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Technical Appendix to "Revisiting the Fiscal Theory of Sovereign Risk from a DSGE Viewpoint"

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A FONCs for Households and Firms

A sequence of budget constraints is given by:

$$R_{t-1}\left[D_{t-1}^{n}+B_{t-1}^{n}\Gamma\left(-sp_{t-1}\right)\left(1-\delta_{t}\right)\right]+W_{t}N_{t}+PR_{t} \quad \geq \quad \int_{0}^{1}P_{t}\left(i\right)C_{t}\left(i\right)di+D_{t}^{n}+B_{t}^{n}, \tag{A.1}$$

As shown in Eq.(4) in the text, the optimal allocation of any given expenditure within goods yields is $C_{t}\left(i\right)=\left[\frac{P_{t}(i)}{P_{t}}\right]^{-\varepsilon}C_{t}$. Plugging this into Eq.(A.1) yields:

$$\int_{0}^{1}P_{t}\left(i\right)C_{t}\left(i\right)di=P_{t}C_{t}. \tag{A.2}$$

Plugging Eq.(A.2) into Eq.(A.1) yields Eq.(5) in the text.

Representative household maximizes Eq.(1) in the text subject to Eq.(5) in the text. The FONCs are given by:

$$\lambda_{t} = \frac{1}{P_{t}C_{t}}, \tag{A.3}$$

$$\lambda_{t} = \frac{N_{t}^{\varphi}}{W_{t}} \tag{A.4}$$

$$\lambda_{t} = \beta\lambda_{t+1}R_{t} \tag{A.5}$$

$$\lambda_{t} = \beta\lambda_{t+1}R_{t}\mathrm{E}_{t}\left(1-\delta_{t+1}\right)\left\{\Gamma\left(-sp_{t}\right)+B_{t}\Gamma'\left(-sp_{t}\right)\left[B\left(1-R\right)\right]^{-1}\right\} \tag{A.6}$$

Combining Eqs.(A.3) and (A.5) yields $\beta\mathrm{E}_{t}\left(\frac{P_{t}C_{t}}{P_{t+1}C_{t+1}}\right)=\frac{1}{R_{t}}$ which is Eq.(6) in the text while combining Eqs.(A.3) and (A.4) yields $C_{t}N_{t}^{\varphi}=\frac{W_{t}}{P_{t}}$ which is Eq.(7) in the text. Combining Eqs.(A.3), (A.5) and (A.6) yields:

$$\beta\mathrm{E}_{t}\left(\frac{P_{t}C_{t}}{P_{t+1}C_{t+1}}\right)=\frac{1}{R_{t}\mathrm{E}_{t}\left(1-\delta_{t+1}\right)\left\{\Gamma\left(-sp_{t}\right)+B_{t}\Gamma'\left(-sp_{t}\right)\left[B\left(1-R\right)\right]^{-1}\right\}},$$

which is Eq.(8) in the text.

Under Calvo–Yun-style price-setting behavior, the pricing rules are given by:

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)\tilde{P}_t^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{A.7}$$

The maximization problems faced by firms given by:

$$\max_{\tilde{P}_t} \mathrm{E}_t \left[\sum_{k=0}^{\infty} (\theta\beta)^k (P_{t+k}C_{t+k})^{-1} \tilde{Y}_{t+k} \left(\tilde{P}_t - MC_{H,t+k}^n\right)\right]$$

This problem's FONC is given by:

$$\mathrm{E}_t \left[\sum_{k=0}^{\infty} (\theta\beta)^k (P_{t+k}C_{t+k})^{-1} \tilde{Y}_{t+k} \left(\tilde{P}_t - \frac{\varepsilon}{\varepsilon-1} MC_{H,t+k}^n\right)\right] = 0, \tag{A.8}$$

which can be rewritten as:

$$\tilde{P}_t = \frac{\mathrm{E}_t \left(\sum_{k=0}^{\infty} \theta^k \beta^k \tilde{Y}_{t+k} \frac{\varepsilon}{\varepsilon-1} P_{t+k} MC_{t+k}\right)}{\mathrm{E}_t \left(\sum_{k=0}^{\infty} \theta^k \beta^k \tilde{Y}_{t+k}\right)}. \tag{A.9}$$

This is Eq.(24) in the text itself.

B Deriving the Welfare Costs

We derive welfare criteria for policy authorities in the text which includes welfare cost function Eq.(35) in the text and transitory component Υ_0. First of all, we derive second-order approximated utility function following Gali[3]. Second, we derive second-order approximated AS equation following Benigno and Woodford[2]. Third, to eliminate linear terms in these second -order approximated utility function and AS equation, we derive second order solvency condition following Benigno and Woodford[1]. Then we eliminate those linear terms following Benigno and Woodford[1].

B.1 Second-order Approximation of Utility Function

Second-order approximation of period utility function $U_t \equiv \ln C_t - \frac{1}{1+\varphi} N_t^{1+\varphi}$ is given by:

$$\frac{U_t - U}{U_C C} = \frac{\Phi}{\varsigma_C} y_t - \left[\frac{1-\Phi}{\varsigma_C} z_t + \frac{(1-\Phi)(1+\varphi)}{2\varsigma_C} y_t^2 - \frac{(1-\Phi)(1+\varphi)}{\varsigma_C} y_t a_t\right] + o\left(\| \xi \|^3\right), \tag{B.1}$$

where we use the facts that $c_t = \varsigma_C^{-1} y_t - \frac{\varsigma_G}{\varsigma_C} g_t$ and $n_t = y_t + z_t - a_t$. Here, z_t is $o\left(\| \xi \|^2\right)$.

