Technical Appendix to: "How Important is Fiscal Policy Cooperation in a Currency Union?

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A Details on Derivation of the Model

A.1 Households

Preferences of the representative household in countries H and F are given by:

$$\begin{aligned}\mathcal{U} &\equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \left(\ln C_t - \frac{1}{1+\varphi} N_t^{1+\varphi} \right), \\ \mathcal{U}^* &\equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \left(\ln C_t^* - \frac{1}{1+\varphi} \left(N_t^*\right)^{1+\varphi} \right), \end{aligned} \tag{A.1}$$

where C_t^* denotes consumption in country F, $N_t^* \equiv N_{F,t} + N_{\mathcal{N},t}^*$ denotes hours of work in country F, $N_{F,t} \equiv \int_1^2 N_{F,t}(f)\, df$ and $N_{\mathcal{N},t}^* \equiv \int_1^2 N_{\mathcal{N},t}^*(f)\, df$ denote hours of work to produce tradables produced in country F and nontradables produced in country F, respectively. The first equality in Eq.(A.1) is Eq.(1) in the text.

More precisely, private consumption is a composite index defined by:

$$\begin{aligned} C_t &\equiv \left[\gamma^{\frac{1}{\eta}} C_{\mathcal{T},t}^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} C_{\mathcal{N},t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \\ C_t^* &\equiv \left[\gamma^{\frac{1}{\eta}} \left(C_{\mathcal{T},t}^*\right)^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} \left(C_{\mathcal{N},t}^*\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \end{aligned} \tag{A.2}$$

with $C_{\mathcal{N},t} \equiv \left[\int_0^1 C_{\mathcal{N},t}(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}$, $C_{\mathcal{N},t}^* \equiv \left[\int_1^2 C_{\mathcal{N},t}^*(f)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}$ $C_{H,t} \equiv \left[\int_0^1 C_{H,t}(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}$ and $C_{F,t} \equiv \left[\int_1^2 C_{F,t}(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}$, where the index $\{h, f\}$ denotes a variable that is specific to agents h and f, $C_{\mathcal{T},t}^*$ denotes the consumption index for tradables in country F, and $C_{\mathcal{N},t}^*$ denotes an index of consumption across the nontradable goods produced in country F. The first equality in Eq.(A.2) is Eq.(2) in the text.

A sequence of budget constraints is given by:

$$\begin{aligned}
D_t^n + W_t N_t + S_t &\geq \int_0^1 P_{H,t}(h)\, C_{H,t}(h)\, dh + \int_1^2 P_{F,t}(f)\, C_{F,t}(f)\, df \\
&\quad + \int_0^1 P_{\mathcal{N},t}(h)\, C_{\mathcal{N},t}(h)\, dh + \mathrm{E}_t Q_{t,t+1} D_{t+1}^n, \\
D_t^{n*} + W_t^* N_t^* + S_t^* &\geq \int_0^1 P_{H,t}(h)\, C_{H,t}^*(h)\, df + \int_1^2 P_{F,t}(f)\, C_{F,t}^*(f)\, df \\
&\quad + \int_0^1 P_{\mathcal{N},t}^*(f)\, C_{\mathcal{N},t}^*(f)\, df + \mathrm{E}_t Q_{t,t+1} D_{t+1}^{n*}, \qquad \text{(A.3)}
\end{aligned}$$

with $P_{H,t} \equiv \left[\int_0^1 P_{H,t}(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}}$, $P_{F,t} \equiv \left[\int_1^2 P_{F,t}(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}$ and $P_{\mathcal{N},t} \equiv \left[\int_0^1 P_{\mathcal{N},t}(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}}$, where $P_{\mathcal{N},t}^* \equiv \left[\int_1^2 P_{\mathcal{N},t}^*(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}$ denotes the price index of nontradables produced in country F and S_t^* denotes the lump sum taxes in country F.

The optimal allocation of any given expenditure within each category of goods yields the following demand functions:

$$\begin{aligned}
C_{H,t}(h) &= \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta} C_{H,t}, \quad ; \quad C_{F,t}(f) = \left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta} C_{F,t}, \\
C_{H,t}^*(h) &= \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta} C_{H,t}^*, \quad ; \quad C_{F,t}^*(f) = \left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta} C_{F,t}^*, \\
C_{\mathcal{N},t}(h) &= \left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)^{-\theta} C_{\mathcal{N},t}, \quad ; \quad C_{\mathcal{N},t}^*(f) = \left(\frac{P_{\mathcal{N},t}^*(f)}{P_{\mathcal{N},t}^*}\right)^{-\theta} C_{\mathcal{N},t}^* \qquad \text{(A.4)}
\end{aligned}$$

These equalities imply that $\int_0^1 P_{H,t}(h)\, C_{H,t}(h)\, dh = P_{H,t} C_{H,t}$, $\int_1^2 P_{F,t}(f)\, C_{F,t}(f)\, df = P_{F,t} C_{F,t}$, $\int_0^1 P_{\mathcal{N},t}(h)\, C_{\mathcal{N},t}(h)\, dh = P_{\mathcal{N},t} C_{\mathcal{N},t}$ and $\int_1^2 P_{\mathcal{N},t}^*(f)\, C_{\mathcal{N},t}^*(f)\, df = P_{\mathcal{N},t}^* C_{\mathcal{N},t}^*$.

Total consumption expenditures by households in countries H and F are given by:

$$\begin{aligned}
P_{H,t} C_{H,t} + P_{F,t} C_{F,t} + P_{\mathcal{N},t} C_{\mathcal{N},t} &= P_t C_t, \\
P_{F,t} C_{F,t}^* + P_{H,t} C_{H,t}^* + P_{\mathcal{N},t}^* C_{\mathcal{N},t}^* &= P_t^* C_t^*.
\end{aligned}$$

Combining Eq.(A.3) and these equalities, we obtain:

$$\begin{aligned}
D_t^n + W_t N_t + S_t &\geq P_t C_t + \mathrm{E}_t Q_{t,t+1} D_{t+1}^n, \\
D_t^{n*} + W_t^* N_t^* + S_t^* &\geq P_t^* C_t^* + \mathrm{E}_t Q_{t,t+1} D_{t+1}^{n*}, \qquad \text{(A.5)}
\end{aligned}$$

where the first equality in Eq.(A.5) is Eq.(3) in the text.

Combining Eq.(A.4) and aggregators, we have:

$$C_{H,t} = \frac{1}{2}\left(\frac{P_{H,t}}{P_{\mathcal{T},t}}\right)^{-1} C_{\mathcal{T},t}, \quad ; \quad C_{F,t} = \frac{1}{2}\left(\frac{P_{F,t}}{P_{\mathcal{T},t}}\right)^{-1} C_{\mathcal{T},t},$$

$$C^*_{H,t} = \frac{1}{2}\left(\frac{P_{H,t}}{P_{\mathcal{T},t}}\right)^{-1} C^*_{\mathcal{T},t}, \quad ; \quad C^*_{F,t} = \frac{1}{2}\left(\frac{P_{F,t}}{P_{\mathcal{T},t}}\right)^{-1} C^*_{\mathcal{T},t},$$
$$C_{\mathcal{T},t} = \gamma\left(\frac{P_{\mathcal{T},t}}{P_t}\right)^{-\eta} C_t, \quad ; \quad C_{\mathcal{N},t} = (1-\gamma)\left(\frac{P_{\mathcal{N},t}}{P_t}\right)^{-\eta} C_t,$$
$$C^*_{\mathcal{T},t} = \gamma\left(\frac{P_{\mathcal{T},t}}{P^*_t}\right)^{-\eta} C^*_t, \quad ; \quad C^*_{\mathcal{N},t} = (1-\gamma)\left(\frac{P^*_{\mathcal{N},t}}{P^*_t}\right)^{-\eta} C^*_t. \tag{A.6}$$

The first, second, fifth and sixth equalities in Eq.(A.6) are Eq.(5) in the text.

CPIs are given by:

$$P_t \equiv \left[\gamma P_{\mathcal{T},t}^{1-\eta} + (1-\gamma) P_{\mathcal{N},t}^{1-\eta}\right]^{\frac{1}{1-\eta}},$$
$$P^*_t \equiv \left[\gamma P_{\mathcal{T},t}^{1-\eta} + (1-\gamma)\left(P^*_{\mathcal{N},t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}, \tag{A.7}$$

where P^*_t denotes the CPI in country F. The first equality in Eq.(A.7) is Eq.(4) in the text.

The representative household maximizes Eq.(A.1) subject to Eq.(A.5). Optimality conditions are given by:

$$\delta \mathrm{E}_t\left(\frac{C_{t+1}^{-1} P_t}{C_t^{-1} P_{t+1}}\right) = \frac{1}{R_t}, \quad ; \quad \delta \mathrm{E}_t\left[\frac{\left(C^*_{t+1}\right)^{-1} P^*_t}{\left(C^*_t\right)^{-1} P^*_{t+1}}\right] = \frac{1}{R_t}, \tag{A.8}$$
$$C_t N_t^{\varphi} = \frac{W_t}{P_t}, \quad ; \quad C^*_t \left(N^*_t\right)^{\varphi} = \frac{W^*_t}{P^*_t}. \tag{A.9}$$

The RHS of Eq.(A.8) is an intertemporal optimality condition in country F, whereas the RHS of Eq.(A.9) is an intratemporal optimality condition in country F. The LHS of both Eqs.(A.8) and (A.9) are Eq.(6) in the text.

Combining and iterating Eq.(A.8) with an initial condition, we have the following optimal risk-sharing condition:

$$C_t = \vartheta C^*_t \mathsf{Q}_t, \tag{A.10}$$

which is Eq.(7) in the text. When $C_{-1} = C^*_{-1} = P_{-1} = P^*_{-1} = 1$, we have $\vartheta = 1$.

A.2 Firms

Each producer can use a linear technology to produce a differentiated good as follows:

$$Y_{H,t}(h) = A_{H,t} N_{H,t}(h), \quad ; \quad Y_{\mathcal{N},t}(h) = A_{\mathcal{N},t} N_{\mathcal{N},t}(h),$$
$$Y_{F,t}(f) = A_{F,t} N_{F,t}(f), \quad ; \quad Y^*_{\mathcal{N},t}(f) = A^*_{\mathcal{N},t} N^*_{\mathcal{N},t}(f), \tag{A.11}$$

with $Y_{H,t} \equiv \left(\int_0^1 Y_{H,t}(h)^{\frac{\theta-1}{\theta}} dh\right)^{\frac{\theta}{\theta-1}}$, $Y_{F,t} \equiv \left(\int_1^2 Y_{F,t}(f)^{\frac{\theta-1}{\theta}} df\right)^{\frac{\theta}{\theta-1}}$, $Y_{\mathcal{N},t} \equiv \left(\int_0^1 Y_{\mathcal{N},t}(h)^{\frac{\theta-1}{\theta}} dh\right)^{\frac{\theta}{\theta-1}}$ and $Y^*_{\mathcal{N},t} \equiv \left(\int_1^2 Y^*_{\mathcal{N},t}(f)^{\frac{\theta-1}{\theta}} df\right)^{\frac{\theta}{\theta-1}}$, where $A_{F,t}$ and

$A^*_{\mathcal{N},t}$ denote stochastic productivity shifters associated with tradables and nontradables produced in country F, respectively. The first equalities in Eq.(A.11) are Eq.(8) in the text.

Using Dixit–Stiglitz aggregators, Eq.(A.11) can be rewritten as:

$$Y_{H,t} = \frac{A_{H,t}N_{H,t}}{\int_0^1 \frac{Y_{H,t}(h)}{Y_{H,t}}dh} \quad ; \quad Y_{\mathcal{N},t} = \frac{A_{\mathcal{N},t}N_{\mathcal{N},t}}{\int_0^1 \frac{Y_{\mathcal{N},t}(h)}{Y_{\mathcal{N},t}}dh},$$
$$Y_{F,t} = \frac{A_{F,t}N_{F,t}}{\int_1^2 \frac{Y_{F,t}(f)}{Y_{F,t}}df} \quad ; \quad Y^*_{\mathcal{N},t} = \frac{A^*_{\mathcal{N},t}N^*_{\mathcal{N},t}}{\int_1^2 \frac{Y^*_{\mathcal{N},t}(f)}{Y^*_{\mathcal{N},t}}df}. \tag{A.12}$$

Under Calvo–Yun-style price-setting behavior, the pricing rules are given by:

$$\begin{aligned}
P_{H,t} &= \left[\alpha P^{1-\theta}_{H,t-1} + (1-\alpha)\tilde{P}^{1-\theta}_{H,t}\right]^{\frac{1}{1-\theta}}, \\
P_{\mathcal{N},t} &= \left[\alpha P^{1-\theta}_{N,t-1} + (1-\alpha)\tilde{P}^{1-\theta}_{\mathcal{N},t}\right]^{\frac{1}{1-\theta}}, \\
P_{F,t} &= \left[\alpha P^{1-\theta}_{F,t-1} + (1-\alpha)\tilde{P}^{1-\theta}_{F,t}\right]^{\frac{1}{1-\theta}}, \\
P^*_{\mathcal{N},t} &= \left[\alpha \left(P^*_{N,t-1}\right)^{1-\theta} + (1-\alpha)\left(\tilde{P}^*_{\mathcal{N},t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}},
\end{aligned} \tag{A.13}$$

where $\tilde{P}_{F,t}$ and $\tilde{P}^*_{\mathcal{N},t}$ are the prices chosen by firms when they obtain the chance to change prices associated with tradables and nontradables produced in country F, respectively.

The maximization problems faced by firms are as follows:

$$\max_{\tilde{P}_{H,t}} \mathrm{E}_t \left[\sum_{k=0}^{\infty} (\alpha\delta)^k \left(P_{t+k}C_{t+k}\right)^{-1} \tilde{Y}_{H,t+k}\left(\tilde{P}_{H,t} - MC^n_{H,t+k}\right)\right],$$
$$\max_{\tilde{P}_{\mathcal{N},t}} \mathrm{E}_t \left[\sum_{k=0}^{\infty} (\alpha\delta)^k \left(P_{t+k}C_{t+k}\right)^{-1} \tilde{Y}_{\mathcal{N},t+k}\left(\tilde{P}_{\mathcal{N},t} - MC^n_{\mathcal{N},t+k}\right)\right],$$
$$\max_{\tilde{P}_{F,t}} \mathrm{E}_t \left[\sum_{k=0}^{\infty} (\alpha\delta)^k \left(P^*_{t+k}C^*_{t+k}\right)^{-1} \tilde{Y}_{F,t+k}\left(\tilde{P}_{F,t} - MC^n_{F,t+k}\right)\right],$$
$$\max_{\tilde{P}^*_{\mathcal{N},t}} \mathrm{E}_t \left[\sum_{k=0}^{\infty} (\alpha\delta)^k \left(P^*_{t+k}C^*_{t+k}\right)^{-1} \tilde{Y}^*_{\mathcal{N},t+k}\left(\tilde{P}^*_{\mathcal{N},t} - MC^{*n}_{\mathcal{N},t+k}\right)\right],$$

$\tilde{Y}_{H,t+k} \equiv \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}}\right)^{-\theta} Y_{H,t+k}$ and $\tilde{Y}_{\mathcal{N},t+k} \equiv \left(\frac{\tilde{P}_{\mathcal{N},t}}{P_{\mathcal{N},t+k}}\right)^{-\theta} Y_{\mathcal{N},t+k}$, where $\tilde{Y}_{F,t+k} \equiv \left(\frac{\tilde{P}_{F,t}}{P_{F,t+k}}\right)^{-\theta} Y_{F,t+k}$ and $\tilde{Y}^*_{\mathcal{N},t+k} \equiv \left(\frac{\tilde{P}^*_{\mathcal{N},t}}{Y^*_{\mathcal{N},t+k}}\right)^{-\theta} Y^*_{\mathcal{N},t+k}$ denote the total demands when the prices change, and $MC^n_{F,t} \equiv \frac{W^*_t}{(1-\tau)A_{F,t}}$ and $MC^{*n}_{\mathcal{N},t} \equiv \frac{W^*_t}{(1-\tau)A^*_{\mathcal{N},t}}$ denote the marginal costs associated with tradables and nontradables produced in country F, respectively.

The FONCs are as follows:

$$
\begin{aligned}
\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\alpha\delta)^k\left(P_{t+k}C_{t+k}\right)^{-1}\tilde{Y}_{H,t+k}\left(\tilde{P}_{H,t}-\zeta MC^n_{H,t+k}\right)\right] &= 0,\\
\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\alpha\delta)^k\left(P_{t+k}C_{t+k}\right)^{-1}\tilde{Y}_{\mathcal{N},t+k}\left(\tilde{P}_{\mathcal{N},t}-\zeta MC^n_{\mathcal{N},t+k}\right)\right] &= 0,\\
\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\alpha\delta)^k\left(P^*_{t+k}C^*_{t+k}\right)^{-1}\tilde{Y}_{F,t+k}\left(\tilde{P}_{F,t}-\zeta MC^n_{F,t+k}\right)\right] &= 0,\\
\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\alpha\delta)^k\left(P^*_{t+k}C^*_{t+k}\right)^{-1}\tilde{Y}^*_{\mathcal{N},t+k}\left(\tilde{P}^*_{\mathcal{N},t}-\zeta MC^{n*}_{\mathcal{N},t+k}\right)\right] &= 0.
\end{aligned}
\tag{A.14}
$$

The first and the second equalities in Eq.(A.14) are Eq.(9) in the text.

We define the real marginal costs as:

$$
\begin{aligned}
MC_{H,t} \equiv \frac{MC^n_{H,t}}{P_{P,t}} \quad &; \quad MC_{\mathcal{N},t} \equiv \frac{MC^n_{\mathcal{N},t}}{P_{P,t}},\\
MC_{F,t} \equiv \frac{MC^n_{F,t}}{P^*_{P,t}} \quad &; \quad MC^*_{\mathcal{N},t} \equiv \frac{(MC^*)^n_{\mathcal{N},t}}{P^*_{P,t}},
\end{aligned}
\tag{A.15}
$$

with $P^*_P \equiv \frac{P_{F,t}Y_{F,t}+P^*_{\mathcal{N},t}Y^*_{\mathcal{N},t}}{Y_{F,t}+Y^*_{\mathcal{N},t}}$.

Combining the first equalities of Eqs.(A.14) and (A.15) yields:

$$
\mathrm{E}_t\left\{\sum_{k=0}^{\infty}(\alpha\delta)^k\left[\tilde{\mathsf{X}}^{-(\theta-1)}_{H,t+k}\mathsf{X}^{-(\eta-1)}_{T,t+k}-\zeta\tilde{\mathsf{X}}^{-\theta}_{H,t+k}\mathsf{X}^{-1}_{H,t+k}\mathsf{X}^{-\eta}_{\mathcal{T},t+k}\mathsf{X}_{P,t+k}MC_{H,t+k}\right]\right\}=0,
\tag{A.16}
$$

with $\tilde{\mathsf{X}}_{H,t+k} \equiv \frac{\tilde{P}_{H,t}}{P_{H,t+k}}$, $\mathsf{X}_{H,t+k} \equiv \frac{P_{H,t+k}}{P_{\mathcal{T},t+k}}$, $\mathsf{X}_{\mathcal{T},t+k} \equiv \frac{P_{\mathcal{T},t+k}}{P_{t+k}}$ and $\mathsf{X}_{P,t+k} \equiv \frac{P_{P,t+k}}{P_{t+k}}$.

