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Appendix to "The Effects of Money-financed Fiscal Stimulus in a Small Open Economy"

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A Derivation of the Model

Following Gali and Monacelli[5], who developed a simpler small open economy model than Gali and Monacelli[4], we extend the closed economy model of Gali[2] to a small open economy model.[1][2] The presentation of the model and notation closely parallel the model proposed by Gali[2].

A.1 Households

The small open economy has a representative household with a continuum of members indexed by $j \in [0,1]$.

The household utility function is

$$\sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left[\mathcal{U}\left(C_t, L_t, N_t; Z_t\right)\right], \tag{A.1}$$

with $C_t \equiv \frac{1}{(1-\nu)^{1-\nu}\nu^{\nu}} C_{H,t}^{1-\nu} C_{F,t}^{\nu}$, $P_{H,t} \equiv \left[\int_0^1 P_{H,t}(j)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$, $P_{F,t} \equiv \left[\int_0^1 P_{F,t}(j)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$, where $C_{H,t} \equiv \left[\int_0^1 C_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$ is the index of domestic goods consumption, $C_{F,t} \equiv \left[\int_0^1 C_{F,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$ is the quantity of a composite foreign good consumed, $\epsilon > 0$ is the elasticity of substitution between goods, $N_t \equiv \int_0^1 N_t(j)\, dj$ is the hours of labor.

Period utility is

$$\mathcal{U}\left(C_t, L_t, N_t; Z_t\right) \equiv \left[U\left(C_t, L_t\right) - V\left(N_t\right)\right] Z_t,$$

with $V(\cdot)$ increasing and convex and $U(\cdot)$ increasing and concave.

Optimal allocation of any given expenditure within each category of goods yields the following demand functions:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} C_{H,t} \quad ; \quad C_{F,t}(j) = \left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{-\epsilon} C_{F,t}, \tag{A.2}$$

for all j. The optimal allocation of expenditures between domestic and foreign goods implies that

$$C_{H,t} = (1-\nu)\mathcal{S}_t^{\nu} C_t \quad ; \quad C_{F,t} = \nu \mathcal{S}_t^{-(1-\nu)} C_t. \tag{A.3}$$

The sequence of budget constraints is:

$$P_t C_t + B_{H,t} + \mathcal{E}_t B_{H,t}^* + M_t = B_{H,t-1}\left(1+i_{t-1}\right) + \mathcal{E}_t B_{H,t-1}^*\left(1+i_{t-1}^*\right) + M_{t-1} + W_t N_t - P_t TR_t + D_t,$$

where $B_{H,t}$ denotes the nominal riskless one-period domestic government bond in units of domestic currency held by domestic households, $B_{H,t}^*$ is the nominal riskless one-period foreign government bond in units of foreign currency held by domestic households, i_t^* is the foreign nominal interest rate, W_t is the nominal wage, and D_t is the nominal dividends paid by firms.

Dividing both sides of the previous expression by CPI P_t yields

$$\begin{aligned} C_t + \mathcal{B}_{H,t} + \mathcal{Q}_t \mathcal{B}_{H,t}^* + L_t &= \Pi_t^{-1} \mathcal{B}_{H,t-1}\left(1+i_{t-1}\right) + \left(\Pi_t^*\right)^{-1} \mathcal{Q}_t \mathcal{B}_{H,t-1}^*\left(1+i_{t-1}^*\right) \\ &\quad + \Pi_t^{-1} L_{t-1} + \frac{W_t}{P_t} N_t - TR_t + \frac{D_t}{P_t}, \end{aligned} \tag{A.4}$$

[1]See the corresponding author's website, https://www.econ.nagoya-cu.ac.jp/~eiji_okano/papers_e.html, for details of the derivation.

[2]Unlike Gali and Monacelli[4] who develop the seminal New Keynesian small open economy model, the model in Gali[2] consists not only of households and producers but also a government issuing government debt and money to finance expenditure. Thus, as discussed in Section 2, we follow Gali and Monacelli[5], who provide a simpler aggregation of individual behavior than Gali and Monacelli[4] to avoid complicating our model.

where $\mathcal{B}_{H,t} \equiv \frac{B_{H,t}}{P_t}$ denotes real domestic government debt, $\mathcal{B}^*_{H,t} \equiv \frac{B^*_{H,t}}{P^*_t}$ is real foreign government debt, $\mathcal{Q}_t \equiv \frac{\mathcal{E}_t P^*_t}{P_t}$ is the real exchange rate (the ratio of the CPI expressed in domestic currency).

Assuming complete international financial markets, the equilibrium price (in units of domestic currency) of a riskless bond denominated in foreign currency is $\mathcal{E}_t \left(1+i^*_t\right)^{-1} = \mathrm{E}_t \left(Q_{t,t+1}\mathcal{E}_{t+1}\right)$, where $\mathrm{E}_t \left(Q_{t,t+1}\right)$ denotes the price of a one-period discount bond paying off one unit of the domestic currency. We can combine the previous pricing equation with the domestic bond pricing equation, $\left(1+i_t\right)^{-1} = \mathrm{E}_t \left(Q_{t,t+1}\right)$ to obtain a version of the uncovered interest parity (UIP) condition:

$$\mathrm{E}_t \left\{ Q_{t,t+1} \left[(1+i_t) - (1+i^*_t) \left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right) \right] \right\} = 0. \tag{A.5}$$

We define $\mathcal{A}_t \equiv \left[(1+i_{t-1}) \mathcal{B}_{H,t-1} + \mathcal{Q}_{t-1} \mathcal{B}^*_{H,t-1} \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \left(1+i^*_{t-1}\right) + L_{t-1} \right] \Pi_t^{-1}$ as the representative household's real financial wealth at the beginning of the period t. Thus, we can rewrite Eq. (A.4) as follows:

$$C_t + \frac{1}{1+i_t} \mathcal{A}_{t+1} \Pi_{t+1} + L_t \left(1 - \frac{1}{1+i_t} \right) = \mathcal{A}_t + \frac{W_t}{P_t} N_t - TR_t + \frac{D_t}{P_t}, \tag{A.6}$$

where we assume a standard solvency constraint $\lim_{k\to\infty} \Lambda_{t,t+k} \mathcal{A}_{t+k} \geq 0$ with $\Lambda_{t,t+k} \equiv \prod_{j=0}^{k-1} \mathcal{R}^{-1}_{t+j}$ as the domestic discount factor, and $\mathcal{R}_t \equiv (1+i_t) \Pi^{-1}_{t+1}$, thus ruling out a Ponzi scheme.

