# On-line Appendices to "Revisiting the Fiscal Theory of Sovereign Risk from a DSGE Viewpoint"

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#### Appendices

#### A Nonstochastic Steady State

We focus on equilibria where the state variables follow paths that are close to a deterministic stationary equilibrium, in which  $\Pi_t = 1$  and  $\frac{\tilde{P}_t}{P_t} = 1$ . Because this steady state is nonstochastic, the productivity has unit values; i.e., A = 1. We assume that the default rate in the steady state is zero; i.e.,  $\delta = 0$ .

In this steady state, the gross nominal interest rate is equal to the inverse of the subjective discount factor, as follows:

$$R = \beta^{-1}$$

Eq.(16) can be rewritten as:

$$\tilde{P}_t = \mathcal{E}_t \left( \frac{K_t}{P^{-1} F_t} \right) \tag{A.1}$$

with:

$$K_{t} \equiv \frac{\varepsilon}{\varepsilon - 1} \sum_{k=0}^{\infty} (P_{t+k}C_{t+k})^{-1} \tilde{Y}_{t+k} M C_{t+k}^{n} \quad ; \quad F_{t} \equiv P_{t} \sum_{k=0}^{\infty} (P_{t+k}C_{t+k})^{-1} \tilde{Y}_{t+k},$$

which implies that:

$$K = \frac{\frac{\varepsilon}{\varepsilon - 1} Y M C^n}{\left(1 - \alpha \beta\right) \left(PC\right)} \quad ; \quad F = \frac{PY}{\left(1 - \alpha \beta\right) \left(PC\right)}$$

These equalities imply that:

$$P = \frac{\varepsilon}{\varepsilon - 1} M C^n.$$

Thus, we have:

$$MC = \left(\frac{\varepsilon}{\varepsilon - 1}\right)^{-1}.$$
 (A.2)

Furthermore, Eqs.(17) and (A.2) imply the following:

$$CN^{\varphi} = \frac{1-\tau}{\frac{\varepsilon}{\varepsilon-1}}.$$
(A.3)

Eq.(A.3) implies the familiar expression:

$$(1-\tau) U_C = \frac{\varepsilon}{\varepsilon - 1} U_N.$$

Note that because  $\tau \in (0,1)$  and  $\varepsilon > 1$ , this steady state is distorted.

Eq.(11) yields the following:

$$B\left(\frac{1-\beta}{\beta}\right) = SP,\tag{A.4}$$

with  $B \equiv \frac{B^n}{P}$ .

Note that  $R = R^H$  because of  $\delta = 0$  and  $R^G = R\Gamma(0)$ . Plugging this into Eq.(??) yields:

$$C^{-1}R\Gamma(0)B = C^{-1}SP + \frac{\beta}{\Gamma(0)}C^{-1}SP + \left(\frac{\beta}{\Gamma(0)}\right)^{2}C^{-1}SP + \dots = \frac{1}{1 - \beta[\Gamma(0)]^{-1}}C^{-1}SP,$$

which implies:

$$\Gamma(0) B\beta^{-1} = \frac{1}{1 - \beta [\Gamma(0)]^{-1}} SP.$$
(A.5)

Plugging Eq.(5) into this equality yields:

$$\Gamma\left(0\right) = \frac{1-\beta}{1-\beta\left[\Gamma\left(0\right)\right]^{-1}},$$

which implies that  $\Gamma(0) = 1$ . Thus, our assumption that  $\delta = 0$  is consistent with  $\Gamma(0) = 1$ . Because of  $\Gamma(0) = 1$ ,  $R^G = R$ . Thus,

$$R^G = R^H. (A.6)$$

In the steady state, Eq.(13) reduces to:

$$1 = \frac{\frac{1}{\left(1 - \beta \frac{R^{H}}{R^{G}}\right)} \left(C^{-1}SP\right)}{C^{-1}RB}.$$
(A.7)

Note that the RHS in Eq.(A.7) corresponds to the steady-state value of  $\Psi$ . That is,  $\Psi = 1$  is applied in the steady state. This implies that the default rate is zero in the steady state.

