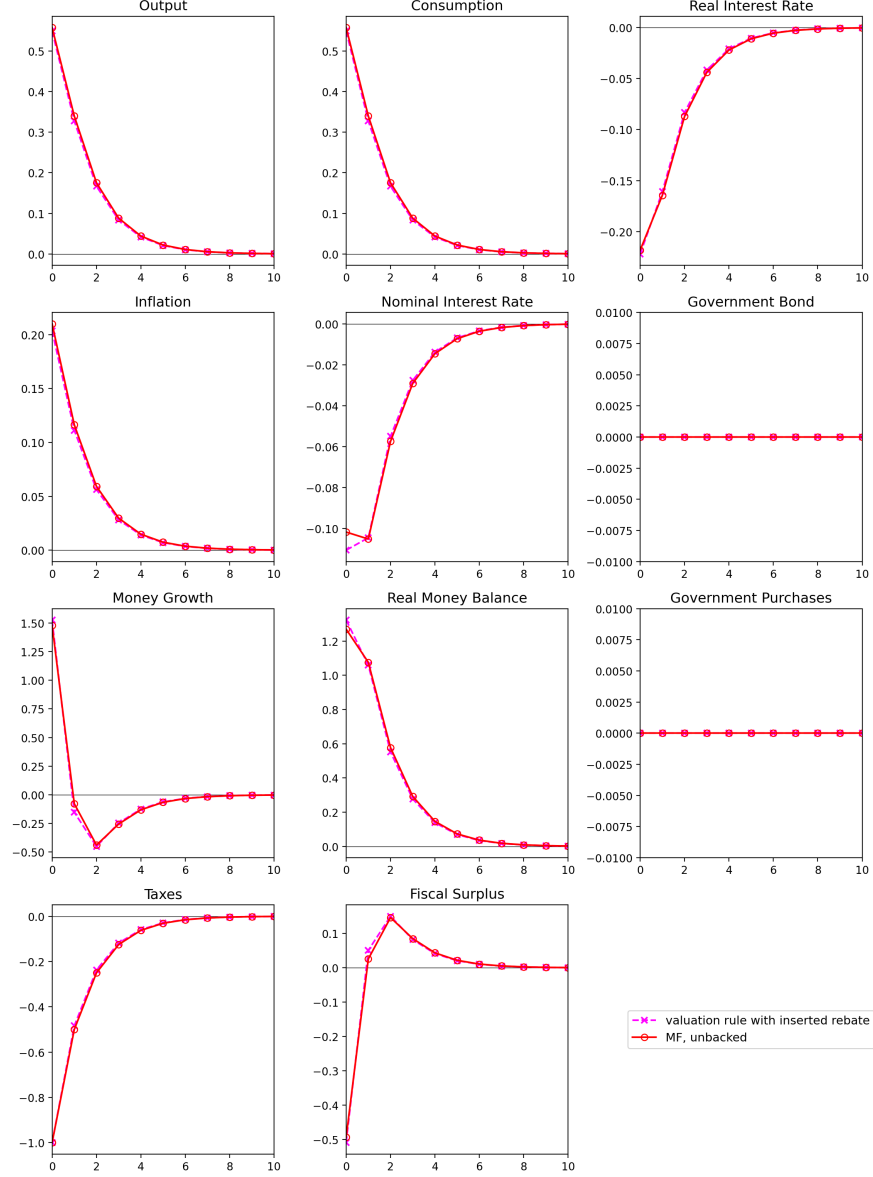
Note: Gradual backing follows (16); the discounted consolidated liability converges to zero at every ψ_q shown, so no permanent net-wealth transfer arises. The government-purchases effect is flat at 2.688–2.689 across the plotted connected determinate region. Within the plotted range, the equilibrium becomes indeterminate above the numerically located threshold, $\psi_q \in (0.1434, 0.1435)$. The immediate-backing (valuation-rule) outcomes (1.48 for purchases; 0 for the rebate) are plotted as isolated markers because they are a distinct implementation, not the limit of the feedback rule.