Households maximize Eq. (A.1), and subject to Eq. (A.6), and have the following optimality conditions:

$$U_{c,t} = \beta (1+i_t) \Pi^{-1}_{t+1} U_{c,t+1} \frac{Z_{t+1}}{Z_t}, \tag{A.7}$$

$$\frac{W_t}{P_t} = \frac{V_{n,t}}{U_{c,t}}, \tag{A.8}$$

$$h\left(\frac{L_t}{C_t}\right) = \frac{i_t}{1+i_t}, \tag{A.9}$$

with $h\left(\frac{L_t}{C_t}\right) \equiv \frac{U_{l,t}}{U_{c,t}}$. $h\left(\frac{L}{C}\right) \equiv \frac{U_l}{U_c}$ is a continuously decreasing function that satisfies $h\left(\bar{\chi}\right) = 0$ for some $0 < \bar{\chi} < \infty$, which guarantees that the demand for real money balances is bounded as the interest rate approaches zero, with a satiation point attained at $L = \bar{\chi} C$. Eqs. (A.7), (A.8), and (A.9) are the consumption Euler equation and the intertemporal optimality condition that determines labor supply under the assumption of a competitive labor market and money demand schedule, respectively. These optimality conditions require transversality condition $\lim_{k\to\infty} \Lambda_{t,t+k} \mathcal{A}_{t+k} = 0$.

A.2 International Risk-sharing Condition

Given the assumption of a complete financial market, a condition analogous to Eq. (A.4) must hold for a representative household in a foreign country. Combining this condition with Eq. (A.7) with the UIP and the definition of the real exchange rate, we obtain the international risk-sharing condition

$$U^{-1}_{c,t} = \vartheta \left(U^*_{c,t}\right)^{-1} \mathcal{Q}_t \frac{Z_t}{Z^*_t}, \tag{A.10}$$

where ϑ is a constant that depends on the initial condition.

We assume the LOOP; that is, $P_{F,t}(j) = \mathcal{E}_t P^*_{F,t}(j)$ for all j, where $P^*_{F,t}(j)$ denotes the price of foreign goods j in units of foreign currency. Integrating all goods, we obtain

$$P_{F,t} = \mathcal{E}_t P^*_{F,t}, \tag{A.11}$$

where $P^*_{F,t}$ denotes the foreign currency price of the foreign goods. Our treatment of the rest of the world as an (approximately) closed economy (with goods produced in a small open economy representing a negligible fraction of the world's consumption basket) implies that the foreign price index coincides with the foreign currency price of foreign goods, that is, $P^*_t = P^*_{F,t}$.

By substituting the definition of CPI into that of the real exchange rate, we have:

$$\mathcal{Q}_t = \mathcal{S}_t^{1-\nu}, \tag{A.12}$$

This implies that the assumption of complete markets at the international level leads to a simple relationship between consumption at home and abroad and TOT. By substituting Eq. (A.12) into Eq. (A.10), we have that

$$U_{c,t}^{-1} = \vartheta \left(U^*_{c,t}\right)^{-1} \mathcal{S}_t^{1-\nu} \frac{Z_t}{Z^*_t}. \tag{A.13}$$

A.3 Domestic Producers

A typical domestic firm produces a differential good by using the technology.

$$Y_t(j) = N_t(j)^{1-\alpha},$$

where $Y_t(j)$ is the output of generic goods j and α denotes the index of decreasing returns to labor. The index for the aggregate domestic output is $Y_t \equiv \left[\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$. By integrating the previous expression, we obtain:

$$N_t^{1-\alpha} = Y_t \left[\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\frac{\epsilon}{1-\alpha}} dj\right]^{1-\alpha}, \tag{A.14}$$

where $\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\frac{\epsilon}{1-\alpha}} dj$ denotes price dispersion.

In each period, a subset of firms of measure $1-\theta$, with $\theta \in [0,1]$ being an index of price rigidities drawn randomly from the population, reoptimizes the price of their good subject to a sequence of isoelastic demand schedules for the latter. The remaining θ firms maintain their prices unchanged. That is, firms are subject to Calvo pricing. Prices are set in domestic currency, the domestic and export markets share the same price, and LOOP also applies to exports.

The first-order necessary condition (FONC) for domestic producers is

$$\sum_{k=0}^{\infty} \theta^k \left[\Lambda_{t,t+k} \left(\frac{1}{P_{t+k}}\right) Y_{t+k|t} \left(\tilde{P}_{H,t} - \mathcal{M} MC^n_{t+k|t}\right)\right] = 0, \tag{A.15}$$

where $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$ denotes the constant (desired) price markup, $Y_{t+k|t} \equiv Y_t \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}}\right)^{-\epsilon}$ is output in period $t+k$ for a firm that last reset its price in period t, $\tilde{P}_{H,t}$ is the price set in period t by firms reoptimizing their price in that period, $MC^n_{t+k|t}$ is the nominal marginal cost in period $t+k$ for a firm that last reset its price in period t, and $MC^n_t \equiv W_t \left(\frac{N_t^\alpha}{1-\alpha}\right)$ is the nominal marginal cost.

A.4 Demand for Exports and Global Shocks

The demand for exports of domestic goods j is

$$EX_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} EX_t, \tag{A.16}$$

where EX_t is the aggregate export index.

Following Gali and Monacelli[5], the aggregate exports are

$$EX_t = \nu \mathcal{S}_t Y_t^*, \tag{A.17}$$

where Y_t^* denotes (per-capita) world output.

A.5 Government

Similar to Gali[2], we assume that the government (consisting of fiscal and monetary authorities acting in a coordinated manner) finances its expenditures through lump-sum taxes and issuing a riskless nominal one-period bond with a nominal interest rate and (non-interest-bearing) money. Thus, the consolidated budget constraint is:

$$P_{H,t}G_t + B_{t-1}(1 + i_{t-1}) = P_t TR_t + B_t + \Delta M_t, \tag{A.18}$$

where $B_t \equiv B_{H,t} + B_{F,t}$, $B_{F,t}$ denotes the nominal riskless one-period domestic government bond in units of domestic currency held by foreign households, Δ is the difference operator, and $G_t \equiv \left(\int_0^1 G_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$ denotes the (real) government expenditure index.

Like Eq. (A.2), the optimal allocation of government expenditures is

$$G_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} G_t. \tag{A.19}$$

We assume that government expenditures are fully allocated to domestically produced goods.

By dividing both sides of Eq. (A.18) by CPI P_t yields

$$\mathcal{S}_t^{-\nu} G_t + \mathcal{B}_{t-1}\mathcal{R}_{t-1} = TR_t + \mathcal{B}_t + \frac{\Delta M_t}{P_t}, \tag{A.20}$$

where $\frac{\Delta M_t}{P_t}$ represents the seigniorage in period t, that is, the purchasing power of the newly issued money. Similar to Gali[2], the analysis below focuses on equilibrium near a steady state with zero inflation, no trend growth, and constant government expenditure, taxes, and debt. The constancy of real balances requires $\Delta M = 0$, hence, zero seigniorage in the steady state. Note that the variables without time scripts are steady-state values of these variables.

