Who Holds the Duration? Central Bank Holdings, Insurance Value and Government Bond Losses*

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Abstract

When yields rose from 2022, the Bank of Japan held about half the market value of the government's coupon bonds but bore less than a third of their cumulative net holding-period losses. The difference lay in what it held: its bonds were much shorter than those it left with other investors. We ask whether that long end had also carried the insurance value of the debt, its tendency to pay off when times are bad, before yields rose. Combining security-level returns and holdings over 2002–2026 with a stochastic discount factor estimated from Japanese asset prices, we find that the duration left outside was also where the insurance value of those bonds sat. Over 2013–2021, bonds with more than twelve years remaining were 36 per cent of the market value of the coupon portfolio held outside the Bank but contributed 79 per cent of its covariance with the discount factor, a concentration that appears with every discount factor we use, including ones estimated on pre-2022 data. From 2022 the same maturity segment bore 77 per cent of that portfolio's price losses. Composition, not only size, shaped how the losses were divided: at observed prices and with the Bank's aggregate holding share unchanged, a uniform allocation across securities would have shifted a further 35 trillion yen of losses to the Bank. A holding share reveals neither where government debt insures its holders nor where its losses fall.

Keywords: government debt; central bank balance sheet; maturity structure; stochastic discount factor; fiscal risk; Japan.
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1 Introduction

Central bank holdings of government debt are usually summarised by a quantity: the share of the debt the central bank holds. A quantity is not an exposure. Government bonds differ in maturity, so two central banks holding the same share of the debt can leave very different amounts of interest-rate exposure with the other holders. How much of the government's duration stays outside the central bank largely determines who bears the losses when yields rise. It may also determine which holders receive the insurance that the debt provides when yields fall in bad times. The question matters again now that central banks are shrinking their holdings.

Japan provides an unusually sharp setting in which to distinguish holdings from exposure. At the end of 2021 the Bank of Japan held 49 per cent of the market value of the coupon bonds issued by the Japanese government. Its portfolio had a modified duration of 6.5 years, against 11.2 years for the bonds held outside it. When yields rose over 2022–2026, the Bank's portfolio accounted for 31 per cent of the cumulative net losses on those bonds. Half the bonds did not mean half the duration, nor half the losses. Because the Bank is part of the consolidated government, its own losses are internal to it; what the Bank held determined how much of the revaluation of the debt fell on other holders, among them private investors and the public pension funds.

The duration left outside is not only a source of losses. Long bonds gain when yields fall, and if yields fall in bad states and inflation does not offset the price gains, government debt is a hedge: its real return is high when investors value income most, and it carries a safety premium that lowers the government's expected cost of borrowing (Pal Mustafi and Preston, 2026). Whether the bonds left outside the Bank were a hedge of this kind cannot be read from their maturity. It depends on how their returns co-moved with the state of the economy, which we measure with a stochastic discount factor (SDF) estimated from Japanese asset prices. This paper asks where in the maturity structure the duration left outside the Bank sat, whether that segment carried the insurance value of the coupon bonds held outside the Bank before yields rose, and whether it bore the losses when they did.[1]

To answer these questions, we build a security-level panel of the returns, amounts outstanding and Bank of Japan holdings of marketable Japanese government bonds (JGBs) from September 2002 to September 2026.[2] The panel tells us where the duration sat. The SDF tells us whether it insured its holders before 2022, and the realised returns from 2022 show where the losses fell when yields rose.

[1]The holding structure we observe reflects the Bank's purchases together with issuance, redemptions, the ageing of the debt stock and the choices of other holders; we do not attribute it to the purchases alone.

[2]The duration comparisons refer to coupon JGBs held by the Bank and by other holders, not to the consolidated government's liabilities, which also include bills and the reserves banks hold at the Bank.

The answer is that one segment did all three: it held most of the duration, carried most of the insurance value and bore most of the losses. The duration left outside the Bank was concentrated in bonds with more than twelve years to maturity. Over 2013–2021, the years of large-scale purchases before yields rose, these bonds were 36 per cent of the market value of the coupon bonds held outside the Bank and 70 per cent of their duration exposure (market value times modified duration), and contributed 79 per cent of the covariance of those bonds' real return with the SDF. From 2022 they accounted for 77 per cent of the price losses on those bonds.

A concentration of duration need not be a concentration of insurance value. Duration measures the size of a bond's response to yields; insurance value depends on how its real return covaries with bad states. In Japan, positive correlations with the SDF in each maturity bucket beyond three years, combined with the greater volatility of long-bond returns, placed most of that value in the long end. Over 2013–2021 the concentration appears with every SDF we use, including SDFs estimated only on data before 2022.

The allocation, not only the quantity, mattered. At observed prices, keeping the Bank's aggregate share at its observed value but spreading it evenly across issues would have moved a further 35 trillion yen of losses from the other holders to the Bank. Applied to 2013–2021, the same reassignment would have lowered the covariance of the coupon bonds held outside the Bank with the SDF by about a tenth.

For the consolidated government, the liabilities held outside the Bank carried a safety premium before 2022: they insured their holders, and the government was compensated with a lower expected cost of borrowing. Once the Bank is consolidated with the government, the reserves that banks hold at the Bank are government liabilities too. Including them, the benchmark risk premium is -30 basis points a year, which lowers, to first order, the primary surplus needed to keep the debt ratio constant by about 0.42 per cent of GDP a year. Its sign, though not its size, survives estimating the SDF on data before 2022 alone.

Where the losses fell does not tell us whether the debt still insured its holders after 2022; that depends on how real returns covaried with bad states in those years. In the benchmark estimate the premium is close to zero over January 2022 to June 2025 and turns positive, at $+22$ basis points, once the months to September 2026 are included. Once the SDF is re-estimated on each bootstrap sample, the interval for the change includes zero, and the change is much smaller when the SDF is estimated from 1991, so we treat it as suggestive.

Related literature. The paper first contributes to work that treats the central bank's balance sheet as part of the government's. Consolidating the Treasury and the central bank turns the maturity of the debt held by the public into a policy object (Greenwood et al., 2014). Purchases financed with reserves shorten the maturity of the consolidated debt

and expose the central bank to losses when rates rise (Office for Budget Responsibility, 2021; Cecchetti and Hilscher, 2025; Bleaney, 2025), and the floating-rate reserves make the consolidated funding cost a fiscal risk (Del Negro and Sims, 2015; Hall and Reis, 2015; Reis, 2022). For Japan, the public sector as a whole, including the public pension funds, has been described as borrowing short to hold long-duration risky assets (Chien et al., 2025, 2026). A related literature asks how removing duration from the market moves yields, in preferred-habitat models (Greenwood and Vayanos, 2014; Vayanos and Vila, 2021) and in the evidence on asset purchases (D'Amico and King, 2013), and how the supply of government debt affects its price (Krishnamurthy and Vissing-Jorgensen, 2012; Fukunaga et al., 2015). This work concerns the consolidated government's funding and its gains and losses as a whole, or the price effects of the duration removed. We take the realised allocation as given and measure, security by security, which duration was left with the holders outside the central bank and how the losses divided between the two sides.

The paper also contributes to the study of government debt as insurance. In the fiscal theory of the price level, whether surpluses back the debt or its real value absorbs fiscal shocks shapes how debt returns move with the state of the economy (Leeper, 1991; Sims, 1994; Woodford, 1995; Cochrane, 2023). Asset-pricing treatments value those returns with an SDF (Corhay et al., 2023; Jiang et al., 2024; Gómez-Cram et al., 2024), and Pal Mustafi and Preston (2026) turn the covariance between the SDF and the return on government debt into a measure of its insurance value for the United States and the United Kingdom. The covariance of nominal bonds with stocks varies with the inflation environment (Campbell et al., 2017, 2020), and government debt can carry a hedging premium (Acharya and Laarits, 2023). We connect these pricing measures to the maturity composition of holdings. At observed prices and a given central bank holding share, we measure how alternative allocations change the insurance value of the bonds left outside the central bank and the division of losses between holders. We do so for a third economy, with a domestic SDF and with the central bank's reserves among the liabilities priced. Studies of Japanese public debt have asked instead whether it can keep rising without a crisis (Doi et al., 2011; Hoshi and Ito, 2014; Braun and Joines, 2015), and a broader literature whether low interest rates make public debt cheap (Blanchard, 2019); neither asks which holders carry the debt's duration.

Section 2 defines the objects we measure, and Section 3 describes the data and the SDF. Section 4 reports where the duration and the insurance value sat and how large the insurance value was, Section 5 how the losses were divided, and Section 6 whether the insurance value changed after 2022. Section 7 concludes.

2 Framework

This section defines what the paper measures: the liabilities left outside the Bank of Japan once the Bank is consolidated with the government (Section 2.1), the interest-rate exposure embedded in the bonds among them and its realised results (Section 2.2), and the insurance value of the liabilities (Section 2.3). Only the last requires an SDF.

2.1 Consolidation and the object being measured

We treat the central bank as part of the government, as Pal Mustafi and Preston (2026, footnote 4) do, because it remits its net earnings to the Treasury. Seen from the consolidated government, the bonds the central bank holds are internal claims that cancel, and what remains outside is the government's liability to everyone else.[3] When the central bank holds little, consolidation changes little. In Japan it changes a great deal. Once the Bank of Japan's bond holdings are netted out, the consolidated government's interest-bearing liabilities include, besides the bonds held outside the Bank, the reserves that financial institutions hold at the Bank as current-account balances. These are an overnight liability that earns the rate the Bank pays on them, and they were the largest item on the Bank's liability side during the large-scale purchases.