Combining the definition of the marginal cost and Eq.(A.9), we have:

$$
\begin{aligned}
MC_{H,t} = \frac{C_tN_t^{\varphi}P_t}{(1-\tau)P_{P,t}A_{H,t}}, \quad &; \quad MC_{\mathcal{N},t} = \frac{C_tN_t^{\varphi}P_t}{(1-\tau)P_{P,t}A_{\mathcal{N},t}},\\
MC_{F,t} = \frac{C^*_t\left(N^*_t\right)^{\varphi}P^*_t}{(1-\tau)P^*_{P,t}A_{F,t}}, \quad &; \quad MC^*_{\mathcal{N},t} = \frac{C^*_t\left(N^*_t\right)^{\varphi}P^*_t}{(1-\tau)P^*_{P,t}A^*_{\mathcal{N},t}}.
\end{aligned}
\tag{A.17}
$$

We define the country-wide real marginal cost as:

$$
\begin{aligned}
MC_t &\equiv \frac{MC_{H,t}Y_{H,t}+MC_{\mathcal{N},t}Y_{\mathcal{N},t}}{Y_{H,t}+Y_{\mathcal{N},t}},\\
MC^*_t &\equiv \frac{MC_{F,t}Y_{F,t}+MC^*_{\mathcal{N},t}Y^*_{\mathcal{N},t}}{Y_{H,t}+Y_{\mathcal{N},t}}.
\end{aligned}
$$

A.3 Local Government

The government expenditure index is given by:

$$G_{H,t}\equiv\left(\int_0^1 G_{H,t}(h)^{\frac{\theta-1}{\theta}}dh\right)^{\frac{\theta}{\theta-1}},\quad;\quad G_{\mathcal{N},t}\equiv\left(\int_0^1 G_{\mathcal{N},t}(h)^{\frac{\theta-1}{\theta}}df\right)^{\frac{\theta}{\theta-1}},$$
$$G_{F,t}\equiv\left(\int_1^2 G_{F,t}(f)^{\frac{\theta-1}{\theta}}dh\right)^{\frac{\theta}{\theta-1}},\quad;\quad G^*_{\mathcal{N},t}\equiv\left(\int_1^2 G^*_{\mathcal{N},t}(f)^{\frac{\theta-1}{\theta}}df\right)^{\frac{\theta}{\theta-1}},$$

where $G_{F,t}$ and $G^*_{\mathcal{N},t}$ denote government expenditure on tradables and non-tradables produced in country F, respectively. For simplicity, we assume that government purchases are fully allocated to a domestically produced good. For any given level of public consumption, the government allocates expenditures across goods in order to minimize total cost. This yields the following set of government demand schedules, analogous to those associated with private consumption.

$$G_{H,t}(h)=\left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta}G_{H,t}\quad;\quad G_{\mathcal{N},t}(h)=\left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)^{-\theta}G_{\mathcal{N},t},$$
$$G_{F,t}(f)=\left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta}G_{F,t}\quad;\quad G^*_{\mathcal{N},t}=\left(\frac{P^*_{\mathcal{N},t}(f)}{P^*_{\mathcal{N},t}}\right)^{-\theta}G^*_{\mathcal{N},t}.\quad\text{(A.18)}$$

The flow government budget constraints are given by:

$$\begin{aligned}B_t^n &= R_{t-1}B_{t-1}^n-\left\{\int_0^1 P_{H,t}(h)\left[\tau Y_{H,t}(h)-G_{H,t}(h)\right]dh\right.\\ &\quad\left.+\int_0^1 P_{\mathcal{N},t}(h)\left[\tau Y_{\mathcal{N},t}(h)-G_{\mathcal{N},t}(h)\right]dh\right\}\\ B_t^{n*} &= R_{t-1}B_{t-1}^{n*}-\left\{\int_1^2 P_{F,t}(h)\left[\tau Y_{F,t}(f)-G_{F,t}(h)\right]dh\right.\\ &\quad\left.+\int_1^2 P^*_{\mathcal{N},t}(h)\left[\tau Y^*_{\mathcal{N},t}(h)-G^*_{\mathcal{N},t}(h)\right]dh\right\}\end{aligned}\quad\text{(A.19)}$$

where $B_t^{n*}\equiv P_t^*B_t^*$ denote the nominal risk-free bonds issued by local government in country F and B_t^* denote the real risk-free bonds issued by local government in country F, respectively.

Combining the definition of prices and output, we have:

$$Y_{H,t}(h)=\left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta}Y_{H,t},\quad;\quad Y_{\mathcal{N},t}(h)=\left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)^{-\theta}Y_{\mathcal{N},t},$$
$$Y_{F,t}(f)=\left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta}Y_{F,t},\quad;\quad Y^*_{H,t}(h)=\left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta}Y^*_{H,t}.\text{(A.20)}$$

Substituting Eqs.(A.18) and (A.20) into Eq.(A.19), we have:

$$\begin{aligned}B_t^n &= R_{t-1}B_{t-1}^n-\left[\tau\left(P_{H,t}Y_{H,t}+P_{\mathcal{N},t}Y_{\mathcal{N},t}\right)-\left(P_{H,t}G_{H,t}+P_{\mathcal{N},t}G_{\mathcal{N},t}\right)\right],\\ B_t^{n*} &= R_{t-1}B_{t-1}^{n*}-\left[\tau\left(P_{F,t}Y_{F,t}+P^*_{\mathcal{N},t}Y^*_{\mathcal{N},t}\right)-\left(P_{F,t}G_{F,t}+P^*_{\mathcal{N},t}G^*_{\mathcal{N},t}\right)\right].\end{aligned}$$

These equalities can be rewritten as:

$$
\begin{aligned}
B_t^n &= R_{t-1}B_{t-1}^n - \left[\tau P_{P,t}\left(Y_{H,t}+Y_{\mathcal{N},t}\right) - P_{G,t}\left(G_{H,t}+G_{\mathcal{N},t}\right)\right], \\
B_t^{n*} &= R_{t-1}B_{t-1}^{n*} - \left[\tau P_{P,t}^*\left(Y_{F,t}+Y_{\mathcal{N},t}^*\right) - P_{G,t}^*\left(G_{F,t}+G_{\mathcal{N},t}^*\right)\right],
\end{aligned}
\tag{A.21}
$$

with $P_{G,t}^* \equiv \frac{P_{F,t}G_{F,t}+P_{\mathcal{N},t}^*G_{\mathcal{N},t}^*}{G_{F,t}+G_{\mathcal{N},t}^*}$. The first equality in Eq.(A.21) is Eq.(10) in the text.

Eq.(A.21) yields the consolidated government budget constraint which is given by:

$$
\begin{aligned}
\frac{1}{2}\left(B_t^n+B_t^{n*}\right) &= R_{t-1}\frac{1}{2}\left(B_{t-1}^n+B_{t-1}^{n*}\right) - \frac{1}{2}\left\{\left[\tau P_{P,t}\left(Y_{H,t}+Y_{\mathcal{N},t}\right) - P_{G,t}\left(G_{H,t}+G_{\mathcal{N},t}\right)\right],\right. \\
&\quad \left. + \left[\tau P_{P,t}^*\left(Y_{F,t}+Y_{\mathcal{N},t}^*\right) - P_{G,t}^*\left(G_{F,t}+G_{\mathcal{N},t}^*\right)\right]\right\}.
\end{aligned}
\tag{A.22}
$$

The appropriate transversality conditions for government assets are given by:

$$
\lim_{k\to\infty} \mathrm{E}_t Q_{t,k}\frac{1}{2}\left(B_k^n+B_k^{*n}\right) = 0,
$$

which appears in footnote 11 in the text.

Starting from Eq.(A.22) with the appropriate transversality condition, the resulting consolidated intertemporal budget constraint can be written as:

$$
\begin{aligned}
&\frac{1}{2}R_{t-1}\left[\frac{C_t^{-1}}{\Pi_t}B_{t-1}\right. \\
&\quad \left. +\frac{\left(C_t^*\right)^{-1}}{\Pi_t^*}B_{t-1}^*\right] = \frac{1}{2}\mathrm{E}_t\left\{\sum_{k=0}^{\infty}\delta^k\left[\frac{P_{P,t+k}\tau\left(Y_{H,t+k}+Y_{\mathcal{N},t+k}\right) - P_{G,t+k}\left(G_{H,t+k}+G_{\mathcal{N},t+k}\right)}{C_{t+k}P_{t+k}}\right.\right. \\
&\qquad \left.\left. + \frac{P_{P,t+k}^*\tau\left(Y_{F,t+k}+Y_{\mathcal{N},t+k}\right) - P_{G,t+k}^*\left(G_{F,t+k}+G_{\mathcal{N},t+k}^*\right)}{C_{t+k}^*P_{t+k}^*}\right]\right\},
\end{aligned}
$$

with $\Pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}$ being the gross CPI inflation rate in country F. This can be rewritten as:

$$
\begin{aligned}
\frac{1}{2}R_{t-1}\left[\frac{C_t^{-1}}{\Pi_t}B_{t-1}+\frac{\left(C_t^*\right)^{-1}}{\Pi_t^*}B_{t-1}^*\right] &= \frac{P_{P,t}\tau\left(Y_{H,t}+Y_{\mathcal{N},t}\right) - P_{G,t}\left(G_{H,t}+G_{\mathcal{N},t}\right)}{P_tC_t} \\
&\quad + \frac{P_{P,t}^*\tau\left(Y_{F,t}+Y_{\mathcal{N},t}^*\right) - P_{G,t}^*\left(G_{F,t}+G_{\mathcal{N},t}^*\right)}{P_t^*C_t^*} \\
&\quad + \delta\mathrm{E}_tR_t\left(\frac{C_{t+1}^{-1}}{\Pi_{t+1}}B_t+\frac{\left(C_{t+1}^*\right)^{-1}}{\Pi_{t+1}^*}B_t^*\right).
\end{aligned}
\tag{A.23}
$$

A.4 Market Clearing

Market clearing conditions for tradables are given by:

$$\begin{aligned} Y_{H,t}(h) &= C_{H,t}(h) + C^*_{H,t}(h) + G_{H,t}(h), \\ Y_{F,t}(f) &= C_{F,t}(f) + C^*_{F,t}(f) + G_{F,t}(f). \end{aligned} \tag{A.24}$$

The first equality in Eq.(A.24) is the LHS of Eq.(11) in the text.

As for nontradables, equilibrium requires that:

$$\begin{aligned} Y_{\mathcal{N},t}(h) &= C_{\mathcal{N},t}(h) + G_{\mathcal{N},t}(h), \\ Y^*_{\mathcal{N},t}(f) &= C^*_{\mathcal{N},t}(f) + G^*_{\mathcal{N},t}(f). \end{aligned} \tag{A.25}$$

The first equality in Eq.(A.25) is the RHS of Eq.(11) in the text.

The market in country H for tradables clears when domestic demand is given by Eq.(A.24). As for nontradables, equilibrium requires Eq.(A.25).

Using Eqs.(A.4), (A.10) and (A.18), Eq.(A.24) can be rewritten as:

$$\begin{aligned} Y_{H,t}(h) &= \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta} \left\{\frac{\gamma}{2}\left(\frac{P_{H,t}}{P_{\mathcal{T},t}}\right)^{-1} C_t \left[\left(\frac{P_{\mathcal{T},t}}{P_t}\right)^{-\eta} + \left(\frac{P_{\mathcal{T},t}}{P^*_t}\right)^{-\eta} \mathsf{Q}_t^{-1}\right] + G_{H,t}\right\}, \\ Y_{F,t}(f) &= \left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta} \left\{\frac{\gamma}{2}\left(\frac{P_{F,t}}{P_{\mathcal{T},t}}\right)^{-1} C_t \left[\left(\frac{P_{\mathcal{T},t}}{P_t}\right)^{-\eta} + \left(\frac{P_{\mathcal{T},t}}{P^*_t}\right)^{-\eta} \mathsf{Q}_t^{-1}\right] + G_{F,t}\right\}, \end{aligned}$$

where we use the fact that $C^*_t = \frac{C_t}{\mathsf{Q}_t}$, which is derived from Eq.(A.10). Combining these equalities and Eqs.(A.4), (A.10) and (A.18), Eq.(A.24) can be rewritten as:

$$\begin{aligned} Y_{H,t} &= \frac{\gamma}{2}\left(\frac{P_{H,t}}{P_{\mathcal{T},t}}\right)^{-1} C_t \left[\left(\frac{P_{\mathcal{T},t}}{P_t}\right)^{-\eta} + \left(\frac{P_{\mathcal{T},t}}{P^*_t}\right)^{-\eta} \mathsf{Q}_t^{-1}\right] + G_{H,t}, \\ Y_{F,t} &= \frac{\gamma}{2}\left(\frac{P_{F,t}}{P_{\mathcal{T},t}}\right)^{-1} C_t \left[\left(\frac{P_{\mathcal{T},t}}{P_t}\right)^{-\eta} + \left(\frac{P_{\mathcal{T},t}}{P^*_t}\right)^{-\eta} \mathsf{Q}_t^{-1}\right] + G_{F,t}. \end{aligned} \tag{A.26}$$

Using Eqs.(A.4), (A.10) and (A.18), Eq.(A.25) can be rewritten as:

$$\begin{aligned} Y_{\mathcal{N},t}(h) &= \left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)^{-\theta} \left[(1-\gamma)\left(\frac{P_{\mathcal{N},t}}{P_t}\right)^{-\eta} C_t + G_{\mathcal{N},t}\right], \\ Y^*_{\mathcal{N},t}(f) &= \left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)^{-\theta} \left[(1-\gamma)\left(\frac{P^*_{\mathcal{N},t}}{P^*_t}\right)^{-\eta} C_t \mathsf{Q}_t^{-1} + G^*_{\mathcal{N},t}\right]. \end{aligned}$$

Combining these equalities and Eqs.(A.4), (A.10) and (A.18), Eq.(A.25) can be rewritten as:

$$\begin{aligned} Y_{\mathcal{N},t} &= (1-\gamma)\left(\frac{P_{\mathcal{N},t}}{P_t}\right)^{-\eta} C_t + G_{\mathcal{N},t}, \\ Y^*_{\mathcal{N},t} &= (1-\gamma)\left(\frac{P^*_{\mathcal{N},t}}{P^*_t}\right)^{-\eta} C_t \mathsf{Q}_t^{-1} + G^*_{\mathcal{N},t}. \end{aligned} \tag{A.27}$$

Eq.(A.26) implies that:

$$\frac{Y_{H,t}-G_{H,t}}{Y_{F,t}-G_{F,t}} = \mathsf{T}_t,$$

where $\mathsf{T}_t \equiv \frac{P_{F,t}}{P_{H,t}}$ denotes the terms of trade (TOT).

Eq.(A.27) implies that:

$$\frac{Y_{\mathcal{N},t}-G_{\mathcal{N},t}}{Y^*_{\mathcal{N},t}-G^*_{\mathcal{N},t}} = \mathsf{N}_t^{\eta}\mathsf{Q}_t^{-(\eta-1)},$$

where $\mathsf{N}_t \equiv \frac{P^*_{\mathcal{N},t}}{P_{\mathcal{N},t}}$ denotes the nontradables price difference between countries H and F (NPD).

Finally, we define country-wide output and government expenditure as:

$$Y_t \equiv \frac{P_{H,t}}{P_{P,t}}Y_{H,t} + \frac{P_{\mathcal{N},t}}{P_{P,t}}Y_{\mathcal{N},t} \quad ; \quad Y^*_t \equiv \frac{P_{F,t}}{P^*_{P,t}}Y_{F,t} + \frac{P_{\mathcal{N},t}}{P^*_{P,t}}Y_{\mathcal{N},t}, \tag{A.28}$$

$$G_t \equiv \frac{P_{H,t}}{P_{G,t}}G_{H,t} + \frac{P_{\mathcal{N},t}}{P_{G,t}}G_{\mathcal{N},t} \quad ; \quad G^*_t \equiv \frac{P_{F,t}}{P_{G,t}}G_{F,t} + \frac{P^*_{\mathcal{N},t}}{P^*_{G,t}}G_{\mathcal{N},t}. \tag{A.29}$$

The LHS equalities in Eqs.(A.28) and (A.29) are Eq.(12) in the text.

A.5 Net Exports

Following Gali and Monacelli[3], we define net exports in country H as follows:

$$NX_t \equiv Y_t - \frac{P_t}{P_{P,t}}C_t - \frac{P_{G,t}}{P_{P,t}}G_t, \tag{A.30}$$

where NX_t denotes net exports in country H.

B Nonstochastic Steady State

We focus on equilibria where the state variables follow paths that are close to a deterministic stationary equilibrium, in which $\Pi_{H,t} = \Pi_{\mathcal{N}} = \Pi_{F,t} = \Pi^*_{\mathcal{N},t} = 1$ with $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$, $\Pi_{\mathcal{N},t} \equiv \frac{P_{\mathcal{N},t}}{P_{\mathcal{N},t-1}}$, $\Pi_{F,t} \equiv \frac{P_{F,t}}{P_{F,t-1}}$ and $\Pi^*_{\mathcal{N},t} \equiv \frac{P^*_{\mathcal{N},t}}{P^*_{\mathcal{N},t-1}}$ where variables without the subscript indicating the period denote their nonstochastic steady state value. These imply that the PPI inflation rate is zero in this steady state. Note that $\tilde{\mathsf{X}}_H = \tilde{\mathsf{X}}_{\mathcal{N}} = \tilde{\mathsf{X}}_F = \tilde{\mathsf{X}}^*_{\mathcal{N}} = 1$ is applied in this steady state with $\tilde{\mathsf{X}}_{H,t} \equiv \frac{\tilde{P}_{H,t}}{P_{H,t}}$, $\tilde{\mathsf{X}}_{\mathcal{N},t} \equiv \frac{\tilde{P}_{\mathcal{N},t}}{P_{\mathcal{N},t}}$, $\tilde{\mathsf{X}}_{F,t} \equiv \frac{\tilde{P}_{F,t}}{P_{F,t}}$ and $\tilde{\mathsf{X}}^*_{\mathcal{N},t} \equiv \frac{\tilde{P}^*_{\mathcal{N},t}}{P^*_{\mathcal{N},t}}$. Because this steady state is nonstochastic, all productivities are unit values, i.e., $A_H = A_{\mathcal{N}} = A_F = A^*_{\mathcal{N}} = 1$. In addition, we assume that $G_H = G_F$, $G_{\mathcal{N}} = G^*_{\mathcal{N}}$ and $B = B^*$ in this steady state.