Let define $u \equiv \sum_{t=0}^{\infty} \beta^t \frac{U_t - U}{U_C C}$. Plugging Eq.(B.1) into this definition yields:

$$\begin{aligned} u \quad = \quad & \sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left[\frac{\Phi}{\varsigma_C} y_t - \frac{(1-\Phi)\varepsilon}{2\varsigma_C \kappa} \pi_t^2 - \frac{(1-\Phi)(1+\varphi)}{2\varsigma_C} y_t^2 + \frac{(1-\Phi)(1+\varphi)}{\varsigma_C} y_t a_t\right] \\ & + \text{t.i.p.} + o\left(\| \xi \|^3\right), \end{aligned} \tag{B.2}$$

where we use the fact that:

$$\sum_{t=0}^{\infty} \beta^t z_t = \frac{\varepsilon}{2\kappa} \sum_{t=0}^{\infty} \beta^t \pi_t^2. \tag{B.3}$$

See Chapter 6 in Woodford[7].

B.2 Second-order Approximation of AS Equation

Let define $K_t \equiv \sum_{k=0}^{\infty} \theta^k \beta^k \tilde{Y}_{t+k} \frac{\varepsilon}{\varepsilon-1} P_{t+k} MC_{t+k}$ and $F_t \equiv \sum_{k=0}^{\infty} \theta^k \beta^k \tilde{Y}_{t+k}$ where K_t and F_t are the numerator and the denominator in the RHS of Eq.(24) in the text. Log-linearizing those definitions are given by:

$$\begin{aligned} k_t &= -\varepsilon \tilde{x}_t + (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \mathrm{E}_t \left(\tilde{k}_{t,t+k} \right) - \frac{\theta\varepsilon}{1-\theta} \pi_t, \\ f_t &= -\varepsilon \tilde{x}_t + (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \mathrm{E}_t \left(\tilde{f}_{t,t+k} \right) - \frac{\theta\varepsilon}{1-\theta} \pi_t, \end{aligned} \tag{B.4}$$

with $\tilde{k}_{t,t+k} \equiv -\varsigma_G c_{t+k} + \varsigma_G g_{t+k} + mc_{t+k} + \varepsilon \sum_{s=1}^{k} \pi_{t+s}$ and $\tilde{f}_{t,t+k} \equiv -\varsigma_G c_{t+k} + \varsigma_G g_{t+k} + (\varepsilon-1) \sum_{s=1}^{k} \pi_{t+s}$.

Subtracting the second equality from the first equality in Eq.(B.4) yields:

$$k_t - f_t = \tilde{k}_t - \tilde{f}_t \tag{B.5}$$

where we define $\tilde{k}_t \equiv (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \mathrm{E}_t \left(\tilde{k}_{t,t+k} \right)$ and $\tilde{f}_t \equiv (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \mathrm{E}_t \left(\tilde{f}_{t,t+k} \right)$.

Second-order approximation of the definitions $\tilde{k}_{t,t}$ and $\tilde{f}_{t,t}$ are given by:

$$\begin{aligned} \tilde{k}_t &= \tilde{k}_t + \frac{1}{2} \tilde{k}_t^2 + o\left(\| \xi \|^3 \right) \\ &= (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \left(\tilde{k}_{t,t+k} + \frac{1}{2} \tilde{k}_{t,t+k} \right) + o\left(\| \xi \|^3 \right) \\ \tilde{f}_t &= \tilde{f}_t + \frac{1}{2} \tilde{f}_t^2 + o\left(\| \xi \|^3 \right) \\ &= (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \left(\tilde{f}_{t,t+k} + \frac{1}{2} \tilde{f}_{t,t+k} \right) + o\left(\| \xi \|^3 \right) \end{aligned} \tag{B.6}$$

Plugging Eq.(B.6) into Eq.(B.5) yields:

$$\begin{aligned} k_t - f_t &= (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \left[\left(\tilde{k}_{t,t+k} - \tilde{f}_{t,t+k} \right) + \frac{1}{2} \left(\tilde{k}_{t,t+k}^2 - \tilde{f}_{t,t+k}^2 \right) \right] - \frac{(1-\theta\beta)\,\theta}{2(1-\theta)} \pi_t \mathcal{Z}_t \\ &+ o\left(\| \xi \|^3 \right) \end{aligned} \tag{B.7}$$

with $\mathcal{Z}_t \equiv \sum_{k=0}^{\infty} (\theta\beta)^k \left(\tilde{k}_{t,t+k}^2 + \tilde{f}_{t,t+k}^2 \right)$ where we use the fact that $\tilde{k}_t - \tilde{f}_t = \frac{\theta}{1-\theta} \pi_t$ which can be derived by log-linearizing the definitions K_t and F_t.

The first term of Eq.(B.7) can be rewritten as:

$$\sum_{k=0}^{\infty} (\theta\beta)^k \left(\tilde{k}_{t,t+k} - \tilde{f}_{t,t+k} \right) = \sum_{k=0}^{\infty} (\theta\beta)^k mc_{t+k} + \frac{1}{1-\theta\beta} \mathcal{P}_t \tag{B.8}$$

with $\mathcal{P}_t \equiv \sum_{k=1}^{\infty} (\theta\beta)^k \pi_{t+k}$.