Figure E.3: Dynamic Effects of a Lump-Sum Rebate under the *MF* Fiscal Stimulus in Normal Times



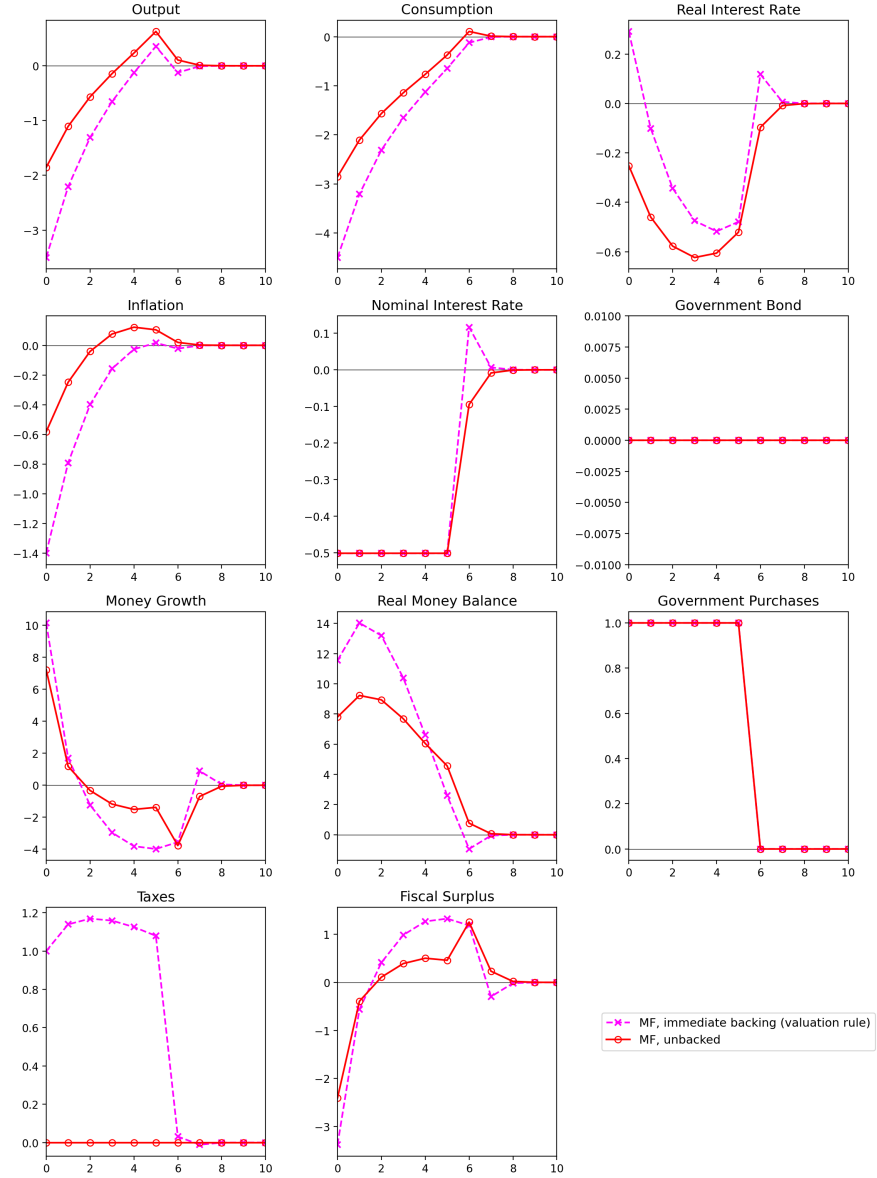
Note: Eleven panels plot the impulse responses of output, consumption, the real interest rate, inflation, the nominal interest rate, government bonds, money growth, real money balances, government purchases, taxes, and the fiscal surplus to a lump-sum rebate in normal times, over eleven quarters. The unbacked *MF* regime is drawn as a solid line with circles; under the immediate-backing valuation regime the rebate has no effect at all, so its path is a flat line at zero, drawn as a dashed line with crosses, in every panel.