In this steady state, the gross nominal interest rate is equal to the inverse of the subjective discount factor, as follows:

$$R = \delta^{-1}.$$

Eq.(A.14) can be rewritten as:

$$\begin{aligned} \tilde{P}_{H,t} &= \mathrm{E}_t\left(\frac{K_{H,t}}{P_{H,t}^{-1}F_{H,t}}\right) \ ; \ \tilde{P}_{\mathcal{N},t} = \mathrm{E}_t\left(\frac{K_{\mathcal{N},t}}{P_{\mathcal{N},t}^{-1}F_{\mathcal{N},t}}\right) \\ \tilde{P}_{F,t} &= \mathrm{E}_t\left(\frac{K_{F,t}}{P_{F,t}^{-1}F_{F,t}}\right) \ ; \ \tilde{P}^*_{\mathcal{N},t} = \mathrm{E}_t\left(\frac{K^*_{\mathcal{N},t}}{P^*_{\mathcal{N},t}F^*_{\mathcal{N},t}}\right), \end{aligned} \tag{B.1}$$

with:

$$\begin{aligned} K_{H,t} &\equiv \zeta\sum_{k=0}^{\infty}(\alpha\delta)^k\left(P_{t+k}C_{t+k}\right)^{-1}\tilde{Y}_{H,t+k}MC^n_{H,t+k} \quad ; \quad F_{H,t} \equiv P_{H,t}\sum_{k=0}^{\infty}(\alpha\delta)^k\left(P_{t+k}C_{t+k}\right)^{-1}\tilde{Y}_{H,t+k} \\ K_{\mathcal{N},t} &\equiv \zeta\sum_{k=0}^{\infty}(\alpha\delta)^k\left(P_{t+k}C_{t+k}\right)^{-1}\tilde{Y}_{\mathcal{N},t+k}MC^n_{\mathcal{N},t+k} \quad ; \quad F_{\mathcal{N},t} \equiv P_{\mathcal{N},t}\sum_{k=0}^{\infty}(\alpha\delta)^k\left(P_{t+k}C_{t+k}\right)^{-1}\tilde{Y}_{\mathcal{N},t+k} \\ K_{F,t} &\equiv \zeta\sum_{k=0}^{\infty}(\alpha\delta)^k\left(P^*_{t+k}C^*_{t+k}\right)^{-1}\tilde{Y}_{F,t+k}MC^{n*}_{F,t+k} \quad ; \quad F_{F,t} \equiv P_{F,t}\sum_{k=0}^{\infty}(\alpha\delta)^k\left(P^*_{t+k}C^*_{t+k}\right)^{-1}\tilde{Y}_{F,t+k} \\ K^*_{\mathcal{N},t} &\equiv \zeta\sum_{k=0}^{\infty}(\alpha\delta)^k\left(P^*_{t+k}C^*_{t+k}\right)^{-1}\tilde{Y}^*_{\mathcal{N},t+k}MC^{n*}_{H,t+k} \quad ; \quad F^*_{\mathcal{N},t} \equiv \sum_{k=0}^{\infty}(\alpha\delta)^k\left(P^*_{t+k}C^*_{t+k}\right)^{-1}\tilde{Y}^*_{\mathcal{N},t+k}. \end{aligned} \tag{B.2}$$

Eq.(B.2) implies that:

$$\begin{aligned} K_H &= \frac{\zeta Y_H MC^n_H}{(1-\alpha\delta)(PC)} \quad ; \quad F_H = \frac{P_H Y_H}{(1-\alpha\delta)(PC)} \\ K_{\mathcal{N}} &= \frac{\zeta Y_{\mathcal{N}} MC^n_{\mathcal{N}}}{(1-\alpha\delta)(PC)} \quad ; \quad F_{\mathcal{N}} = \frac{P_{\mathcal{N}} Y_{\mathcal{N}}}{(1-\alpha\delta)(PC)} \\ K_F &= \frac{\zeta Y_F MC^n_F}{(1-\alpha\delta)(P^*C^*)} \quad ; \quad F_F = \frac{P_F Y_F}{(1-\alpha\delta)(P^*C^*)} \\ K^*_{\mathcal{N}} &= \frac{\zeta Y^*_{\mathcal{N}} MC^{n*}_{\mathcal{N}}}{(1-\alpha\delta)(P^*C^*)} \quad ; \quad F_{\mathcal{N}} = \frac{P^*_{\mathcal{N}} Y^*_{\mathcal{N}}}{(1-\alpha\delta)(P^*C^*)}. \end{aligned}$$

These equalities and Eq.(B.1) imply that:

$$P_H = \zeta MC^n \ ; \ P_{\mathcal{N}} = \zeta MC^n \ ; \ P_F = \zeta MC^{n*} \ ; \ P^*_{\mathcal{N}} = \zeta MC^{n*}, \tag{B.3}$$

where we use the equalities as follows:

$$MC^n_H = MC^n_{\mathcal{N}} \equiv MC^n \ ; \ MC^n_F = MC^{n*}_{\mathcal{N}} \equiv MC^{n*},$$

which are implied by Eq.(A.17). These equalities imply that:

$$P_H = P_{\mathcal{N}},$$
$$P_F = P_{\mathcal{N}}^*.$$

Combining these equalities and the definition of $P_{P,t}$, $P_{P,t}^*$, $P_{G,t}$ and $P_{G,t}^*$, we have:

$$\begin{aligned} P_P &= P_H = P_{\mathcal{N}} = P_G, \\ P_P^* &= P_F = P_{\mathcal{N}}^* = P_G^*. \end{aligned} \tag{B.4}$$

Following Gali and Monacelli[3], we assume that PPP (purchasing power parity) holds in the steady state, which means that:

$$\mathsf{Q} = 1. \tag{B.5}$$

Eqs.(B.4) and (B.5) imply the following:

$$P = P^* = P_{\mathcal{T}} = P_{\mathcal{N}} = P_{\mathcal{N}}^* = P_H = P_F = P_P = P_P^* = P_G = P_G^*. \tag{B.6}$$

Note that because $P_F = P_H$ and $P_{\mathcal{N}} = P_{\mathcal{N}}^*$, we have:

$$\mathsf{T} = \mathsf{N} = 1. \tag{B.7}$$

Because of Eqs.(B.3), Eq.(B.4) can be rewritten as:

$$MC^n = MC^{n*}.$$

Thus, we have:

$$MC = MC^* = \zeta^{-1},$$

with $MC \equiv \frac{MC^n}{P}$ and $MC^* \equiv \frac{MC^{n*}}{P}$.

Furthermore, Eqs.(A.17) and (B.4) imply the following:

$$CN^{\varphi} = C^* \left(N^*\right)^{\varphi} = \frac{1-\tau}{\zeta}. \tag{B.8}$$

Eq.(B.8) implies the familiar expression:

$$\begin{aligned} (1-\tau)\, U_C\left(C\right) &= \zeta U_N\left(N\right), \\ (1-\tau)\, U_C\left(C^*\right) &= \zeta U_N\left(N^*\right). \end{aligned} \tag{B.9}$$

Note that because $\tau \in (0,1)$ and $\theta > 1$, this steady state is distorted.

Eq.(A.26) can be rewritten as:

$$Y_H = \gamma C + G_H \quad ; \quad Y_F = \gamma C + G_F, \tag{B.10}$$

by using Eq.(B.6). Because $G_H = G_F$, $Y_H = Y_F$. As with Eq.(B.6), Eq.(A.29) can be rewritten as:

$$Y_{\mathcal{N}} = (1-\gamma)\, C + G_{\mathcal{N}} \quad ; \quad Y_{\mathcal{N}}^* = (1-\gamma)\, C + G_{\mathcal{N}}^*. \tag{B.11}$$

Because $G_{\mathcal{N}} = G^*_{\mathcal{N}}$, $Y_{\mathcal{N}} = Y^*_{\mathcal{N}}$.

Eq.(B.6) and Eq.(12) in the text imply the following:

$$\begin{aligned} Y &= Y_H + Y_{\mathcal{N}} \quad ; \quad Y^* = Y_F + Y^*_{\mathcal{N}}, \\ G &= G_H + G_{\mathcal{N}} \quad ; \quad G^* = G_F + G^*_{\mathcal{N}}. \end{aligned} \tag{B.12}$$

Combining Eqs.(B.11) and (B.12), we have:

$$Y = C + G \quad ; \quad Y^* = C + G^*. \tag{B.13}$$

Because $G_H = G_F$ and $G_{\mathcal{N}} = G^*_{\mathcal{N}}$, Eq.(B.12) implies $G = G^*$. Thus,

$$Y = Y^*.$$

Eqs.(A.10) and (B.5) imply that:

$$C = C^*. \tag{B.14}$$

Eqs.(B.8) and (B.14) imply the following:

$$N = N^*.$$

Eq.(A.21) yields the following:

$$B\left(\frac{1-\delta}{\delta}\right) = \tau Y - G, \tag{B.15}$$

with $B \equiv \frac{B^n}{P}$. This equality implies $B = B^*$.

We assume $B > 0$; thus, another transversality condition for local government is given by:

$$\lim_{k\to\infty} \mathrm{E}_t \left[\delta^{k-t} U_C\left(C\right) RB\right] = 0, \tag{B.16}$$

which appears in footnote 11 in the text.

C Log-linearization of the Model

C.1 Aggregate Demand and Output

Log-linearizing Eq.(7) in the text, we obtain the following:

$$c_t^R = \mathsf{q}_t, \tag{C.1}$$

where q_t denotes the logarithmic CPI differential between the two countries.

Log-linearizing Eq.(A.7) and rearranging yields:

$$\mathsf{q}_t = (1-\gamma)\,\mathsf{n}_t. \tag{C.2}$$

Log-linearizing and manipulating Eq.(A.7), we obtain:

$$\pi_t = \gamma \pi_{\mathcal{T},t} + (1-\gamma)\,\pi_{\mathcal{N},t}, \tag{C.3}$$

with $\pi_{\mathcal{T},t} = \frac{1}{2}\pi_{H,t} + \frac{1}{2}\pi_{F,t}$ which is derived by log-linearizing the definition of the price index of tradables, where π_t denotes the CPI inflation rate in country H, $\pi_{\mathcal{T},t}$ denotes the tradable goods price inflation rate, $\pi_{H,t}$ and $\pi_{F,t}$ denote the inflation rates of tradables produced in countries H and F, respectively, and $\pi_{\mathcal{N},t}$ denotes the inflation rate of nontradables produced in country H.

Log-linearizing the definition of PPI, we have:

$$p_{P,t} = \gamma p_{H,t} + (1-\gamma)\,p_{\mathcal{N},t}. \tag{C.4}$$

This equality implies that:

$$\pi_{P,t} = \gamma \pi_{H,t} + (1-\gamma)\,\pi_{\mathcal{N},t}, \tag{C.5}$$

where $\pi_{P,t}$ denotes the PPI inflation rate in country H.

Log-linearizing Eq.(A.28), we have:

$$\begin{aligned} y_t &= \gamma y_{H,t} + \gamma p_{H,t} - \gamma p_{P,t} + (1-\gamma)\,y_{\mathcal{N},t} + (1-\gamma)\,p_{\mathcal{N},t} - (1-\gamma)\,p_{P,t} \\ &= \gamma y_{H,t} + (1-\gamma)\,y_{\mathcal{N},t} + \gamma p_{H,t} + (1-\gamma)\,p_{\mathcal{N},t} - p_{P,t}. \end{aligned}$$

Substituting Eq.(C.4) into this equality, we have:

$$\begin{aligned} y_t &= \gamma y_{H,t} + (1-\gamma)\,y_{\mathcal{N},t} + p_{P,t} - p_{P,t}, \\ &= \gamma y_{H,t} + (1-\gamma)\,y_{\mathcal{N},t}. \end{aligned} \tag{C.6}$$

Log-linearizing the definition of the average price of goods purchased by the government in country H yields:

$$p_{G,t} = \gamma p_{H,t} + (1-\gamma)\,p_{\mathcal{N},t}, \tag{C.7}$$

which implies that $p_{P,t} = p_{G,t}$.

Combining the log-linearized LHS of Eq.(A.29) and Eq.(C.7), we have:

$$g_t = \gamma g_{H,t} + (1-\gamma)\,g_{\mathcal{N},t}. \tag{C.8}$$

Log-linearizing the first equalities of Eqs.(A.26) and (A.27) and substituting these equalities into Eq.(C.6), we have:

$$y_t = (1-\sigma_G)\,c_t + \frac{(1-\sigma_G)\,\gamma}{2}\mathsf{t}_t + \frac{(1-\sigma_G)\,\psi}{2}\mathsf{n}_t + \sigma_G g_t. \tag{C.9}$$

Subtracting the counterpart of Eq.(C.9) in country F from Eq.(C.9), we have:

$$y_t^R = \gamma\,(1-\sigma_G)\,\mathsf{t}_t + (1-\gamma)\,\varpi\,(1-\sigma_G)\,\mathsf{n}_t + \sigma_G g_t^R. \tag{C.10}$$

Log-linearizing Eq.(A.23), we have:

$$\begin{aligned} b_t &= \mathrm{E}_t c_{t+1} - c_t - \frac{1}{\delta}\pi_t + \mathrm{E}_t \pi_{t+1} + \frac{1}{\delta}\hat{r}_{t-1} - \hat{r}_t + \frac{1}{\delta} b_{t-1} + \left(\frac{1-\delta}{\delta}\right)\frac{\gamma}{2}\mathsf{t}_t \\ &\quad - \frac{\tau}{\sigma_B} y_t + \frac{\sigma_G}{\sigma_B} g_t. \end{aligned} \tag{C.11}$$

Combining Eqs.(C.3), (C.6), (C.8), (C.10), (C.11) and the counterpart of Eq.(C.11), we have:

$$\begin{aligned} y_t^W &= \frac{\beta_W}{1-\sigma_G}\mathrm{E}_t y_{t+1}^W + \beta_W \mathrm{E}_t \pi_{t+1}^W - \beta_W \hat{r}_t + \frac{\beta_W}{\delta}\hat{r}_{t-1} - \beta_W b_t^W \\ &\quad + \frac{\beta_W}{\delta} b_{t-1}^W - \frac{\beta_W}{\delta}\pi_t^W + \sigma_G \nu_W g_t^W, \end{aligned} \tag{C.12}$$

$$\begin{aligned} y_t^R &= -\beta_R \delta b_t^R + \beta_R (1-\gamma)\upsilon \mathsf{n}_t - \beta_R (1-\gamma)\mathsf{n}_{t-1} + \beta_R b_{t-1}^R \\ &\quad + \sigma_G \nu_R g_t^R. \end{aligned} \tag{C.13}$$

where $\hat{r}_t \equiv \frac{dR_t}{R}$ denotes the deviation of the nominal interest rate from its steady-state value; π_t denotes the CPI inflation rate in country H; n_t denotes the logarithmic nontradables price disparity between countries H and F (NPD) with $\mathsf{N}_t \equiv \frac{P^*_{\mathcal{N},t}}{P_{\mathcal{N},t}}$, $\beta_W \equiv \frac{(1-\sigma_G)\sigma_B}{\sigma_B+(1-\sigma_G)\tau}$, $\beta_R \equiv \frac{\sigma_B(1-\sigma_G)}{(1-\sigma_G)\delta\tau-(1-\delta)\sigma_B}$, $\upsilon \equiv 1-(1-\delta)\varpi$, $\varpi \equiv 1+(\eta-1)\gamma$, $\nu_W \equiv \frac{[\sigma_B(1-\rho_G)+1-\sigma_G]}{\sigma_B+(1-\sigma_G)\tau}$, $\nu_R \equiv \frac{[(1-\sigma_G)\delta-(1-\delta)\sigma_B]}{(1-\sigma_G)\delta\tau-(1-\delta)\sigma_B}$, $\beta_{\mathcal{T}} \equiv 1-\frac{\beta_W\rho_{\mathcal{T}}}{1-\sigma_G}$, $\beta_{\mathcal{N}} \equiv 1-\frac{\beta_W\rho_{\mathcal{N}}}{1-\sigma_G}$, $\bar{\beta} \equiv \frac{(1-\sigma_G)(1+\varphi)}{\lambda}$, $\lambda \equiv 1+(1-\sigma_G)\varphi$, $\varsigma_W \equiv \nu_W + \frac{\beta_W\rho_G}{(1-\sigma_G)\lambda} - \frac{1}{\lambda}$ and $\varsigma_R \equiv \nu_R - \frac{1}{\lambda}$, $\sigma_B \equiv \frac{B}{Y}$ and $\sigma_G \equiv \frac{G}{Y}$ being the steady-state ratio of government bonds to output and the steady-state ratio of government expenditure to output, respectively; and $\rho_G < 1$, $\rho_{\mathcal{T}} < 1$ and $\rho_{\mathcal{N}} < 1$ being the coefficient associated with exogenous processes on government expenditure, on the productivity shifter of tradables and on the productivity of nontradables, respectively.

Log-linearizing Eq.(A.30) and substituting Eq.(C.9) yields:

$$\widehat{nx}_t = \frac{(1-\sigma_G)\psi}{2}\mathsf{n}_t,$$

with $\psi \equiv (1-\gamma)\gamma(\eta-1)$ where $\widehat{nx}_t \equiv \frac{dNX_t}{Y}$ denotes the percentage deviation of the net exports in country H from the steady-state value of output. Note that this equality becomes $\widehat{nx}_t = 0$ which implies that balanced trade is definitely applied, under our benchmark parameterization, $\eta = 1$.

C.2 Aggregate Supply and Inflation

Log-linearizing Eq.(A.16), we have:

$$\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\alpha\delta)^k\left(\tilde{\mathsf{x}}_{H,t+k} + \mathsf{x}_{H,t+k} + \mathsf{x}_{\mathcal{T},t+k} - \mathsf{x}_{P,t+k} - mc_{H,t+k}\right)\right] = 0,$$

with $\tilde{\mathsf{x}}_{H,t+k} \equiv \ln \tilde{\mathsf{X}}_{H,t+k}$, $\mathsf{x}_{H,t+k} \equiv \ln \mathsf{X}_{H,t+k}$, $\mathsf{x}_{\mathcal{T},t+k} \equiv \ln \mathsf{X}_{\mathcal{T},t+k}$ and $\mathsf{x}_{P,t+k} \equiv \ln \mathsf{X}_{P,t+k}$.

Using the fact that $\tilde{\mathsf{x}}_{H,t+k} = \mathsf{x}_{H,t} - \sum_{s=1}^{k} \pi_{H,t+s}$, this can be rewritten as:

$$\mathrm{E}_t \left[\sum_{k=0}^{\infty} (\alpha\delta)^k \left(\tilde{\mathsf{x}}_{H,t} - \sum_{s=1}^{k} \pi_{H,t+s} + \mathsf{x}_{H,t+k} + \mathsf{x}_{\mathcal{T},t+k} - \mathsf{x}_{P+k,t} - mc_{H,t+k} \right) \right] = 0.$$

Furthermore, using the fact that $\sum_{k=0}^{\infty} (\alpha\delta)^k \sum_{s=1}^{k} \pi_{H,t+s} = \frac{1}{1-\alpha\delta} \sum_{k=1}^{\infty} (\alpha\delta)^k \pi_{H,t+k}$, this can be rewritten as:

$$\begin{aligned} \frac{1}{1-\alpha\delta} \tilde{\mathsf{x}}_{H,t} - \frac{1}{1-\alpha\delta} \mathrm{E}_t \sum_{k=1}^{\infty} (\alpha\delta)^k \pi_{H,t+k} + \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \mathsf{x}_{H,t+k} + \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \mathsf{x}_{\mathcal{T},t+k}, \\ -\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \mathsf{x}_{P,t+k} - \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k mc_{H,t+k} = 0. \end{aligned}$$

Rearranging this, we have:

$$\begin{aligned} \tilde{\mathsf{x}}_{H,t} &= \sum_{k=1}^{\infty} (\alpha\delta)^k \pi_{H,t+k} - (1-\alpha\delta) \sum_{k=0}^{\infty} (\alpha\delta)^k \mathsf{x}_{H,t+k} - (1-\alpha\delta) \sum_{k=0}^{\infty} (\alpha\delta)^k \mathsf{x}_{\mathcal{T},t+k}, \\ &\quad + (1-\alpha\delta) \sum_{k=0}^{\infty} (\alpha\delta)^k \mathsf{x}_{P+k,t} + (1-\alpha\delta) \sum_{k=0}^{\infty} (\alpha\delta)^k mc_{H,t+k}, \\ &= \alpha\delta\pi_{H,t+1} - (1-\alpha\delta)\mathsf{x}_{H,t} - (1-\alpha\delta)\mathsf{x}_{T,t} + (1-\alpha\delta)\mathsf{x}_{P,t}, \\ &\quad + (1-\alpha\delta) mc_{H,t} + \alpha\delta\tilde{\mathsf{x}}_{H,t+1}. \end{aligned} \tag{C.14}$$

Log-linearizing the first equality of Eq.(A.13), we have:

$$\tilde{\mathsf{x}}_{H,t} = \frac{\alpha}{1-\alpha} \pi_{H,t}. \tag{C.15}$$

Combining Eqs.(C.14) and (C.15) yields:

$$\begin{aligned} \pi_{H,t} &= \delta\pi_{H,t+1} - \kappa\mathsf{x}_{H,t} - \kappa\mathsf{x}_{T,t} + \kappa\mathsf{x}_{P,t} + \kappa mc_{H,t}, \\ &= \delta\pi_{H,t+1} + (1-\gamma)\kappa p_{\mathcal{N},t} - (1-\gamma)\kappa p_{H,t} + \kappa mc_{H,t}. \end{aligned}$$

Taking the conditional expectation at t, the second equality can be rewritten as:

$$\pi_{H,t} = \delta \mathrm{E}_t \pi_{H,t+1} + \kappa(1-\gamma) p_{\mathcal{N},t} - \kappa(1-\gamma) p_{H,t} + \kappa mc_{H,t}. \tag{C.16}$$

with $\kappa \equiv \frac{(1-\alpha)(1-\alpha\delta)}{\alpha}$. Similar to Eq.(C.16), the log-linearized second equality of Eq.(A.14) is given by:

$$\pi_{\mathcal{N},t} = \delta \mathrm{E}_t \pi_{N,t+1} - \kappa\gamma p_{\mathcal{N},t} + \kappa\gamma p_{H,t} + \kappa mc_{\mathcal{N},t}. \tag{C.17}$$

Other FONCs for firms can be log-linearized similarly.