### **B** Log-linearization of the Model

Log-linearizing Eq.(9) yields:

$$\hat{r}_t = \hat{r}_t^H - \mathcal{E}_t \left( \delta_{t+1} \right). \tag{B.1}$$

Log-linearizing Eq.(11) yields:

$$b_{t} = \frac{1}{\beta}\hat{r}_{t-1} - \frac{1}{\beta}\delta_{t} - \frac{1}{\beta}\pi_{t} + \frac{1}{\beta}b_{t-1} - \frac{1-\beta}{\beta}sp_{t} - \frac{\phi}{\beta}sp_{t-1},$$
(B.2)

where we use the log-linearized definition of the government debt coupon rate  $\hat{r}_t^G = \hat{r}_t - \phi s p_t$  with  $\hat{r}_t^G \equiv \frac{dR_t^G}{R^G}$ . Putting Eqs.(B.1) and (10) into Eq.(B.2) yields Eq.(24).

The log-linearized fiscal surplus is given by:

$$sp_t = \frac{\beta\tau}{(1-\beta)\varsigma_B}\hat{\tau}_t + \frac{\beta\tau}{(1-\beta)\varsigma_B}y_t - \frac{\beta\varsigma_G}{(1-\beta)\varsigma_B}g_t.$$
 (B.3)

Log-linearizing Eq.(14) yields:

$$c_{t} = E_{t}(c_{t+1}) - \beta \hat{r}_{t} + E_{t}(\pi_{t+1}) - \frac{\omega_{\phi}}{1 - \beta} b_{t} + E_{t}(\delta_{t+1}) - \frac{\omega_{sp}}{\beta (1 - \beta)} sp_{t} + \frac{1}{\beta} \hat{r}_{t-1} - \frac{1}{\beta} \pi_{t} + \frac{1}{\beta} b_{t-1} - \frac{1}{\beta} \delta_{t} - \frac{\phi}{\beta} sp_{t-1},$$
(B.4)

with  $\omega_{sp} \equiv (1-\beta)^2 - \phi \omega_{\gamma} \beta$ , where we use the log-linearized definition of the government debt coupon rate. Eq.(B.4) is our log-linearized Euler equation.

Log-linearizing Eq.(15) yields:

$$n_t = y_t - a_t. \tag{B.5}$$

 $Z_t$  disappears in Eq.(B.5) because of  $o\left(\|\xi\|^2\right)$ . By log-linearizing Eq.(16), we have:

$$\pi_t = \beta \mathcal{E}_t \left( \pi_{t+1} \right) + \kappa m c_t, \tag{B.6}$$

which is the fundamental equality of our NKPC.

Log-linearizing Eq.(17) yields:

$$mc_t = c_t + \varphi y_t + \frac{\tau}{1 - \tau} \hat{\tau}_t - (1 + \varphi) a_t.$$
(B.7)

By log-linearizing Eq.(18), we obtain:

$$y_t = \varsigma_C c_t + \varsigma_G g_t. \tag{B.8}$$

Putting Eq.(B.8) into Eq.(B.4) yields:

$$y_{t} = \mathbf{E}_{t}(y_{t+1}) - \varsigma_{C}\hat{r}_{t} + \varsigma_{C}\mathbf{E}_{t}(\pi_{t+1}) - \frac{\varsigma_{C}\omega_{\phi}}{1-\beta}b_{t} + \varsigma_{C}\mathbf{E}_{t}(\delta_{t+1}) + \frac{\varsigma_{C}}{\beta}\hat{r}_{t-1} - \frac{\varsigma_{C}}{\beta}\pi_{t} + \frac{\varsigma_{C}}{\beta}b_{t-1} + \frac{\varsigma_{C}}{\beta}\delta_{t} - \frac{\varsigma_{C}\omega_{sp}}{\beta(1-\beta)}sp_{t} - \frac{\phi\varsigma_{C}}{\beta}sp_{t-1} + \varsigma_{G}(1-\rho_{G})g_{t}.$$
(B.9)