Let define $\widetilde{kk}_{t,t+k} \equiv -\varsigma_G c_{t+k} + \varsigma_G g_{t+k} + mc_{t+k}$ and $\widetilde{ff}_{t,t+k} \equiv -\varsigma_G c_{t+k} + \varsigma_G g_{t+k}$. Now, the second term of Eq.(B.7) can be rewritten as:

$$\begin{aligned} \frac{1}{2} \left(\tilde{k}_{t,t+k}^2 - \tilde{f}_{t,t+k}^2 \right) &= \frac{1}{2} \left(\widetilde{kk}_{t,t+k}^2 - \widetilde{ff}_{t,t+k}^2 \right) \\ &+ \sum_{k=0}^{\infty} (\theta\beta)^k \pi_{t+k} \mathcal{N}_{t+k} \\ &+ \frac{2\theta-1}{(1-\theta\beta)\,2} \sum_{k=0}^{\infty} (\theta\beta)^k \pi_{t+k} \left(\pi_{t+k} + 2\mathcal{P}_{t+h} \right). \end{aligned} \tag{B.9}$$

Let define $\widetilde{kk}_{t,t+k} \equiv -\varsigma_G c_{t+k} + \varsigma_G g_{t+k} + mc_{t+k}$ and $\widetilde{ff}_{t,t+k} \equiv -\varsigma_G c_{t+k} + \varsigma_G g_{t+k}$. By using this definition, the definitions of $\tilde{k}_{t,t+k}$ and $\tilde{f}_{t,t+k}$ can be rewritten as:

$$\begin{aligned}\tilde{k}_{t,t+k} &= \widetilde{kk}_{t,t+k} + \varepsilon \sum_{s=1}^{k} \pi_{t+s} \\ \tilde{f}_{t,t+k} &= \widetilde{ff}_{t,t+k} + (\varepsilon - 1) \sum_{s=1}^{k} \pi_{t+s} \end{aligned} \tag{B.10}$$

Plugging Eqs.(B.8)–(B.10) into Eq.(B.7) yields:

$$\begin{aligned}\tilde{k}_t - \tilde{f}_t &= \sum_{k=0}^{\infty} (\theta\beta)^k \mathrm{E}_t \left\{ (1-\theta\beta) \left[\left(\widetilde{kk}_{t,t+k} - \widetilde{ff}_{t,t+k} \right) + \frac{1}{2} \left(\widetilde{kk}^2_{t,t+k} - \widetilde{ff}^2_{t,t+k} \right) \right] \right\} \\ &+ \sum_{k=1}^{\infty} (\theta\beta)^k \pi_{t+k} + (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \pi_{t+k} \mathcal{N}_{t+k} \\ &+ \frac{2\theta - 1}{2} \sum_{k=0}^{\infty} (\theta\beta)^k (\pi_{t+k} + 2\mathcal{P}_{t+k}) - \frac{(1-\theta\beta)\theta}{2(1-\theta)} \pi_t \mathcal{Z}_t + o\left(\| \xi \|^3\right) \end{aligned} \tag{B.11}$$

with $\mathcal{N}_{t+k} \equiv \sum_{k=0}^{\infty} (\theta\beta)^k \left[\varepsilon \widetilde{kk}_{t,t+k+1} + (1-\varepsilon) \widetilde{ff}_{t,t+k+1} \right]$.

The FONC for firms can be rewritten as:

$$\frac{1}{1-\theta} \left(1 - \theta \Pi_t^{\varepsilon-1}\right) = \left(\frac{F_t}{K_t} \right)^{\varepsilon - 1} \tag{B.12}$$

By second-order approximation, Eq.(B.12) can be rewritten as:

$$\pi_t + \frac{\varepsilon - 1}{2(1-\theta)} \pi_t^2 = \frac{1-\theta}{\theta} (k_t - f_t) + o\left(\| \xi \|^3\right) \tag{B.13}$$

Plugging Eq.(B.13) into Eq.(B.11) yields:

$$\begin{aligned}\pi_t + \frac{\varepsilon - 1}{(1-\theta)2} \pi_t^2 + \frac{1-\theta\beta}{2} \pi_t \mathcal{Z}_t &= \kappa \left[\left(\widetilde{kk}_{t,t} - \widetilde{ff}_{t,t} \right) + \frac{1}{2} \left(\widetilde{kk}^2_{t,t} - \widetilde{ff}^2_{t,t} \right) \right] + \beta \mathrm{E}_t \left[\pi_{t+1} \right. \\ &\left. + \frac{1-\theta\beta}{2} \pi_{t+1} \mathcal{Z}_{t+1} + \frac{2\varepsilon - 1}{2} \pi^2_{t+1} + \frac{\theta(\varepsilon - 1)}{(1-\theta)2} \pi^2_{t+1} \right],\end{aligned}$$

which acn be rewritten as:

$$\nu_t = \kappa \left[\widetilde{kk}_{t,t} - \widetilde{ff}_{t,t} + \frac{1}{2} \left(\widetilde{kk}^2_{t,t} - \widetilde{ff}^2_{t,t} \right) \right] + \beta \nu_{t+1} + \frac{\varepsilon}{2} \pi_t^2, \tag{B.14}$$

by using the definition $\nu_t = \pi_t + \frac{\varepsilon-1}{(1-\theta)2} \pi_t^2 + \frac{1-\theta\beta}{2} \pi_t \mathcal{Z}_t + \frac{\varepsilon}{2} \pi_t^2$.