Two definitions of the liability portfolio therefore matter. The first, "JGBs only", is the JGBs held outside the Bank of Japan, the Japanese analogue of the US and UK portfolios in Pal Mustafi and Preston (2026). The second, "consolidated", adds the current-account balances, with market-value weights and their remuneration as their nominal return. We use it as the benchmark because it is the closest counterpart we can price to the consolidated government's obligations in the budget constraint of Section 2.3. It still covers only part of them: banknotes and the non-marketable bonds sold to households are outside it. We call the JGBs held outside the Bank, the "JGBs only" liability, the outside portfolio, its coupon bonds the outside coupon portfolio, and their holders the outside holders. The duration, its insurance value by maturity and the losses are measured on the outside coupon portfolio, which carries almost all the duration; the aggregate insurance value is measured for both liability portfolios. We do not study the realised cost of servicing the reserves, which moves with the policy rate through their remuneration (Hall and Reis, 2015; Reis, 2022; Congressional Budget Office, 2022).

[3] In the US application of Pal Mustafi and Preston (2026), the series of Hall and Sargent (2011, 2025) nets the holdings of the Federal Reserve, government agencies and trust funds out of the debt; in the UK application only the Bank of England's holdings are netted out. We follow the UK application (Section 3.1).

2.2 Duration exposure and realised holding-period results

We measure the interest-rate exposure of a portfolio by its modified duration, the first-order sensitivity of its value to a parallel shift in yields. The duration exposure of a portfolio or of a maturity segment at a date is its market value times its modified duration, the yen counterpart of dollar duration. A segment's share of a portfolio's exposure then equals its contribution $w_{b,t}D_{b,t}$ to the portfolio's modified duration divided by that duration, where $w_{b,t}$ is the segment's market-value share and $D_{b,t}$ its modified duration; over a period we report the ratio of the period averages of these contributions. Duration describes the bonds themselves. We do not observe the holders' derivatives, other assets or liabilities, so we measure the exposure embedded in the bonds, not the net exposure of the institutions that hold them.

The realised result of that exposure is the holding-period result: the change in price plus coupon income over a month on the market value held at the end of the previous month. A negative result is a net loss, and its price component a price loss. Computed from observed prices, the result also reflects the convexity and non-parallel shifts of the yield curve that duration ignores.[4] Neither duration nor the holding-period result requires an SDF. Because the Bank is part of the consolidated government, its losses net out within the consolidated government, whereas the losses of the outside holders are market-value gains to the consolidated government on its liabilities. The division of the losses between the two therefore measures how much of the revaluation of the debt fell on holders other than the central bank, a group that includes the public pension funds (Section 3.1).

2.3 Insurance value and the stochastic discount factor

Duration says how much the value of the liabilities moves with yields, not whether those movements are costly or valuable to the government. That depends on when they occur. Consider a government whose nominal liabilities are held by investors with a real stochastic discount factor M_{t+1}. Let $\tilde{R}^g_{t+1}$ be the gross nominal holding-period return on the market-value-weighted portfolio of those liabilities and Π_{t+1} gross inflation. The government budget constraint written in returns is $(\tilde{R}^g_t/\Pi_t)\, b_{t-1} = s_t + b_t$, where b_t is the real market value of the liabilities and s_t the real primary surplus (Corhay et al., 2023). Investors price the liabilities, so $1 = \mathbb{E}_t[M_{t+1}\tilde{R}^g_{t+1}/\Pi_{t+1}]$, and with $R_{0,t} = 1/\mathbb{E}_t M_{t+1}$ the zero-beta rate, the expected real return on the portfolio in excess of that rate is

$$\mathbb{E}_t\left[\frac{\tilde{R}^g_{t+1}}{\Pi_{t+1}}\right] - R_{0,t} = -R_{0,t}\ \mathrm{Cov}_t\left(M_{t+1}, \frac{\tilde{R}^g_{t+1}}{\Pi_{t+1}}\right). \tag{1}$$

[4]The holding-period result is a mark-to-market investment result, not an accounting measure of unrealised losses. It locates gains and losses across holders and is not the consolidated government's net fiscal loss, since JGBs held by the Bank are internal claims; nor do mark-to-market changes in bonds carried at amortised cost enter remittances to the Treasury as they occur.

The covariance on the right is the object we measure. The SDF is that of the investors who price the assets; the insurance value is a property of the payoffs of the liabilities, not of the circumstances of any particular holder. If it is positive, the liabilities pay off in states that investors value, they carry a negative risk premium, and the government borrows at an expected real return below the zero-beta rate: the debt has insurance value. If it is negative, the government pays a risk premium. The covariance is a property of the whole portfolio and of the real return, so inflation surprises enter with the same sign as price falls.[5]

The sign matters for fiscal sustainability. The debt-to-GDP ratio grows in expectation at the expected real return on the liabilities net of real growth, so a risk premium raises the primary surplus required to keep the ratio constant by the premium times the ratio, relative to a liability priced at the zero-beta rate, and a safety premium lowers it. Section 4.2 applies this conversion. It concerns expected returns in investors' pricing, not a flow that appears in the budget.

Consolidation changes the measured covariance. Current-account balances earned a rate close to zero over most of the sample, so their real return co-moves with the SDF mainly through inflation. Adding them dilutes the co-movement of the bonds with the SDF. Section 4.2 reports how much insurance value remains for the consolidated liability.

Duration exposure and insurance value share a determinant, but they are not the same object. A longer bond has a larger price response to a change in yields, so a portfolio's exposure rises with its duration. Its covariance with the SDF can rise with duration too, when yield changes move bonds of all maturities in the same direction in the same states. Whether it does is an empirical question, and Section 4.3 examines it. The two objects also differ in timing: the insurance value is a covariance measured over a period, positive only if, in that period, bad states came with high real returns, as when yields fall, whereas the realised result is the loss recorded in one episode of rising yields. Nothing in the framework says that long bonds always insure.

3 Data and measurement

Measuring the covariance in (1) requires the real return on the liabilities held outside the central bank and an SDF that prices it. For the United States, Pal Mustafi and Preston (2026) take the return from the series of Hall and Sargent (2011). No such series exists for

[5]Pal Mustafi and Preston (2026) show, in a model in which monetary policy follows a Taylor rule and the surplus follows a fiscal rule subject to deficit shocks, that the sign of the covariance distinguishes the two regimes of Leeper (1991): it is zero when surpluses adjust to stabilise the debt and negative when the real value of the liabilities absorbs deficit shocks in the states investors dislike. More generally, the covariance is positive when bad states bring lower yields, as when the central bank meets adverse demand shocks by cutting rates. A shift towards supply shocks that the central bank accommodates can also make debt risky (Pal Mustafi and Preston, 2026, Section 6.4), so we do not read the sign as a test of the fiscal regime.

Japan, so we construct one from security-level data; the same panel gives the holdings and the mark-to-market results by holder. Appendix A reports the checks against official records and the sensitivity of the results to the construction choices, and Appendix B the estimation details.

3.1 Government liabilities by security and holder

Securities and prices. The portfolio consists of the marketable JGBs: Treasury bills and the bonds other than bills, which for brevity we call coupon JGBs although they include the three-year discount bonds.[6] Month-end prices are the reference statistics of the Japan Securities Dealers Association (JSDA), the average of dealer quotes for each issue.[7] The panel covers 1,165 issues and 95,203 issue-months from September 2002 to September 2026, with 328 issues outstanding in an average month. The return of each issue follows its contractual terms: its coupon dates and accrual, the applied rates of the floating-rate notes and the Ministry of Finance's index ratios for the inflation-indexed bonds (Appendix A).

Amounts outstanding and holdings. The Ministry of Finance publishes the amounts sold at auction, delivered in liquidity-supply auctions (reopenings of existing issues) and bought back for cancellation, but not a monthly series of amounts outstanding by issue. We reconstruct it from these flows and from scheduled redemptions; the reconstruction matches the Ministry's issue-level register at March 2026 for every coupon bond up to small differences.[8] The Bank of Japan publishes its holdings of each issue, and we use the observation closest to each month-end.[9] The outside holding of each issue is the amount outstanding minus the Bank's holding. As in the UK application of Pal Mustafi and Preston (2026), we net out only the central bank, so the holdings of other public entities, chiefly the public pension funds, remain in the outside portfolio.[10] Treasury bills enter as

[6] Fixed-coupon bonds of two to forty years, including the five- and ten-year climate transition bonds; fifteen-year floating-rate notes; ten-year inflation-indexed bonds; and three-year discount bonds. Non-marketable bonds sold to households, separately traded principal and coupon strips, and when-issued trades are excluded.

[7] The JSDA statistic dated on a business day reports quotes collected at three p.m. on the previous business day. We use the statistic dated on the last business day of the month; the equity and yield data are dated on that day.

[8] Before April 2012 the Ministry reports some buybacks only as totals per operation; we allocate each total across the eligible issues in proportion to their holdings outside the Bank (205 operations, 18.8 trillion yen). Appendix Table A1 reports alternative allocations.

[9] Until 2005 the publication was less frequent, and for 41 months the nearest observation is dated after the month-end (Appendix A).

[10] Netting out general-government holdings as well makes little difference (Appendix Table A1). Bonds underwritten by the former Trust Fund Bureau before 2001 were not auctioned and are outside the reconstruction, which therefore covers 68 to 75 per cent of the medium- and long-term bonds outstanding over 2003–2007, and 92 per cent or more from 2013 (Appendix A).

one segment: the outstanding stock minus the Bank's holdings, with the equal-weighted return of the bills quoted by the JSDA.