Putting the definition of the OGTL ino Eq.(B.9) yields Eq.(??).Putting Eqs.(B.7) and (B.8) into Eq.(B.6), we

$$\pi_t = \beta \mathbf{E}_t \left( \pi_{t+1} \right) + \frac{\kappa \left[ 1 + \varphi \varsigma_C \right]}{1 - \varsigma_G} y_t + \frac{\kappa \tau}{1 - \tau} \hat{\tau}_t - \frac{\kappa \varsigma_G}{1 - \varsigma_G} g_t - \kappa \left( 1 + \varphi \right) a_t.$$
(B.10)

Eq.(B.10) stems from the firms' FONC. Eq.(14) does not have any notable features. Putting the definition of the OGTL into Eq.(B.10) yields Eq.(??).

### C Derivation of the Second-order Approximated Utility Function

Following Gali[20], the second-order approximated utility function is given by:

$$\sum_{t=0}^{\infty} \beta^{t} \mathbf{E}_{0} \left( \frac{U_{t} - U}{U_{C}C} \right) = \sum_{t=0}^{\infty} \beta^{t} \mathbf{E}_{0} \left[ \frac{\Phi}{1 - \varsigma_{G}} y_{t} - \frac{(1 - \Phi)(1 + \varphi)}{\varsigma_{C}2} y_{t}^{2} + \frac{(1 - \Phi)(1 + \varphi)}{1 - \varsigma_{G}} y_{t} a_{t} - \frac{(1 - \Phi)\varepsilon}{\varsigma_{C}2\kappa} \pi_{t}^{2} \right] + \text{t.i.p.} + o\left( \left\| \xi \right\|^{3} \right),$$
(C.1)

where t.i.p. denotes the terms independent of policy,  $o\left(\|\xi\|^3\right)$  are the terms of order three or higher, and  $\Phi \equiv 1 - \frac{1-\tau}{\frac{\varepsilon}{\varepsilon-1}}$  denotes the steady-state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor. On the RHS, there are linear terms  $\sum_{t=0}^{\infty} \beta^t E_0\left(\frac{\Phi}{1-\varsigma_G}y_t\right)$  generating the welfare reversal.<sup>1</sup> To avoid welfare reversal, we need to eliminate the linear terms on the RHS in Eq.(C.1). Following Benigno and Woodford[3] and Benigno and Woodford[4], the linear terms are rewritten as follows:

$$\begin{split} \sum_{t=0}^{\infty} \beta^{t} \mathbf{E}_{0} \left( \frac{\Phi}{1-\varsigma_{G}} y_{t} \right) &= -\sum_{t=0}^{\infty} \beta^{t} \mathbf{E}_{0} \left[ \frac{\Phi \left[ \left( 1-\tau \right) \left( 1+\omega_{g} \right) \omega_{\nu 1} - \tau \omega_{\omega 1} \right]}{2\Gamma\varsigma_{C}^{2}} y_{t}^{2} \\ &- \frac{\Phi \left[ \omega_{\omega 2} \tau - \left( 1-\tau \right) \left( 1+\omega_{g} \right) \omega_{\nu 3} \right]}{\Gamma\varsigma_{C}^{2}} y_{t} g_{t} - \frac{\Phi \left( 1-\tau \right) \left( 1+\omega_{g} \right) \omega_{\nu 4}}{\Theta\varsigma_{C}} y_{t} a_{t} \\ &+ \frac{\Phi \left( 1-\tau \right) \left( 1+\omega_{g} \right) \varepsilon \left( 1+\varphi \right)}{2\Theta\kappa} \pi_{t}^{2} \right] + \Upsilon_{0} + o \left( \left\| \xi \right\|^{3} \right), \end{split}$$