A.6 The Market-clearing Condition

The market-clearing condition is:

$$Y_t(j) = C_{H,t}(j) + EX_t(j) + G_t(j).$$

Plugging Eqs. (A.2), (A.3), (A.16), (A.17), and (A.19) in the previous expression, we obtain

$$Y_t = (1-\nu)\mathcal{S}_t^{\nu} C_t + \nu \mathcal{S}_t Y_t^* + G_t, \tag{A.21}$$

where we use the optimal allocation of the output $Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} Y_t$. Note that we assume $Y_t^* = C_t^*$, where C_t^* denotes (per capita) world consumption.

A.7 Trade Balance

Similar to Gali and Monaceli[4], we define the real trade balance as:

$$\frac{NX_t}{P_{H,t}} \equiv Y_t - \mathcal{S}_t^{\nu} C_t - G_t, \tag{A.22}$$

where NX_t denotes the nominal trade balance.

A.8 The Steady State

The analysis below considers the equilibrium in the neighborhood of a steady state with zero inflation and zero government expenditure; that is, $\Pi = 1$ and $G = 0$, similar to Gali[2]. We also assume that $Z = Z^* = 1$.

Eqs. (A.5), (A.7), (A.8), and (A.9) imply the following in the steady state.

$$\begin{aligned} i &= i^*, \\ i &= \rho, \\ (1-\alpha) U_c &= V_n N^{\alpha} \mathcal{M}, \\ h\left(\frac{L}{C}\right) &= \frac{\rho}{1+\rho}, \end{aligned}$$

where the former two equalities imply no changes in the nominal exchange rate in the steady state, whereas the latter two equalities are identical to the steady-state conditions in Gali[2]. In this steady state, the world output equals world consumption, $Y^* = C^*$ and $C = C^* = Y$ are applicable. In addition, $\mathcal{S} = 1$; that is, the TOT (and the real exchange rate) is uniquely pinned down and is unity in the perfect foresight steady state. We obtain the condition $\mathcal{S} = 1$ by following Gali and Monacelli's[3] method.[3] This feature of the steady state, which implies that PPP $\mathcal{Q}_t = 1$ is applicable in the long run, is important for our equilibrium dynamics.

B Relationship between the Trade balance and Net Foreign Assets

Plugging Eq.(A.5) into the sequence of the budget constraint yields

$$P_t C_t + B_{H,t} + \mathcal{E}_t B_{H,t}^* + \Delta M_t = \left(B_{H,t-1} + \mathcal{E}_{t-1} B_{H,t-1}^*\right)(1 + i_{t-1}) + W_t N_t + D_t - P_t TR_t,$$

which can be rewritten as

$$P_t TR_t = -P_t C_t - B_{H,t} - \mathcal{E}_t B_{H,t}^* - \Delta M_t + \left(B_{H,t-1} + \mathcal{E}_{t-1} B_{H,t-1}^*\right)(1 + i_{t-1}) + W_t N_t + D_t. \tag{B.1}$$

Eq.(A.18) can be rewritten as

$$P_t TR_t = P_{H,t} G_t + B_{t-1}(1 + i_{t-1}) - B_t - \Delta M_t.$$

Combinig the previous expression with Eq.(B.1) yields

$$\left(\mathcal{E}_{t-1} B_{H,t-1}^* - B_{F,t-1}\right)(1 + i_{t-1}) + W_t N_t + D_t = P_t C_t + P_{H,t} G_t + \left(\mathcal{E}_t B_{H,t}^* - B_{F,t}\right). \tag{B.2}$$

[3]Additionally, to obtain this condition with certainty, we assume that the steady-state wedge between the marginal rate of substitution from consumption to leisure and the marginal product of labor is common worldwide. See Benigno and Woodford[?] for further details.

In the LHS in Eq.(B.2), the sum of the second and third terms in the LHS is the distribution of the nominal income, so that

$$P_{H,t}Y_t = W_t N_t + D_t.$$

Substituting the previous expression into Eq.(B.2) yields

$$\left(\mathcal{E}_{t-1}B^*_{H,t-1} - B_{F,t-1}\right)(1+i_{t-1}) + P_{H,t}Y_t = P_t C_t + P_{H,t}G_t + \left(\mathcal{E}_t B^*_{H,t} - B_{F,t}\right),$$

which can be rewritten as

$$\left(\mathcal{E}_{t-1}B^*_{H,t-1} - B_{F,t-1}\right) i_{t-1} + P_{H,t}Y_t = P_t C_t + P_{H,t}G_t + \left(\mathcal{E}_t B^*_{H,t} - B_{F,t}\right) - \left(\mathcal{E}_{t-1}B^*_{H,t-1} - B_{F,t-1}\right).$$

dividing both sides of the previous expression P_t yields

$$\frac{\left(\mathcal{E}_{t-1}B^*_{H,t-1} - B_{F,t-1}\right) i_{t-1}}{P_t} + \frac{P_{H,t}}{P_t}Y_t = C_t + \frac{P_{H,t}}{P_t}G_t + CA_t, \tag{B.3}$$

where $CA_t \equiv \frac{\left(\mathcal{E}_t B^*_{H,t} - B_{F,t}\right) - \left(\mathcal{E}_{t-1}B^*_{H,t-1} - B_{F,t-1}\right)}{P_t}$ denotes real current account.

Eq.(A.22) can be rewritten as

$$NX_t = \frac{P_{H,t}}{P_t}Y_t - C_t - \frac{P_{H,t}}{P_t}G_t.$$

Substituting the previous expression into Eq.(B.3) yields

$$\frac{\left(\mathcal{E}_{t-1}B^*_{H,t-1} - B_{F,t-1}\right) i_{t-1}}{P_t} + NX_t = CA_t. \tag{B.4}$$