Substituting Eqs.(C.17) and (C.16) into Eq.(C.5), we have a PPI-based inflation dynamics equation as follows:

$$\pi_{P,t} = \delta \mathrm{E}_t \pi_{P,t+1} + \kappa mc_t, \tag{C.18}$$

where we use $mc_t = \gamma mc_{H,t} + (1-\gamma)\, mc_{\mathcal{N},t}$ which is derived by log-linearizing the definition of country-wide marginal cost.

Combining Eq.(C.17) and its counterpart for country F, the nontradables inflation differential is given by:

$$\pi^R_{\mathcal{N},t} = \delta \mathrm{E}_t \pi^R_{N,t+1} + \kappa\gamma \mathsf{n}_t - \kappa\gamma \mathsf{t}_t + \kappa mc^R_{\mathcal{N},t}, \tag{C.19}$$

with

$$\pi^R_{\mathcal{N},t} \equiv -\left(\mathsf{n}_t - \mathsf{n}_{t-1}\right), \tag{C.20}$$

being relative nontradables inflation.

By log-linearizing the first equalities in Eq.(A.12) and combining it with Eq.(C.6), we have:

$$y_t = \gamma a_{H,t} + (1-\gamma)\, a_{\mathcal{N},t} + n_t, \tag{C.21}$$

where we also use the log-linearized definition of hours of work, $n_t = \gamma n_{H,t} + (1-\gamma)\, n_{\mathcal{N},t}$.

Combining log-linearized Eq.(A.17), and Eqs.(C.9) and (C.21), we have:

$$\begin{aligned} mc_{H,t} &= \frac{\lambda}{1-\sigma_G} y_t - \frac{\psi}{2}\mathsf{n}_t - (1+\varphi\gamma)\, a_{H,t} - (1-\gamma)\,\varphi a_{\mathcal{N},t} - \frac{\sigma_G}{1-\sigma_G} g_t, \\ mc_{\mathcal{N},t} &= \frac{\lambda}{1-\sigma_G} y_t - \frac{\psi}{2}\mathsf{n}_t - \varphi\gamma a_{H,t} - [1+(1-\gamma)\,\varphi]\, a_{\mathcal{N},t} - \frac{\sigma_G}{1-\sigma_G} g_t. \end{aligned} \tag{C.22}$$

Substituting Eq.(C.22) into the log-linearized definition of the marginal cost $mc_t = \gamma mc_{H,t} + (1-\gamma)\, mc_{\mathcal{N},t}$, we have:

$$mc_t = \frac{\lambda}{1-\sigma_G} y_t - \frac{\psi}{2}\mathsf{n}_t - (1+\varphi)\,\gamma a_{H,t} - (1+\varphi)(1-\gamma)\, a_{\mathcal{N},t} - \frac{\sigma_G}{1-\sigma_G} g_t. \tag{C.23}$$

Combining the second equality in Eq.(C.22) and its counterpart for country F, the logarithmic marginal cost differential associated with nontradables is given by:

$$\begin{aligned} mc^R_{\mathcal{N},t} &= \frac{\lambda}{1-\sigma_G} y^R_t - \psi \mathsf{n}_t - \varphi\gamma a_{H,t} + \varphi\gamma a_{F,t} - [1+(1-\gamma)\,\varphi]\, a_{\mathcal{N},t}, \\ &\quad + [1+(1-\gamma)\,\varphi]\, a^*_{\mathcal{N},t} - \frac{\sigma_G}{1-\sigma_G} g^R_t. \end{aligned} \tag{C.24}$$

C.3 Marginal Cost and Output Gap

Following Gali[3], we define the relationship between output, its natural level and the output gap as:

$$y_t \equiv \bar{y}_t + \tilde{y}_t, \tag{C.25}$$

where $\tilde{y}_t$ denotes the logarithmic output gap measured from its natural level, and $\bar{y}_t$ denotes the logarithmic natural output level. Under the long-run equilibrium, $\tilde{y}_t = 0$ must hold.[1]

When the fiscal authorities design their policies to reduce the distortion generated by monopolistically competitive markets, real marginal costs under the long-run equilibrium are constant, and their logarithm is given by $mc_t = 0$. In addition, under the long-run equilibrium, PPP is applied.[2] Thus, the logarithmic NPD under the long-run equilibrium is given by $\mathsf{n}_t = 0$.

Combining these facts, Eq.(C.23) implies that:

$$\bar{y}_t = \bar{\beta}\gamma a_{H,t} + \bar{\beta}\left(1-\gamma\right) a_{\mathcal{N},t} + \frac{\sigma_G}{\lambda} g_t. \tag{C.26}$$

Combining Eqs.(C.12), (C.13), (C.25) and (C.26) can be rewritten as:

$$\begin{aligned}
\tilde{y}_t^W &= \frac{\beta_W}{1-\sigma_G}\mathrm{E}_t\tilde{y}_{t+1}^W - \beta_W\hat{r}_t + \beta_W\mathrm{E}_t\pi_{t+1}^W + \frac{\beta_W}{\delta}\hat{r}_{t-1} - \beta_W b_t^W + \frac{\beta_W}{\delta}b_{t-1}^W, \\
&\quad - \frac{\beta_W}{\delta}\pi_t^W - \frac{\gamma\bar{\beta}\beta_{\mathcal{T}}}{2}a_{H,t} - \frac{\left(1-\gamma\right)\bar{\beta}\beta_{\mathcal{N}}}{2}a_{\mathcal{N},t} - \frac{\gamma\bar{\beta}\beta_{\mathcal{T}}}{2}a_{F,t}, \\
&\quad - \frac{\left(1-\gamma\right)\bar{\beta}\beta_{\mathcal{N}}}{2}a_{\mathcal{N},t}^* + \sigma_G\varsigma_W g_t^W, \qquad\qquad (\mathrm{C.27}) \\
\tilde{y}_t^R &= -\beta_R\delta b_t^R + \beta_R\left(1-\gamma\right)\upsilon\mathsf{n}_t - \beta_R\left(1-\gamma\right)\mathsf{n}_{t-1} + \beta_R b_{t-1}^R - \bar{\beta}\gamma a_{H,t}, \\
&\quad + \bar{\beta}\gamma a_{F,t} - \bar{\beta}\left(1-\gamma\right)a_{\mathcal{N},t} + \bar{\beta}\left(1-\gamma\right)a_{\mathcal{N},t}^* + \varsigma_R\sigma_G g_t^R, \qquad\qquad (\mathrm{C.28})
\end{aligned}$$

where $\tilde{y}_t \equiv y_t - \bar{y}_t$ denotes the logarithmic output gap measured from its natural level in country H and $\bar{y}_t$ denotes the logarithmic natural output level in country H, which becomes $\tilde{y}_t = 0$ under the long-run equilibrium. These equalities are Eqs.(13) and (14) in the text, respectively.

Combining Eqs.(C.18), (C.23), (C.25) and (C.26) we have:

$$\pi_{P,t} = \delta\mathrm{E}_t\pi_{P,t+1} + \frac{\kappa\lambda}{1-\sigma_G}\tilde{y}_t - \frac{\psi\kappa}{2}\mathsf{n}_t, \tag{C.29}$$

which is Eq.(15) in the text. Similar to Eq.(C.29), we have the counterpart of Eq.(C.29) in country F.

[1] Following Gali[3], nominal rigidities disappear in the long-run equilibrium.
[2] Following Gali[3], we assume a steady state where PPP is applied.

C.4 NKRD

Combining Eqs.(C.19), (C.23), (C.25) and (C.26), we have:

$$\begin{aligned}\pi^R_{\mathcal{N},t} &= \delta \mathrm{E}_t \pi^R_{N,t+1} + \kappa\varphi\tilde{y}^R_t + \kappa \mathsf{n}_t - \kappa\varphi\gamma\left(1-\bar{\beta}\right) a_{H,t} + \kappa\varphi\gamma\left(1-\bar{\beta}\right) a_{F,t}, \\ &\quad - \kappa\left[1+\varphi\left(1-\gamma\right)\left(1-\bar{\beta}\right)\right] a_{\mathcal{N},t} + \kappa\left[1+\varphi\left(1-\gamma\right)\left(1-\bar{\beta}\right)\right] a^*_{\mathcal{N},t}, \\ &\quad - \frac{\kappa\sigma_G}{1-\sigma_G}\left(1-\frac{\varphi}{\lambda}\right) g^R_t, \end{aligned} \tag{C.30}$$

which is Eq.(16) in the text.

D Welfare Criterion

Following Gali and Monacelli[3], Gali[2] and Benigno and Woodford[1], we show the derivation of the welfare criterion in the text based on the second-order approximated utility function of Eq.(A.1) in the present appendix. $\eta = 1$ is assumed through the present appendix.

This section consists of four subsections. Subsection D.1 presents the second-order Taylor expansion of the utility function. Subsection D.2 presents the second-order approximation of the FONCs for firms. Subsection D.3 eliminates the linear term and completes the derivation of the welfare criterion. Subsection D.4 discusses other details regarding the coefficients and the NKPC in terms of the welfare-relevant output gap.

D.1 Step 1: The Second-order Taylor Expansion of the Utility Function

The second-order Taylor expansion of the period utility function in Eq.(1) in the text is given by:

$$\begin{aligned}\frac{U_t - U}{U_C C} &= C^{-1}\left[C\left(c_t + \frac{1}{2}c_t^2\right) + \frac{1}{2}\frac{U_{CC}}{U_C}C^2 c_t^2 - \frac{U_N}{U_C}N\left(n_t + \frac{1}{2}n_t^2\right) + \frac{1}{2}\frac{U_{NN}}{U_C}N^2 n_t^2\right] \\ &\quad + o\left(\|\xi\|^3\right) \\ &= c_t + \frac{1}{2}c_t^2 + \frac{1}{2}\frac{U_{CC}}{U_C}C c_t^2 - \frac{U_N}{U_C}\frac{N}{C}\left(n_t + \frac{1}{2}n_t^2\right) + \frac{U_{NN}}{U_C}\frac{N^2}{C}n_t^2 \\ &\quad + o\left(\|\xi\|^3\right) \end{aligned} \tag{D.1}$$

where we assume that utility is separable by consumption and hours of work, i.e., $U_{CN} = 0$. Plugging $U_C = C^{-1}$, $U_{CC} = -C^{-2}$, $U_N = N^{\varphi}$ and $U_{NN} = \varphi N^{\varphi-1}$ into Eq.(D.1), we have:

$$\begin{aligned}\frac{U_t - U}{U_C C} &= c_t - \frac{U_N N}{U_C C}\left(n_t + \frac{1+\varphi}{2}n_t^2\right) + o\left(\|\xi\|^3\right) \\ &= \Phi\frac{N}{C}n_t + c_t - \frac{N}{C}\left[n_t + \frac{1+\varphi}{2}\left(1+\Phi\right)n_t^2\right] + o\left(\|\xi\|^3\right), \end{aligned} \tag{D.2}$$

with $\Phi \equiv 1 - \frac{1-\tau}{\zeta}$ denoting the steady state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor where we use the fact that $1 - \Phi = \frac{1-\tau}{\zeta}$ and Eq.(B.9).

Likewise, we have:

$$\frac{U_t^* - U}{U_C C} = \Phi \frac{N}{C} n_t^* + c_t^* - \frac{N}{C}\left[n_t^* + \frac{1+\varphi}{2}(1+\Phi)(n_t^*)^2\right] + o\left(\|\xi\|^3\right). \quad \text{(D.3)}$$

Eq.(A.12) can be rewritten as:

$$N_{H,t} = \frac{Y_{H,t}\mathsf{D}_{H,t}}{A_{H,t}} \quad N_{\mathcal{N},t} = \frac{Y_{\mathcal{N},t}\mathsf{D}_{\mathcal{N},t}}{A_{\mathcal{N},t}},$$

with $\mathsf{D}_{H,t} \equiv \int_0^1 \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta} dh$ and $\mathsf{D}_{\mathcal{N},t} \equiv \int_0^1 \left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)^{-\theta} dh$ where we use the fact that $\frac{\int_0^1 Y_{H,t}(h)dh}{Y_{H,t}} = \mathsf{D}_{H,t}$ and $\frac{\int_0^1 Y_{\mathcal{N},t}(h)dh}{Y_{\mathcal{N},t}} = \mathsf{D}_{\mathcal{N},t}$.

Log-linearizing these equalities, we obtain:

$$n_{H,t} = y_{H,t} + \mathsf{d}_{H,t} - a_{H,t}, \quad ; \quad n_{\mathcal{N},t} = y_{\mathcal{N},t} + \mathsf{d}_{\mathcal{N},t} - a_{\mathcal{N},t}$$

Combining these equalities with Eq.(C.6) and the log-linearized definition of country level hours of work, $n_t = \gamma n_{H,t} + (1-\gamma) n_{\mathcal{N},t}$ yields:

$$n_t = y_t + \gamma \mathsf{d}_{H,t} + (1-\gamma)\mathsf{d}_{\mathsf{N},t} - a_t. \quad \text{(D.4)}$$

Let $P_{P,t}(h) \equiv \frac{P_{H,t}(h)Y_{H,t}(h)+P_{\mathcal{N},t}(h)Y_{\mathcal{N},t}(h)}{Y_{H,t}(h)+Y_{\mathcal{N},t}(h)}$ and $P_{P,t}^*(f) \equiv \frac{P_{F,t}(f)Y_{F,t}(f)+P_{\mathcal{N},t}^*(f)Y_{\mathcal{N},t}^*(f)}{Y_{F,t}(h)+Y_{\mathcal{N},t}^*(h)}$. These yield $p_{P,t}(h) = \gamma p_{H,t} + (1-\gamma) p_{\mathcal{N},t}(h)$ and $p_{P,t}^*(f) = \gamma p_{f,t} + (1-\gamma) p_{\mathcal{N},t}^*(f)$ by log-linearizing. Taking these equalities, Eq.(D.4) can be rewritten as:

$$\begin{aligned}
n_t &= y_t + \gamma \ln \mathrm{E}_h \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta} + (1-\gamma)\ln \mathrm{E}_h \left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)^{-\theta} - a_t, \\
&= y_t - \theta \mathrm{E}_h \left[\gamma \ln\left(\frac{P_{H,t}(h)}{P_{H,t}}\right) + \ln(1-\gamma)\left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)\right] - a_t, \\
&= y_t - \theta \mathrm{E}_h \left[\gamma\left(p_{H,t}(h) - p_{H,t}\right) + (1-\gamma)\left(p_{\mathcal{N},t}(h) - p_{\mathcal{N},t}\right)\right] - a_t, \\
&= y_t - \theta \ln \mathrm{E}_h \left(\frac{P_{P,t}(h)}{P_{P,t}}\right) - a_t, \\
&= y_t + \ln \int_0^1 \left(\frac{P_{P,t}(h)}{P_{P,t}}\right)^{-\theta} dh - a_t, \\
&= y_t + \mathsf{d}_t - a_t, \qquad \text{(D.5)}
\end{aligned}$$

with $\mathsf{D}_t \equiv \int_0^1 \left(\frac{P_{P,t}(h)}{P_{P,t}}\right)^{-\theta} dh$. Likewise, we have:

$$n_t^* = y_t^* + \mathsf{d}_t^* - a_t. \quad \text{(D.6)}$$

Substituting Eqs.(D.5) and (D.6) into Eqs.(D.2) and (D.3), we have:

$$
\begin{aligned}
\frac{U_t - U}{U_C C} &= \frac{\Phi}{1-\sigma_G} y_t + c_t - \frac{1}{1-\sigma_G}\left[y_t + (1+\Phi)\,\mathsf{d}_t + \frac{(1+\varphi)(1+\Phi)}{2}\left(y_t^2 - 2y_t a_t\right)\right], \\
&\quad + \text{t.i.p.} + o\left(\|\xi\|^3\right), \\
\frac{U_t^* - U}{U_C C} &= \frac{\Phi}{1-\sigma_G} y_t^* + c_t^* - \frac{1}{1-\sigma_G}\left\{y_t^* + (1+\Phi)\,\mathsf{d}_t^* + \frac{(1+\varphi)(1+\Phi)}{2}\left[(y_t^*)^2 - 2y_t^* a_t^*\right]\right\}, \\
&\quad + \text{t.i.p.} + o\left(\|\xi\|^3\right), \qquad \text{(D.7)}
\end{aligned}
$$

with $a_t \equiv \gamma a_{H,t} + (1-\gamma)\, a_{\mathcal{N},t}$ and $a_t^* \equiv \gamma a_{F,t} + (1-\gamma)\, a_{\mathcal{N},t}^*$ where we use the fact that $\frac{N}{C} = (1-\sigma_G)^{-1}$ because $N = Y$.

Combining Eqs.(C.1), (C.2), (C.9) and (C.10), we have:

$$
c_t = \frac{1}{1-\sigma_G} y_t + \frac{1}{1-\sigma_G} y_t^* - c_t^* + \frac{2\sigma_G}{1-\sigma_G} g_t^W.
$$

Combining Eq.(D.7) and this equality yields:

$$
\begin{aligned}
\frac{U_t^W - U}{U_C C} &= \frac{\Phi}{1-\sigma_G} y_t^W - \frac{1}{(1-\sigma_G)\,2}\left\{(1+\Phi)\left(\mathsf{d}_t + \mathsf{d}_t^*\right) + \frac{(1+\varphi)(1+\Phi)}{2}\left[y_t^2\right.\right., \\
&\quad \left.\left. -2y_t a_t + (y_t^*)^2 - 2y_t^* a_t^*\right]\right\} + \text{t.i.p.} + o\left(\|\xi\|^3\right). \qquad \text{(D.8)}
\end{aligned}
$$

Let $\hat{p}_{P,t}(h) \equiv p_{P,t}(h) - p_{P,t}$. As derived by Gali and Monacelli[3], note that:

$$
\begin{aligned}
\left(\frac{P_{P,t}(h)}{P_{P,t}}\right)^{1-\theta} &= \exp\left[(1-\theta)\,\hat{p}_{P,t}(h)\right], \\
&= 1 - (1-\theta)\,\hat{p}_{P,t}(h) + \frac{(1-\theta)^2}{2}\hat{p}_{P,t}(h) + o\left(\|\xi\|^3\right) \qquad \text{(D.9)}
\end{aligned}
$$

In the symmetric equilibrium, we have $\frac{P_{P,t}(h)}{P_{P,t}} = 1$. This implies:

$$
\mathrm{E}_h\left(\frac{P_{P,t}(h)}{P_{P,t}}\right)^{1-\theta} = 1. \qquad \text{(D.10)}
$$

Combining Eqs.(D.9) and (D.10), we have:

$$
\mathrm{E}_h \hat{p}_{P,t}(h) = \frac{\theta - 1}{2}\mathrm{E}_h \hat{p}_{P,t}(h)^2. \qquad \text{(D.11)}
$$

In addition, the second-order approximation to $\left(\frac{P_{P,t}(h)}{P_{P,t}}\right)^{-\theta}$ yields:

$$
\left(\frac{P_{P,t}(h)}{P_{P,t}}\right)^{-\theta} = 1 - \theta\hat{p}_{P,t}(h) + \frac{\theta^2}{2}\hat{p}_{P,t}(h)^2 + o\left(\|\xi\|^3\right).
$$

This equality implies:

$$\mathsf{D}_t = 1 - \theta \mathrm{E}_h \hat{p}_{P,t}(h) + \frac{\theta^2}{2} \mathrm{E}_h \hat{p}_{P,t}(h)^2 + o\left(\|\xi\|^3\right).$$

Substituting Eq.(D.11) into this equality, we have:

$$\begin{aligned} \mathsf{D}_t &= 1 + \frac{\theta}{2} \mathrm{E}_h \hat{p}_{P,t}(h)^2 + o\left(\|\xi\|^3\right) \\ &= 1 + \frac{\theta}{2} \mathrm{var}_h\left(\hat{p}_{P,t}(h)\right) + o\left(\|\xi\|^3\right). \end{aligned}$$

This equality implies:

$$\mathsf{d}_t = \frac{\theta}{2} \mathrm{var}_h\left(p_{P,t}(h)\right) + o\left(\|\xi\|^3\right), \tag{D.12}$$

which clearly corresponds to the equality derived by Gali and Monacelli[3].