with  $\omega_g \equiv \frac{G}{SP} = \frac{\beta_{\varsigma G}}{(1-\beta)\varsigma_B}$ ,  $\Theta \equiv (1+\omega_g)(1-\tau)[1+\varsigma_C\varphi]+\tau [1-\varsigma_C(1+\omega_g)]$ ,  $\omega_{\nu 1} \equiv \varsigma_C\varphi [\varsigma_C(1+2\varphi)+2(2-\varsigma_G)]$ ,  $\omega_{\omega 1} \equiv (1+\varsigma_G)[1-\varsigma_C(1+\omega_g)]$ ,  $\omega_{\omega 2} \equiv \varsigma_C [\varsigma_G(1+\omega_g)+\omega_g]-2\varsigma_G$ ,  $\omega_{\nu 3} \equiv 1-\varsigma_C \{\varsigma_G(1-2\varsigma_G)-\varphi [\varsigma_G(2-\varsigma_G)-2]\}$ , and  $\omega_{\nu 4} \equiv \varphi_{\varsigma C} [1+2(1+\varphi)]+(1+\varphi)(2-\varsigma_G)$ , where  $\Upsilon_0 \equiv -\frac{\tau\Phi}{\Gamma(1-\beta)}\omega + \frac{(1-\tau)(1+\omega_g)\Phi}{\Theta\kappa}\nu$  denotes a transitory component.

Let define  $\Lambda_x \equiv \frac{\omega_{u1}}{\Theta \varsigma_C^2}$  and  $\Lambda_\pi \equiv \frac{\varepsilon [\Phi(1-\tau)(1+\omega_g)(1+\varphi)\varsigma_C+\Theta(1-\Phi)]}{\Theta \kappa \varsigma_C}$ ,  $\omega_{u1} \equiv \Phi [(1-\tau)(1+\omega_g)\omega_{\nu 1}-\tau \omega_{\omega 1}] + (1-\Phi)(1+\varphi)\varsigma_C\Theta$ . Then the previous expression can be rewritten as:

$$L_t \equiv \frac{\Lambda_x}{2} x_t^2 + \frac{\Lambda_\pi}{2} \pi_t^2,$$

which is period welfare costs in Eq.(15) in the text. Note that  $x_t \equiv y_t - y_t^*$  with  $y_t^* \equiv \frac{\omega_{u2}}{\omega_{u1}} a_t + \frac{\omega_{u3}}{\omega_{u1}} g_t$ ,  $\omega_{u2} \equiv \varsigma_C \left[ \Phi \left( 1 - \tau \right) \left( 1 + \omega_g \right) \omega_{\nu 4} + \left( 1 - \Phi \right) \left( 1 + \varphi \right) \Theta \right], \ \omega_{u3} \equiv \Phi \left[ \omega_{\omega 2} \tau - \left( 1 - \tau \right) \left( 1 + \omega_g \right) \omega_{\nu 3} \right].$ 

# D Counter Factual Exercise on the Price Level Targeting in an Endowment Economy

In the endowment economy model,  $Y_t = Y$  is applicable so that Eqs.(C.9) which is prototype of the NKIS and (C.3) which is log-linearized fiscal surplus are replaced by:

$$\delta_{t} = \hat{r}_{t-1}^{G} - \pi_{t} + b_{t-1} - (1-\beta) sp_{t} + \frac{\beta (1-\rho_{G}) \varsigma_{G}}{1-\varsigma_{G}} g_{t} + \beta \mathbf{E}_{t} (\pi_{t+1}) - \beta \hat{r}_{t}^{H}$$

<sup>&</sup>lt;sup>1</sup>The presence of linear terms generally leads to the incorrect evaluation of welfare, with a simple example of this result proposed by Kim and Kim[6]. Tesar[7] used the log-linearization method and derived the paradoxical result that an incomplete-markets economy produces a higher level of welfare than the complete-markets economy. Kim and Kim[6] point out that the reversal of welfare ordering implies approximation errors owing to the linearization.

$$+\beta \mathbf{E}_t \left(\delta_{t+1}\right) - \beta b_t,\tag{D.1}$$

$$sp_t = \frac{\beta\tau}{(1-\beta)\varsigma_B}\hat{\tau}_t - \frac{\beta\varsigma_G}{(1-\beta)\varsigma_B}g_t.$$
 (D.2)

Figure 1 shows responses in the endowment economy model under the price level targeting. Figure 2 shows responses in the endowment economy model under the price level targeting although  $\phi = 0.33$ .