The first term on the LHS of Eq.(B.4), and the definition of the current account can be rewritten as:

$$\begin{aligned}
\frac{\left(\mathcal{E}_{t-1}B^*_{H,t-1} - B_{F,t-1}\right) i_{t-1}}{P_t} &= \left(\frac{\mathcal{E}_{t-1}P^*_{t-1}}{P_{t-1}}\frac{B^*_{H,t-1}}{P^*_{t-1}}\frac{P_{t-1}}{P_t} - \frac{B_{F,t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t}\right) i_{t-1} \\
&= \left(\mathcal{Q}_{t-1}\mathcal{B}^*_{H,t-1} - \mathcal{B}_{F,t-1}\right) i_{t-1}\Pi_t^{-1}, \\
CA_t &= \frac{\mathcal{E}_t P^*_t}{P_t}\frac{B^*_{H,t}}{P^*_t} - \frac{B_{F,t}}{P_t} \\
&\quad - \left(\frac{\mathcal{E}_{t-1}P^*_{t-1}}{P_{t-1}}\frac{B^*_{H,t-1}}{P^*_{t-1}}\frac{P_{t-1}}{P_t} - \frac{B_{F,t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t}\right) \\
&= \mathcal{Q}_t\mathcal{B}^*_{H,t} - \mathcal{B}_{F,t} - \left(\mathcal{Q}_{t-1}\mathcal{B}^*_{H,t-1} - \mathcal{B}_{F,t-1}\right)\Pi_t^{-1}.
\end{aligned}$$

Following Ferrero et al.[1], we assume that debt issued by the government in a small open economy is just traded internationally, so that $B^*_{H,t} = 0$ for all t. Then, the previous equalities can be rewritten as:

$$\frac{\left(\mathcal{E}_{t-1}B^*_{H,t-1} - B_{F,t-1}\right) i_{t-1}}{P_t} = -\mathcal{B}_{F,t-1}i_{t-1}\Pi_t^{-1}, \tag{B.5}$$

$$CA_t = -\left(\mathcal{B}_{F,t} - \mathcal{B}_{F,t-1}\Pi_t^{-1}\right). \tag{B.6}$$

Plugging Eqs.(B.5) into Eq.(B.4) yields

$$NX_t = CA_t + \mathcal{B}_{F,t-1}i_{t-1}\Pi_t^{-1}. \tag{B.7}$$

B.1 The Steady State

Because $Y = C$ and $G = 0$, Eq.(A.22) implies the following.

$$NX = 0.$$

Because of $NX = 0$, Eq.(B.7) implies $\mathcal{B}_F \rho = 0$, such that

$$\mathcal{B}_F = 0.$$

Then, Eq.(B.6) implies the following.

$$CA = 0.$$

B.2 Log-linearization of Eqs.(B.6) and (B.7)

Log-linearization of Eq.(B.6) is given by

$$\widehat{ca}_t = -\hat{b}_{F,t} + \hat{b}_{F,t-1},$$

where $\widehat{ca}_t \equiv \ln\left(\frac{CA_t}{Y}\right)$ denotes the ratio of the current account to the steady-state output and $\hat{b}_{F,t} \equiv \ln\left(\frac{\mathcal{B}_{F,t}}{Y}\right)$ denotes the ratio of the real government bonds held by foreign households to the steady-state output. Let $\widehat{nfa}_t \equiv -\hat{b}_{F,t}$ which is the ratio of net foreign assets to steady-state output. Then, the previous expression can be rewritten as:

$$\widehat{ca}_t = \widehat{nfa}_t - \widehat{nfa}_{t-1}. \tag{B.8}$$

Log-linearization of Eq.(B.7) is given by

$$\widehat{nx}_t = \widehat{ca}_t - \rho\widehat{nfa}_{t-1}. \tag{B.9}$$

where $\widehat{nfa}_t \equiv -\hat{b}_{F,t}$, Plugging Eq.(B.8) into Eq.(B.9) yields

$$\widehat{nx}_t = \widehat{nfa}_t - (1+\rho)\,\widehat{nfa}_{t-1}, \tag{B.10}$$

which implies that net exports are the remainder after deducting interest income from the changes in net foreign assets.

C Responses to Fiscal Stimulus in Normal Times

This section shows the effects of fiscal stimulus in normal times, and Figures 1 and 2 illustrate the responses under an *MF* fiscal stimulus to a tax cut and an increase in government expenditure, respectively, while Figures 3 and 4 depict the responses under a *DF* fiscal stimulus to a tax cut and an increase in government expenditure, respectively. The red line with circles and the blue line with diamonds are the responses in a small open economy in which we set ν to 0.4, that is, our benchmark, and in a closed economy in which we set the openness ν to zero.

C.1 *MF* Fiscal Stimulus

C.1.1 Response to a Tax Cut

Output and CPI inflation increase after a tax cut in a small open economy, as in a closed economy, and increases in both are higher than in a closed economy (Panels 1 and 3 in Figure 1). A tax cut applies pressure that increases CPI inflation. Because of PPP, in the long run, the nominal exchange rate depreciates, and import prices increase. The import price has no nominal rigidity, and an increase in the import price is higher than the domestic price, so the TOT worsens(Panel 7, Figure 1). Because the TOT worsens the output increases. In addition, an increase in the import price increases CPI inflation in a small open economy so an increase in CPI inflation is higher than that in a closed economy.

However, in contrast to a closed economy, the nominal interest rate increases and money growth declines, contrary to intuition. (Panels 5 and 6 in Figure 1). To understand the reason for reduced money growth, we focus on the (logarithmic) government budget constraint Eq.(9)in the text, which implies that an increase in CPI inflation mitigates the burden of redeeming government debt through a decrease in the real consumption interest rate. In other words, an increase in CPI inflation yields government revenue from the "inflation tax." Then, we define $\widehat{sp}_t \equiv \widehat{tr}_t - \hat{g}_t + (1+\rho)\, b\pi_t$ as the fiscal surplus with inflation tax, and we can rewrite Eq.(9) in the text as:

$$\widehat{sp}_t \quad = \quad -\left[\hat{b}_t - (1+\rho)\,\hat{b}_{t-1} - (1+\rho)\,b\hat{i}_{t-1}\right] - \chi \Delta m_t,$$

This implies that an increase in CPI inflation applies downward pressure, reducing newly issued government debt $\hat{b}_t - \left[(1+\rho)\,\hat{b}_{t-1} + (1+\rho)\,b\hat{i}_{t-1}\right]$ and money growth by increasing the fiscal surplus with inflation tax. Under *MF* fiscal stimulus, the balance of real government debt remains unchanged. Thus, an increase in CPI inflation provides an incentive to reduce money growth because of the increase in fiscal surplus with inflation tax (Panel 6, Figure 1). The real money-balance schedule Eq.(7) shows that a decrease in the real money balance accompanies an increase in the nominal interest rate (Panel 5, Figure 1). Because the nominal interest rate increases, the decrease in the real consumption interest rate is limited, and the increase in consumption is not as vigorous, unlike consumption in a closed economy.