Portfolio returns and duration. The nominal return on the "JGBs only" portfolio is the market-value-weighted average of the issue returns, with weights given by the holdings outside the Bank at the end of the previous month; bills are 18 per cent of it on average. The "consolidated" portfolio adds the current-account balances from the Bank's balance sheet. Their return is zero through October 2008 and, from November 2008, when the Bank's remuneration of excess balances began, the monthly average of the uncollateralised overnight call rate.[11] The balances were 6 per cent of the consolidated portfolio at the end of 2012 and 36 per cent on average from 2013. For each issue and month we compute the modified duration at the end of the previous month: exactly for the fixed-coupon nominal bonds, which are 95 per cent of the outside coupon portfolio by market value, and by approximation for the other instruments (Appendix A). Real returns deflate nominal returns by the all-items consumer price index (CPI).[12] All series end in September 2026.

3.2 The stochastic discount factor

We follow the parametric approach of Pal Mustafi and Preston (2026), which builds on Di Tella et al. (2026). The zero-beta rate is an affine function of L demeaned instruments Z_t, and the real SDF is linear in the demeaned returns of K factors F_{t+1},

$$R_{0,t} = \gamma_0 + \sum_{l=1}^{L} \gamma_l Z_{l,t}, \qquad M_{t+1} = R_{0,t}^{-1} + \sum_{j=1}^{K} \delta_j \left(F_{j,t+1} - \bar{F}_j \right), \tag{2}$$

where $\bar{F}_j$ is the sample mean of the factor.[13] The factors are the real market return in excess of the zero-beta rate and the Japanese size (SMB, small minus big) and value (HML, high minus low book-to-market) factors of Fama and French (1993), as constructed for international markets by Fama and French (2012), converted into yen and deflated by the CPI. The $L = 5$ instruments are the real one-month bill rate, the twelve-month average of past CPI inflation, the term spread, the earnings yield of the Tokyo Stock Exchange and the credit spread of A-rated corporate bonds; each is known at the start of the

[11] Applying the tiered rates of 2016–2024 to the published tier balances, and allowing for the special remuneration schemes, changes the risk premia by at most about 0.1 basis points (Appendix Table A1).

[12] The linked series on the 2020 base, published alongside the 2025 base; Appendix Table A1 reports the latter.

[13] With sample demeaning, (2) is an empirical linear SDF rather than an exact conditional one; the restrictions below are imposed through their unconditional moments with a finite set of instruments.

month whose return it helps price.[14] The test assets are the 25 size-and-book-to-market portfolios for Japan from Kenneth French's library, the 33 Tokyo Stock Exchange industry portfolios, the market portfolio and the one-month bill, $N = 60$ real returns in yen.[15] As in Pal Mustafi and Preston (2026), no longer-term government bonds are among the test assets, so the SDF prices government debt from other assets.

The parameters $\theta = (\gamma, \delta)$, with $\gamma = (\gamma_0, \ldots, \gamma_L)$ and $\delta = (\delta_1, \ldots, \delta_K)$, are estimated by two-step generalised method of moments (GMM) from the conditions that the SDF prices the test assets, whose gross real returns form the vector R_{t+1}, and the zero-beta rate,

$$\begin{aligned} &\mathbb{E}_t\left[M_{t+1}R_{0,t} - 1\right] = 0, \qquad \mathbb{E}_t\left[w_\theta' R_{t+1} - R_{0,t}\right] = 0, \\ &\mathbb{E}_t\left[M_{t+1}R_{i,t+1} - 1\right] = 0, \qquad i = 1, \ldots, N, \end{aligned} \tag{3}$$

where w_θ is the minimum-variance portfolio of the test assets with zero loading on the factors (Di Tella et al., 2026). Interacting each condition with a constant and the instruments gives 372 moments, more than the $T = 289$ months from September 2002 to September 2026, so the long-run covariance matrix of the moments, estimated with six Newey and West (1987) lags, is singular. We therefore shrink its correlation matrix towards the identity, with an intensity chosen from the data by the formula of Ledoit and Wolf (2004) ($a = 0.093$); fixed intensities from 0.05 to 1 give the same signs and similar magnitudes.[16] The benchmark SDF has a standard deviation of 0.177 and implies an equity premium of 8.9 per cent a year (Appendix Table B2).

We use three further SDFs as checks. The first is the nonparametric SDF of Hansen and Jagannathan (1991), the non-negative M_{t+1} of minimum variance that prices the test assets exactly in sample; with 60 assets and 289 months, its behaviour in a finite sample can be sensitive to the choice of test assets. The second is the US parametric SDF converted into yen, the device Pal Mustafi and Preston (2026) use for the United Kingdom, which is valid if markets are complete and integrated (Backus et al., 2001):

$$M_{t+1}^{JP} = M_{t+1}^{US} \frac{S_t}{S_{t+1}} \frac{\Pi_{t+1}^{JP}}{\Pi_{t+1}^{US}}, \tag{4}$$

[14]The term spread is the ten-year JGB yield minus the three-month bill yield; the earnings yield is that of the Exchange's first section and, from April 2022, of its Prime market; the credit spread is on five-year bonds. The earnings yield replaces the inverse of the cyclically adjusted price–earnings ratio used for the United States, which cannot be constructed for Japan before September 2010. The CPI-based instruments use the index for the second month before the return month, the latest published when that month starts; using the preceding month, as Pal Mustafi and Preston (2026) do, changes the premia by at most 2.3 basis points.

[15]The 25 portfolios, the market and the factors are converted into yen at the month-end exchange rate. The industry returns are the Exchange's official dividend-inclusive returns from January 2016 and, before that, the price change plus one twelfth of the trailing dividend yield (Appendix Table A1).

[16]Applied to the US data of Pal Mustafi and Preston (2026), the same procedure closely matches the reported moments of their SDF, its factor loadings and the implied equity premium (Appendix Table B1).

where S_t is the yen price of a dollar. Its correlation with the Japanese parametric SDF is 0.54. The third is the US nonparametric SDF converted in the same way.

3.3 Inference

With $R^g_{t+1} = \tilde{R}^g_{t+1}/\Pi_{t+1}$ the gross real return on the liability portfolio, we report the annualised risk premium implied by the SDF,

$$RP = -12\,\frac{\mathrm{Cov}(M_{t+1}, R^g_{t+1})}{\mathbb{E}[M_{t+1}]}, \tag{5}$$

computed within a period from the sample covariance and the sample mean of M.[17] A negative value is a safety premium. We report it for the full sample, for the sub-periods before and after January 2022, the start of the year in which inflation and yields rose, and for their difference; the break is not chosen from the covariances. Covariances are reported multiplied by 10^4.

The tables report two kinds of uncertainty. The standard errors in parentheses, used in the literature, treat the estimated SDF as given: they are Newey and West (1987) standard errors (six lags) of the mean of $(M_{t+1} - \bar{M})(R^g_{t+1} - \bar{R}^g)$, scaled as in (5) with the mean of the SDF held fixed, and we call them fixed-SDF standard errors. They do not account for the fact that the SDF is itself estimated from 289 months of asset returns. The intervals in brackets do. For the benchmark SDF we draw $B = 999$ block-bootstrap samples of circular six-month blocks (Politis and Romano, 1992), separately before and after January 2022, redrawing the asset returns, factors, instruments and liability returns together. On each sample we re-estimate both GMM steps, including the shrinkage, and we report 90 per cent percentile intervals.[18] The holdings and the mark-to-market results do not use the SDF; their uncertainty is that of the construction (Appendix A). In all tables, totals, shares and changes are computed before rounding.

4 Who held the duration, and where the insurance value sat

This section answers the first two parts of the paper's question. Section 4.1 shows which maturities, and how much duration, were left outside the Bank. Section 4.2 asks whether

[17] The sample covariance within a period estimates the unconditional covariance over the period, which includes the covariance of the conditional means; the ratio is the covariance-based premium for the period, as in Pal Mustafi and Preston (2026), not the period average of the conditional premium in (1). The covariance divides by the number of months.

[18] For the contribution of the long-maturity bonds in Section 4.3, a second bootstrap draws the blocks within 2002–2012, 2013–2021 and 2022–2026, so that every sample contains the 108 months of 2013–2021. For the change in the premium, twelve-month blocks and a version that keeps the first-step estimates fixed are in Appendix Table C1.

the liabilities held outside the Bank carried insurance value at all, and Section 4.3 locates it within the maturity structure of the outside coupon portfolio.

4.1 Who held which maturities

The Bank of Japan's share of the market value of coupon JGBs rose from 12 per cent at the end of 2012 to 49 per cent at the end of 2021 (Figure 1, panel A). Its share of the duration exposure rose by less, from 7 to 35 per cent, because the Bank held mostly short- and medium-term bonds. At the end of 2021 it held 61 per cent of the coupon bonds with less than three years to maturity[19], 68 per cent of those with three to seven years and 42 per cent of those with seven to twelve years, but 26 per cent of those with twelve to 22 years and 29 per cent of those beyond 22 years (Appendix Figure C1).

The durations of the two portfolios diverged accordingly (panel B). The modified duration of the outside coupon portfolio rose from 5.0 years in 2002 to 11.2 at the end of 2021, whereas the Bank's rose from 4.4 to 6.5. Over 2022–2026 the two averaged 10.6 and 5.8 years, and 43 per cent of the outside coupon portfolio's market value had more than twelve years to maturity.

[19]We use five buckets of remaining maturity: 0–3, 3–7, 7–12, 12–22 and more than 22 years. Beyond ten years the portfolio contains only bonds issued at fifteen years or more.