Lemma 1

$$\sum_{t=0}^{\infty} \delta^t \mathrm{var}_h\left(p_{P,t}(h)\right) = \frac{1}{\kappa} \sum_{t=0}^{\infty} \delta^t \pi_{P,t}^2$$

Proof: See Woodford[6], p 399–400.

Substituting *Lemma 1*, Eqs.(D.12) and (D.8) into the definition of welfare in the text, we have:

$$\begin{aligned} \mathcal{W}^W = \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t & \left[\frac{\Phi}{1-\sigma_G} \tilde{y}_t^W - \frac{(1+\Phi)\theta}{(1-\sigma_G)4} \pi_{P,t}^2 - \frac{(1+\Phi)\theta}{(1-\sigma_G)4} \left(\pi_{P,t}^*\right)^2 \right. \\ & \left. - \frac{(1+\varphi)(1+\Phi)}{(1-\sigma_G)4} (y_t - a_t)^2 - \frac{(1+\varphi)(1+\Phi)}{(1-\sigma_G)4} (y_t^* - a_t^*)^2 \right] \\ & + \text{t.i.p.} + o\left(\|\xi\|^3\right). \end{aligned} \tag{D.13}$$

Note that $\mathrm{E}_0 \sum_{k=0}^{\infty} \delta^t \frac{U_t^W - U}{U_C C} = \mathcal{W}^W$ because $U_C C = 1$ and $U = U^*$.

D.2 Step 2: The Second-order Approximation of the FONCs for Firms

Substituting the first and the second equalities in Eq.(B.1) into the first and the second equalities in Eq.(A.13), we have:

$$\begin{aligned} \frac{1}{1-\alpha}\left(1 - \alpha \Pi_{H,t}^{\theta-1}\right) &= \left(\frac{F_{H,t}}{K_{H,t}}\right)^{\theta-1} \\ \frac{1}{1-\alpha}\left(1 - \alpha \Pi_{\mathcal{N},t}^{\theta-1}\right) &= \left(\frac{F_{\mathcal{N},t}}{K_{\mathcal{N},t}}\right)^{\theta-1} \end{aligned} \tag{D.14}$$

Taking logarithms on both sides in Eq.(D.14), we have:

$$\begin{aligned}
-\log\left(\frac{1}{1-\alpha}-\frac{\alpha}{1-\alpha}\Pi_{H,t}^{\theta-1}\right) &= (\theta-1)\left(\log K_{H,t}-\log F_{H,t}\right) \\
-\log\left(\frac{1}{1-\alpha}-\frac{\alpha}{1-\alpha}\Pi_{\mathcal{N},t}^{\theta-1}\right) &= (\theta-1)\left(\log K_{\mathcal{N},t}-\log F_{\mathcal{N},t}\right) \quad \text{(D.15)}
\end{aligned}$$

The first-order approximation of the LHS in Eq.(D.15) is given by:

$$\begin{aligned}
-\log\left(\frac{1}{1-\alpha}-\frac{\alpha}{1-\alpha}\Pi_{H,t}^{\theta-1}\right) &= \frac{(\theta-1)\,\alpha}{1-\alpha}\pi_{H,t}+o\left(\|\xi\|^2\right) \\
-\log\left(\frac{1}{1-\alpha}-\frac{\alpha}{1-\alpha}\Pi_{\mathcal{N},t}^{\theta-1}\right) &= \frac{(\theta-1)\,\alpha}{1-\alpha}\pi_{\mathcal{N},t}+o\left(\|\xi\|^2\right) \quad \text{(D.16)}
\end{aligned}$$

A weighted average of the two equalities in Eq.(D.16) is given by:

$$\frac{(\theta-1)\,\alpha}{1-\alpha}\gamma\pi_{H,t}+\frac{(\theta-1)\,\alpha}{1-\alpha}(1-\gamma)\,\pi_{\mathcal{N},t}=\frac{(\theta-1)\,\alpha}{1-\alpha}\pi_{P,t},$$

where we use Eq.(C.5). The second-order approximation of the RHS of this equality yields:

$$\frac{(\theta-1)\,\alpha}{1-\alpha}\pi_{P,t}=\frac{(\theta-1)\,\alpha}{1-\alpha}\pi_{P,t}+\frac{(\theta-1)\,\alpha 3}{(1-\alpha)\,4}\pi_{P,t}^2+o\left(\|\xi\|^3\right).$$

Combining this equality and Eq.(D.15), we have:

$$\frac{(\theta-1)\,\alpha}{1-\alpha}\pi_{P,t}+\frac{(\theta-1)\,\alpha 3}{(1-\alpha)\,4}\pi_{P,t}^2=(\theta-1)\,(k_t-f_t)+o\left(\|\xi\|^3\right), \quad \text{(D.17)}$$

with $k_t \equiv \gamma k_{H,t}+(1-\gamma)\,k_{\mathcal{N},t}$ and $f_t \equiv \gamma f_{H,t}+(1-\gamma)\,f_{\mathcal{N},t}$ where we use the fact that $K=F$. Note that $K_{H,t}$, $K_{\mathcal{N},t}$, $F_{H,t}$ and $F_{\mathcal{N},t}$ are $\left(\|\xi\|^2\right)$.

Log-linearizing the first and the second equality in the LHS in Eq.(B.2) and combining them, we have:

$$k_t=\tilde{k}_t-\frac{\theta\alpha}{1-\alpha}\pi_{P,t}, \quad \text{(D.18)}$$

with $\tilde{k}_t \equiv (1-\alpha\delta)\,\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\,\tilde{k}_{t,t+k}$ and $\tilde{k}_{t,t+k} \equiv (1+\varphi)\,y_{t+k}-\frac{(1+\varphi)\sigma_G}{\lambda}a_{t+k}+\frac{(1+\varphi)\sigma_G}{\lambda}g_{t+k}+\theta\sum_{s=1}^{k}\pi_{P,t+s}$.

Log-linearizing the first and the second equality on the RHS in Eq.(B.2) and combining them, we have:

$$f_t=\tilde{f}_t-\frac{\theta\alpha}{1-\alpha}\pi_{P,t}, \quad \text{(D.19)}$$

with $\tilde{f}_t \equiv (1-\alpha\delta)\,\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\,\tilde{f}_{t,t+k}$ and $\tilde{f}_{t,t+k} \equiv -\frac{\sigma_G}{1-\sigma_G}y_{t+k}-\frac{(1+\varphi)\sigma_G}{\lambda}a_{t+k}+\frac{(1+\varphi)\sigma_G}{\lambda}g_{t+k}+(\theta-1)\sum_{s=1}^{k}\pi_{P,t+s}$.

Subtracting Eq.(D.19) from Eq.(D.18) yields:

$$k_t - f_t = \tilde{k}_t - \tilde{f}_t. \tag{D.20}$$

Substituting Eq.(D.20) into Eq.(D.17) yields:

$$\frac{(\theta-1)\,\alpha}{1-\alpha}\pi_{P,t} + \frac{(\theta-1)\,\alpha 3}{(1-\alpha)\,4}\pi_{P,t}^2 = (\theta-1)\left(\tilde{k}_t - \tilde{f}_t\right) + o\left(\|\xi\|^3\right). \tag{D.21}$$

An arbitrary variable V_t can be approximated as:

$$\begin{aligned} V_t &= e^{\ln V_t} \\ &= e^{\ln V} + e^{\ln V}\left(\ln V_t - \ln V\right) + \frac{1}{2}e^{\ln V}\left(\ln V_t - \ln V\right)^2 + o\left(\|\xi\|^3\right) \\ &= V\left(1 + v_t + \frac{1}{2}v_t^2\right) + o\left(\|\xi\|^3\right). \end{aligned}$$

Thus, we have the second-order approximation of $\tilde{k}_t$ and $\tilde{f}_t$ as follows:

$$\begin{aligned} \tilde{k}_t &= \tilde{k}_t + \frac{1}{2}\tilde{k}_t^2 \\ &= (1-\alpha\delta)\,\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\left(\tilde{k}_{t,t+k} + \frac{1}{2}\tilde{k}_{t,t+k}^2\right) \qquad \text{(D.22)} \\ \tilde{f}_t &= \tilde{f}_t + \frac{1}{2}\tilde{f}_t^2 \\ &= (1-\alpha\delta)\,\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\left(\tilde{f}_{t,t+k} + \frac{1}{2}\tilde{f}_{t,t+k}^2\right) \qquad \text{(D.23)} \end{aligned}$$

Subtracting Eq.(D.23) from Eq.(D.22) yields:

$$\begin{aligned} \tilde{k}_t - \tilde{f}_t &= (1-\alpha\delta)\,\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\left[\left(\tilde{k}_{t,t+k} - \tilde{f}_{t,t+k}\right) + \frac{1}{2}\left(\tilde{k}_{t,t+k}^2 - \tilde{f}_{t,t+k}^2\right)\right] \\ &\quad - \frac{(1-\alpha\delta)\,\alpha}{2\,(1-\alpha)}\pi_{P,t}Z_t + o\left(\|\xi\|^3\right) \qquad \text{(D.24)} \end{aligned}$$

with

$$Z_t \equiv \mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\left(\tilde{k}_{t,t+k} + \tilde{f}_{t,t+k}\right). \tag{D.25}$$

The first term on the RHS in Eq.(D.24) can be rewritten as:

$$\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\left(\tilde{k}_{t,t+k} - \tilde{f}_{t,t+k}\right) = \mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\frac{\lambda}{1-\sigma_G}\tilde{y}_{t+k} + \frac{1}{1-\alpha\delta}\mathcal{P}_{P,t} \tag{D.26}$$

with $\mathcal{P}_{P,t} \equiv \mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\pi_{P,t+k}$.

The second term on the RHS in Eq.(D.24) can be rewritten as:

$$
\begin{aligned}
\frac{1}{2}\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\left(\tilde{k}_{t,t+k}^2-\tilde{f}_{t,t+k}^2\right) &= \frac{1}{2}\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\left(\widetilde{kk}_{t,t+k}^2-\widetilde{ff}_{t,t+k}^2\right)\\
&+\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\sum_{s=1}^{k}\pi_{P,t+k}\left[\theta\widetilde{kk}_{t,t+k}-(\theta-1)\widetilde{ff}_{t,t+k}\right]\\
&+\frac{1}{2}\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\left(\sum_{s=1}^{k}\pi_{P,t+s}\right)^2\left[\theta^2-(\theta-1)^2\right] \quad \text{(D.27)}
\end{aligned}
$$

with

$$
\begin{aligned}
\widetilde{kk}_{t,t+k} &\equiv (1+\varphi)\tilde{y}_{t+k}-\frac{\sigma_G(1+\varphi)}{\lambda}a_{t+k}+\frac{\sigma_G(1+\varphi)}{\lambda}g_{t+k},\\
\widetilde{ff}_{t,t+k} &\equiv -\frac{\sigma_G}{1-\sigma_G}\tilde{y}_{t+k}-\frac{\sigma_G(1+\varphi)}{\lambda}a_{t+k}+\frac{\sigma_G(1+\varphi)}{\lambda}g_{t+k}. \quad \text{(D.28)}
\end{aligned}
$$

The last term on the RHS in Eq.(D.27) can be rewritten as:

$$
\frac{1}{2}\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\left(\sum_{s=1}^{k}\pi_{P,t+s}\right)^2\left[\theta^2-(\theta-1)^2\right]=\frac{2\theta-1}{(1-\alpha\delta)2}\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\pi_{P,t+k}\left(\pi_{P,t+k}+2\mathcal{P}_{P,t+k}\right). \tag{D.29}
$$

Furthermore, the second term on the RHS in Eq.(D.29) can be rewritten as:

$$
\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\sum_{s=1}^{k}\pi_{P,t+s}\left[\theta\widetilde{kk}_{t,t+k}-(\theta-1)\widetilde{ff}_{t,t+k}\right]=\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\pi_{P,t+k}J_{t+k}, \tag{D.30}
$$

with

$$
J_t\equiv\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\left[\theta\widetilde{kk}_{t,t+k}-(\theta-1)\widetilde{ff}_{t,t+k}\right]. \tag{D.31}
$$

Substituting Eqs.(D.29) and (D.30) into Eq.(D.28) yields:

$$
\begin{aligned}
\frac{1}{2}\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\left(\tilde{k}_{t,t+k}^2-\tilde{f}_{t,t+k}^2\right) &= \frac{1}{2}\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\left(\widetilde{kk}_{t,t+k}^2-\widetilde{ff}_{t,t+k}^2\right)\\
&+\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\pi_{P,t+k}J_{t+k} \quad \text{(D.32)}\\
&+\frac{2\theta-1}{(1-\alpha\delta)2}\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\pi_{P,t+k}\left(\pi_{P,t+k}+2\mathcal{P}_{P,t+k}\right).
\end{aligned}
$$

Substituting Eqs.(D.25) and (D.31) into Eq.(D.24), we have:

$$\begin{aligned}\tilde{k}_t - \tilde{f}_t &= \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left\{ (1-\alpha\delta) \left[\left(\widetilde{kk}_{t,t+k} - \widetilde{ff}_{t,t+k} \right) + \frac{1}{2} \left(\widetilde{kk}^2_{t,t+k} - \widetilde{ff}^2_{t,t+k} \right) \right] \right\} \\ &\quad + \mathrm{E}_t \sum_{k=1}^{\infty} (\alpha\delta)^k \pi_{P,t+k} + (1-\alpha\delta)\, \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \pi_{P,t+k} J_{t+k} \\ &\quad + \frac{2\theta-1}{2} \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left(\pi_{P,t+k} + 2\mathcal{P}_{P,t+k} \right) - \frac{(1-\alpha\delta)\,\alpha}{2(1-\alpha)} \pi_{P,t} Z_t + o\left(\|\xi\|^3 \right).\end{aligned} \tag{D.33}$$

Substituting Eq.(D.21) into Eq.(D.33) to eliminate the term $\tilde{k}_t - \tilde{f}_t$ in the LHS in Eq.(D.32) yields:

$$\begin{aligned}\pi_{P,t} + \frac{3}{4}\pi^2_{P,t} + \frac{1-\alpha\delta}{2}\pi_{P,t} Z_t &= \kappa \left(\widetilde{kk}_{t,t+k} - \widetilde{ff}_{t,t+k} \right) + \frac{(1-\alpha)(1-\alpha\delta)}{2\alpha} \left(\widetilde{kk}^2_{t,t+k} - \widetilde{ff}^2_{t,t+k} \right) \\ &\quad + (1-\alpha)\,\delta \mathrm{E}_t \pi_{P,t+1} + (1-\alpha)(1-\alpha\delta)\, \mathrm{E}_t \pi_{P,t+1} J_{t+1} \\ &\quad + \frac{(1-\alpha)(2\theta-1)}{2} \delta \mathrm{E}_t \pi_{P,t+1} \left(\pi_{P,t+1} + 2\mathcal{P}_{P,t+1} \right) \\ &\quad + \alpha\delta \mathrm{E}_t \left(\pi_{P,t+1} + \frac{3}{4}\pi^2_{P,t+1} + \frac{1-\alpha\delta}{2}\pi_{P,t+1} Z_{t+1} \right) \\ &\quad + o\left(\|\xi\|^3 \right).\end{aligned} \tag{D.34}$$

Eq.(D.31) can be rewritten as:

$$J_t = \frac{1}{2}\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left[\widetilde{kk}_{t,t+k} - \widetilde{ff}_{t,t+k} + (2\theta-1) \left(\widetilde{kk}_{t,t+k} - \widetilde{ff}_{t,t+k} \right) \right]. \tag{D.35}$$

Substituting Eqs.(D.18), (D.19) and (D.28) into Eq.(D.25), we have:

$$\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left(\widetilde{kk}_{t,t+k} - \widetilde{ff}_{t,t+k} \right) = Z_t - \frac{2\theta-1}{1-\alpha\delta} \mathcal{P}_{P,t}. \tag{D.36}$$

Substituting Eq.(D.28) into Eq.(D.26) yields:

$$\begin{aligned}\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left(\widetilde{kk}_{t,t+k} - \widetilde{ff}_{t,t+k} \right) &= \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left(\tilde{k}_{t,t+k} - \tilde{f}_{t,t+k} \right) \\ &\quad - \frac{1}{1-\alpha\delta} \mathcal{P}_{P,t},\end{aligned} \tag{D.37}$$

where we use the fact that

$$\widetilde{kk}_{t,t} - \widetilde{ff}_{t,t} = \frac{\lambda}{1-\sigma_G} \tilde{y}_t. \tag{D.38}$$

Substituting Eqs.(D.36) and (D.37) into Eq.(D.35) yields:

$$J_t = \frac{1}{2}\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left[Z_t + (2\theta - 1)\left(\tilde{k}_{t,t+k} - \tilde{f}_{t,t+k}\right) - \frac{2(\theta-1)}{1-\alpha\delta}\mathcal{P}_{P,t} \right]. \quad \text{(D.39)}$$

In the first order, Eq.(D.21) can be rewritten as:

$$\pi_{P,t} = \kappa \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left(\tilde{k}_{t,t+k} - \tilde{f}_{t,t+k}\right) + o\left(\|\xi\|^2\right).$$

Substituting this equality into Eq.(D.39), we have:

$$J_t = \frac{1}{2} Z_t + \frac{\alpha(2\theta-1)}{2\kappa}\pi_{P,t} - \frac{2\theta-1}{1-\alpha\delta}\mathcal{P}_{P,t}.$$

Substituting this equality into Eq.(D.34) yields:

$$\begin{aligned}
\pi_{P,t} + \frac{3}{4}\pi_{P,t}^2 + \frac{1-\alpha\delta}{2}\pi_{P,t} Z_t \quad = \quad & \kappa \left[\widetilde{kk}_{t,t} - \widetilde{ff}_{t,t} + \frac{1}{2}\left(\widetilde{kk}_{t,t}^2 - \widetilde{ff}_{t,t}^2\right)\right] \\
& + \delta \mathrm{E}_t \pi_{P,t+1} + \frac{(1-\alpha\delta)\delta}{2}\mathrm{E}_t \pi_{P,t+1} Z_{t+1} \\
& + \frac{3\delta}{4}\mathrm{E}_t \pi_{P,t+1}^2 + \frac{\delta\Theta}{4}\mathrm{E}_t \pi_{P,t+1}^2 \\
& + o\left(\|\xi\|^3\right).
\end{aligned}$$

with $\Theta \equiv \alpha(4\theta - 1) - 3$. Adding $\frac{\Theta}{4}\pi_{P,t}^2$ to both sides in this equality, we have:

$$\mathcal{M}_t = \kappa \left[\widetilde{kk}_{t,t} - \widetilde{ff}_{t,t} + \frac{1}{2}\left(\widetilde{kk}_{t,t}^2 - \widetilde{ff}_{t,t}^2\right)\right] + \delta \mathrm{E}_t \mathcal{M}_{t+1} + \frac{\Theta}{4}\pi_{P,t}^2, \quad \text{(D.40)}$$

with $\mathcal{M}_t \equiv \pi_{P,t} + \frac{3}{4}\pi_{P,t}^2 + \frac{1-\alpha\delta}{2}\pi_{P,t} Z_t + \frac{\Theta}{4}\pi_{P,t}^2$. Substituting Eq.(D.38) into Eq.(D.40), we have:

$$\pi_{P,t} = \delta \pi_{P,t+1} + \frac{\lambda}{1-\sigma_G}\tilde{y}_t + o\left(\|\xi\|^2\right)$$

in the first order. Thus, Eq.(D.40) corresponds to the second-order approximated NKPC in Benigno and Woodford[1]. Iterating Eq.(D.40) forward, we have:

$$\mathcal{M} = \kappa \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \left[\widetilde{kk}_{t,t} - \widetilde{ff}_{t,t} + \frac{1}{2}\left(\widetilde{kk}_{t,t}^2 - \widetilde{ff}_{t,t}^2\right)\right] + \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \frac{\Theta}{4}\pi_{P,t}^2, \quad \text{(D.41)}$$

with $\mathcal{M} \equiv \mathcal{M}_0$ where we take an expectation in period zero.