The increase in consumption in a small open economy is smaller and does not seem to contribute significantly to boosting output. Instead, the worsening of the TOT contributes to increasing output. Given the worsening in the TOT stemming from the increase in import prices, domestic goods production becomes vigorous, while households must pay more for foreign goods. Under our parameterization, namely, $\sigma = 1$ together with the assumption of perfect substitution between foreign and domestic goods, the increase in domestic goods production is canceled out by the increase in the purchases of foreign goods. Balanced trade was then achieved. Balanced trade is applicable throughout our analysis if a demand shock does not hit the small open economy.

C.1.2 Response to an Increase in Government Expenditure

Next, we discuss the dynamic responses to an increase in government expenditure, which are similar to a tax cut under *MF* fiscal stimulus. Output rises, as in a closed economy, but with a smaller increase in consumption and a higher increase in CPI inflation than in a closed economy (Panels 1 and 3, Figure 2). The nominal exchange rate depreciation accompanies the increase in CPI inflation due to PPP in the long run and import inflation increases. This increase in import inflation coincides with a worsening TOT (Panel 8, Figure 2). This worsening in TOT increases output, and the output is higher than in a closed economy (Panel 1, Figure 2). Thus, the *MF* fiscal stimulus is more effective in a small open economy than in a closed economy, regardless of whether the stimulus is a tax cut or an increase in government expenditure.

Notably, money growth decreases in period zero, similar to the tax cut under the *MF* scheme in a small open economy (Panel 6, Figure 2). The significant increase in CPI inflation finances the increase in government expenditure through an increase in fiscal surplus with inflation tax rather than money growth.

C.2 *DF* Fiscal Stimulus

C.2.1 Response To a Tax Cut

Similar to a closed economy, a tax cut under the *DF* scheme, regardless of whether the DIT or the CIT, does not affect any variables besides the fiscal variables, such as real government debt, the fiscal surplus with inflation tax, and taxes (Figure 3). As Gali[2] notes, this neutrality result is well known and a consequence of Ricardian equivalence, given any assumption of lump-sum taxes and Ricardian fiscal policy. Any short-run tax reduction is matched by future tax increases, leaving their present discounted value unchanged and households' intertemporal budget constraint unaffected. As the tax cut and increase in government debt affect no other equilibrium conditions, all variables, besides the fiscal variables, remain unchanged in response to a tax cut under the *DF* scheme.

C.2.2 Response To an Increase in Government Expenditure

In a small open economy, under the *DF* scheme with the DIT, the responses of the output, domestic inflation, nominal interest rate, and money growth are the same as those in a closed economy (Panels 1, 4, 5, and 6). As mentioned in Section 4.1.2 in the text, this result is consistent with Gali and Monacelli[3], who show that the equilibrium dynamics in a small open economy is isomorphic to that of a closed economy under the DIT. The nominal exchange rate moves one-for-one with the TOT, given that domestic prices are fully stabilized under the DIT, as Eq.(12) in the text implies. Thus, under the DIT, the dynamics of output, domestic inflation, and so forth in a small open economy are identical to those in a closed economy.

The responses under the *DF* scheme with CIT are significantly different from those in a closed economy. There is still crowding out in consumption in response to the increase in government expenditure, but the decrease in consumption in the small open economy is smaller than in the closed economy. Given this smaller decrease in consumption, the increase in output in the small open economy is about 5.88 times larger than in a closed economy in period zero, and cumulative output is higher than in a closed economy (Panel 1, Figure 4). In a small open economy, CPI and domestic inflation are distinct, and any response includes changes in the TOT and nominal exchange rate. While domestic inflation is stable in a closed economy because it is identical to CPI inflation in a closed economy, domestic inflation increases in a small open economy and import inflation decreases to cancel out any increase in domestic inflation (Panel 4, Figure 4).

In a closed economy, an increase in government expenditure requires stabilization of the average markup to stabilize domestic inflation (which is identical to CPI inflation). To stabilize the average markup, the pressure to increase output in response to an increase in government expenditure is canceled by the substantial decrease in consumption. Thus, the nominal interest rate is hiked, and money growth falls (Panels 5 and 6, Figure 4). However, in the *DF* scheme with CIT, an increase in domestic inflation is possible, and this increase improves the TOT with little increase in the nominal interest rate (Panels 4 and 8, Figure 4). This improvement in the TOT coincides with a decline in import inflation, which completely stabilizes CPI inflation. Glaring changes in the nominal interest rate or money growth are not necessary in a small open economy, unlike in a closed economy (Panels 5 and 6, Figure 4). Thus, the crowding-out of consumption is smaller in a closed economy, and the output increases more.

Together with less crowding-out of consumption, another notable feature of small open economies is their responses to the increase in government expenditure under the *DF* scheme. A small open economy will increase government debt and taxes, but less so than a closed economy. Furthermore, while money growth decreases in a closed economy, the money growth response in a small open economy is small. Thus, unlike in a closed economy, increases in government debt and taxes are suppressed in a small open economy.

Gali[2] highlights the effectiveness of the *MF* fiscal stimulus, especially a tax cut under the *MF* scheme, to boost output, and argues that the increase in government expenditure under the *DF* scheme is strongly subdued compared to the increase in government expenditure under the *MF* fiscal stimulus. In a small open economy, because of the smaller crowding-out of consumption, the effectiveness of the increase in government expenditure under the *DF* scheme with CIT is slightly improved, although the *MF* fiscal stimulus is still more effective.[4][5]

D The Model and Responses in the Imperfect Pass-through Environment

D.1 Modification of the Model to Generate Imperfect Pass-through

As Eq. (A.11) is not necessarily available in an imperfect pass-through environment, Eq. (A.12) is replaced by

$$\mathcal{Q}_t = \Psi_t \mathcal{S}_t^{1-\nu}. \tag{D.1}$$

Eq. (A.13) is replaced as follows:

$$U_{c,t}^{-1} = \vartheta \left(U_{c,t}^{*}\right)^{-1} \Psi_t \mathcal{S}_t^{1-\nu} \frac{Z_t}{Z_t^{*}}, \tag{D.2}$$

which is obtained by substituting Eq. (D.1) into Eq. (A.10). By log-linearizing Eq. (D.2), we obtain Eq. (24) in the text.

Foreign retailers face the following maximization problem.

$$\max_{\tilde{P}_{F,t}} \sum_{k=0}^{\infty} \theta_F^k \left\{ \Lambda_{t,t+k}^{*} \left(\frac{1}{P_{t+k}^{*}} \right) \left[\frac{\tilde{P}_{F,t}}{\mathcal{E}_t} - P_{F,t+k}^{*} \left(1-\tau_F\right) \right] C_{F,t+k|t} \right\},$$

[4]Under the *MF* scheme, cumulative output is 2.69% in a closed economy while it is 2.86% in a small open economy. Under the *DF* scheme, the cumulative output is 0.25%, whereas it is 0.87% (*DF* scheme with CIT) in a small open economy. An increase in the cumulative output under the *DF* scheme with CIT in a small open economy is higher than that under the *DF* scheme in a closed economy. However, in a small open economy, the increase in the cumulative output under the *MF* scheme is completely higher than that under the *DF* scheme with the CIT.