Then, we get:

$$\nu = \kappa \sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left[\widetilde{kk}_{t,t} - \widetilde{ff}_{t,t} + \frac{1}{2} \left(\widetilde{kk}^2_{t,t} - \widetilde{ff}^2_{t,t} \right) + \frac{\varepsilon}{2\kappa} \pi_t^2 \right] \tag{B.15}$$

Second order approximation of Eq.(26) in the text $MC_t = \frac{C_t N_t^{\varphi}}{(1-\tau_t) A_t}$ is given by:

$$\begin{aligned} mc_t &= \frac{MC_C}{MC} C c_t + \frac{MC_N}{MC} N n_t + \frac{MC_\tau}{MC} \tau \hat{\tau}_t + \frac{MC_A}{MC} a_t + \frac{1}{2} c_t^2 + \frac{\varphi}{2} n_t^2 + \frac{MC_\tau}{MC} \frac{\tau}{2} \hat{\tau}_t^2 \\ &+ \frac{\varphi(\varphi - 1)}{2} n_t^2 + \varphi c_t n_t + \frac{MC_C}{MC} C \frac{MC_{C\tau}}{MC_C} \tau c_t \hat{\tau}_t - \varphi n_t a_t - c_t a_t + \frac{MC_N}{MC} N \frac{MC_{N\tau}}{MC_N} \tau n_t \hat{\tau}_t \\ &+ \frac{MC_A}{MC} \frac{MC_{A\tau}}{MC_A} \tau \hat{\tau}_t a_t + \frac{MC_\tau MC_{\tau\tau}}{MC MC_\tau} \frac{\tau^2}{2} \hat{\tau}_t^2 + \text{s.o.t.i.p.} + o\left(\| \xi \|^3\right) \end{aligned} \tag{B.16}$$

By using the definition of $\widetilde{kk}_{t,t+k}$ and $\widetilde{ff}_{t,t+k}$, we have:

$$\begin{aligned}\widetilde{kk}_{t,t}-\widetilde{ff}_{t,t}+\frac{1}{2}\left(\widetilde{kk}_{t,t}^{2}-\widetilde{ff}_{t,t}^{2}\right) &= mc_t+\frac{1}{2}\left[\left(-\varsigma_G c_t+\varsigma_G g_t+mc_t\right)^2-\left(-\varsigma_G c_t+\varsigma_G g_t\right)^2\right]+\text{s.o.t.i.p.}\\ &= mc_t+\frac{1}{2}mc_t^2-\varsigma_G c_t mc_t+\varsigma_G g_t mc_t+\text{s.o.t.i.p..}\end{aligned}$$

Let define $\widetilde{kf}_t \equiv mc_t+\frac{1}{2}mc_t^2-\varsigma_G c_t mc_t+\varsigma_G g_t mc_t$. Then we have:

$$\widetilde{kf}_t=\widetilde{kk}_{t,t}-\widetilde{ff}_{t,t}+\frac{1}{2}\left(\widetilde{kk}_{t,t}^{2}-\widetilde{ff}_{t,t}^{2}\right). \tag{B.17}$$

Plugging Eq.(B.16) into the definition of $\widetilde{kf}_t$ yields:

$$\begin{aligned}\widetilde{kf}_t &= c_t+\varphi n_t+\frac{\tau}{1-\tau}\hat{\tau}_t-a_t+\left(1-\varsigma_G\right)c_t^2+\varphi^2 n_t^2+\frac{\tau\left[\left(1-\tau\right)\left(1+\epsilon_\tau\right)+\tau\right]}{2\left(1-\tau\right)^2}\hat{\tau}_t^2\\ &+\varphi\left(2-\varsigma_G\right)c_t n_t+\frac{\left(1-\tau\right)\epsilon_C+\tau\varsigma_C}{1-\tau}c_t\hat{\tau}_t-2\varphi n_t a_t-\left(2-\varsigma_G\right)c_t a_t\\ &+\frac{\varphi\left[\left(1-\tau\right)\epsilon_N+\tau\right]}{1-\tau}n_t\hat{\tau}_t-\frac{\left(1-\tau\right)\epsilon_A+\tau}{1-\tau}\hat{\tau}_t a_t+\varsigma_G c_t g_t+\varphi\varsigma_G n_t g_t+\frac{\varsigma_G\tau}{1-\tau}\hat{\tau}_t g_t\\ &-\varsigma_G g_t a_t+\text{s.o.t.i.p.}+o\left(\|\xi\|^3\right),\end{aligned} \tag{B.18}$$

with $\epsilon_\tau \equiv \frac{MC_{\tau\tau}}{MC_\tau}\tau$, $\epsilon_C \equiv \frac{MC_{C\tau}}{MC_C}\tau$, $\epsilon_N \equiv \frac{MC_{N\tau}}{MC_N}\tau$ and $\epsilon_A \equiv \frac{MC_{A\tau}}{MC_A}\tau$.

Eq.(28) in the text can be rewritten as:

$$C_t=Y_t-G_t, \tag{B.19}$$

which can be second-order approximated as:

$$\begin{aligned}C\left(Y_t,G_t\right) &= C+C_Y Y y_t+C_G G g_t+C_Y Y\frac{1}{2}\left(1+\frac{C_{YY}}{C_Y}YY\right)y_t^2+\frac{C_{YG}}{C_Y}YGy_t g_t\\ &+\text{s.o.t.i.p.}+o\left(\|\xi\|^3\right).\end{aligned}$$