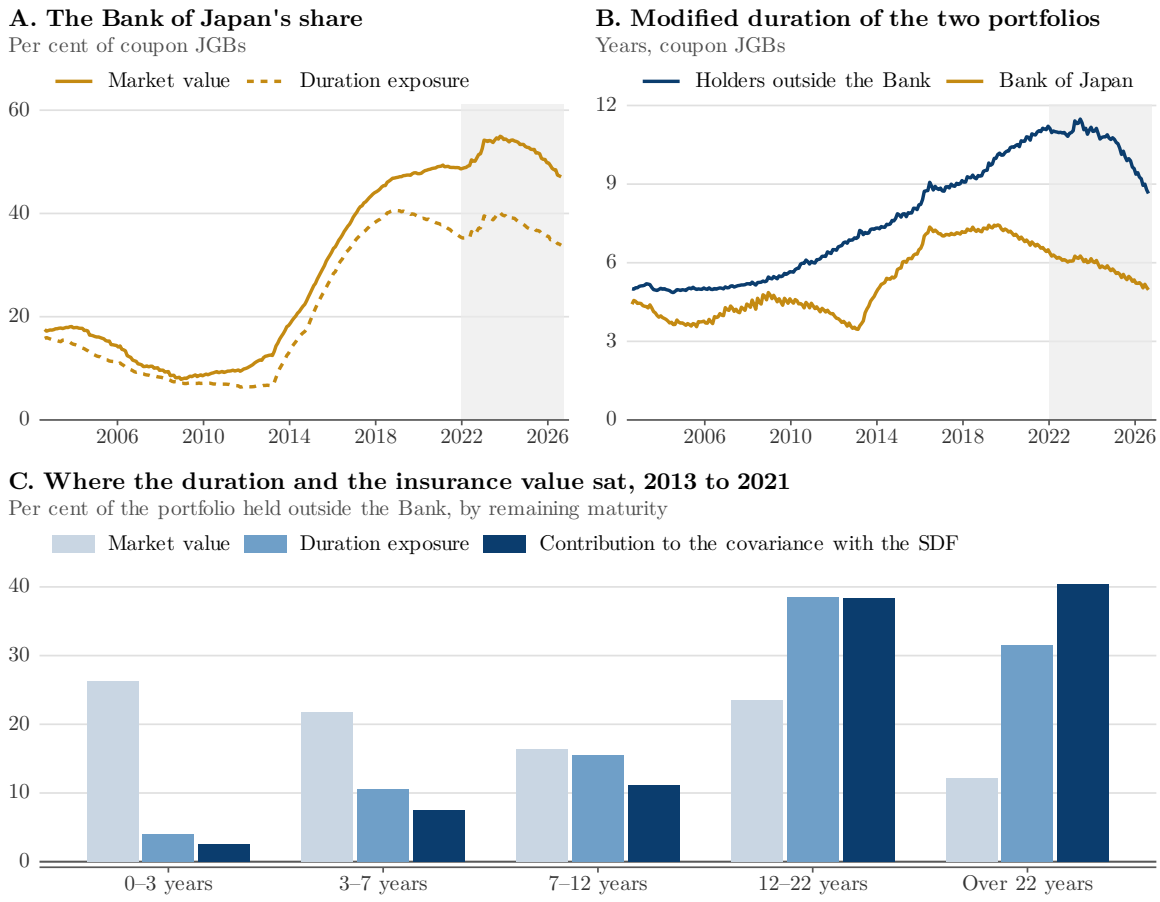


Figure 1: Holdings, duration and insurance value by remaining maturity

Notes: Panel A: the Bank of Japan's share of coupon JGBs by market value and by duration exposure (market value times modified duration). Panel B: modified duration of the coupon JGBs held outside the Bank and of the Bank's holdings. Panel C: over January 2013 to December 2021, each remaining-maturity bucket's average share of the market value of the outside coupon portfolio, its period share of the portfolio's duration exposure (Section 2.2), and its share of the covariance of that portfolio's real return with the benchmark SDF (Section 4.3). The grey background in panels A and B marks the period from January 2022.

4.2 The aggregate insurance value before 2022

Did the liabilities held outside the Bank carry insurance value at all? Table 1 reports the risk premium (5) implied by the four SDFs for the two definitions of the liability, over the full sample, the two sub-periods and the change between them; Section 6 discusses the last two columns.

Before 2022 every SDF assigns a safety premium to the liabilities held outside the Bank. For the benchmark SDF the premium is −30 basis points a year on the consolidated liability and −37 on JGBs alone: the liabilities paid off, on average, in states that the SDF

prices highly.[20] With the SDF re-estimated in the bootstrap, the 90 per cent intervals are $[-50, -1]$ and $[-62, -1]$. Every parametric specification we estimate also gives a negative premium, including specifications estimated from 1991 on the smaller set of assets and instruments available that early (Section 6.2),[21] as Pal Mustafi and Preston (2026) find for the United States and the United Kingdom. Estimated only on data to December 2021, the benchmark and the smaller specification give premia about a third as large (-11 and -9 basis points for the consolidated liability): the sign of the pre-2022 premium does not depend on the years of losses, but its size does.

Table 1: Risk premium on government liabilities implied by alternative SDFs

SDF	Liability	Full sample	To 2021:12	From 2022:01	Change
Japanese parametric (benchmark)	Consolidated	−25 (8)	−30 (8)	+22 (25)	+51 (26)
			[−50, −1]	[−27, 88]	[−16, 127]
	JGBs only	−32 (11)	−37 (10)	+22 (36)	+59 (37)
			[−62, −1]	[−56, 125]	[−36, 171]
US parametric, in yen	Consolidated	−8 (5)	−12 (5)	+17 (10)	+29 (11)
	JGBs only	−8 (6)	−14 (5)	+31 (17)	+44 (17)
Japanese nonparametric	Consolidated	−57 (28)	−39 (26)	−54 (82)	−15 (86)
	JGBs only	−78 (36)	−45 (30)	−115 (126)	−69 (130)
US nonparametric, in yen	Consolidated	−16 (17)	−26 (13)	+29 (43)	+55 (44)
	JGBs only	−18 (24)	−31 (16)	+38 (71)	+69 (73)

Notes: Annualised risk premium $RP = -12\ \mathrm{Cov}(M, R^g)/\mathbb{E}[M]$ in basis points, computed within each period. A negative value is a safety premium. Parentheses: Newey–West (six lags) standard errors that treat the estimated SDF as given; the standard error of the change is the square root of the sum of the two sub-period variances. Brackets (benchmark SDF only): 90 per cent percentile intervals from $B = 999$ block-bootstrap samples (six-month circular blocks drawn within each sub-period) on each of which the SDF is re-estimated in both GMM steps. "Consolidated" is the market-value-weighted portfolio of JGBs held outside the Bank and current-account balances at the Bank of Japan; "JGBs only" excludes the balances.

Including the current-account balances reduces the safety premium by about a fifth, from -37 to -30 basis points, because the balances earned close to zero (Section 2.3); the bonds are the main source of the hedge.

Multiplied by the debt ratio, the safety premium gives, to first order, the reduction in the primary surplus needed to keep that ratio constant (Section 2.3): before 2022, about

[20] Appendix C.2 reports how the SDF prices the liability return itself, relative to the one-month bill.

[21] The 25 size-and-book-to-market portfolios, the market and the call rate, with the real call rate, trailing inflation and the term spread as instruments.

0.42 per cent of GDP a year for the consolidated liability and 0.42 per cent for JGBs alone.[22]

4.3 Where in the portfolio the insurance value sat

Where within the bonds did the insurance value sit, and did the long end that the Bank left outside carry it? Bills, which mature within a year, and overnight current-account balances add little duration, so we answer with the outside coupon portfolio. We split it into five buckets of remaining maturity and decompose the covariance of its real return with the benchmark SDF into the contributions of the buckets,

$$\operatorname{Cov}\left(M, \sum_b w_{b,t} r_{b,t}\right) = \sum_b \operatorname{Cov}\left(M, w_{b,t} r_{b,t}\right), \tag{6}$$

where $w_{b,t}$ is the bucket's market-value share at the end of month $t-1$ and $r_{b,t}$ its net real holding-period return in month t. If the covariance were spread in proportion to market value, each bucket's contribution would match its weight; if it followed duration, it would match the bucket's share of the duration exposure. Table 2 compares the three for 2002–2021 and for 2013–2021, and panel C of Figure 1 plots them for 2013–2021. The second period spans the large-scale purchases that began in April 2013 and ends before yields rose; in it the two portfolios diverged most.

Over 2002–2021, the covariance followed duration rather than market value. Bonds beyond twelve years were 26 per cent of the portfolio by market value and 57 per cent of its duration exposure, and they contributed 53 per cent of its covariance with the SDF; bonds under three years were 29, 6 and 3 per cent. Beyond three years the correlation between the SDF and the bucket returns is similar across maturities, between 0.28 and 0.32. The similar correlations and the greater volatility of long-bond returns are consistent with a concentration that follows duration, rather than with a markedly stronger correlation of long-bond returns with the SDF. The correlation at the maturity targeted under yield-curve control is no lower than at adjacent maturities.[23]

[22] The debt ratio of the consolidated liability averages 1.42 before 2022. The two figures are close because the current-account balances raise the debt ratio by about a quarter while lowering the premium by about a fifth. The debt ratio is the market value of the marketable JGBs held outside the Bank, plus the current-account balances for the consolidated liability, at the start of each month, divided by nominal GDP interpolated from quarterly data. The figure is a first-order scale for the liabilities we price: banknotes, non-marketable bonds and the government's financial assets are outside the portfolio.