[5]We have to be aware of that these results do not include the reputational costs of a government that frequently uses money financing. These unknown costs would make the *MF* fiscal stimulus unworthy.

s.t.

$$C_{F,t+k|t} \equiv \left(\frac{\tilde{P}_{F,t}}{P_{F,t+k}}\right)^{-\epsilon} C_{F,t+k},$$

where $\tilde{P}_{F,t}$ is the price set in period t by foreign retailers reoptimizing their prices in that period, $\Lambda^*_{t,t+k}$ is the foreign discount factor, and τ_F denotes the export subsidiary rate. The foreign government pays an export subsidiary. This subsidiary's role is analogous to that of the employment subsidiary in Gali and Monacelli[4].

The FONC for foreign retailers is given by

$$\sum_{k=0}^{\infty} \theta_F^k \left\{ \Lambda^*_{t,t+k} \left(\frac{1}{P^*_{t+k}}\right) \left[\frac{\tilde{P}_{F,t}}{\mathcal{E}_t} - \mathcal{M}\left(1-\tau_F\right) P^*_{F,t+k}\right] C_{F,t+k|t} \right\} = 0. \tag{D.3}$$

By log-linearizing Eq. (D.3), we obtain the following log-linearized FONC for foreign retailers:

$$\tilde{p}_{F,t} - e_t = \left(1-\theta_F\beta\right) \sum_{k=0}^{\infty} \left(\theta_F\beta\right)^k \left[\psi_{t+k} + \left(p_{F,t+k} - e_{t+k}\right)\right].$$

By rearranging the previous expression, Eq.(27) in the text.

Under an imperfect pass-through, Eq. (A.21) is not available and is replaced by the following expression:

$$EX_t = \nu \mathcal{S}_t \Psi_t Y_t^*. \tag{D.4}$$

Eq. (A.21) is replaced by

$$Y_t = \left(1-\nu\right) \mathcal{S}_t^{\nu} C_t + \nu \mathcal{S}_t \Psi_t Y_t^* + G_t, \tag{D.5}$$

because Eq. (D.4), we replace Eq. (A.17). Log-linearization of Eq. (D.5) yields Eq. (26) in the text.

D.2 The Steady State

FONC for foreign retailers (Eq. (D.3) suggests the following.

$$\Psi = \left[\mathcal{M}\left(1-\tau_F\right)\right]^{-1},$$

which implies that as long as $\tau_F = \frac{1}{\epsilon}$ is chosen,

$$\Psi = 1, \tag{D.6}$$

is applicable. Eq. (D.6) implies that LOOP is applicable in the steady state. If there is no subsidiary, that is, $\tau_F = 0$, Eq. (D.6) is replaced by $\Psi = \mathcal{M}^{-1} < 1$ implying that monopolistic competitive power remains in the steady state and that the LOOP is no longer applicable, even in the steady state.

Eq. (D.6) is essential to form the steady-state relationship $C = C^*$. In fact, Eq. (A.16) implies that, in the steady state,

$$U_c^{-1} = \vartheta \left(U_c^*\right)^{-1} \Psi.$$

The previous expression implies that the steady-state relationship $C = C^*$ is no longer applicable if $\Psi \neq 1$ is not applicable. To attain $C = C^*$ in a steady state with an imperfect pass-through, the assumption $\tau_F = \frac{1}{\epsilon}$ is essential. Thus, we adopt foreign retailers instead of the local retailers in Monacelli[6].[6] Owing to the assumption $\tau_F = \frac{1}{\epsilon}$, all the steady-state conditions shown in Section 4.1 are inherited in the steady state in an imperfect pass-through environment as well as $\Psi = 1$. Note that our strategy is not novel and has already been developed by Gali and Monacelli[3].

In addition, $\mathcal{S} = 1$ is applicable. That is, the TOT (along with the real exchange rate) is pinned down uniquely and is unity in the perfect foresight steady state, even with an imperfect pass-through.

D.3 Responses to an Increase in the Government Expenditure

See Figures 5 and 6.

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[6] Monacelli[6] seems to implicitly assume the domestic government subsidizes both domestic producers and local retailers importing foreign goods. We follow Gali[2], in which the domestic government does not subsidize. As long as we adopt local retailers similar to Monacelli[6], they are not subsidized by the domestic government, and Eq. (A.19) decreases to $\Psi = \mathcal{M}^{-1} < 1$ in steady state. In this case, $C = C^*$ is no longer available, and the steady-state conditions are complicated. Thus, we assume that the foreign government subsidizes (i.e., foreign retailers are subsidized by the foreign government).