Because of $\frac{C_t-C}{C}=c_t$, this can be rewritten as:

$$\begin{aligned}c_t &= C_Y\varsigma_C^{-1}y_t+C_G\varsigma_C^{-1}\varsigma_G g_t+C_Y\varsigma_C^{-1}\frac{1}{2}\left(1-\frac{C_{YY}}{C_Y}Y\right)y_t^2+\frac{C_{YG}}{C_Y}Y\varsigma_C^{-1}\varsigma_G y_t g_t\\ &+\text{s.o.t.i.p.}+o\left(\|\xi\|^3\right), \qquad \text{(B.20)}\\ &= \frac{1}{\varsigma_C}y_t-\frac{\varsigma_G}{\varsigma_C}g_t-\frac{\varsigma_G}{2\varsigma_C}y_t^2+\frac{\varsigma_G}{\varsigma_G^2}y_t g_t+\text{s.o.t.i.p.}+o\left(\|\xi\|^3\right)\end{aligned}$$

Second-order approximation of Eq.(22) in the text $N_t=\frac{Y_t Z_t}{A_t}$ can be rewritten as:

$$n_t=y_t-a_t+\frac{1}{2}y_t^2+z_t-y_t a_t+\text{s.o.t.i.p.}+o\left(\|\xi\|^3\right). \tag{B.21}$$

Plugging Eqs.(B.21) and (B.21) into Eq.(B.18) yields:

$$\begin{aligned}\widetilde{kf}_t &= \frac{1+\varphi\varsigma_C}{\varsigma_C}y_t+\frac{\tau}{1-\tau}\hat{\tau}_t+\varphi z_t+\frac{\omega_{\nu 1}}{2\varsigma_C^2}y_t^2+\frac{\omega_{\nu_2}}{2\left(1-\tau\right)^2}\hat{\tau}_t^2+\frac{\omega_{\nu 3}}{\varsigma_C^2}y_t g_t\\ &-\frac{\omega_{\nu_4}}{\varsigma_C}y_t a_t-\frac{\omega_{\nu 5}}{\left(1-\tau\right)\varsigma_C}\hat{\tau}_t g_t-\frac{\omega_{\nu 6}}{1-\tau}\hat{\tau}_t a_t+\frac{\omega_{\nu 7}}{\left(1-\tau\right)\varsigma_C}y_t\hat{\tau}_t+\text{t.i.p.}\\ &+o\left(\|\xi\|^3\right),\end{aligned} \tag{B.22}$$

with $\omega_{\nu 1} \equiv \varsigma_C \varphi \left[\varsigma_C (1+2\varphi) + 2(2-\varsigma_G)\right] - \varsigma_G$, $\omega_{\nu 2} \equiv \tau \left[1 + \epsilon_\tau (1-\tau)\right]$, $\omega_{\nu 3} \equiv 1 - \varsigma_C \left\{\varsigma_G (1-2\varsigma_G) - \varphi \left[\varsigma_G (2-\varsigma_G) - 2\right]\right\}$, $\omega_{\nu 4} \equiv \varphi \varsigma_C \left[1 + 2(1+\varphi)\right] + (1+\varphi)(2-\varsigma_G)$, $\omega_{\nu 5} \equiv \varsigma_G \epsilon_C (1-\tau)$, $\omega_{\nu 6} \equiv (1-\tau)(\varphi \epsilon_N + \epsilon_A) + \tau (1+\varphi)$ and $\omega_{\nu 7} \equiv (1-\tau)\left[\epsilon_C + \varphi \varsigma_C \epsilon_N\right] + \tau \varsigma_C (1+\varphi)$.

Plugging Eq.(B.22) into Eq.(B.15), we have:

$$\begin{aligned} \nu &= \kappa \sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left[\frac{1+\varphi\varsigma_C}{\varsigma_C} y_t + \frac{\tau}{1-\tau} \hat{\tau}_t + \frac{\omega_{\nu 1}}{2\varsigma_C^2} y_t^2 + \frac{\omega_{\nu 2}}{2\varsigma_C^2} \hat{\tau}_t^2 + \frac{\omega_{\nu 3}}{\varsigma_C^2} y_t g_t - \frac{\omega_{\nu 4}}{\varsigma_C} y_t a_t \right. \\ & \left. - \frac{\omega_{\nu 5}}{(1-\tau)\varsigma_C} \hat{\tau}_t g_t - \frac{\omega_{\nu 6}}{1-\tau} \hat{\tau}_t a_t + \frac{\omega_{\nu 7}}{(1-\tau)\varsigma_C} y_t \hat{\tau}_t + \frac{\varepsilon (1+\varphi)}{2\kappa} \pi_t^2 \right] + \text{t.i.p.}, \end{aligned} \tag{B.23}$$

where we use Eq.(B.3).

B.3 Second-order Approximation of Solvency Condition

Let define:

$$\mathcal{W}_t \equiv \sum_{t=0}^{\infty} \beta^t \mathrm{E}_t \left(C_t^{-1} SP_t \right) \tag{B.24}$$

with $\mathcal{W}_t = (1-\delta_t) C_t^{-1} R_t^G B_t \Pi_t^{-1}$.

First, we take a second-order approximation of $C_t^{-1} SP_t$ as follows:

$$C_t^{-1} SP_t = C^{-1} SP \left(1 - c_t + sp_t + \frac{1}{2} c_t^2 + c_t sp_t \right) + \text{s.o.t.i.p.} \tag{B.25}$$

Second-order approximation of the definition of $SP_t \equiv \tau_t Y_t - G_t$ is given by:

$$\begin{aligned} sp_t = (1+\omega_g)\hat{\tau}_t + (1+\omega_g) y_t - \omega_g g_t + \frac{1+\omega_g}{2}\hat{\tau}_t^2 + \frac{1+\omega_g}{2} y_t^2 + (1+\omega_g) y_t \hat{\tau}_t + \text{s.o.t.i.p.} \\ + o\left(\| \xi \|^3\right), \end{aligned} \tag{B.26}$$

with $\omega_g \equiv \frac{G}{SP}$ where we use the fact that $sp_t = \frac{SP_t - SP}{SP}$.