Eq.(D.28) implies:

$$\begin{aligned}
\widetilde{kk}_{t,t}^2 - \widetilde{ff}_{t,t}^2 \quad = \quad & \tilde{\omega}_1 \tilde{y}_t^2 - \frac{2\sigma_G(1+\varphi)}{(1-\sigma_G)\lambda} y_t a_t + \frac{2\sigma_G(1+\varphi)}{(1-\sigma_G)\lambda} y_t g_t + \text{t.i.p.} \\
& = \tilde{\omega}_1 \left[\tilde{y}_t - \tilde{\omega}_4 \tilde{\omega}_2 a_t - \frac{\sigma_G}{\lambda}\tilde{\omega}_3 g_t \right]^2,
\end{aligned}$$

with $\tilde{\omega}_1 \equiv \frac{\varsigma}{(1-\sigma_G)^2}$, $\tilde{\omega}_2 \equiv 1+\frac{\sigma_G}{\varsigma}$, $\tilde{\omega}_3 \equiv 1-\frac{(1-\sigma_G)(1+\varphi)}{\varsigma}$ and $\tilde{\omega}_4 \equiv \frac{(1-\sigma_G)(1+\varphi)}{\lambda}$. Substituting this equality and Eq.(D.38) into Eq.(D.41), we have:

$$\begin{aligned}\mathcal{M} &= \kappa \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \left\{ \frac{\lambda}{1-\sigma_G} \tilde{y}_t + \frac{\tilde{\omega}_1}{2} \left[y_t - \tilde{\omega}_4 \tilde{\omega}_2 a_t - \frac{\sigma_G}{\lambda} \tilde{\omega}_3 g_t \right]^2 + \frac{\Theta}{4} \pi_{P,t}^2 \right\} \\ &\quad + \text{t.i.p.} + o\left(\|\xi\|^3 \right). \end{aligned} \tag{D.42}$$

The counterpart of Eq.(D.42) in country F is derived similarly as:

$$\begin{aligned}\mathcal{M}^* &= \kappa \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \left\{ \frac{\lambda}{1-\sigma_G} \tilde{y}_t^* + \frac{\tilde{\omega}_1}{2} \left[y_t^* - \tilde{\omega}_4 \tilde{\omega}_2 a_t^* - \frac{\sigma_G}{\lambda} \tilde{\omega}_3 g_t^* \right]^2 + \frac{\Theta}{4} \left(\pi_{P,t}^* \right)^2 \right\} \\ &\quad + \text{t.i.p.} + o\left(\|\xi\|^3 \right). \end{aligned}$$

Combining this equality and Eq.(D.42) yields:

$$\begin{aligned}\mathcal{M}^W &= \kappa \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \left\{ \frac{\lambda}{1-\sigma_G} \tilde{y}_t^W + \frac{\tilde{\omega}_1}{4} \left[\left(y_t - \tilde{\omega}_4 \tilde{\omega}_2 a_t - \frac{\sigma_G}{\lambda} \tilde{\omega}_3 g_t \right)^2 + \left(y_t^* \right.\right.\right. \\ &\quad \left.\left.\left. - \tilde{\omega}_4 \tilde{\omega}_2 a_t^* - \frac{\sigma_G}{\lambda} \tilde{\omega}_3 g_t^* \right)^2 \right] + \frac{\Theta}{8} \left[\pi_{P,t}^2 + \left(\pi_{P,t}^* \right)^2 \right] \right\} \\ &\quad + \text{t.i.p.} + o\left(\|\xi\|^3 \right). \end{aligned} \tag{D.43}$$

D.3 Step 3: Elimination of the Linear Term and Completing the Derivation

Multiplying Φ by both sides in Eq.(D.43), and subtracting this from Eq.(D.13), we have:

$$\begin{aligned}\mathcal{W}^W - \Phi \mathcal{M}^W &= \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \left\{ \frac{1}{(1-\sigma_G) 4} \left[(1+\varphi)(1+\Phi) + \kappa \Phi \tilde{\omega}_1 \right] \left[y_t^2 + \left(y_t^* \right)^2 \right] \right. \\ &\quad - (1+\varphi) \left[\frac{2\kappa \Phi (1-\sigma_G)}{\lambda} \tilde{\omega}_2 + \frac{1+\Phi}{(1-\sigma_G) 2} \right] \left(y_t a_t + y_t^* a_t^* \right) \\ &\quad - \frac{2\kappa \Phi \sigma_G}{\lambda} \tilde{\omega}_3 \left(y_t g_t + y_t^* g_t^* \right) + \frac{1}{4} \left[\frac{(1+\Phi)\theta}{(1-\sigma_G)\kappa} + \frac{\Phi \Theta}{2} \right] \left[\pi_{P,t}^2 + \left(\pi_{P,t}^* \right)^2 \right] \\ &\quad \left. + \frac{\Phi (1-\kappa \lambda)}{1-\sigma_G} \tilde{y}_t^W \right\} + \text{t.i.p.} + o\left(\|\xi\|^3 \right). \end{aligned}$$

Rearranging this equality, we have:

$$\begin{aligned}\mathcal{W}^W &= -\mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \left\{ \frac{1}{(1-\sigma_G) 4} \left[(1+\varphi)(1+\Phi) + \kappa \Phi \tilde{\omega}_1 \right] \left[y_t^2 + \left(y_t^* \right)^2 \right] \right. \\ &\quad - (1+\varphi) \left[\frac{2\kappa \Phi (1-\sigma_G)}{\lambda} \tilde{\omega}_2 + \frac{1+\Phi}{(1-\sigma_G) 2} \right] \left(y_t a_t + y_t^* a_t^* \right) \end{aligned}$$

$$-\frac{2\kappa\Phi\sigma_G}{\lambda}\tilde{\omega}_3\left(y_t g_t + y_t^* g_t^*\right) + \frac{1}{4}\left[\frac{(1+\Phi)\,\theta}{(1-\sigma_G)\,\kappa} + \frac{\Phi\Theta}{2}\right]\left[\pi_{P,t}^2 + \left(\pi_{P,t}^*\right)^2\right]\Bigg\}$$
$$+ \mathrm{E}_0\sum_{t=0}^{\infty}\delta^t\frac{\Phi\,(1-\kappa\lambda)}{1-\sigma_G}\tilde{y}_t^W + \Phi\mathcal{M}^W + \text{t.i.p.} + o\left(\|\xi\|^3\right). \quad \text{(D.44)}$$

Note that $\mathcal{M}^W = \mathrm{E}_0\sum_{t=0}^{\infty}\delta^t\frac{\kappa\lambda}{1-\sigma_G}y_t^W + o\left(\|\xi\|^2\right)$ because of Eq.(D.43). Thus, the second and the third terms in Eq.(D.44) can be rewritten as:

$$\begin{aligned}
\mathrm{E}_0\sum_{t=0}^{\infty}\delta^t\frac{\Phi\,(1-\kappa\lambda)}{1-\sigma_G}\tilde{y}_t^W + \Phi\mathcal{M}^W &= \mathrm{E}_t\sum_{k=0}^{\infty}\delta^k\frac{\Phi\,(1-\kappa\lambda)}{1-\sigma_G}\tilde{y}_t^W + \Phi\mathcal{M}^W + \frac{\Phi}{\kappa\lambda}\mathcal{M}^W - \frac{\Phi}{\kappa\lambda}\mathcal{M}^W \\
&= \mathrm{E}_0\sum_{t=0}^{\infty}\delta^t\frac{\Phi\,(1-\kappa\lambda)}{1-\sigma_G}\tilde{y}_t^W - \Phi\left(\frac{1}{\kappa\lambda}-1\right)\mathcal{M}^W + \frac{\Phi}{\kappa\lambda}\mathcal{M}^W \\
&= \frac{\Phi}{\kappa\lambda}\mathcal{M}^W \\
&= \Gamma_0. \qquad \text{(D.45)}
\end{aligned}$$

where $\Gamma_0 \equiv \frac{\Phi}{\kappa\lambda}\pi_{P,0}^W$ denotes a transitory component, which is predetermined. We use this fact to derive the above equality as follows:

$$\begin{aligned}
\pi_{P,0}^W &= \mathrm{E}_0\sum_{t=0}^{\infty}\delta^t\frac{\kappa\lambda}{1-\sigma_G}y_t^W + o\left(\|\xi\|^2\right), \\
&= \mathcal{M}^W,
\end{aligned}$$

which can be derived by iterating the second-order approximated period FONCs for firms, namely, $\pi_{P,t} = \delta\mathrm{E}_t\pi_{P,t+1} + \frac{\lambda}{1-\sigma_G}\tilde{y}_t + o\left(\|\xi\|^2\right)$.

Substituting Eq.(D.45) into Eq.(D.44) and rearranging, we have:

$$\begin{aligned}
\mathcal{W}^W &= -\mathrm{E}_0\sum_{t=0}^{\infty}\delta^t\left\{\frac{1}{(1-\sigma_G)\,4}\omega_1\left[y_t^2 + (y_t^*)^2\right] - \omega_2\left(y_t a_t + y_t^* a_t^*\right),\right. \\
&\quad \left.-\omega_3\left(y_t g_t + y_t^* g_t^*\right) + \frac{1}{4}\omega_4\left[\pi_{P,t}^2 + \left(\pi_{P,t}^*\right)^2\right]\right\} + \Gamma_0 + \text{t.i.p.} + o\left(\|\xi\|^3\right),
\end{aligned}$$

with $\omega_1 \equiv (1+\varphi)(1+\Phi) + \kappa\Phi\tilde{\omega}_1$, $\omega_2 \equiv (1+\varphi)\left[\frac{2\kappa\Phi(1-\sigma_G)\tilde{\omega}_2}{\lambda} + \frac{1+\Phi}{(1-\sigma_G)2}\right]$, $\omega_3 \equiv \frac{2\kappa\Phi\sigma_G\tilde{\omega}_3}{\lambda}$ and $\omega_4 \equiv \frac{(1+\Phi)\theta}{(1-\sigma_G)\kappa} + \frac{\Phi\Theta}{2}$.As shown in this equality, the linear term disappears. By arranging this equality, we have:

$$\begin{aligned}
\mathcal{W}^W &= -\frac{1}{2}\mathrm{E}_0\sum_{t=0}^{\infty}\delta^t\left[\frac{\Lambda_y}{2}\hat{y}_t^2 + \frac{\Lambda_y}{2}\left(\hat{y}_t^*\right)^2 + \frac{\Lambda_\pi}{2}\pi_{P,t}^2 + \frac{\Lambda_\pi}{2}\left(\pi_{P,t}^*\right)^2\right] + \Gamma_0 + \text{t.i.p.} \\
&\quad + o\left(\|\xi\|^3\right),
\end{aligned}$$

with $\Lambda_y \equiv \frac{\chi+\kappa\Phi\varsigma}{(1-\sigma_G)^2}$, $\varsigma \equiv (1+\varphi)^2(1-\sigma_G)^2 - \sigma_G^2$, $\Lambda_\pi \equiv \frac{(1+\Phi)\theta}{(1-\sigma_G)\kappa} + \frac{\Phi\Theta}{2}$, $\chi \equiv (1+\varphi)(1+\Phi)(1-\sigma_G)$, $\varsigma \equiv (1+\varphi)^2(1-\sigma_G)^2 - \sigma_G^2$ and $\Theta \equiv \alpha(4\theta-1) -$

3; and where Ω_1 and Ω_2 are certain coefficients that consist of the structural parameters, $\hat{y}_t \equiv y_t - y_t^e$ denotes the welfare-relevant output gap, $y_t^e \equiv \Omega_1 \gamma a_{H,t} + \Omega_1 (1-\gamma) a_{\mathcal{N},t} + \Omega_2 g_t$ denotes the logarithmic efficient level of output in country H. This equality corresponds to the second-order approximated welfare function in the text.

Note that Eq.(15) in the text can be rewritten as follows:

$$\pi_{P,t} = \delta \mathrm{E}_t \pi_{P,t+1} + \frac{\kappa\lambda}{1-\sigma_G} \hat{y}_t + \varepsilon_t,$$

where $\varepsilon_t \equiv \kappa (1+\varphi) \Omega_3 \gamma a_{H,t} + \kappa (1+\varphi) \Omega_3 (1-\gamma) a_{\mathcal{N},t} - \kappa \sigma_G \Omega_4 g_t$ is a composite cost-push term with Ω_3 and Ω_4 denoting certain coefficients that consist of the structural parameters. This NKPC corresponds to the one derived by Benigno and Woodford (2005). A composite cost-push term indicates the degree to which the exogenous disturbances preclude simultaneous dissolution of the tradeoff between inflation and the welfare-relevant output gap. Thus, the inflation–output gap tradeoffs can no longer be dissolved completely, even if all goods are tradable.

D.4 Other Details on Coefficients and the NKPC in Terms of the Welfare-relevant Output Gap

Note that complicated coefficients associated with the target level of output are as follows:

$$\begin{aligned}
\Omega_1 &\equiv \frac{(1-\sigma_G)(1+\varphi)\left[4\kappa\Phi(1-\sigma_G)^2(\varsigma+\sigma_G) + \lambda\varsigma(1+\Phi)\right]}{(\chi+\kappa\Phi+\varsigma)\lambda\varsigma} \\
\Omega_2 &\equiv \frac{(1-\sigma_G)^2 4\kappa\Phi\sigma_G\left[\varsigma-(1-\sigma_G)(1+\varphi)\right]}{(\chi+\kappa\Phi\varsigma)\lambda\varsigma}.
\end{aligned}$$

The NKPCs in terms of the welfare-relevant output gap are different to the NKPCs in terms of the output gap. Eq.(38) can be rewritten as:

$$\begin{aligned}
\pi_{P,t} &= \delta \mathrm{E}_t \pi_{P,t+1} + \frac{\kappa\lambda}{1-\sigma_G}\tilde{y}_t \\
&= \delta \mathrm{E}_t \pi_{P,t+1} + \frac{\kappa\lambda}{1-\sigma_G}\left(\hat{y}_t + y_t^e - \bar{y}_t\right) \\
&= \delta \mathrm{E}_t \pi_{P,t+1} + \frac{\kappa\lambda}{1-\sigma_G}\hat{y}_t + \frac{\kappa\lambda}{1-\sigma_G}\left(\Omega_1 - \bar{\beta}\right) a_t + \frac{\kappa\lambda}{1-\sigma_G}\left(\Omega_2 - \frac{\sigma_G}{\lambda}\right) g_t \\
&= \delta \mathrm{E}_t \pi_{P,t+1} + \frac{\kappa\lambda}{1-\sigma_G}\hat{y}_t + \kappa(1+\varphi)\Omega_3 a_t + \kappa\sigma_G\Omega_4 g_t \\
&= \delta \mathrm{E}_t \pi_{P,t+1} + \frac{\kappa\lambda}{1-\sigma_G}\hat{y}_t + \varepsilon_t, \qquad \text{(D.46)}
\end{aligned}$$

with $\Omega_3 \equiv \frac{4\kappa\Phi(1-\sigma_G)^2(\varsigma+\sigma_G)+\lambda\varsigma(1+\Phi)}{(\chi+\kappa\Phi+\varsigma)\varsigma}$ and $\Omega_4 \equiv \frac{(1-\sigma_G)4\kappa\Phi[\varsigma-(1-\sigma_G)(1+\varphi)]}{(\chi+\kappa\Phi+\varsigma)\varsigma} - \frac{1}{1-\sigma_G}$. This equality corresponds to Eq.(21) in the text.

E Lagrangian and its FONCs

E.1 Optimal Monetary Policy Alone

The Lagrangian is given by:

$$\begin{aligned}
\pounds &= \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \delta^t \left[L_t^W + \mu_{1,t} \left(\tilde{y}_t^W - \frac{\beta_W}{1-\sigma_G} \tilde{y}_{t+1}^W + \beta_W \hat{r}_t - \beta_W \pi_{t+1}^W - \frac{\beta_W}{\delta} \hat{r}_{t-1} \right. \right. \right. \\
& \left. + \frac{\beta_W}{\delta} \pi_t^W + \beta_W b_t^W \right) + \mu_{2,t} \left[\tilde{y}_t^R - \beta_R (1-\gamma) \upsilon \mathsf{n}_t + \beta_R (1-\gamma) \mathsf{n}_{t-1} \right] \\
& + \mu_{3,t} \left(\pi_{P,t} - \delta \pi_{P,t+1} - \frac{\kappa\lambda}{1-\sigma_G} \tilde{y}_t \right) + \mu_{4,t} \left(\pi_{P,t}^* - \delta \pi_{P,t+1}^* - \frac{\kappa\lambda}{1-\sigma_G} \tilde{y}_t^* \right) \\
& \left. \left. + \mu_{5,t} \left(\mathsf{n}_t - \frac{\delta}{1+\delta+\kappa} \mathsf{n}_{t+1} + \frac{\kappa\varphi}{1+\delta+\kappa} \tilde{y}_t^R - \frac{1}{1+\delta+\kappa} \mathsf{n}_{t-1} \right) \right] \right\},
\end{aligned}$$

because $b_t = b_t^* = 0$ for all t.