Figure E.4: Dynamic Effects of a Counterfactual Rebate under the *MF* Fiscal Stimulus in Normal Times



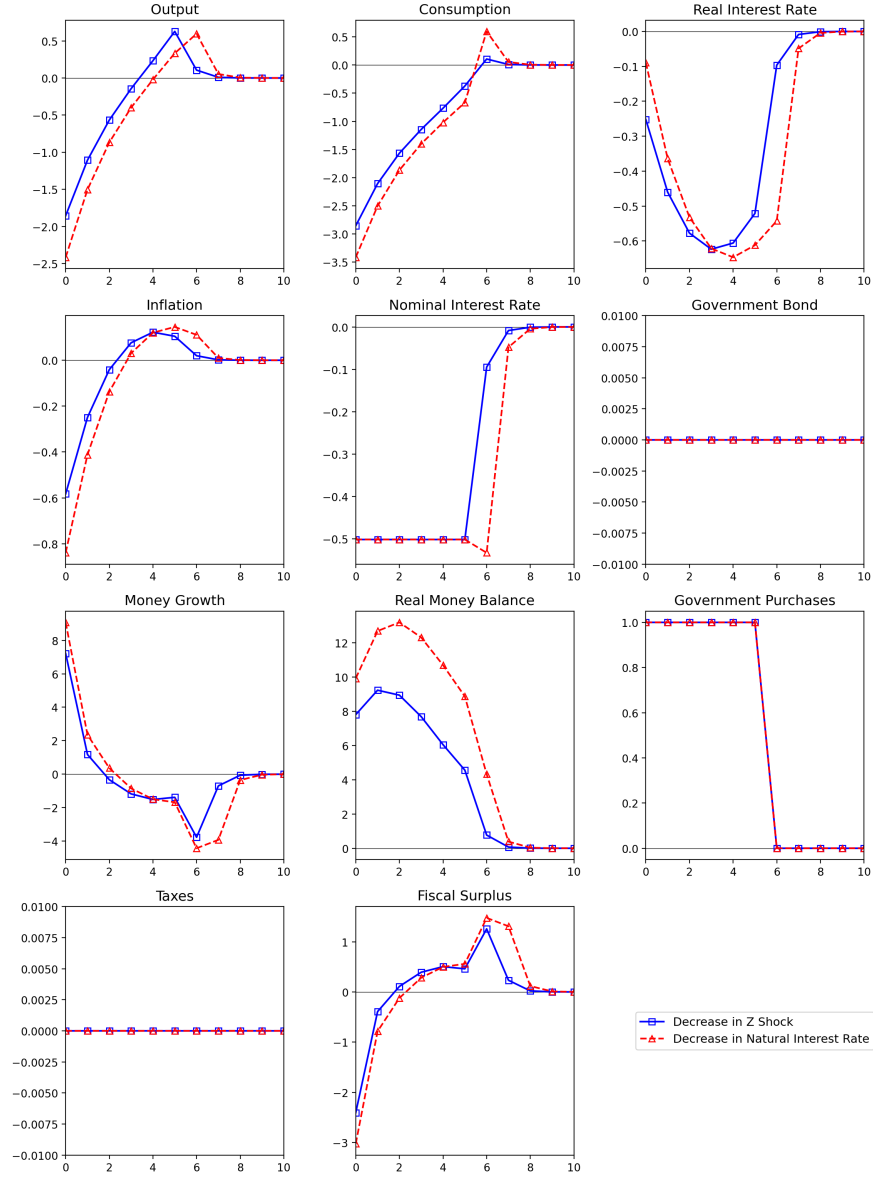
Note: The panels and their ordering are as in Figure E.3. The solid line with circles is the unbacked *MF* regime; the dashed line with crosses is the inserted-rebate valuation-rule counterfactual defined in Appendix D, obtained by adding ωu_t^A to the right-hand side of the valuation closure. Its endogenous money-growth path closely tracks the unbacked path without being constrained to equal it.

Figure E.5: Dynamic Effects of an Increase in Government Purchases under the *MF* Fiscal Stimulus in a Liquidity Trap