Plugging Eq.(B.21) into Eq.(B.25) yields:

$$\begin{aligned} C_t^{-1} SP_t &= \left[1 - \varsigma_C^{-1} y_t + \frac{\varsigma_G}{\varsigma_C} g_t + sp_t + \frac{1+\varsigma_G}{2\varsigma_C^2} y_t^2 - \frac{2\varsigma_G}{\varsigma_C^2} y_t g_t - \varsigma_C y_t sp_t + \frac{\varsigma_G}{\varsigma_C} g_t sp_t \right] \\ & + \text{s.o.t.i.p.} + o\left(\| \xi \|^3\right). \end{aligned} \tag{B.27}$$

Plugging Eq.(B.26) into Eq.(B.27) yields:

$$\begin{aligned} C_t^{-1} SP_t &= C^{-1} SP \left[1 - \frac{1 - \varsigma_C (1+\omega_g)}{\varsigma_C} y_t + \frac{\varsigma_G - \varsigma_C \omega_g}{\varsigma_C} g_t + (1+\omega_g)\hat{\tau}_t + \frac{\omega_{w1}}{2\varsigma_C^2} y_t^2 \right. \\ & \left. + \frac{1+\omega_g}{2} \hat{\tau}_t^2 + \frac{\omega_{w2}}{\varsigma_C^2} y_t g_t - \frac{(1+\omega_g)\varsigma_G}{\varsigma_C} y_t \hat{\tau}_t + \frac{\varsigma_G (1+\omega_G)}{\varsigma_C} \hat{\tau}_t g_t \right] \\ & + \text{s.o.t.i.p.} + o\left(\| \xi \|^3\right), \end{aligned} \tag{B.28}$$

with $\omega_{w1} \equiv (1+\varsigma_G)\left[1 - \varsigma_C (1+\omega_g)\right]$ and $\omega_{w2} \equiv \varsigma_C \left[\varsigma_G (1+\omega_g) + \omega_g\right] - 2\varsigma_G$.

Let define $w_t \equiv \frac{C_t^{-1} SP_t - C^{-1} SP}{C^{-1} SP}$. Combining Eqs.(B.24) and (B.28) and this definition yields:

$$\begin{aligned} w_t &= (1-\beta) \left\{ - \frac{1 - \varsigma_C (1+\omega_g)}{\varsigma_C} y_t + \frac{\varsigma_G - \varsigma_C \omega_g}{\varsigma_C} g_t + (1+\omega_g)\hat{\tau}_t + \frac{\omega_{w1}}{2\varsigma_C^2} y_t^2 + \frac{1+\omega_g}{2} \hat{\tau}_t^2 \right. \\ & \left. + \frac{\omega_{w2}}{\varsigma_C^2} y_t g_t - \frac{(1+\omega_g)\varsigma_G}{\varsigma_C} y_t \hat{\tau}_t + \frac{\varsigma_G (1+\omega_G)}{\varsigma_C} \hat{\tau}_t g_t \right\} + \beta \mathrm{E}_t (\omega_{t+1}) + \text{s.o.t.i.p.} \\ & + o\left(\| \xi \|^3\right). \end{aligned} \tag{B.29}$$

Iterating forward Eq.(B.29) yields:

$$\begin{aligned} w &= (1-\beta)\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left[-\frac{1-\varsigma_C(1+\omega_g)}{\varsigma_C}y_t+(1+\omega_g)\hat{\tau}_t+\frac{\omega_{w1}}{2\varsigma_C^2}y_t^2+\frac{1+\omega_g}{2}\hat{\tau}_t^2\right. \\ &\left. +\frac{\omega_{w2}}{\varsigma_C^2}y_t g_t-\frac{(1+\omega_g)\varsigma_G}{\varsigma_C}y_t\hat{\tau}_t+\frac{\varsigma_G(1+\omega_G)}{\varsigma_C}\hat{\tau}_t g_t\right]+\text{t.i.p.}+o\left(\|\xi\|^3\right) \end{aligned} \tag{B.30}$$

B.4 Eliminating Liner Terms

In the first-order, Eqs.(B.3), (B.23) and (B.30) are given by:

$$\begin{aligned} w &= (1-\beta)\sum_{t=0}^{\infty}\beta^t \mathrm{E}_t\left[-\frac{1-\varsigma_G(1+\omega_g)}{\varsigma_C}y_t+(1+\omega_g)\hat{\tau}_t\right]+\text{t.i.p.}+o\left(\|\xi\|^2\right) \\ \nu &= \kappa\sum_{t=0}^{\infty}\beta^t \mathrm{E}_t\left[\frac{1+\varsigma_C\varphi}{\varsigma_C}y_t+\frac{\tau}{1-\tau}\hat{\tau}_t\right]+\text{t.i.p.}+o\left(\|\xi\|^2\right) \\ u &= \sum_{t=0}^{\infty}\beta^t \mathrm{E}_t\left(\frac{\Phi}{\varsigma_C}y_t\right)+\text{t.i.p.}+o\left(\|\xi\|^2\right). \end{aligned} \tag{B.31}$$