The FONCs are as follows:

$$\begin{aligned}
\frac{\Lambda_\pi}{2} \pi_{P,t} + \frac{\beta_W}{2\delta} (\mu_{1,t} - \mu_{1,t-1}) + (\mu_{3,t} - \mu_{3,t-1}) &= 0 \\
\frac{\Lambda_\pi}{2} \pi_{P,t}^* + \frac{\beta_W}{2\delta} (\mu_{1,t} - \mu_{1,t-1}) + (\mu_{4,t} - \mu_{4,t-1}) &= 0 \\
\frac{\Lambda_y}{2} \hat{y}_t + \frac{1}{2} \mu_{1,t} + \mu_{2,t} - \frac{\lambda\kappa}{(1-\sigma_G)} \mu_{3,t} + \frac{\kappa\varphi}{1+\delta+\kappa} \mu_{5,t} & \\
- \frac{\beta_W}{(1-\sigma_G) 2\delta} \mu_{1,t-1} &= 0 \\
\frac{\Lambda_y}{2} \tilde{y}_t^* + \frac{1}{2} \mu_{1,t} - \mu_{2,t} - \frac{\lambda\kappa}{(1-\sigma_G)} \mu_{4,t} - \frac{\kappa\varphi}{1+\delta+\kappa} \mu_{5,t} & \\
- \frac{\beta_W}{(1-\sigma_G) 2\delta} \mu_{1,t-1} &= 0 \\
-\beta_R (1-\gamma) \upsilon \mu_{2,t} - \mu_{5,t} - \frac{1}{1+\delta+\kappa} \mu_{5,t-1} &= 0 \\
\mu_{1,t} &= 0 \qquad \text{(E.1)}
\end{aligned}$$

Note that the fifth equality in Eq.(E.1) corresponds to the second equality in Eq.(21) in the text. Because of commitment, a lagged Lagrangian multiplier appears.

Combining the first to the fourth and the sixth equalities in Eq.(E.1), we have:

$$\begin{aligned}
\Lambda_\pi \pi_t^W + (\mu_{3,t} - \mu_{3,t-1}) + (\mu_{4,t} - \mu_{4,t-1}) &= 0, \\
\frac{1+\varphi}{1-\sigma_G} \hat{y}_t^W - \frac{\kappa\lambda}{(1-\sigma_G) 2} \mu_{3,t} - \frac{\kappa\lambda}{(1-\sigma_G) 2} \mu_{4,t} &= 0. \qquad \text{(E.2)}
\end{aligned}$$

Combining both equalities in Eq.(E.2) yields:

$$\pi_t^W = -\frac{\Lambda_y (1-\sigma_G)}{\Lambda_\pi \kappa\lambda} \left(\hat{y}_t^W - \hat{y}_{t-1}^W \right),$$

which is Eq.(19) in the text.

Combining the first to the fourth and the sixth equalities in Eq.(E.1), we have:

$$\begin{aligned} \frac{\Lambda_\pi}{2}\pi_{P,t}^R + (\mu_{3,t} - \mu_{3,t-1}) - (\mu_{4,t} - \mu_{4,t-1}) &= 0, \\ \frac{\Lambda_y}{2}\hat{y}_t^R + \mu_{2,t} - \frac{\kappa\lambda}{1-\sigma_G}\mu_{3,t} + \frac{\kappa\lambda}{1-\sigma_G}\mu_{4,t} + \frac{2\kappa\varphi}{1+\delta+\kappa}\mu_{5,t} &= 0. \end{aligned} \tag{E.3}$$

Combining both equalities in Eq.(E.3) yields:

$$\begin{aligned} \pi_{P,t}^R &= -\frac{\Lambda_y(1-\sigma_G)}{\Lambda_\pi\kappa\lambda}\left(\hat{y}_t^R - \hat{y}_{t-1}^R\right) - \frac{2(1-\sigma_G)}{\Lambda_\pi\kappa\lambda}(\mu_{2,t} - \mu_{2,t-1}) \\ &\quad - \frac{(1-\sigma_G)4\kappa\varphi}{\Lambda_\pi\kappa\lambda(1+\delta+\kappa)}(\mu_{5,t} - \mu_{5,t-1}), \end{aligned} \tag{E.4}$$

which is the first equality in Eq.(21) in the text.

E.2 Optimal Monetary and Fiscal Policy

The Lagrangian is given by:

$$\begin{aligned} \pounds &= \mathrm{E}_0\Bigg\{\sum_{t=0}^{\infty}\delta^t\Bigg[L_t^W + \mu_{1,t}\Bigg(\tilde{y}_t^W - \frac{\beta_W}{1-\sigma_G}\tilde{y}_{t+1}^W + \beta_W\hat{r}_t - \beta_W\pi_{t+1}^W - \frac{\beta_W}{\delta}\hat{r}_{t-1} \\ &\quad + \frac{\beta_W}{\delta}\pi_t^W + \beta_W b_t^W - \frac{\beta_W}{\delta}b_{t-1}^W\Bigg) + \mu_{2,t}\left[\tilde{y}_t^R + \beta_R\delta b_t^R - \beta_R(1-\gamma)\upsilon\mathsf{n}_t\right. \\ &\quad \left. + \beta_R(1-\gamma)\mathsf{n}_{t-1}\right] + \mu_{3,t}\left(\pi_{P,t} - \delta\pi_{P,t+1} - \frac{\kappa\lambda}{1-\sigma_G}\tilde{y}_t\right) + \mu_{4,t}\left(\pi_{P,t}^*\right. \\ &\quad \left. - \delta\pi_{P,t+1}^* - \frac{\kappa\lambda}{1-\sigma_G}\tilde{y}_t^*\right) + \mu_{5,t}\left(\mathsf{n}_t - \frac{\delta}{1+\delta+\kappa}\mathsf{n}_{t+1} + \frac{\kappa\varphi}{1+\delta+\kappa}\tilde{y}_t^R\right. \\ &\quad \left. - \frac{1}{1+\delta+\kappa}\mathsf{n}_{t-1}\right)\Bigg]\Bigg\}. \end{aligned}$$

The FONCs of the Lagrangian are given by Eq.(E.1) and the following equalities:

$$\begin{aligned} \frac{\beta_W}{2}\mu_{1,t} + \beta_R\delta\mu_{2,t} &= 0, \\ \frac{\beta_W}{2}\mu_{1,t} - \beta_R\delta\mu_{2,t} &= 0. \end{aligned} \tag{E.5}$$

Combining both equalities in Eq.(E.5), we have:

$$\mu_{2,t} = 0. \tag{E.6}$$

Substituting Eq.(E.6) into Eq.(E.4), we have:

$$\pi_{P,t}^R = -\frac{\Lambda_y(1-\sigma_G)}{\Lambda_\pi\kappa\lambda}\left(\hat{y}_t^R - \hat{y}_{t-1}^R\right) - \frac{(1-\sigma_G)4\kappa\varphi}{\Lambda_\pi\kappa\lambda(1+\delta+\kappa)}(\mu_{5,t} - \mu_{5,t-1}). \tag{E.7}$$

Substituting Eq.(E.6) and the initial condition $\mu_{5,-1} = 0$ into the fifth equality in Eq.(E.1), we have:

$$\mu_{5,t} = 0. \tag{E.8}$$

Substituting Eq.(E.8) into Eq.(E.7) yields:

$$\pi_{P,t}^R = -\frac{\Lambda_y (1 - \sigma_G)}{\Lambda_\pi \kappa \lambda} \left(\hat{y}_t^R - \hat{y}_{t-1}^R \right),$$

which is Eq.(22) in the text.

F Derivation of Social Loss

Using the stable roots obtained by analyzing the determinacy, this section calculates social loss analytically.[3] We assume that the model includes the price shocks that forbid the central bank from being able to stabilize inflation and the output gap simultaneously.

Similar to Eq.(D.46), we have the NKPC in terms of the welfare-relevant output gap in country F as follows:

$$\pi_{P,t}^* = \delta \mathrm{E}_t \pi_{P,t+1} + \frac{\kappa \lambda}{1 - \sigma_G} \hat{y}_t^* + \varepsilon_t^*, \tag{F.1}$$

with $\varepsilon_t^* \equiv \kappa (1 + \varphi) \Omega_3 a_t^* + \kappa \sigma_G \Omega_4 g_t^*$. Combining this equality and Eq.(D.46), we have:

$$\pi_t^W = \delta \mathrm{E}_t \pi_{t+1}^W + \frac{\kappa \lambda}{1 - \sigma_G} \hat{y}_t^W + \varepsilon_t^W.$$

Note that $\varepsilon_t^W = \Omega_5 \gamma \xi_{H,t} + \Omega_5 \gamma \xi_{F,t} + \Omega_5 (1 - \gamma) \xi_{\mathcal{N},t} + \Omega_5 (1 - \gamma) \xi_{\mathcal{N},t}^* - \Omega_6 \xi_{G,t}^W$ with $\Omega_5 \equiv \frac{\kappa(1+\varphi)\Omega_3}{2}$ and $\Omega_6 \equiv \kappa \sigma_G \Omega_4$.

Firstly, we calculate the system of the average block. Substituting Eq.(19) in the text into this equality, we have:

$$\mathrm{E}_t \hat{y}_{t+1}^W = \Omega_7 \delta^{-1} \hat{y}_t^W - \delta^{-1} \hat{y}_{t-1}^W + \Omega_8 \delta^{-1} \varepsilon_t^W, \tag{F.2}$$

with $\Omega_7 \equiv \frac{1-\sigma_G}{\Lambda_\pi \kappa \lambda} \left(1 + \delta + \frac{\Omega_8 \kappa \lambda}{1-\sigma_G} \right)$ and $\Omega_8 \equiv \frac{\Lambda_\pi \kappa \lambda}{\Lambda_y (1-\sigma_G)}$. Its vector form is given by:

$$\begin{bmatrix} \mathrm{E}_t \hat{y}_{t+1}^W \\ \hat{y}_t^W \end{bmatrix} = \mathbf{M} \begin{bmatrix} \hat{y}_t^W \\ \hat{y}_{t-1}^W \end{bmatrix} + \begin{bmatrix} \Omega_8 \delta^{-1} \\ 0 \end{bmatrix} \varepsilon_t^W$$

with $\mathbf{M} \equiv \begin{bmatrix} \Omega_7 \delta^{-1} & -\delta^{-1} \\ 1 & 0 \end{bmatrix}$.

[3] See Monacelli[4] and Walsh[5].

These roots are the solution to the characteristic equations as follows:

$$\Psi^2 - \mathrm{tr}\,(\mathbf{M}) + \det\,(\mathbf{M}) = 0,$$

with $\mathrm{tr}\,(\mathbf{M}) = \Omega_7\delta^{-1} = \Psi_1 + \Psi_2$ and $\det\,(\mathbf{M}) = \delta^{-1} = \Psi_1\Psi_2 > 1$.

Let us suppose $\mid \Psi_1 \mid < 1$. Ψ_2 satisfies:

$$\Psi_1 < 1 < \delta^{-1} < \Psi_2 = \delta^{-1}\Psi_1^{-1}.$$

The pair of solutions to the characteristic equation is as follows:

$$\Psi_{1,2} = \frac{\Omega_7}{2\delta}\left(1 \pm \sqrt{1 - \frac{4\delta}{\Omega_7^2}}\right),$$

with $\Psi_{1,2}$ denoting the pair of solutions to the characteristic equation.

Eq.(F.2) can be rewritten as:

$$\left(1 - \frac{\Omega_7}{\delta}\mathrm{L} + \frac{1}{\delta}\mathrm{L}^2\right)\hat{y}_t^W = \frac{\Omega_8}{\delta}\varepsilon_{t-1}^W, \tag{F.3}$$

where L is the lag operator. The coefficient on the LHS in Eq.(F.3) can be rewritten as:

$$1 - \frac{\Omega_7}{\delta}\mathrm{L} + \frac{1}{\delta}\mathrm{L}^2 = (1 - \Psi_1\mathrm{L})(1 - \Psi_2\mathrm{L}).$$

Substituting this into Eq.(F.3), we have:

$$(1 - \Psi_1\mathrm{L})(1 - \Psi_2\mathrm{L})\,\hat{y}_t^W = \frac{\Omega_8}{\delta}\varepsilon_t^W. \tag{F.4}$$

Because $(1 - \Psi_2\mathrm{L})^{-1} = -\sum_{k=1}^{\infty}(\Psi_2\mathrm{L})^{-k}$, this can be rewritten as:

$$(1 - \Psi_1\mathrm{L})\,\hat{y}_t^W = -\Omega_8\Psi_1\varepsilon_t^W,$$

where we use the fact that $\Psi_2 = \delta^{-1}\Psi_1^{-1}$. Thus, the final form of the solution is given by:

$$\hat{y}_t^W = -\Omega_8\Psi_1\sum_{k=0}^{\infty}\Psi_1^k\varepsilon_{t-k}^W. \tag{F.5}$$

Secondly, we calculate the system of the relative block. Subtracting its counterpart in country F from Eq.(D.46), we have:

$$\pi_t^R = \delta\mathrm{E}_t\pi_{t+1}^R + \frac{\kappa\lambda}{1-\sigma_G}\hat{y}_t^R + \varepsilon_t^R,$$

with $\varepsilon_t^R = 2\Omega_5\gamma\xi_{H,t} - 2\Omega_5\gamma\xi_{F,t} + 2\Omega_5(1-\gamma)\xi_{\mathcal{N},t} - 2\Omega_5(1-\gamma)\xi_{\mathcal{N},t}^* - \Omega_6\xi_{G,t}^R$. Substituting Eq.(22) in the text into this equality and using a similar procedure to derive Eq.(F.5), we have:

$$\hat{y}_t^R = -\Omega_8\Psi_1\sum_{k=0}^{\infty}\Psi_1^k\varepsilon_{t-k}^R. \tag{F.6}$$

Note that $v_t = v_t^W + \frac{1}{2}v_t^R$. Thus, combining Eqs.(F.5) and (F.6), we have:

$$\hat{y}_t = -\Omega_8 \Psi_1 \sum_{k=0}^{\infty} \Psi_1^k \varepsilon_{t-k}. \tag{F.7}$$

Subtracting Eq.(F.7) with a one period lag from Eq.(F.7), we have:

$$(\hat{y}_t - \hat{y}_{t-1}) = \Omega_8 \Psi_1 (1 - \Psi_1) \sum_{k=0}^{\infty} \Psi_1^{k-1} \varepsilon_{t-k} - \Omega_8 \varepsilon_t. \tag{F.8}$$

The FONCs under the optimal monetary and policy regime imply:

$$\pi_{P,t} = -\Omega_8^{-1} (\hat{y}_t - \hat{y}_{t-1}). \tag{F.9}$$

Combining Eqs.(F.7) and (F.8), we have:

$$\pi_{P,t} = -\left[\Psi_1 (1 - \Psi_1) \sum_{k=0}^{\infty} \Psi_1^{k-1} \varepsilon_{t-k} - \varepsilon_t \right]. \tag{F.10}$$

Note that $v_t^* = v_t^W - \frac{1}{2}v_t^R$. Thus, combining Eqs.(F.5) and (F.6), we have:

$$\hat{y}_t^* = -\Omega_8 \Psi_1 \sum_{k=0}^{\infty} \Psi_1^k \varepsilon_{t-k}^*. \tag{F.11}$$

Subtracting Eq.(F.7) with a one period lag from Eq.(F.7) yields:

$$\left(\hat{y}_t^* - \hat{y}_{t-1}^*\right) = \Omega_8 \Psi_1 (1 - \Psi_1) \sum_{k=0}^{\infty} \Psi_1^{k-1} \varepsilon_{t-k}^* - \Omega_8 \varepsilon_t^*. \tag{F.12}$$

The FONCs under the optimal monetary and policy regime imply:

$$\pi_{P,t}^* = -\Omega_8^{-1} \left(\hat{y}_t^* - \hat{y}_{t-1}^*\right). \tag{F.13}$$

Combining Eqs.(F.7) and (F.8), we have:

$$\pi_{P,t}^* = -\left[\Psi_1 (1 - \Psi_1) \sum_{k=0}^{\infty} \Psi_1^{k-1} \varepsilon_{t-k}^* - \varepsilon_t^* \right]. \tag{F.14}$$

Eq.(F.7) implies:

$$\hat{y}_t^2 = (\Omega_8 \Psi_1)^2 \sum_{k=0}^{\infty} \Psi_1^{2k} \varepsilon_{t-k}^2. \tag{F.15}$$

Eq.(F.10) implies:

$$(\pi_{P,t} - \varepsilon_t)^2 = \Psi_1^2 (1 - \Psi_1)^2 \sum_{k=0}^{\infty} \Psi_1^{2(k-1)} \varepsilon_{t-k}^2. \tag{F.16}$$

The LHS in Eq.(F.16) can be rewritten as:

$$(\pi_{P,t} - \varepsilon_t)^2 = \pi_{P,t}^2 - 2\pi_{P,t}\varepsilon_t + \varepsilon_t^2. \tag{F.17}$$

Multiplying by ε_t on both sides of Eq.(F.10), we have:

$$\pi_{P,t}\varepsilon_t = \Psi_1\varepsilon_t^2, \tag{F.18}$$

because the serial correlation of shocks is zero. Combining Eqs.(F.17) and (F.18) yields:

$$(\pi_{P,t} - \varepsilon_t)^2 = \pi_{P,t}^2 + (1 - 2\Psi_1)\,\varepsilon_t^2. \tag{F.19}$$

Combining Eqs.(F.16) and (F.19), we have:

$$\pi_{P,t}^2 = \Psi_1^2 (1 - \Psi_1)^2 \sum_{k=0}^{\infty} \Psi_1^{2(k-1)} \varepsilon_{t-k}^2 - (1 - 2\Psi_1)\,\varepsilon_t^2. \tag{F.20}$$

By using a similar procedure to derive Eqs.(F.15) and (F.20), we have:

$$\begin{aligned}
(\hat{y}_t^*)^2 &= (\Omega_8\Psi_1)^2 \sum_{k=0}^{\infty} \Psi_1^{2k} \left(\varepsilon_{t-k}^*\right)^2, && \text{(F.21)}\\
\left(\pi_{P,t}^*\right)^2 &= \Psi_1^2 (1 - \Psi_1)^2 \sum_{k=0}^{\infty} \Psi_1^{2(k-1)} \left(\varepsilon_{t-k}^*\right)^2 - (1 - 2\Psi_1)\,(\varepsilon_t^*)^2. && \text{(F.22)}
\end{aligned}$$

Under the self-oriented setting, the Lagrangian for country H is given by:

$$\begin{aligned}
\pounds \;=\; & \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \delta^t \left[L_t + \mu_{1,t} \left(\tilde{y}_t^W - \frac{\beta_W}{1-\sigma_G}\tilde{y}_{t+1}^W + \beta_W \hat{r}_t - \beta_W \pi_{t+1}^W - \frac{\beta_W}{\delta}\hat{r}_{t-1} \right.\right.\right.\\
& \left. + \frac{\beta_W}{\delta}\pi_t^W + \beta_W b_t^W - \frac{\beta_W}{\delta} b_{t-1}^W \right) + \mu_{2,t} \left[\tilde{y}_t^R + \beta_R \delta b_t^R - \beta_R (1-\gamma)\, \upsilon \mathsf{n}_t \right.\\
& \left. + \beta_R (1-\gamma)\, \mathsf{n}_{t-1} \right] + \mu_{3,t} \left(\pi_{P,t} - \delta \pi_{P,t+1} - \frac{\kappa\lambda}{1-\sigma_G}\tilde{y}_t \right) + \mu_{4,t} \left(\pi_{P,t}^* \right.\\
& \left. - \delta \pi_{P,t+1}^* - \frac{\kappa\lambda}{1-\sigma_G}\tilde{y}_t^* \right) + \mu_{5,t} \left(\mathsf{n}_t - \frac{\delta}{1+\delta+\kappa}\mathsf{n}_{t+1} + \frac{\kappa\varphi}{1+\delta+\kappa}\tilde{y}_t^R \right.\\
& \left.\left. - \frac{1}{1+\delta+\kappa}\mathsf{n}_{t-1} \right) \right] + \mu_{6,t} \left[\pi_t^W + \frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi}\hat{y}_t^W - \frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi}\hat{y}_{t-1}^W \right]\\
& + \mu_{7,t} \left[\pi_{P,t}^R + \frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi}\hat{y}_t^R - \frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi}\hat{y}_{t-1}^R + \frac{2\,(1-\sigma_G)}{\Lambda_\pi\kappa\lambda}\mu_{2,t} \right.\\
& \left. - \frac{2\,(1-\sigma_G)}{\Lambda_\pi\kappa\lambda}\mu_{2,t-1} + \frac{4\,(1-\sigma_G)\,\kappa\varphi}{\Lambda_\pi\kappa\lambda\,(1+\delta+\kappa)}\mu_{5,t} - \frac{4\,(1-\sigma_G)\,\kappa\varphi}{\Lambda_\pi\kappa\lambda\,(1+\delta+\kappa)}\mu_{5,t-1} \right]\\
& \left. + \mu_{8,t} \left[\mu_{5,t} - (1-\gamma)\,\beta_R \upsilon \mu_{2,t} - \frac{1}{1+\delta+\kappa}\mu_{5,t-1} \right] \right\}.
\end{aligned}$$

Because the central bank conducts optimal monetary policy, the FONCs under the optimal monetary policy alone, Eq.(21) in the text, appear in this Lagrangian as constraints. A similar Lagrangian is given for the government in country F although L_t^* replaces L_t. Note that any exogenous shifters are omitted in this Lagrangian.