# E Empirical Evidence for the Calibrated Unfamiliar Parameters and AR(1) Processes

One of our calibrated parameters, the elasticity of the interest rate spread to the fiscal deficit  $\gamma$ , draws on the following regression:

$$\frac{CR^{Risky} - CR_t}{\bar{X}} = \alpha_0 + \alpha_1 \left(1 - DUM_t\right) df_t + \alpha_2 DUM_t + \alpha_3 DUM_t df_t,$$
(E.1)

where  $CR_t^{risky}$  corresponding to  $R_t^G$  denotes the nominal coupon rate for risky assets,  $CR_t$  the nominal coupon rate for safe assets,  $DUM_t$  is a Greek crisis dummy variable that takes a value of one for the period from May 2010 to June 2012 and zero otherwise (detailed explanation provided below), and  $\bar{X}$  denotes the average of  $CR_t^{Risky} - CR_t$  for the period of  $DUM_t = 1$ .  $\alpha_1$  and  $\alpha_3$  measure how changes in the percentage deviation of the fiscal deficit  $df_t \equiv -sp_t$  widen or narrow the interest rate spread (coupon rate based)  $CR_t^{risky} - CR_t$ . Although these coefficients correspond to  $\gamma$ , we focus on  $\alpha_3$  because it is the elasticity during the severe debt crisis. Specifically,  $\alpha_3$  can be regarded as  $\frac{d(CR_t^{Risky} - CR_t)}{d(df_t)} \frac{1}{CR_t^{Risky} - CR_t}$ , which is consistent with our assumption of  $\gamma$ .

Data are monthly and retrieved from Thomson Datastream, and we use the coupon rate spread between the 10-year government bond for Greece and that for Germany and the real government budget balance in Greece.<sup>2</sup> The sample period is from January 2005 to April 2015. Note that the Athens Olympics were in January 2005, at the beginning of the period when the unhealthy fiscal deficit started. The real government budget balance is seasonally adjusted and Hodrick–Prescott (HP) filtered. We assign  $DUM_t = 1$  during May 2010 to June 2012, otherwise  $DUM_t = 0$ . Note that Greece requested fiscal support from both the International Monetary Fund (IMF) and the ECB in April 2010, May 2010 was the following month, and Greece decided to adopt a reduced budget following the results of the poll in June 2012. That is,  $DUM_t = 1$  is assigned during the severe debt crisis in Greece.

The estimators on  $\alpha_0$ ,  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_3$  are 0.0802, 0.0144, 0.8651, and 1.1736, respectively. The corresponding standard errors are 0.0188, 0.0012, 0.0211, and 0.0955, respectively. All coefficients are significant at the 1% level. The result that  $\alpha_3$  is significant implies that the elasticity of the interest rate spread (coupon rate based) to the fiscal deficit  $\gamma$  is significant during the severe debt crisis when the nominal interest rate rose rapidly, and its elasticity is 1.1736. Thus, we set  $\gamma$  to 1.1736. Because  $\gamma$  is significant during May 2010 to June 2012, we regard the average of the spread  $CR_t^{risky} - CR_t$  as the risk premium, and we find that the interest rate spread for risky assets  $\phi$  is 0.033.

AR(1) processes are also estimated from the data for real GDP, the GDP deflator, nominal government expenditure and employment in Greece retrieved from IMF World Economic Outlook, and the sample period is from January 2005 to April 2015. Productivity is GDP divided by

 $<sup>^{2}</sup>$ The original data include the nominal government budget balance, which we deflate using the CPI.

employment and real government expenditure is nominal government expenditure divided by the GDP deflator. The generated data are HP filtered. Our results for the persistence of productivity  $\rho_A$  and the persistence of government expenditure are 0.976 and 0.927, respectively, and the innovations for productivity and government expenditure are 0.0316 and 0.0728, respectively, as mentioned in Section 5.1.