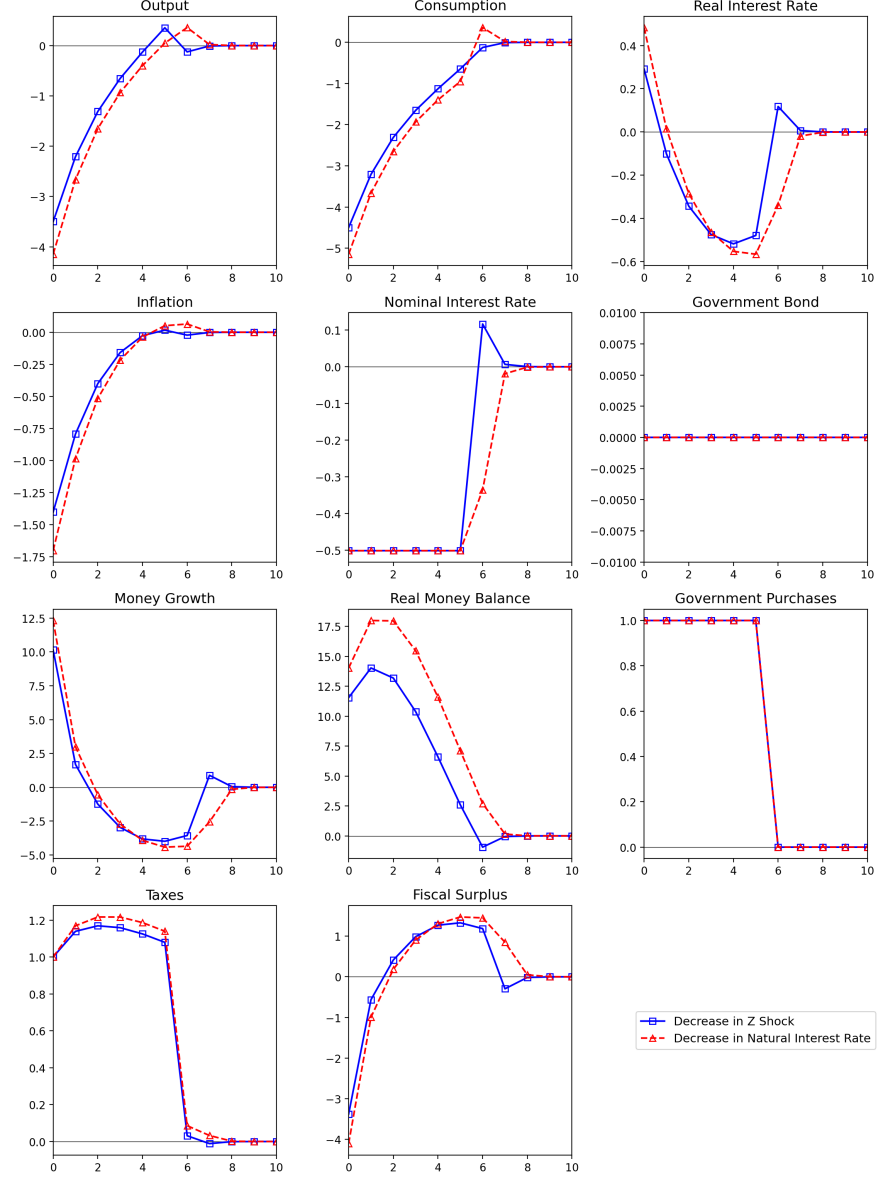
Note: Eleven panels plot the impulse responses of output, consumption, the real interest rate, inflation, the nominal interest rate, government bonds, money growth, real money balances, government purchases, taxes, and the fiscal surplus to a one-percent increase in government purchases over eleven quarters, as in Figure 2, but with the economy in a liquidity trap. The solid line with circles is the unbacked *MF* regime and the dashed line with crosses the immediate-backing valuation regime.

Figure E.6: Dynamic Effects of an Increase in Government Purchases under the Unbacked MF Stimulus in a Liquidity Trap: Z-Shock and Natural-Rate Scenarios