Figure 1: Dynamic Response to a Tax Cut under the *MF* Scheme

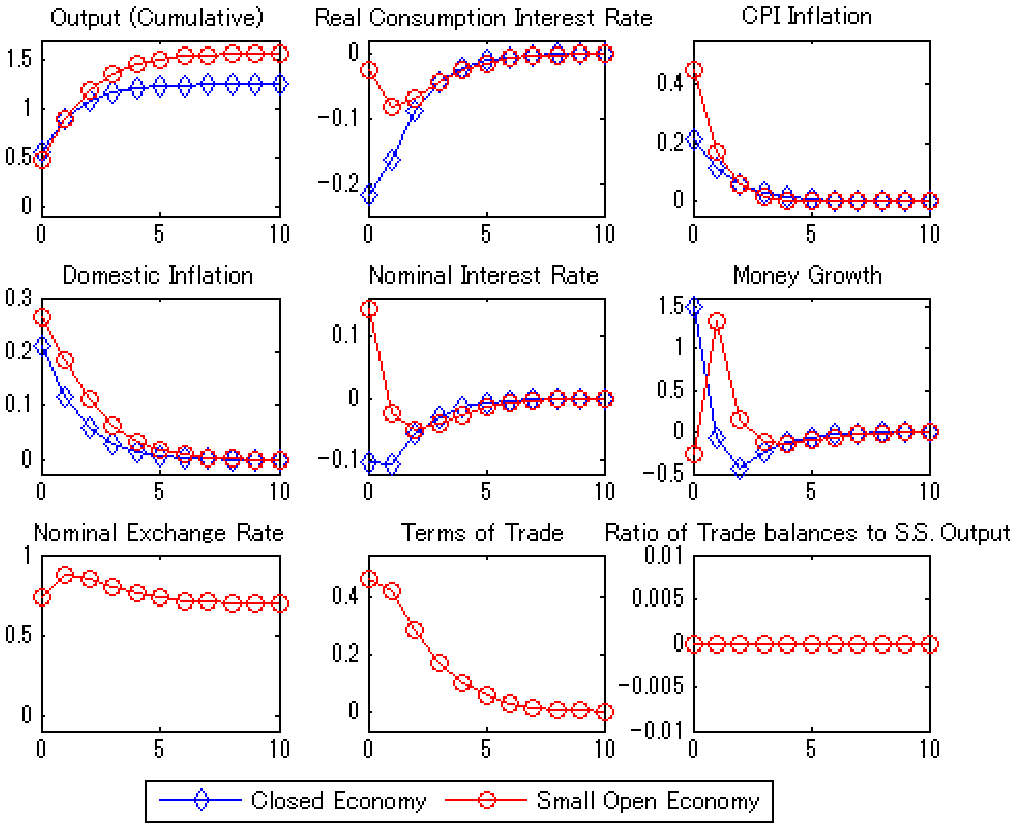

Figure 2: Dynamic Response to an Increase in Government Expenditure under the *MF* Scheme

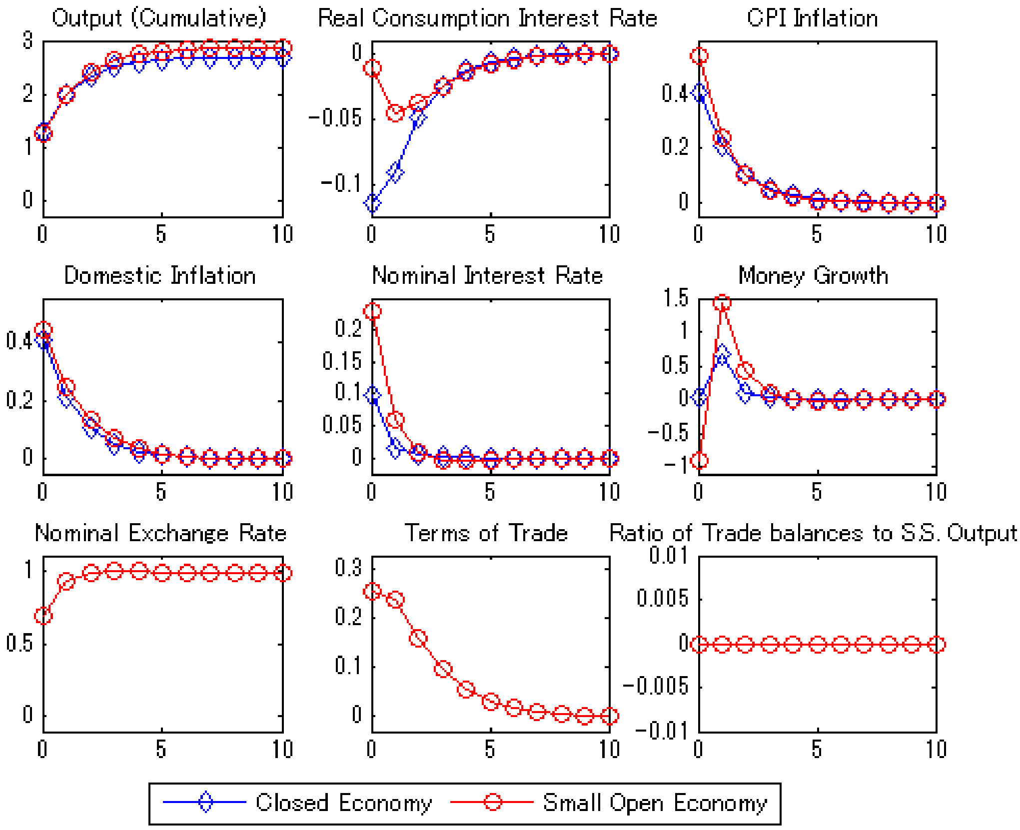

Figure 3: Dynamic Response to a Tax Cut under the *DF* Scheme

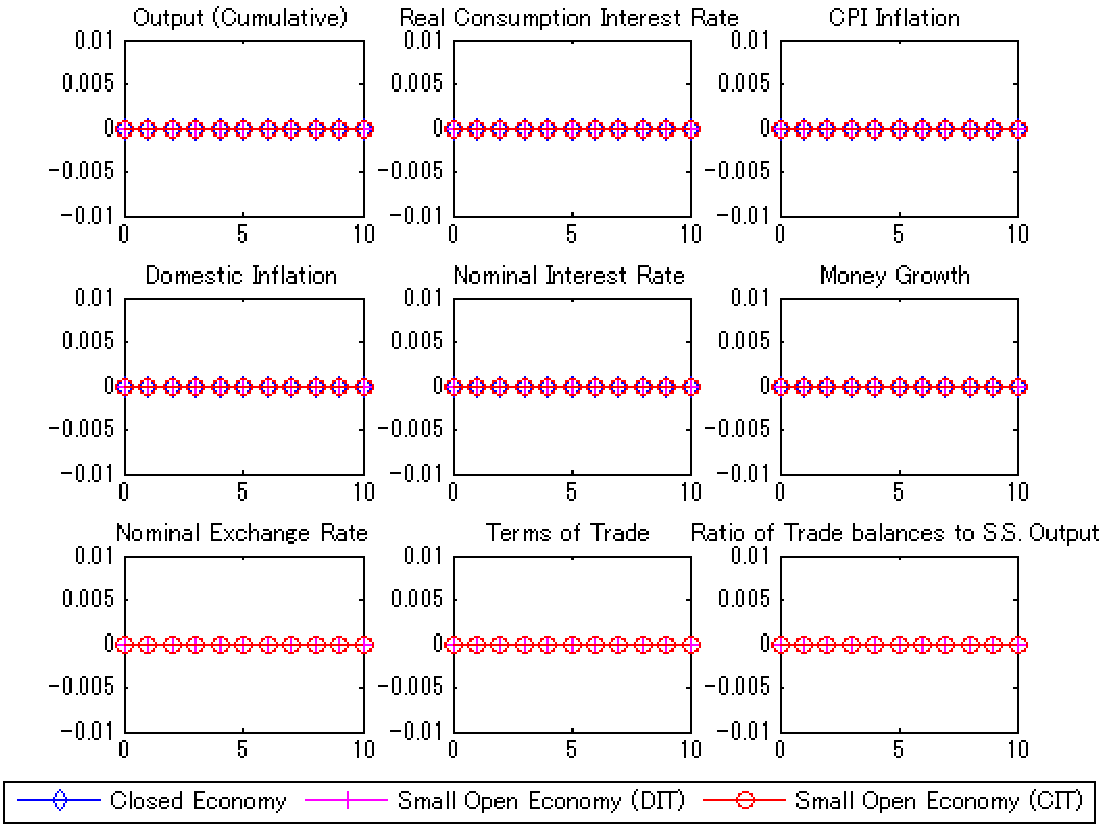

Figure 4: Dynamic Response to an Increase in Government Expenditure under the *DF* Scheme