As we discussed in Section 2.1, our assumption concerning the elasticity of the interest rate spread to the fiscal deficit  $\gamma > 1$  is supported by the data. This is because the t-statistic for the null hypothesis  $\alpha_3 = 1$  against the alternative hypothesis  $\alpha_3 > 1$  is 1.8182, and its corresponding p-value is 0.0359, and thus  $\alpha_3 > 1$  is supported statistically. Note that as mentioned,  $\alpha_3$  corresponds to  $\gamma$ .

### F Empirical Evidence for Government Debt with Interest Payment as an Argument for $\Gamma(\cdot)$

Similar to Eq.(E.1), we estimate the following:

$$\frac{CR^{Risky} - CR_t}{\bar{X}} = \tilde{\alpha}_0 + \tilde{\alpha}_1 \left(1 - DUM_t\right) rb_t + \tilde{\alpha}_2 DUM_t + \tilde{\alpha}_3 DUM_t rb_t$$

where  $\tilde{\alpha}_1$  and  $\tilde{\alpha}_3$  measure how changes in the percentage deviation of government debt with interest payment from its steady-state value  $rb_t \equiv \frac{R_t B_t}{RB} - 1$  widen or narrow the coupon rate spread  $CR_t^{risky} - CR_t$ . Thus,  $\tilde{\alpha}_1$  and  $\tilde{\alpha}_3$  correspond to  $\gamma$ . Data are quarterly and retrieved from Thomson Datastream, and we use the sum of government debt and the government interest payment divided by the CPI in Greece. The generated data are HP filtered. The sample period runs from Q1, 2005 to Q1, 2015 because data on government debt and interest payment are available in quarterly frequency. We assign  $DUM_t = 1$  during Q2, 2010 to Q2, 2012, otherwise  $DUM_t = 0$ . The estimation procedure is the same as Eq.(E.1).

The estimators on  $\tilde{\alpha}_0$ ,  $\tilde{\alpha}_1$ ,  $\tilde{\alpha}_2$ , and  $\tilde{\alpha}_3$  are 0.0518, -1.5522, 0.9727, and 1.5428, respectively. The corresponding standard errors are 0.0687, 1.5492, 0.0735, and 1.8895, respectively. That  $\tilde{\alpha}_1$  and  $\tilde{\alpha}_3$  are not significant means that  $\gamma$  cannot be estimated if we assume that the argument for  $\Gamma(\cdot)$  is government debt with interest payment in Greece. This estimation result and the result on Appendix D imply that the (negative) fiscal surplus as an argument for  $\Gamma(\cdot)$  is plausible, although government debt with interest payment as an argument for  $\Gamma(\cdot)$  is not plausible.

### G Empirical Evidence for Price Stickiness

Following Gali and Gertler<sup>[5]</sup> and Benigno and Lopez-Salido<sup>[2]</sup>, we estimate an equation as follows:

$$\mathbf{E}_{t} \left[ \theta \pi_{t} - \theta 0.99 \pi_{t+1} - (1 - \theta) \left( 1 - \theta 0.99 \right) m c_{t} \right] = 0.$$
 (G.1)

The estimation method is the generalized method of moments developed by Hansen[?]. We use quarterly data for Greece for the GDP deflator and nominal unit labor cost retrieved from Thomson Datastream, both seasonally adjusted. The sample period runs from Q1, 2005 to Q3, 2015. The rate of change in the GDP deflator is regarded as the data series for inflation  $\pi_t$ . We deflate the nominal unit labor cost by the GDP deflator to generate the real unit labor cost. Finally, we calculate the percentage deviation of the marginal cost from its steady-state value following  $mc_t = \frac{MC_t - MC_t^{HP}}{MC_t^{HP}}$ , where  $MC_t^{HP}$  is the HP-filtered real marginal cost.