The government in country H chooses the sequence $\{\pi_{P,t}, \hat{y}_t, \mathsf{n}_t, b_t\}_{t=0}^{\infty}$ while the government in country F chooses the sequence $\left\{\pi_{P,t}^*, \hat{y}_t^*, \mathsf{n}_t, b_t^*\right\}_{t=0}^{\infty}$ under commitment. The FONCs are given by:

$$\begin{aligned}
\Lambda_\pi \pi_{P,t} + \frac{\beta_W}{2\delta}\mu_{1,t} + \mu_{3,t} - \frac{\beta_W}{2\delta}\mu_{1,t-1} - \mu_{3,t-1} + \frac{1}{2}\mu_{6,t} + \mu_{7,t} &= 0 \\
\Lambda_y \hat{y}_t + \frac{1}{2}\mu_{1,t} + \mu_{2,t} - \frac{\kappa\lambda}{1-\sigma_G}\mu_{3,t} + \frac{\kappa\varphi}{1+\delta+\kappa}\mu_{5,t} - \frac{\beta_W}{(1-\sigma_G)\,2\delta}\mu_{1,t-1} & \\
+\frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi 2}\mu_{6,t} + \frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi}\mu_{7,t} &= 0 \\
-\beta_R\,(1-\gamma)\,\upsilon\mu_{2,t} + \mu_{5,t} - \frac{1}{1+\delta+\kappa}\mu_{5,t-1} &= 0 \\
\frac{\beta_W}{2}\mu_{1,t} + \beta_R\delta\mu_{2,t} &= 0 \\
\Lambda_\pi \pi_{P,t}^* + \frac{\beta_W}{2\delta}\mu_{1,t} + \mu_{4,t} - \frac{\beta_W}{2\delta}\mu_{1,t-1} - \mu_{4,t-1} + \frac{1}{2}\mu_{6,t} - \mu_{7,t} &= 0 \\
\Lambda_y \hat{y}_t^* + \frac{1}{2}\mu_{1,t} + \mu_{2,t} - \frac{\kappa\lambda}{1-\sigma_G}\mu_{4,t} - \frac{\kappa\varphi}{1+\delta+\kappa}\mu_{5,t} - \frac{\beta_W}{(1-\sigma_G)\,2\delta}\mu_{1,t-1} & \\
+\frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi 2}\mu_{6,t} - \frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi}\mu_{7,t} &= 0 \\
\frac{\beta_W}{2}\mu_{1,t} - \beta_R\delta\mu_{2,t} &= 0.
\end{aligned} \tag{F.23}$$

The fourth and the seventh equalities in Eq.(F.23) imply:

$$\mu_{1,t} = 0 \ ; \ \mu_{2,t} = 0. \tag{F.24}$$

The third equality in Eq.(F.23) and Eq.(F.24) imply:

$$\mu_{5,t} = 0, \tag{F.25}$$

given the initial condition $\mu_{5,-1} = 0$. Substituting Eqs.(F.24) and (F.25) into the first, the second, the fifth and the sixth equalities in Eq.(F.23) yields:

$$\Lambda_\pi \pi_{P,t} + (\mu_{3,t} - \mu_{3,t-1}) + \frac{1}{2}\mu_{6,t} + \mu_{7,t} = 0 \tag{F.26}$$

$$\Lambda_y \hat{y}_t - \frac{\kappa\lambda}{1-\sigma_G}\mu_{3,t} + \frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi 2}\mu_{6,t} + \frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi}\mu_{7,t} = 0 \tag{F.27}$$

$$\Lambda_\pi \pi_{P,t}^* + (\mu_{4,t} - \mu_{4,t-1}) + \frac{1}{2}\mu_{6,t} - \mu_{7,t} = 0 \tag{F.28}$$

$$\Lambda_y \hat{y}_t^* - \frac{\kappa\lambda}{1-\sigma_G}\mu_{4,t} + \frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi 2}\mu_{6,t} - \frac{(1-\sigma_G)\,\Lambda_y}{\kappa\lambda\Lambda_\pi}\mu_{7,t} = 0 \tag{F.29}$$

Eqs.(F.26) and (F.28) imply the following:

$$\frac{1}{2}\mu_{6,t} = -\Lambda_\pi \pi_t^W - \frac{1}{2}(\mu_{3,t} - \mu_{3,t-1}) - \frac{1}{2}(\mu_{4,t} - \mu_{4,t-1}) \quad \text{(F.30)}$$

$$\mu_{7,t} = -\frac{\Lambda_\pi}{2}\pi_{P,t}^R - \frac{1}{2}(\mu_{3,t} - \mu_{3,t-1}) + \frac{1}{2}(\mu_{4,t} - \mu_{4,t-1}) \quad \text{(F.31)}$$

Combining Eq.(F.30) and Eq.(19) in the text, we have:

$$\frac{1}{2}\mu_{6,t} = \frac{(1-\sigma_G)\Lambda_y}{\kappa\lambda}\left(\hat{y}_t^W - \hat{y}_{t-1}^W\right) - \frac{1}{2}(\mu_{3,t} - \mu_{3,t-1}) - \frac{1}{2}\mu_{4,t}. \quad \text{(F.32)}$$

Combining Eqs.(F.27) and (F.29), we have:

$$-\frac{1}{2}(\mu_{3,t} - \mu_{3,t-1}) - \frac{1}{2}(\mu_{4,t} - \mu_{4,t-1}) = -\frac{(1-\sigma_G)\Lambda_y}{\kappa\lambda}\left(\hat{y}_t^W - y_{t-1}^W\right) - \frac{(1-\sigma_G)^2\Lambda_y}{(\kappa\lambda)^2\Lambda_\pi}(\mu_{6,t} - \mu_{6,t-1}).$$

Substituting this equality into Eq.(F.32) yields:

$$\left[\frac{1}{2} + \frac{(1-\sigma_G)^2\Lambda_y}{(\kappa\lambda)^2\Lambda_\pi}\right]\mu_{6,t} = \frac{(1-\sigma_G)^2\Lambda_y}{(\kappa\lambda)^2\Lambda_\pi}\mu_{6,t-1}.$$

This equality implies the following:

$$\mu_{6,t} = 0, \quad \text{(F.33)}$$

given the initial condition $\mu_{6,-1} = 0$.

Substituting Eqs.(F.24) and (F.25) into Eq.(21) in the text, we have:

$$\pi_{P,t}^R = -\frac{(1-\sigma_G)\Lambda_y}{\kappa\lambda\Lambda_\pi}\hat{y}_t^R + \frac{(1-\sigma_G)\Lambda_y}{\kappa\lambda\Lambda_\pi}\hat{y}_{t-1}^R.$$

Combining this equality and Eq.(F.31) yields:

$$\mu_{7,t} = \frac{(1-\sigma_G)\Lambda_y}{\kappa\lambda 2}\left(\hat{y}_t^R - \hat{y}_{t-1}^R\right) - \frac{1}{2}(\mu_{3,t} - \mu_{3,t-1}) + \frac{1}{2}(\mu_{4,t} - \mu_{4,t-1}). \quad \text{(F.34)}$$

Combining Eqs.(F.27) and (F.29), we have:

$$-\frac{1}{2}(\mu_{3,t} - \mu_{3,t-1}) + \frac{1}{2}(\mu_{4,t} - \mu_{4,t-1}) = -\frac{(1-\sigma_G)\Lambda_y}{2\kappa\mu_{4,t}\lambda}\left(\hat{y}_t^R - \hat{y}_{t-1}^R\right) - \frac{(1-\sigma_G)^2\Lambda_y}{\kappa\lambda\Lambda_\pi}(\mu_{7,t} - \mu_{7,t-1}).$$

Combining this equality and Eq.(F.34), we have:

$$\mu_{7,t} = -\frac{(1-\sigma_G)^2\Lambda_y}{\kappa\lambda\Lambda_\pi}(\mu_{7,t} - \mu_{7,t-1}),$$

which implies the following

$$\mu_{7,t} = 0, \quad \text{(F.35)}$$

given the initial condition $\mu_{7,-1} = 0$.

Substituting Eqs.(F.33) and (F.35) into Eqs.(F.26)–(F.29) yields:

$$\begin{aligned}
\pi_{P,t} &= -\frac{1}{\Lambda_\pi}\left(\mu_{3,t} - \mu_{3,t-1}\right) \\
\left(\mu_{3,t} - \mu_{3,t-1}\right) &= \frac{\left(1-\sigma_G\right)\Lambda_y}{\kappa\lambda}\left(\hat{y}_t - \hat{y}_{t-1}\right) \\
\pi^*_{P,t} &= -\frac{1}{\Lambda_\pi}\left(\mu_{4,t} - \mu_{4,t-1}\right) \\
\left(\mu_{4,t} - \mu_{4,t-1}\right) &= \frac{\left(1-\sigma_G\right)\Lambda_y}{\kappa\lambda}\left(\hat{y}^*_t - \hat{y}^*_{t-1}\right). \qquad \text{(F.36)}
\end{aligned}$$

Combining the first and the second equalities in Eq.(F.36) yields:

$$\pi_{P,t} = -\frac{\left(1-\sigma_G\right)\Lambda_y}{\kappa\lambda\Lambda_\pi}\left(\hat{y}_t - \hat{y}_{t-1}\right). \qquad \text{(F.37)}$$

Combining the third and the fourth equalities in Eq.(F.36) yields:

$$\pi^*_{P,t} = -\frac{\left(1-\sigma_G\right)\Lambda_y}{\kappa\lambda\Lambda_\pi}\left(\hat{y}^*_t - \hat{y}^*_{t-1}\right). \qquad \text{(F.38)}$$

Combining Eqs.(F.37) and (F.38) yields:

$$\begin{aligned}
\pi^W_t &= -\frac{\left(1-\sigma_G\right)\Lambda_y}{\kappa\lambda\Lambda_\pi}\left(\hat{y}^W_t - \hat{y}^W_{t-1}\right), \\
\pi^R_t &= -\frac{\left(1-\sigma_G\right)\Lambda_y}{\kappa\lambda\Lambda_\pi}\left(\hat{y}^R_t - \hat{y}^R_{t-1}\right),
\end{aligned}$$

which correspond to Eqs.(22) and (25) in the text, respectively. Thus, the optimality conditions for self-oriented fiscal authorities are the same as the one under the optimal monetary and fiscal policy regime. This implies that the social loss is the same between the optimal monetary and fiscal policy regime under the cooperative setting and the self-oriented fiscal authorities with optimal monetary policy.

The definitions of the composite cost push terms imply the following:

$$\begin{aligned}
\varepsilon^2_t &= \Omega_9\xi^2_{H,t} + \Omega_{10}\xi^2_{\mathcal{N},t} + \Omega_{11}\xi^2_{G,t} \\
\left(\varepsilon^*_t\right)^2 &= \Omega_9\xi^2_{F,t} + \Omega_{10}\left(\xi^*_{\mathcal{N},t}\right)^2 + \Omega_{11}\left(\xi^*_{G,t}\right)^2,
\end{aligned}$$

with $\Omega_9 \equiv \left[\kappa\left(1+\varphi\right)\Omega_3\gamma\right]^2$, $\Omega_{10} \equiv \left[\kappa\left(1+\varphi\right)\Omega_3\left(1-\gamma\right)\right]^2$ and $\Omega_{11} \equiv \left(\kappa\sigma_G\Omega_4\right)^2$.

Substituting these equalities into Eqs.(F.15), (F.20) to (F.22) yields:

$$\begin{aligned}
\hat{y}^2_t &= \left(\Omega_8\Psi_1\right)^2\sum_{k=0}^{\infty}\Psi_1^{2k}\left(\Omega_9\xi^2_{H,t-k} + \Omega_{10}\xi^2_{\mathcal{N},t-k} + \Omega_{11}\xi^2_{G,t-k}\right) \\
\pi^2_{P,t} &= \Psi_1^2\left(1-\Psi_1\right)^2\sum_{k=0}^{\infty}\Psi_1^{2(k-1)}\left(\Omega_9\xi^2_{H,t-k} + \Omega_{10}\xi^2_{\mathcal{N},t-k} + \Omega_{11}\xi^2_{G,t-k}\right)
\end{aligned}$$

$$
\begin{aligned}
& \quad - (1-2\Psi_1)\left(\Omega_9\xi_{H,t}^2+\Omega_{10}\xi_{\mathcal{N},t}^2+\Omega_{11}\xi_{G,t}^2\right) \\
(\hat{y}_t^*)^2 &= (\Omega_8\Psi_1)^2\sum_{k=0}^{\infty}\Psi_1^{2k}\left[\Omega_9\xi_{F,t-k}^2+\Omega_{10}\left(\xi_{\mathcal{N},t-k}^*\right)^2+\Omega_{11}\left(\xi_{G,t-k}^*\right)^2\right] \\
\left(\pi_{P,t}^*\right)^2 &= \Psi_1^2(1-\Psi_1)^2\sum_{k=0}^{\infty}\Psi_1^{2(k-1)}\left[\Omega_9\xi_{F,t-k}^2+\Omega_{10}\left(\xi_{\mathcal{N},t-k}^*\right)^2+\Omega_{11}\left(\xi_{G,t-k}^*\right)^2\right] \\
& \quad - (1-2\Psi_1)\left[\Omega_9\xi_{F,t}^2+\Omega_{10}\left(\xi_{\mathcal{N},t}^*\right)^2+\Omega_{11}\left(\xi_{G,t}^*\right)^2\right] \qquad \text{(F.39)}
\end{aligned}
$$

Substituting Eq.(F.39) into Eq.(18) in the text, we have:

$$
\begin{aligned}
L_t^W &= \frac{\Psi_1}{2\left(1-\Psi_1^2\right)}\left[\frac{\Lambda_y\Omega_8\Psi_1}{2}+\Lambda_\pi(1-\Psi_1)\right]\{\Omega_9\left[\mathrm{var}\left(\xi_{H,t}\right)+\mathrm{var}\left(\xi_{F,t}\right)\right] \\
& \quad +\Omega_{10}\left[\mathrm{var}\left(\xi_{\mathcal{N},t}\right)+\mathrm{var}\left(\xi_{\mathcal{N},t}^*\right)\right]+\Omega_{11}\left[\mathrm{var}\left(\xi_{G,t}\right)+\mathrm{var}\left(\xi_{G,t}^*\right)\right]\},
\end{aligned}
$$

where we take the expectation in period zero on both sides. Substituting this equality into:

$$
\begin{aligned}
\mathsf{q}_t &= (1-\gamma)\left\{\varphi\gamma\left(1-\bar{\beta}\right)a_{H,t}+\left[1+\varphi(1-\gamma)\left(1-\bar{\beta}\right)\right]a_{\mathcal{N},t}-\varphi\gamma\left(1-\bar{\beta}\right)a_{F,t}\right. \\
& \quad \left.-\left[1+\varphi(1-\gamma)\left(1-\bar{\beta}\right)\right]a_{\mathcal{N},t}^*-\frac{\varphi\sigma_G}{1+\varphi}g_t^R\right\},
\end{aligned}
$$

which is derived from Eq.(16) in the text with $\alpha=0$ and $\tilde{y}_t=\tilde{y}_t^*=0$ yields:

$$
\begin{aligned}
\mathcal{L}^W &= \frac{\Psi_1}{(1-\delta)2\left(1-\Psi_1^2\right)}\left[\frac{\Lambda_y\Omega_8\Psi_1}{2}+\Lambda_\pi(1-\Psi_1)\right]\{\Omega_9\left[\mathrm{var}\left(\xi_{H,t}\right)+\mathrm{var}\left(\xi_{F,t}\right)\right] \\
& \quad +\Omega_{10}\left[\mathrm{var}\left(\xi_{\mathcal{N},t}\right)+\mathrm{var}\left(\xi_{\mathcal{N},t}^*\right)\right]+\Omega_{11}\left[\mathrm{var}\left(\xi_{G,t}\right)+\mathrm{var}\left(\xi_{G,t}^*\right)\right]\}. \qquad \text{(F.40)}
\end{aligned}
$$

Substituting Eq.(F.39) into Eq.(18) in the text, we have:

$$
\begin{aligned}
L_t^{NC} &= \frac{\Psi_1}{2\left(1-\Psi_1^2\right)}\left[\frac{\Lambda_y\Omega_8\Psi_1}{2}+\Lambda_\pi(1-\Psi_1)\right]\left[\Omega_9\mathrm{var}\left(\xi_{H,t}\right)+\Omega_{10}\mathrm{var}\left(\xi_{\mathcal{N},t}\right)\right. \\
& \quad \left.+\Omega_{11}\mathrm{var}\left(\xi_{G,t}\right)\right],
\end{aligned}
$$

where we take the expectation in period zero on both sides. Substituting this equality into the definition of the respective loss in country H in the text yields:

$$
\begin{aligned}
\mathcal{L}_t^{NC} &= \frac{\Psi_1}{(1-\delta)2\left(1-\Psi_1^2\right)}\left[\frac{\Lambda_y\Omega_8\Psi_1}{2}+\Lambda_\pi(1-\Psi_1)\right]\left[\Omega_9\mathrm{var}\left(\xi_{H,t}\right)+\Omega_{10}\mathrm{var}\left(\xi_{\mathcal{N},t}\right)\right. \\
& \quad \left.+\Omega_{11}\mathrm{var}\left(\xi_{G,t}\right)\right].
\end{aligned}
$$

Substituting this equality and its counterpart in country F into the definition of the union-wide social loss brought about by self-oriented fiscal authorities in both countries, we have:

$$
\begin{aligned}
\mathcal{L}^{NCW} &= \frac{\Psi_1}{(1-\delta)2\left(1-\Psi_1^2\right)}\left[\frac{\Lambda_y\Omega_8\Psi_1}{2}+\Lambda_\pi(1-\Psi_1)\right]\{\Omega_9\left[\mathrm{var}\left(\xi_{H,t}\right)+\mathrm{var}\left(\xi_{F,t}\right)\right] \\
& \quad +\Omega_{10}\left[\mathrm{var}\left(\xi_{\mathcal{N},t}\right)+\mathrm{var}\left(\xi_{\mathcal{N},t}^*\right)\right]+\Omega_{11}\left[\mathrm{var}\left(\xi_{G,t}\right)+\mathrm{var}\left(\xi_{G,t}^*\right)\right]\},
\end{aligned}
$$

which implies that $\mathcal{L}^W=\mathcal{L}^{NCW}$. Furthermore, this equality and Eq.(F.40) correspond to the equality on page 25 of the text.

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