[23] Yield-curve control, which pinned the ten-year yield from September 2016, could in principle have removed the information in the covariance at the targeted maturity. Between September 2016 and December 2021 the correlation between the SDF and the real return in the seven-to-twelve-year bucket (0.40) is not lower than in the adjacent buckets (0.30 and 0.37), so we do not find a lower sample correlation at the targeted maturity, although a comparison of correlations cannot rule such censoring out.

Table 2: Maturity segments of the coupon JGB portfolio held outside the Bank: weight, duration, volatility, correlation with the SDF, and contribution to the covariance

	September 2002 to December 2021					January 2013 to December 2021				
Remaining maturity	Weight (%)	Dur. share (%)	SD (%)	Corr.	Contrib. (%)	Weight (%)	Dur. share (%)	SD (%)	Corr.	Contrib. (%)
0–3 years	29	6	1.1	0.08	3	26	4	1.1	0.08	3
3–7 years	26	16	1.6	0.28	18	22	11	1.3	0.20	8
7–12 years	19	21	2.7	0.32	26	16	16	2.2	0.25	11
12–22 years	18	34	3.9	0.29	30	24	38	4.0	0.28	38
Over 22 years	7	23	7.7	0.28	23	12	31	7.1	0.31	40
Over 12 years	26	57			53	36	70			79
Total	100	100			100	100	100			100

Notes: Coupon JGBs held outside the Bank by remaining maturity at the end of the previous month; the shares are within the portfolio held outside the Bank. "Weight" is the period mean of the bucket's share of the portfolio's market value. "Dur. share" is the period share of the portfolio's duration exposure (Section 2.2). "SD" is the annualised standard deviation of the bucket's real holding-period return. "Corr." is the correlation between the benchmark SDF and that return. "Contrib." is the bucket's share of the covariance between the SDF and the portfolio's real return, $\mathrm{Cov}(M, w_{b,t}r_{b,t})/\sum_b \mathrm{Cov}(M, w_{b,t}r_{b,t})$ in (6). "Over 12 years" adds the two longest buckets before rounding. With time-varying weights a contribution does not decompose exactly into weight, volatility and correlation.

Over 2013–2021 the concentration was stronger. As the outside holdings shifted towards the long end, bonds beyond twelve years became 36 per cent of the portfolio by market value and 70 per cent of its duration exposure, and they contributed 79 per cent of its covariance with the SDF. In this period the correlation with the SDF also rises somewhat with maturity, from 0.20 to 0.31 beyond three years, consistent with the long end contributing more than its share of the duration exposure.

The shares are ratios of sample covariances whose common denominator is estimated with error (Section 4.2), so we also assess the long end's contribution in levels, re-estimating the SDF on each bootstrap sample. Over 2013–2021 the covariance contributed by bonds beyond twelve years is 2.89, with a 90 per cent interval of $[0.06, 5.43]$. Its excess over what the segment's market-value weight alone would imply is 1.59, with an interval of $[0.01, 2.92]$ (Appendix C.3).

The share is similar across SDFs. Estimated only on data to December 2021, so that it uses no asset prices from the years of losses, the benchmark specification gives the long end 73 per cent of the covariance over 2013–2021. Across the 16 SDFs and specifications in Appendix C.3, which include the nonparametric and converted US SDFs and fixed shrinkage intensities, the share lies between 66 and 80 per cent over 2013–2021

and between 42 and 66 per cent over 2002–2021. It also stays well above the segment's market-value weight when the weights are held at their period averages (76 per cent over 2013–2021) and when the cutoff of the long segment is moved to ten, fifteen or twenty years.

The long end, of which the Bank held relatively little, thus carried most of the insurance value of the outside coupon portfolio over 2013–2021, and a disproportionate share of it over 2002–2021. Section 5 turns to the losses on this segment when yields rose.

5 Mark-to-market losses and alternative allocations

This section turns from the insurance value to the realised losses: how the exposure located in Section 4 was realised when yields rose from 2022, and how much of the split of the losses between the Bank and the outside holders reflects the allocation of the bonds rather than the size of the Bank's holdings.

5.1 Losses by holder and maturity

For each bond and month we multiply the market value each holder held at the end of the previous month by the bond's holding-period return, split into the change in price and coupon income. Figure 2 plots the results by maturity, and Table 3 reports them. Between January 2022 and September 2026 the outside coupon portfolio lost 125.1 trillion yen, a price loss of 151.2 trillion partly offset by 26.1 trillion of coupon income, and the Bank lost 56.1 trillion. Although the Bank held 52 per cent of the coupon bonds on average over the period, it bore 31 per cent of the net loss.[24]

The gap between the Bank's share of the holdings and its share of the losses follows duration: over the period the outside coupon portfolio averaged a modified duration of 10.6 years and the Bank's 5.8 (Section 4.1), and the compounded holding-period return was −21.8 per cent on the outside coupon portfolio and −10.2 per cent on the Bank's.[25] Of the outside price loss, 77 per cent arose on bonds with more than twelve years to maturity, the segment that had carried most of the insurance value over 2013–2021.

[24] Over the fiscal year to March 2026, the price change on the Bank's coupon-JGB holdings in the panel is −16.8 trillion yen. Over the same year the unrealised loss on JGBs in the Bank's financial statements, market value minus book value, rose from 28.6 to 45.4 trillion yen (Bank of Japan, 2026, reference table 9), by about the same amount, although the two measures differ in definition: the price change includes the pull to par of bonds the Bank records at amortised cost, and the Bank's figure also covers Treasury bills.

[25] These are returns on portfolios whose composition changes every month, not changes in the value of a fixed stock of bonds.

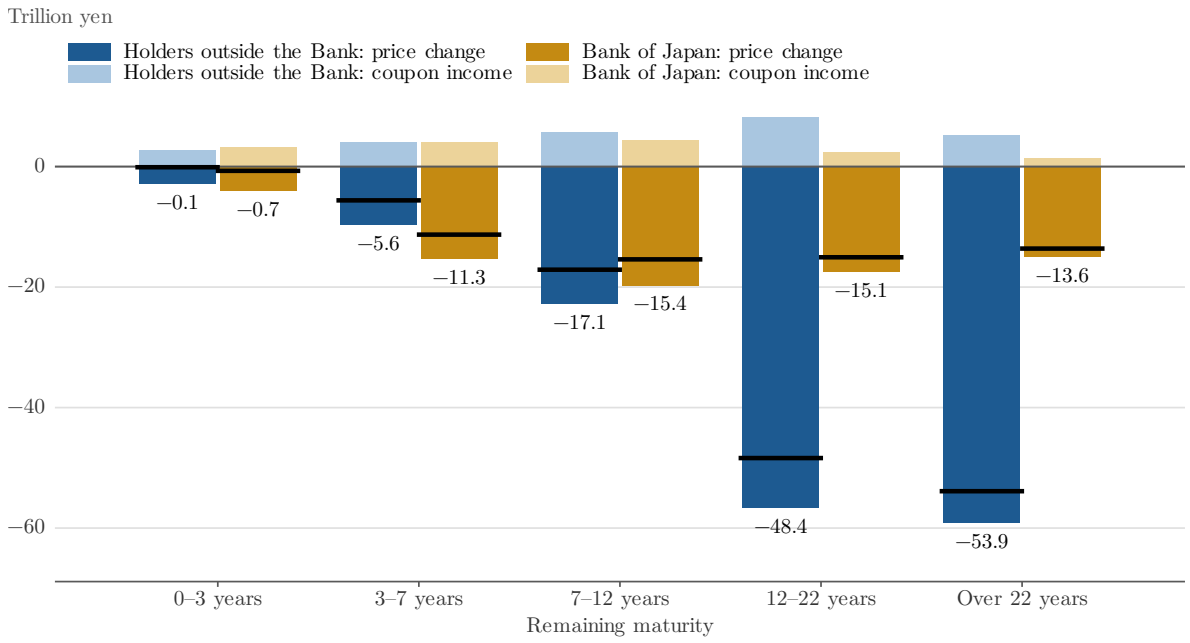


Figure 2: Mark-to-market results by holder and remaining maturity, January 2022 to September 2026

Notes: Cumulative nominal holding-period results on coupon JGBs, by holder and by remaining maturity at the end of the previous month, in trillion yen. Bars: price change (below zero) and coupon income (above zero); dashes and labels: totals (Table 3).

Table 3: Mark-to-market holding-period results on coupon JGBs by holder and remaining maturity, January 2022 to September 2026

	Outside the Bank			Bank of Japan			Average market value	
Remaining maturity	Total	Price	Income	Total	Price	Income	Outside	Bank
0–3 years	−0.1	−2.9	+2.8	−0.7	−4.0	+3.3	103	196
3–7 years	−5.6	−9.7	+4.1	−11.3	−15.3	+4.1	87	172
7–12 years	−17.1	−22.9	+5.7	−15.4	−19.7	+4.3	97	108
12–22 years	−48.4	−56.6	+8.2	−15.1	−17.5	+2.4	136	44
Over 22 years	−53.9	−59.1	+5.2	−13.6	−14.9	+1.3	81	23
Total	−125.1	−151.2	+26.1	−56.1	−71.5	+15.5	504	542

Notes: Trillion yen. Cumulative sum over months of the market value held at the end of the previous month times the nominal holding-period return of each bond, aggregated by holder and by the bond's remaining maturity at the end of the previous month. "Total" is the net holding-period result, the sum of "Price" and "Income". "Price" is the component due to the change in the clean price (including the index ratio of inflation-indexed bonds; at maturity, the redemption amount minus the opening price); "Income" is coupon payments and the change in accrued interest. "Average market value" is the average over the period of the market value held at the end of the previous month. Treasury bills are excluded. Buckets are by remaining maturity, not modified duration.