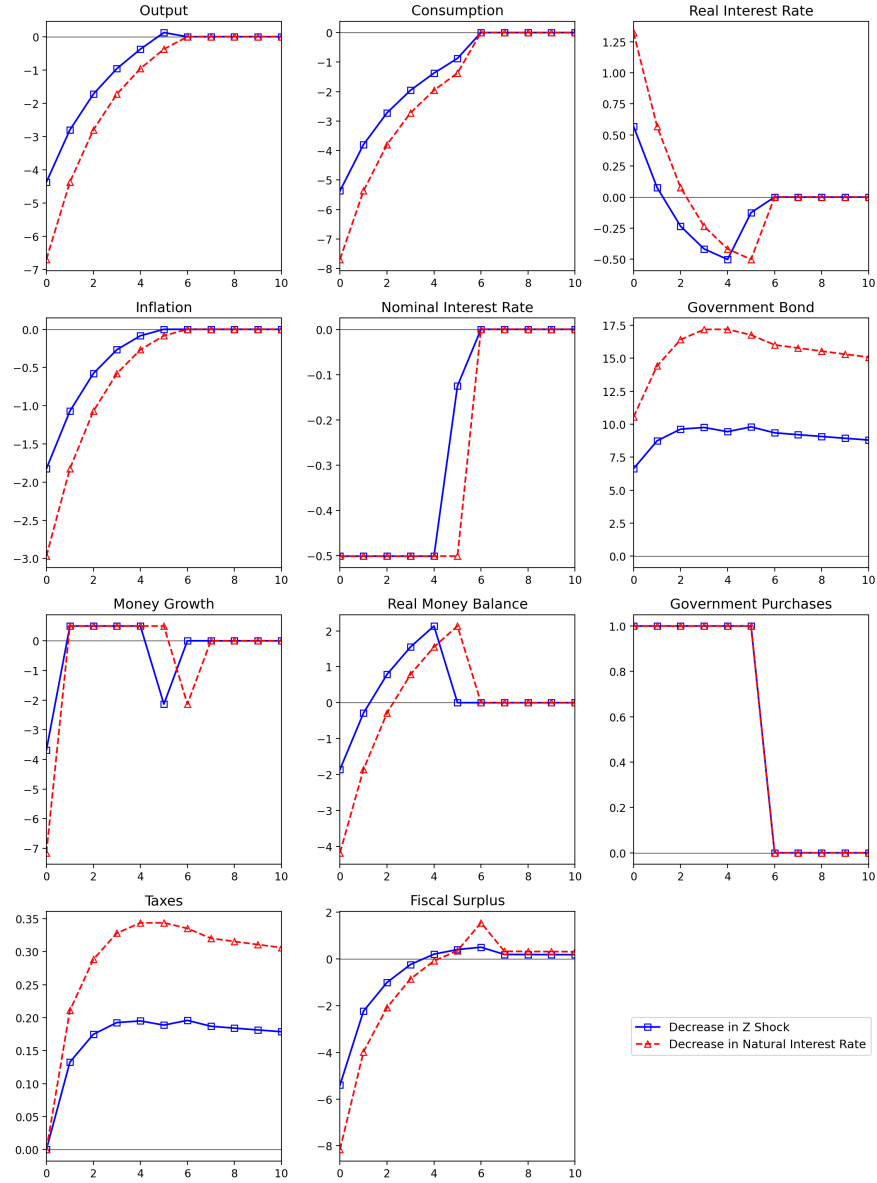
Note: The panels and their ordering are as in Figure 2, for the unbacked MF regime in a liquidity trap. The solid line with squares is the response to a decrease in the Z shock and the dashed line with triangles the response to a decrease in the natural interest rate. The natural-rate scenario produces more contractionary responses of output, consumption, and inflation, while the exogenous government-purchases path and the variables fixed by the financing closure remain unchanged.

Figure E.7: Dynamic Effects of an Increase in Government Purchases under the Immediate-Backing MF Stimulus in a Liquidity Trap: Z-Shock and Natural-Rate Scenarios



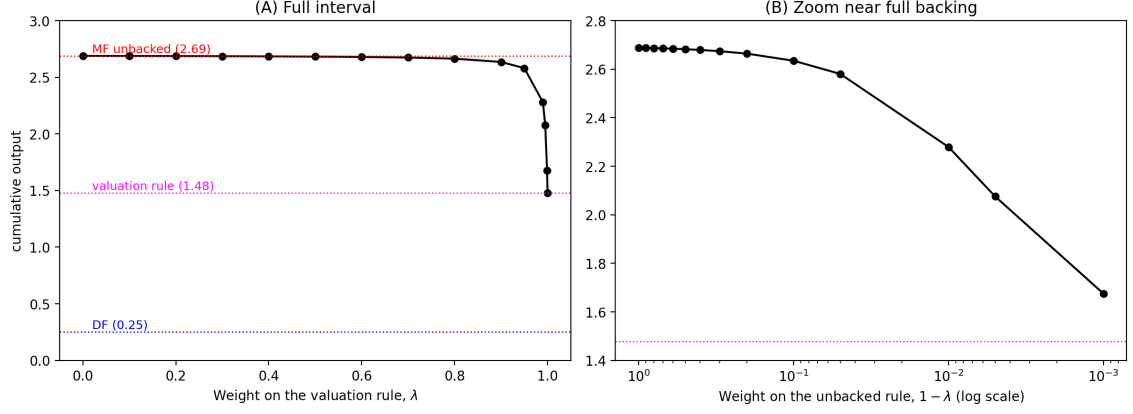
Note: Eleven panels plot the same variables and ordering as Figure E.6 (output, consumption, the real interest rate, inflation, the nominal interest rate, government bonds, money growth, real money balances, government purchases, taxes, and the fiscal surplus), over eleven quarters, here for the immediate-backing MF regime in a liquidity trap. The solid line with squares is the Z-shock scenario and the dashed line with triangles the natural-rate scenario.