To estimate,  $\pi_{t-1}$ ,  $\pi_{t-2}$ ,  $mc_{t-1}$ , and  $mc_{t-2}$  are designated as instrumental variables. We use heteroscedasticity and autocorrelation-consistent standard errors. The spectral estimation method is the quadratic spectral kernel, and the bandwidth parameter is selected using the Andrews[?] procedure. The J-statistic for the validity of overidentifying restrictions is 2.03, and the associated p-value is 0.56. This suggests that the above equation is successfully estimated.

As estimation results, we obtain the estimator 0.705 and standard error 0.206. Because the p-value is 0.001, our estimator is significant at the 1% level.

## H Empirical Evidence for the Relationship between the Redemption Yield and the Coupon Rate

We estimate an equation as follows:

$$r_t^H = \beta_0 + \beta_1 r_t^G,$$

where  $r_t^H$  and  $r_t^G$  denote the yield and the coupon rate on benchmark 10-year government bonds, respectively. Here, the coupon rate is the monthly average. We use monthly data for the PIIGS i.e., Portugal, Italy, Ireland, Greece, and Spain—and Germany and the US, and retrieve the data from Thomson Datastream. The sample period runs from January 2005 to September 2015. We verify  $\beta_0 = 0$  and  $\beta_1 = 1$ , which implies that the yield equals the coupon rate on average. Our results for  $\beta_0$  in Portugal, Italy, Ireland, Greece, Spain, Germany, and the US are 9.501, 0.353, -5.419, 7.939, 0.353, -0.176, and 0.129, respectively, and the corresponding standard errors are 4.349, 0.542, 2.718, 3.898, 0.542, 0.131, and 0.089, respectively. The estimator for  $\beta_0$  in Portugal, Ireland, and Greece is significant at the 5% level, while the remainder are not significant. Our results for  $\beta_1$  in Portugal, Italy, Ireland, Greece, Spain, Germany, and the US are -0.919, 0.893, 2.204, 0.350, 0.893, 1.020, and 0.960, respectively, and the standard errors are 0.852, 0.126, 0.659, 1.0418, 0.126, 1.020, and 0.960, respectively. We cannot reject that  $\beta_1 = 1$  in Italy, Ireland, Spain, Germany, and the US and the estimators are significant at the 1% level, while the estimator on  $\beta_1$ in Portugal and Greece is not significant.

We also conduct F-tests for the null hypothesis that  $\beta_0 = 0$  and  $\beta_1 = 1$ , and obtain F-statistics of 2.670, 0.567, 3.036, 5.187, 0.567, 2.584, and 1.082 for Portugal, Italy, Ireland, Greece, Spain, Germany, and the US, respectively. The p-values are 0.073, 0.568, 0.052, 0.007, 0.569, 0.079 and 0.342 for Portugal, Italy, Ireland, Greece, Spain, Germany, and the US, respectively. Because the F-statistics in Greece are significant at the 1% level, we cannot accept our hypothesis  $r_t^H = r_t^G$  for Greece.

Summarizing our results, the hypothesis  $\beta_0 = 0$  and  $\beta_1 = 1$  is supported in Italy, Spain, Germany, and the US. That is, roughly speaking, the yield is consistent with the coupon rate on benchmark 10-year government bonds in these countries. However, in Portugal, Ireland, and Greece, the yield is not consistent with the coupon rate on the benchmark 10-year government bond.

An important issue is that this empirical analysis draws on data for 10-year government bonds whereas our model includes only one-period bonds. To confirm the robustness of the empirical results, we re-estimate the above equation using the data on government bonds with maturities of 2 and 5 years. Unfortunately, coupon rate data on government bonds with maturities shorter than 10 years are not available for Greece. We find that the results remain almost unchanged if we use government bonds with a shorter maturity (an exception is Spain). The results obtained are not provided in this paper but are available from the authors upon request. For a notable approach to incorporating long-term debt into quantitative analyses of sovereign debt and default, see Chatterjee and Eyingungor[1]. We defer this to future research.

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Figure 1: Responses to an Increase in the Government Expenditure



Figure 2: Responses to an Increase in the Government Expenditure ( $\phi = 0.33$ )