Thus, we formularize to eliminate linear terms on y_t and $\hat{\tau}_t$ in the last equality in Eq.(B.31) as follows:

$$\begin{aligned} \frac{\Phi}{\varsigma_C} &= \vartheta_1\left[-\frac{1-\varsigma_C(1+\omega_g)}{\varsigma_C}\right]+\vartheta_2\left[\frac{1+\varsigma_C\varphi}{\varsigma_C}\right] \\ 0 &= \vartheta_1(1+\omega_g)+\vartheta_2\left(\frac{\tau}{1-\tau}\right) \end{aligned}$$

where ϑ_1 and ϑ_2 are undetermined coefficients. Here, $\frac{\Phi}{\varsigma_C}$ and 0 are coefficients on y_t and $\hat{\tau}_t$ in the last equality in Eq.(B.31) while $-\frac{1-\varsigma_C(1+\omega_g)}{\varsigma_C}$ and $(1+\omega_g)$ are coefficients on those in the first equality in Eq.(B.31), respectively and and $\frac{1+\varsigma_C\varphi}{\varsigma_C}$ and $\frac{\tau}{1-\tau}$ are coefficients on those in the second equality in Eq.(B.31), respectively.

By solving this system, we get:

$$\vartheta_1 = -\frac{\tau\Phi}{\Gamma} \tag{B.32}$$

$$\vartheta_2 = \frac{(1-\tau)(1+\omega_g)\Phi}{\Gamma} \tag{B.33}$$

with $\Gamma\equiv(1+\omega_g)(1-\tau)(1+\varsigma_C\varphi)+\tau[1-\varsigma_C(1+\omega_g)]$.

By using the facts that $-\frac{1-\varsigma_C(1+\omega_g)}{\varsigma_C}$ and $\frac{1+\varsigma_C\varphi}{\varsigma_C}$ are coefficients on y_t on Eqs.(B.23) and (B.30), the linear term y_t on Eq.(B.3) is given by:

$$\begin{aligned} \sum_{t=0}^{\infty}\beta^t\frac{\Phi}{\varsigma_C}\mathrm{E}_t(y_t) &= \vartheta_1(1-\beta)^{-1}w+\theta_2\kappa^{-1}\nu-\theta_1\sum_{t=0}^{\infty}\beta^t\mathrm{E}_0\left[\frac{\omega_{w1}}{2\varsigma_C^2}y_t^2+\frac{1+\omega_g}{2}\hat{\tau}_t^2+\frac{\omega_{w2}}{\varsigma_C^2}y_t g_t\right. \\ &\left.-\frac{(1+\omega_g)\varsigma_G}{\varsigma_C}y_t\hat{\tau}_t+\frac{\varsigma_G(1+\omega_G)}{\varsigma_C}\hat{\tau}_t g_t\right]-\vartheta_2\sum_{t=0}^{\infty}\beta^t\mathrm{E}_0\left[\frac{\omega_{\nu1}}{2\varsigma_C^2}y_t^2+\frac{\omega_{\nu2}}{2\varsigma_C^2}\hat{\tau}_t^2 t\right. \\ &+\frac{\omega_{\nu3}}{\varsigma_C^2}y_t g_t-\frac{\omega_{\nu4}}{\varsigma_C}y_t a_t-\frac{\omega_{\nu5}}{(1-\tau)\varsigma_C}\hat{\tau}_t g_t-\frac{\omega_{\nu6}}{1-\tau}\hat{\tau}_t a_t+\frac{\omega_{\nu7}}{(1-\tau)\varsigma_C}y_t\hat{\tau}_t \\ &\left.+\frac{\varepsilon(1+\varphi)}{2\kappa}\pi_t^2\right]+\Upsilon_0+\text{t.i.p.}, \end{aligned} \tag{B.34}$$

where $\Upsilon_0 \equiv -\frac{\tau\Phi}{\Gamma(1-\beta)}w + \frac{(1-\tau)(1+\omega_g)\Phi}{\Gamma\kappa}\nu$.

By plugging Eqs.(B.32) and (B.33) into Eq.(B.34), we get:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t \frac{\Phi}{\varsigma_C}\mathrm{E}_0(y_t) &= -\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left\{\frac{\Phi\left[(1-\tau)(1+\omega_g)\omega_{\nu 1}-\omega_{w1}\right]}{2\Gamma\varsigma_C^2}y_t^2\right.\\ &\quad -\frac{\Phi\left[\omega_{w2}\tau-(1-\tau)(1+\omega_g)\omega_{\nu 3}\right]}{\Gamma\varsigma_C^2}y_t g_t - \frac{\Phi(1-\tau)(1+\omega_g)\omega_{\nu 4}}{\Gamma\varsigma_C}y_t a_t\\ &\quad \left.+\frac{(1-\tau)(1+\omega_g)\Phi\varepsilon(1+\varphi)}{2\Gamma\kappa}\pi_t^2\right\} + \Upsilon_0 + \text{t.i.p.} + o\left(\|\xi\|^3\right). \qquad \text{(B.35)}\end{aligned}$$

Plugging Eq.(B.35) into Eq.(B.2) yields:

$$u = -\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left[\frac{\Lambda_x}{2}(y_t - y_t^*)^2 + \frac{\Lambda_\pi}{2}\pi_t^2\right] + \Upsilon_0 + \text{t.i.p.} + o\left(\|\xi\|^3\right),$$

which is second-order approximated utility function without linear terms and terms in parentheses corresponds to Eq.(35) in the text.