5.2 Size against composition

The Bank held half the market, and it held mostly bonds within seven years of maturity. To separate the two, we hold every bond's price and return at their observed values and reassign the result on each bond in each month between the two holders under alternative holding structures that keep the Bank's aggregate share at its observed value. The calculation asks who would have borne the observed price changes under a different distribution of the bonds, not what the price changes would have been.

In the first structure the Bank holds its aggregate share uniformly across issues, so the two portfolios have the same duration, 8.1 years on average, against 10.6 and 5.8 years in fact (Table 4, panel A). The outside holders would have lost 89.7 trillion yen instead of 125.1, and the Bank 91.5 trillion instead of 56.1: changing only the allocation moves 35 trillion yen of losses to the Bank and lowers the outside share of the net loss from 69 to 50 per cent. A uniform allocation also changes which issues are held within each maturity, so the second structure keeps the Bank's holdings within each bucket in proportion to its actual holdings and only equalises its share across buckets (Appendix A). It moves 34 trillion yen, close to the first, so the allocation across maturities accounts for most of the difference.[26] Under these reassignments, the allocation of the losses follows the allocation of duration.

Panel B applies the same structures to the years before 2022, with the SDF also held fixed. Over 2013–2021 the uniform allocation shortens the outside coupon portfolio from 9.0 to 8.0 years and lowers the covariance of its real return with the SDF from 3.67 to 3.29; the premium on the consolidated liability over 2013–2021 moves from −25 to −23 basis points (from −30 to −28 over 2002–2021). The insurance value, like the losses, moved roughly in proportion to the outside duration: the covariance fell by 10 per cent against 11 per cent for the duration over 2013–2021, and the outside losses by 28 against 23 per cent from 2022. The effect is smaller before 2022 because the Bank held less and the two portfolios' durations were closer.

[26] A comparison with the holding structure at the end of 2012, which also changes the size of the Bank's holdings, is in Appendix A.

Table 4: Losses and insurance value under alternative holding structures

A. Mark-to-market results, January 2022 to September 2026 (trillion yen)

	Outside the Bank		Bank of Japan	Shares (%)		Modified duration	
Holding structure	Total	Price	Total	Outside	>12 years	Outside	Bank
Actual holdings	−125.1	−151.2	−56.1	69	77	10.6	5.8
Uniform, by issue	−89.7	−109.8	−91.5	50	66	8.1	8.1
Uniform, by maturity bucket	−91.2	−114.3	−89.9	50	66	8.2	8.0

B. Insurance value before 2022 (benchmark SDF)

	2013:01–2021:12				2002:09–2021:12
	Duration	Covariance	Risk premium (basis points per year)		
Holding structure	outside	($\times 10^4$)	Outside coupon	Consolidated	Consolidated
Actual holdings	9.0	3.67	−44	−25	−30
Uniform, by issue	8.0	3.29	−39	−23	−28
Uniform, by maturity bucket	8.0	3.23	−38	−22	−27

Notes: Each row of panel A reassigns the observed monthly nominal holding-period result of every coupon JGB (market value of the issue at the end of the previous month times its return) between the holders outside the Bank and the Bank of Japan according to the stated holding structure, holding prices and returns fixed; the Bank's aggregate market-value share in each month is the observed one in every row. "Uniform, by issue" gives the Bank its actual aggregate share of coupon JGBs in each month, applied to every issue. "Uniform, by maturity bucket" sets the Bank's market-value share in every maturity bucket equal to its actual aggregate market-value share in each month, allocated within the bucket in proportion to its actual holdings (Appendix A). "Total" and "Price" are defined as in Table 3. "Outside" under shares is the outside holders' share of the total net loss; ">12 years" is the share of their price loss arising on bonds with more than twelve years to maturity. "Modified duration" is the average over the months of the modified duration of each portfolio under the structure, in years. The total net loss is the same in every row. Panel B applies the same structures to the months before 2022, with prices, returns and the benchmark SDF held fixed: "Duration outside" is the average modified duration of the coupon portfolio held outside the Bank, in years; "Covariance" is the covariance of the SDF with its real return; the risk premia, $-12\ \mathrm{Cov}(M, R)/\mathbb{E}[M]$, are for that portfolio and for the consolidated liability, whose coupon component is replaced while its bills and current-account balances are unchanged. The outside market value of coupon JGBs is the observed one in every row, so the weights of the consolidated liability are unchanged.

5.3 A comparison with the United States

The United States offers a contrast (Table 5). Using the holding-period returns on marketable Treasury debt by holder constructed by Hall and Sargent (2025), the cumulative return over 2022–2023 was −4.7 per cent for private investors and −4.2 per cent for the

Federal Reserve on the Treasury's measure of its holdings, or −6.5 per cent on its own accounts. The Federal Reserve, whose holdings had been lengthened by, among other things, the maturity extension programme of 2011–2012, thus lost about as much as private investors, or more, although the private series include bills, which damp their losses. Over the same two years the Bank of Japan, at a modified duration of 6.2 years against 11.1 for the outside coupon portfolio, had a cumulative return of −2.4 per cent, against −6.1 per cent outside.[27] The comparison is consistent with losses dividing according to what the central bank holds, not only how much.

Table 5: Cumulative nominal returns by holder: Japan and the United States

	Japan, coupon JGBs		US Treasury debt	
Period	Outside the Bank	Bank of Japan	Private investors	Federal Reserve
2022:01–2023:12	−6.1	−2.4	−4.7	−4.2 (−6.5)
2003:06–2003:09	−3.3	−2.5		

Notes: Per cent. Japan: compounded market-value-weighted monthly nominal holding-period returns on coupon JGBs held by each sector, from the security-level panel. United States: compounded monthly nominal holding-period returns on marketable interest-bearing Treasury debt held by private investors and by the Federal Reserve, from Hall and Sargent (2025), who weight the securities by the holdings recorded in the Treasury's Monthly Statement of the Public Debt (MSPD); in parentheses, the Federal Reserve's return computed by the same authors from the holdings of its own System Open Market Account (SOMA), which records the Federal Reserve's securities directly and is the closer counterpart of the Bank of Japan's published holdings. Unlike the Japanese series, the US series include bills, which are held mostly by private investors. Both Federal Reserve measures are shown because the MSPD and SOMA records differ in coverage and timing. The 2003 row covers the returns from the end of May to the end of September 2003, when the ten-year JGB yield rose sharply.

6 Did the insurance value change after 2022?

The losses in Section 5 locate the exposure to rising yields. Whether the debt held outside the Bank continued to insure its holders depends on how real returns covaried with bad states after 2022. We therefore turn from the location of the losses to the aggregate insurance value over the later period.

[27] The same ordering appears in 2003, before the large-scale purchases, when the ten-year yield rose from 0.5 to 1.4 per cent between the end of May and the end of September. The Bank then held 18 per cent of coupon JGBs at a duration of 4.3 years, against 5.1 for the outside coupon portfolio, and the cumulative returns were −2.5 and −3.3 per cent (Table 5); the gap in losses was smaller, as was the gap in duration.

6.1 Before and after 2022

In the benchmark estimates the premium on the consolidated liability turns from −30 basis points (8) before 2022 to +22 (25) from 2022, a change of +51 basis points (26) (Table 1). For JGBs alone it moves from −37 to +22, a change of +59 (37). The converted US SDF gives the same direction with smaller magnitudes, a change of +29 (11) for the consolidated liability.[28]

Re-estimating the SDF on each bootstrap sample raises the standard error of the change for the consolidated liability from 26 to 44 basis points (Appendix Table C1), and its 90 per cent interval, $[-16, 127]$, includes zero; for JGBs alone the interval is $[-36, 171]$. Even the sign of the post-2022 premium is not determined: its interval is $[-27, 88]$.

6.2 Sensitivity and timing

Unlike the location of the insurance value, the change depends on the sample from which the SDF is estimated and on its test assets (Appendix D.1). The SDF can be estimated from 1991 with a smaller set of assets and instruments. With that set, the pre-2022 premium on the consolidated liability is −31 basis points when the SDF is estimated on 2002–2026 and −21 when it is estimated on 1991–2026, and the change is +28 (36) and +3 (22) basis points respectively; for JGBs alone the change is +7 and −20.

In the benchmark estimates, the change comes partly from nominal returns, which no longer rose in bad states, and partly from inflation, which began to erode the real value of the debt in those states. It arises in the second half of 2022, the second half of 2023 and the first half of 2024, and again in February and March 2026, two months in which bonds and equities rose and then fell together; the block covariances in between, in the second half of 2024 and in 2025, are positive (Appendix D.2). Rolling 36-month covariances show a similar timing (Appendix D.3).

In the point estimates, the weakening was much smaller than in the United States. Over January 2022 to June 2025, the part of the post-2022 period that the US series covers, the Japanese premium was 0 (11) basis points, against −30 before 2022: the safety premium was close to zero by mid-2025, and the move to +22 comes from the later months. Measured in the same way, the US premium rose from −37 to +129 (Appendix D.4). The statistical certainty of the difference between the two changes, −136 basis points, depends on the method of inference (Appendices C.1 and D.4).

In fiscal terms, the benchmark change corresponds, at the full-sample debt ratio of 1.50, to 0.77 per cent of GDP a year, with a bootstrap interval of $[-0.24, 1.91]$ (Appendix D.5).