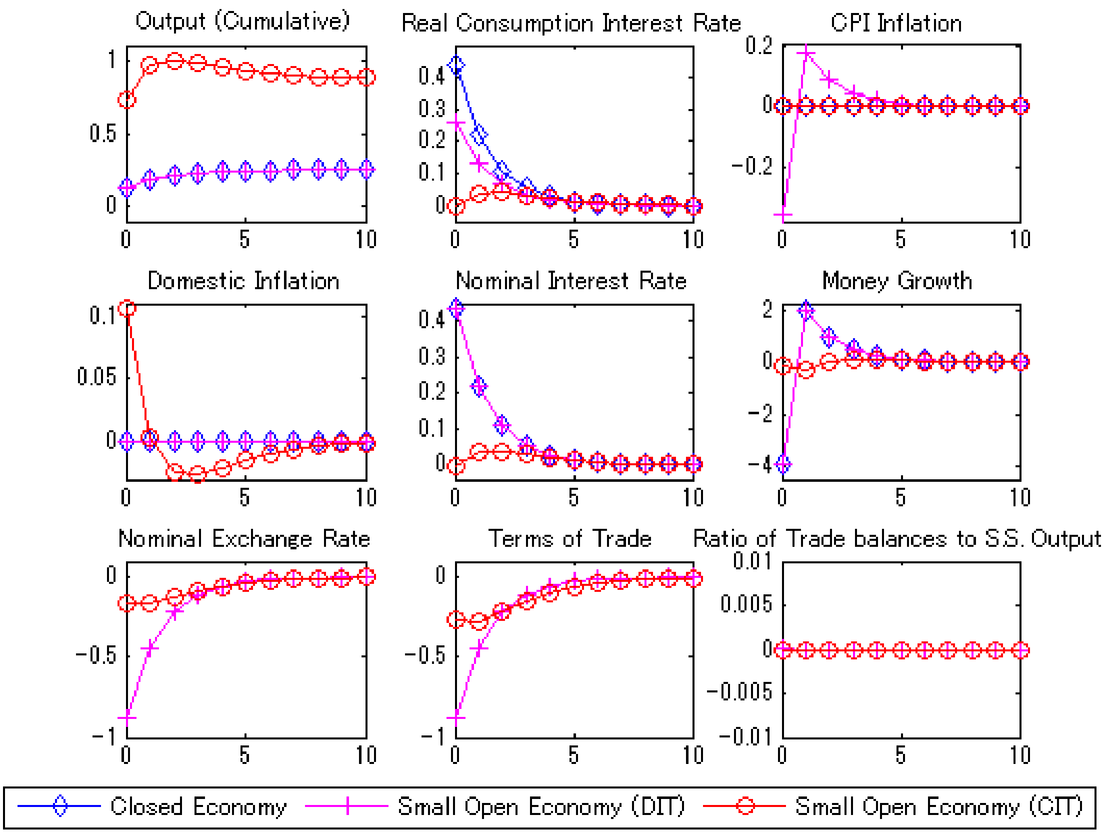

Figure 5: Dynamic Effects of an Increase in Government Expenditure in an Imperfect Pass-through Environment in a Liquidity Trap: Comparing the *MF* scheme with the CIT

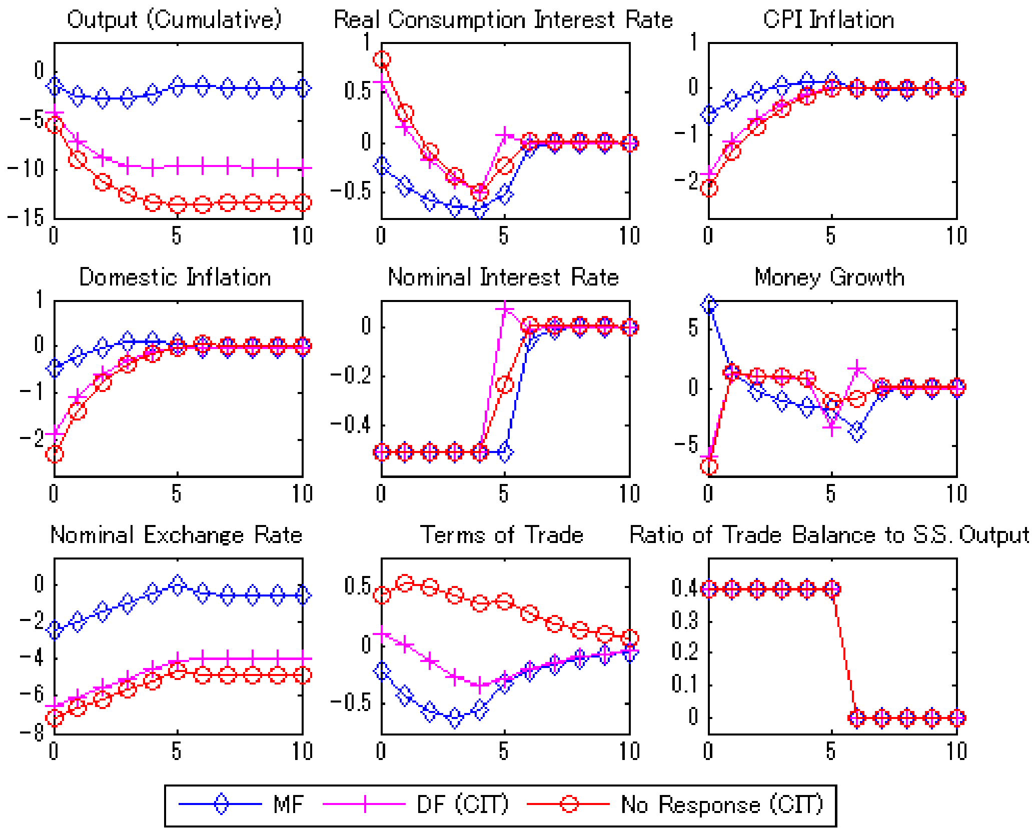

Figure 6: Dynamic Effects of an Increase in Government Expenditure in an Imperfect Pass-through Environment in a Liquidity Trap: Comparing the *MF* scheme with the DIT