C FONCs for Policy Authorities

Under the OM policy, The Lagrangean is given by:

$$\begin{aligned}\pounds &= \sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left\{L_t + \mu_{1,t}\left[x_t - x_{t+1} + \varsigma_C\hat{r}_t - \varsigma_C\pi_{t+1} + \frac{\varsigma_C(1-\beta)}{\beta\phi}\delta_{t+1} - \frac{\varsigma_C}{\beta}\hat{r}_{t-1}\right.\right.\\ &\quad \left.+\frac{\varsigma_C}{\beta}\pi_t - \frac{\varsigma_C\omega_o}{\beta^2\phi}\delta_t + \frac{\varsigma_C\varpi}{\beta}sp_t - \frac{\varsigma_C(\omega_\gamma+\phi\beta)}{\beta^2}sp_{t-1} - \epsilon_{x,t}\right] + \mu_{2,t}\left[\pi_t\right.\\ &\quad \left.-\beta\pi_{t+1} - \frac{\kappa(1+\varphi\varsigma_C)}{\varsigma_C}x_t - \epsilon_{\pi,t}\right] + \mu_{3,t}\left(sp_t - \frac{1}{\varpi}\hat{r}_{t-1} + \frac{\omega_o}{\phi\beta\varpi}\delta_t + \frac{1}{\varpi}\pi_t\right.\\ &\quad \left.\left.-\frac{\omega_\gamma-\phi\beta}{\varpi\beta}sp_{t-1} - \frac{1-\beta}{\phi\varpi}\delta_{t+1}\right) - \mu_{4,t}\left(sp_t - \frac{\beta\tau}{(1-\beta)\varsigma_B}x_t - \epsilon_{sp,t}\right)\right\}.\end{aligned}$$

Note that $\hat{r}_t$ disappears because of $\hat{r}_t = 0$ for all t. The FONCs are given by Eqs.(38)–(42) in the text.

Under the MIS policy, The Lagrangean is given by:

$$\begin{aligned}\pounds &= \sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left\{L_t^R + \mu_{1,t}\left[x_t - x_{t+1} + \varsigma_C\hat{r}_t - \varsigma_C\pi_{t+1} + \frac{\varsigma_C(1-\beta)}{\beta\phi}\delta_{t+1} - \frac{\varsigma_C}{\beta}\hat{r}_{t-1}\right.\right.\\ &\quad \left.+\frac{\varsigma_C}{\beta}\pi_t - \frac{\varsigma_C\omega_o}{\beta^2\phi}\delta_t + \frac{\varsigma_C\varpi}{\beta}sp_t - \frac{\varsigma_C(\omega_\gamma+\phi\beta)}{\beta^2}sp_{t-1} - \epsilon_{x,t}\right] + \mu_{2,t}\left[\pi_t\right.\\ &\quad \left.-\beta\pi_{t+1} - \frac{\kappa(1+\varphi\varsigma_C)}{\varsigma_C}x_t - \epsilon_{\pi,t}\right] + \mu_{3,t}\left(sp_t - \frac{1}{\varpi}\hat{r}_{t-1} + \frac{\omega_o}{\phi\beta\varpi}\delta_t + \frac{1}{\varpi}\pi_t\right.\\ &\quad \left.\left.-\frac{\omega_\gamma-\phi\beta}{\varpi\beta}sp_{t-1} - \frac{1-\beta}{\phi\varpi}\delta_{t+1}\right) - \mu_{4,t}\left(sp_t - \frac{\beta\tau}{(1-\beta)\varsigma_B}x_t - \epsilon_{sp,t}\right)\right\}.\end{aligned}$$

Note that $\hat{r}_t$ disappears because of $\hat{r}_t = 0$ for all t. The FONCs are given by Eq.(40) and Eqs(42)–(45) in the text where we plug Eq.(9) in the text into L_t^R.

Under the OMF policy, The Lagrangean is given by:

$$\pounds = \sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left\{L_t + \mu_{1,t}\left[x_t - x_{t+1} + \varsigma_C\hat{r}_t - \varsigma_C\pi_{t+1} + \frac{\varsigma_C(1-\beta)}{\beta\phi}\delta_{t+1} - \frac{\varsigma_C}{\beta}\hat{r}_{t-1}\right.\right.$$

$$
+\frac{\varsigma_C}{\beta}\pi_t - \frac{\varsigma_C\omega_o}{\beta^2\phi}\delta_t + \frac{\varsigma_C\varpi}{\beta}sp_t - \frac{\varsigma_C(\omega_\gamma+\phi\beta)}{\beta^2}sp_{t-1} - \epsilon_{x,t}\Bigg] + \mu_{2,t}\Big[\pi_t
-\beta\pi_{t+1} - \frac{\kappa(1+\varphi\varsigma_C)}{\varsigma_C}x_t - \frac{\kappa\tau}{1-\tau}\hat{\tau}_t - \epsilon_{\pi,t}\Bigg] + \mu_{3,t}\left(sp_t - \frac{1}{\varpi}\hat{r}_{t-1} + \frac{\omega_o}{\phi\beta\varpi}\delta_t + \frac{1}{\varpi}\pi_t\right.
\left.-\frac{\omega_\gamma-\phi\beta}{\varpi\beta}sp_{t-1} - \frac{1-\beta}{\phi\varpi}\delta_{t+1}\right) + \mu_{4,t}\left[sp_t - \frac{\beta\tau}{(1-\beta)\varsigma_B}\hat{\tau}_t - \frac{\beta\tau}{(1-\beta)\varsigma_B}x_t - \epsilon_{sp,t}\right]\Bigg\}.
$$

The FONCs are given by Eqs.(38)–(42) in the text and $\mu_{2,t} = -\frac{(1-\tau)\beta}{(1-\beta)\varsigma_B\kappa}\mu_{4,t}$.

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