[28] The two nonparametric SDFs disagree with each other: the converted US one moves like the parametric estimates (+55, with a standard error of 44), whereas the Japanese one stays negative in both periods with very wide standard errors.

The estimates point to a weakening of the insurance value after 2022 but do not establish it.

7 Conclusion

A holding share was a poor guide to how much exposure the Bank of Japan left with other holders. The Bank came to hold half of the coupon bonds but about a third of their duration exposure. Over 2013–2021, the long-maturity segment carried most of the insurance value of the outside coupon portfolio; from 2022, it also bore most of that portfolio's price losses. Insurance value and exposure to rising yields were two faces of the same position.

Composition, not only size, shaped how that position was divided. At observed prices and the same aggregate share, an even allocation across bonds would have left the outside holders with less duration, a covariance with the SDF about a tenth smaller over 2013–2021, and 28 per cent less of the losses from 2022. For the government, that insurance lowered its expected cost of borrowing before 2022. The losses after 2022 locate the exposure; they do not show whether the debt still insures its holders. That question remains unresolved: in the point estimates the premium was close to zero over January 2022 to June 2025 and is positive once the months to September 2026 are included, but once the SDF is re-estimated the interval for the change includes zero.

Evaluating central bank holdings therefore requires their maturity composition as well as their scale. As holdings decline, which bonds run off or are sold, together with the maturities of new issuance, will determine the duration left with other holders. Whether that exposure provides insurance will depend on how yields and inflation move in bad states. The holding share alone answers neither question.

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A Data construction: checks and sensitivities

This appendix summarises the checks on the security-level construction of Section 3 and the sensitivity of the results to the construction choices that involve an assumption.

Amounts outstanding. The reconstruction from auction, liquidity-supply and buyback records was checked against the Ministry of Finance's register of amounts outstanding by issue at the end of March 2026: every coupon issue appears in both, and the reconstruction falls short of the register by more than 100 million yen for four issues only (at most 5 billion yen each); all other differences are the Fiscal Investment and Loan Programme bonds that share issue codes with ordinary JGBs. The issue-level amounts delivered in all 460 liquidity-supply auctions match the published totals to within 200 million yen. Against the Bank of Japan's aggregate of medium-, long- and super-long-term JGBs outstanding, the reconstructed face value covers 68 to 75 per cent at the end of March 2003–2007, 92 per cent in March 2013, 96 per cent in March 2015 and 97 to 99 per cent from March 2016. The early gap consists mainly of the bonds underwritten by the former Trust Fund Bureau before 2001, which were not auctioned and are therefore outside the reconstruction, and of retail bonds. The underwritten bonds are excluded from the reconstructed outside portfolio throughout. This is a construction assumption: the Fiscal Loan Fund, which succeeded the Bureau, held them to maturity in the main but also sold some into the market in the early 2000s, and holdings that reached private investors in that way are missing from the portfolio until the bonds matured. The full-period figures from 2002 are therefore subject to the early gap.

Missing prices. Issues without a quote in the following month are dropped from that month's return; the excluded market value never exceeds 0.32 per cent of the total.

Inflation-indexed and floating-rate bonds. The index ratios used for the inflation-indexed bonds are the Ministry's published daily ratios for 3,080 of 3,091 issue-months; three are reconstructed from the published base indices with the contractual rounding rule, which reproduces every one of the 27,197 published ratios in the same base period, and eight (the first three issues, December 2004 to February 2005) are approximated from the CPI. Reconstructing the index ratios from a spliced CPI series instead would depart from the official ratios around CPI base-year changes by up to 37 basis points in a month. The applied rates of the fifteen-year floating-rate notes are the Ministry's published rates for all 7,252 issue-months.

Modified duration. For fixed-coupon nominal bonds the modified duration is computed from the remaining cash-flow schedule and the JSDA's compound yield, with semiannual compounding and the full (dirty) price, including accrued interest, as denominator; a

numerical one-basis-point revaluation reproduces it to within 10^{-4} years. The other instruments are approximated: inflation-indexed bonds by the schedule at a zero real yield, discount bonds by the remaining maturity divided by one plus half the yield, and the fifteen-year floating-rate notes, whose semiannual coupon is linked to the ten-year JGB auction yield (we abstract from the contractual adjustment to that reference rate), by 0.25 years, the average time to the next reset, which treats them as insensitive to a parallel shift beyond that horizon. Computed on fixed-coupon nominal bonds alone, the holders' portfolio durations change by at most 0.4 years in any month (at the end of 2021, 11.4 and 6.5 years), and the share of the outside portfolio's duration beyond twelve years by at most one percentage point.

Holdings in excess of amounts outstanding. In seven issue-months, at the ends of October 2022 and January and February 2023, the Bank's recorded holdings of four ten-year issues near the then benchmark exceed the reconstructed amount outstanding, by 2.9 trillion yen of face value in total. The Bank's securities lending facility, under which it sells bonds with an agreement to repurchase them, could account for such an excess if the lent bonds remain in its recorded holdings; the published data do not allow us to verify this issue by issue. The outside holding is set to zero in those issue-months. The reassignments in Table 4 start from the market value of the Bank's recorded holdings plus the outside holdings, issue by issue, and divide that common base between the two holders. In these seven issue-months the common base is the Bank's recorded holding, which exceeds the reconstructed amount outstanding. Capping the Bank's holding at the reconstructed amount outstanding in the seven issue-months, or dropping them, changes the Bank's share of the total net loss by at most 0.02 percentage points and each of the three reassigned amounts by at most 0.24 trillion yen.

Reassignment by maturity bucket. In the second structure of Table 4, the Bank's target in each bucket and month is its actual aggregate market-value share times the bucket's market value. The target is first allocated across the issues in the bucket in proportion to the Bank's actual holdings, with each issue capped at its market value; any shortfall is then allocated to the uncapped issues in proportion to their market value. The bucket targets are met exactly, with a largest monthly residual below 10^{-9} trillion yen. The cap binds in every month in the two buckets beyond twelve years, where issues held in full account on average for 45 per cent of the Bank's target, and in no month in the shorter buckets.

The holding structure of the end of 2012. A third structure keeps the outside holders' share of every maturity bucket at its value at the end of 2012, before the large-scale purchases began. The outside portfolio would then have incurred 172.9 trillion yen of

the net loss from 2022 and the Bank 8.3 trillion, with average modified durations of 8.6 and 4.3 years. Compared with this structure, the actual holdings left 48 trillion yen less of the loss outside the Bank, and holdings of the actual size spread evenly across issues would have left 83 trillion yen less. The end-2012 structure changes the size of the Bank's holdings as well as their allocation, so it compares the actual allocation with the historical one rather than isolating the effect of the allocation.

Table A1: Construction choices and their effect on the results

Choice	Alternative examined	Effect
Allocation of pre-April-2012 buyback totals across eligible issues in proportion to holdings outside the Bank (205 operations, 18.8 trillion yen)	Allocating each total entirely to the oldest or entirely to the newest eligible issue, or in proportion to total amounts outstanding including Bank holdings	Monthly real return changes by at most 1.8 bp; correlation with benchmark above 0.9999
Bank of Japan holdings taken from the publication nearest to the month-end (41 months in 2002–2005 dated 4–15 days after)	Only publications on or before the month-end	At most 0.64 bp per month; risk premia change by less than 0.01 bp
Only the Bank of Japan netted out of the outside portfolio	General-government holdings also netted, allocated across issues from the Flow of Funds	Mean absolute change 0.87 bp per month; correlation 0.9998; full-sample premium −33 against −32 bp
Remuneration of current-account balances proxied by the overnight call rate	Three-tier rates applied to published tier balances (2016–2024); special remuneration schemes from fiscal years' actual amounts	At most 0.39 bp per month; premia change by less than 0.03 bp. Lending-support scheme: less than 0.01 bp; regional scheme: within ±0.1 bp depending on its allocation within the fiscal year
Twenty-year bonds nos. 2 and 3 (1986–87 issues, matured 2007) excluded because their amounts outstanding cannot be reconstructed	Included with an assumed 500 billion yen outstanding each, net of Bank holdings	At most 0.21 per cent of the outside portfolio's market value; monthly real return changes by at most 0.15 bp
Industry total returns: official dividend-inclusive returns from 2016; price change plus one twelfth of the dividend yield before 2016	Price change plus one twelfth of the dividend yield also from 2016	Using the approximation also from 2016 changes the post-2022 change in the premium from +59 to +62 bp for JGBs only and from +51 to +53 bp for the consolidated liability
Bill return: equal-weighted return of the bills quoted by the JSDA	One-month bill yield at the previous month-end, divided by twelve	Premia change by at most 0.1 bp
Japanese CPI: 2020-base linked series	2025-base linked series	Premia −29, +22, +51 bp (consolidated; main −30, +22, +51); long buckets' share of 2013–21 covariance 81 per cent (main 79); SDF correlation 0.999

Notes: "bp" is basis points. Effects on the risk premia hold the estimated SDF fixed except in the industry-return and CPI-base rows, where the SDF is re-estimated. The industry-return row applies the approximation also after 2016; it does not test the approximation used before 2016.

Sensitivities. Table A1 lists the construction choices that involve an assumption and the effect of the examined alternatives on the monthly real return of the outside portfolio and on the risk premia. The examined alternatives preserve the signs of the main pre- and post-2022 premia and the direction of their changes. The largest effect on these, from the

industry returns, is 2.0 basis points on the post-2022 change for the consolidated liability and 3.1 for JGBs alone; the base year of the CPI moves them by at most 1.3 basis points, and the other construction choices by less than one basis point.