Figure E.8: Dynamic Effects of an Increase in Government Purchases under the *DF* Fiscal Stimulus in a Liquidity Trap: Z-Shock and Natural-Rate Scenarios



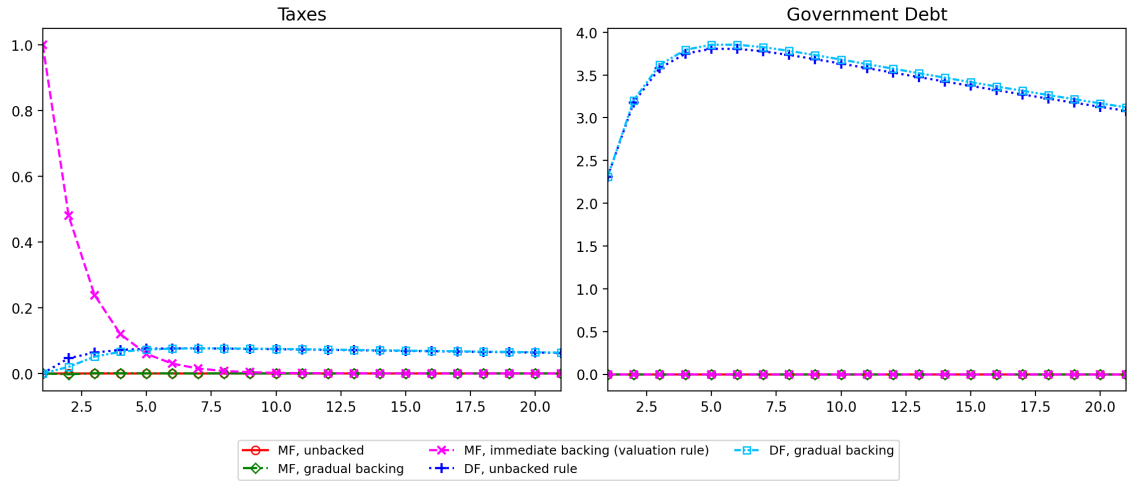
Note: Eleven panels plot the same variables and ordering as Figure E.6 (output, consumption, the real interest rate, inflation, the nominal interest rate, government bonds, money growth, real money balances, government purchases, taxes, and the fiscal surplus), over eleven quarters, here for the *DF* fiscal stimulus in a liquidity trap. The solid line with squares is the *Z*-shock scenario and the dashed line with triangles the natural-rate scenario.

Figure E.9: Cumulative Output under a Reduced-Form Interpolation between the Unbacked and Valuation Rules



Note: The parameter λ is a reduced-form rule weight as in (C.1); it should not be interpreted as a literal share of liabilities that is fiscally backed. $\lambda = 0$ is Galí's benchmark (the unbacked rule), $\lambda = 1$ is the valuation rule. Panel (B) plots the same data against the weight on the unbacked rule, $1 - \lambda$, on a log scale, showing that the decline is concentrated within the last percentage points near the valuation-rule endpoint.

Figure E.10: Fiscal Diagnostics Behind Figure 3: Taxes and Government Debt



Note: Two panels plot the responses of taxes and of government debt to a one-percent increase in government purchases over twenty-one quarters, for the same five regimes underlying Figure 3. The two *DF* rules, which coincide in every macroeconomic variable by Proposition 2, are shown separately here: the unbacked rule as a dotted line with pluses and the gradual-backing rule as a dash-dot-dotted line with squares. The three *MF* regimes use the line styles of Figure 3. Within the *DF* column, switching between the two implementable fiscal rules changes only taxes and debt. Under *MF*, the fiscal closures also generate different macroeconomic paths, as shown in Figure 3.