B Estimation details and parameter estimates

Comparison with the US estimates. Before applying the procedure of Section 3.2 to Japan, we re-estimate the US SDF with the same test assets, factors and instruments as Pal Mustafi and Preston (2026) over July 1926 to December 2024 ($T = 1182$). Table B1 compares the two sets of estimates. The distribution of the parametric SDF (mean 0.995, standard deviation 0.116 against 0.111), the distribution of the nonparametric SDF (standard deviation 0.396 against 0.392), the correlation between the two (0.34 against 0.34), the factor loadings ($\delta_{mkt} = -2.08$ against -1.99) and the implied equity premium (7.2 against 6.8 per cent per year) are all close to the published values. The instrument coefficients γ are not: all our instrument coefficients except γ_{treas} differ from the published ones by more than their standard errors. The differences may reflect the construction of the instruments (the short rate used in the term spread, the source of the long yield, and units). The procedure therefore matches the reported moments of the SDF and the factor loadings rather than the full parameter vector, and we read the Japanese γ estimates with the same caution.

Estimates for Japan. Table B2 reports the parameter estimates for the benchmark specification and three alternatives: one-step GMM, equities only (dropping the one-month bill from the test assets), and the 25 size-and-book-to-market portfolios with the market and the bill ("FF25"), for which the 174 moments are fewer than T and the unshrunk long-run covariance matrix is positive definite, though poorly conditioned (the condition numbers of the correlation matrices of the unshrunk specifications are of order 10^7 to 10^8), so the optimal weighting matrix can be used without shrinkage. The benchmark SDF prices the market with a loading of $\delta_{mkt} = -3.65$ ($t = -10.9$); the HML loading is also precisely estimated ($\delta_{hml} = -2.51$, $t = -6.4$). Among the instruments, only the earnings yield has a t-statistic above two. The one-step estimates have very large standard errors, as expected with 372 moments weighted equally; the equities-only SDF is much less volatile; the FF25 SDF is more volatile and takes negative values in some months. These t-statistics, like the standard errors in the table, treat the first-step estimates as given.

Table B1: Comparison with the US estimates of Pal Mustafi and Preston (2026)

	Parametric SDF		Nonparametric SDF	
	This paper	PMP (2026)	This paper	PMP (2026)
Mean	0.995	0.995	1.000	0.999
Standard deviation	0.116	0.111	0.396	0.392
Skewness	−0.31	−0.41	0.55	0.52
AR(1)	0.09	0.09	−0.02	−0.01
Correlation between the two SDFs	0.34 / 0.34			
δ_{mkt}	−2.08 (0.16)	−1.99		
δ_{smb}	0.38 (0.29)	0.50		
δ_{hml}	−0.77 (0.26)	−0.85		
γ_{treas}	0.57 (0.05)	0.61		
γ_{infl}	−2.30 (0.12)	−1.24		
γ_{ts}	−1.03 (0.32)	−0.04		
γ_{cape}	0.32 (0.02)	0.09		
γ_{bs}	5.11 (0.41)	1.32		

Notes: Monthly data, July 1926 to December 2024. "PMP (2026)" reproduces the values reported in Pal Mustafi and Preston (2026), their Table 1 and Appendix Table A1. Standard errors of the parameter estimates in parentheses. The correlation line reports the correlation between the parametric and nonparametric SDF in this paper and in PMP (2026). Subscripts of γ: *treas*, real bill rate; *infl*, trailing inflation; *ts*, term spread; *cape*, inverse cyclically adjusted price–earnings ratio; *bs*, credit spread. Subscripts of δ: market, size and value factors.

Specifications with three instruments. The specifications with three instruments of Appendix D.1 are estimated with the same procedure on the 25 portfolios, the market and the overnight call rate (its monthly average, as a gross return), with the real call rate, trailing inflation and the ten-year yield minus the call rate as instruments; Table B3 reports their estimates, including the unshrunk optimal-weighting estimate on the long sample that Appendix D.1 omits. Its diagnostics are in the last column. The GMM objectives in the table are computed with each column's own weighting matrix and are not comparable across columns, nor are they standard J tests when the weighting matrix is shrunk. We therefore also report a criterion that does not depend on the weighting matrix, the root-mean-square Euler-equation error $\mathbb{E}[MR_i] - 1$ over the 27 test assets, annualised (multiplied by 12×10^4) and expressed in basis points. It is a moment error, not an error in expected returns. On that criterion the unshrunk estimate prices the test assets much worse than the shrunk and one-step estimates (401 against 125 basis points for the shrunk estimate over its own sample, 580 against 354 over 2002–2026), its market loading is imprecisely estimated ($\delta_{mkt} = -0.94$, $t = -1.0$), and its implied equity

premium over 2002–2026 is only 2.1 per cent per year, against 8.9 for the benchmark. These features are consistent with an imprecisely estimated weighting matrix, with 116 moments estimated from 427 observations, but they may also reflect moment conditions that the data do not satisfy over the long sample; we report the estimate in full and do not use it in the comparisons.

Table B2: Parametric SDF for Japan: parameter estimates

	Benchmark	One-step	Equities only	FF25
γ_{treas}	−0.38 (0.36)	−3.21 (27.93)	−0.10 (0.86)	−0.71 (0.09)
γ_{infl}	−0.28 (0.44)	−1.74 (115.41)	2.04 (2.90)	−0.81 (0.38)
γ_{ts}	−0.38 (1.22)	−0.57 (203.40)	0.30 (7.53)	−5.92 (1.23)
γ_{ep}	−0.10 (0.05)	−0.18 (3.02)	0.05 (0.17)	−0.26 (0.05)
γ_{cs}	−5.29 (5.46)	−9.17 (559.62)	−20.53 (13.30)	2.88 (3.23)
δ_{mkt}	−3.65 (0.33)	−3.32 (2.22)	−1.65 (0.60)	−2.99 (0.62)
δ_{smb}	−1.49 (0.85)	−2.44 (2.52)	−1.89 (1.03)	−5.80 (1.14)
δ_{hml}	−2.51 (0.39)	−2.97 (2.45)	−1.73 (0.65)	−7.58 (0.49)
SD of M	0.177	0.170	0.089	0.252
Test assets	60	60	59	27
Moments	372	372	366	174
Shrinkage a	0.093	–	0.090	–
Equity premium (ann., %)	8.9	7.9	3.8	6.1

Notes: Monthly data, September 2002 to September 2026 ($T = 289$). Standard errors in parentheses, computed from the second-step GMM objective treating the first-step estimates as given. Subscripts of γ: *treas*, real bill rate; *infl*, trailing inflation; *ts*, term spread; *ep*, earnings yield; *cs*, credit spread. Subscripts of δ: market, size and value factors. The benchmark and one-step specifications use 60 test assets (25 size-and-book-to-market portfolios, 33 industry portfolios, the market and the one-month bill); "equities only" drops the bill; "FF25" uses the 25 portfolios, the market and the bill. "Shrinkage a" is the data-chosen intensity with which the correlation matrix of the second-step weighting matrix is shrunk towards the identity; "–" denotes no shrinkage. The equity premium is the annualised premium of the market return implied by the estimated SDF, $-12\,\mathrm{Cov}(M, R_M)/\mathbb{E}[M]$, in per cent.

The shrunk long-sample SDF loads more on the value factor and less on the market than the benchmark (δ_{hml} and δ_{mkt} in Tables B2 and B3); compared with the same specification estimated on 2002–2026, it loads much less on the market and on size, so that the value factor accounts for more of its variation. In the second half of 2023 it therefore reads months in which value stocks rallied and JGBs fell as good states, and November 2023, when the reverse happened, as a bad one, whereas the benchmark SDF moves much less in those months.

Table B3: Parametric SDF with three instruments: parameter estimates by estimation sample and weighting

	2002–26, shrunk	1991–2026, shrunk	1991–2026, one-step	1991–2026, optimal
γ_0	0.9993 (0.0002)	1.0000 (0.0002)	1.0002 (0.0089)	1.0000 (0.0001)
γ_{call}	−0.064 (0.078)	−0.000 (0.062)	0.439 (4.936)	0.149 (0.057)
γ_{infl}	−0.619 (0.164)	−0.197 (0.183)	0.754 (454.449)	0.115 (0.145)
γ_{ts}	0.646 (0.384)	−0.058 (0.385)	−0.803 (1867.969)	−0.590 (0.268)
δ_{mkt}	−3.795 (0.800)	−1.730 (0.755)	−1.818 (1.365)	−0.937 (0.918)
δ_{smb}	−4.797 (1.671)	−0.830 (0.643)	−0.623 (3.372)	0.850 (1.463)
δ_{hml}	−5.716 (1.158)	−4.364 (0.821)	−5.061 (1.297)	−6.519 (0.922)
T	289	427	427	427
Moments	116	116	116	116
Shrinkage a	0.054	0.036	–	–
GMM objective $T\bar{g}'W\bar{g}$	15.5	30.0	0.011	570.0
RMS Euler error, own sample (bp)	189	125	118	401
RMS Euler error, 2002–26 (bp)	189	354	396	580
SD of M	0.232	0.152	0.172	0.207
Corr. with benchmark	0.90	0.79	0.75	0.50
Equity premium (%)	8.5	4.0	4.2	2.1

Notes: Test assets: the 25 size-and-book-to-market portfolios, the market and the overnight call rate (27 real gross returns). Instruments: the real call rate (*call*), trailing twelve-month inflation (*infl*) and the ten-year yield minus the call rate (*ts*); 116 moments. Estimation samples: September 2002 to September 2026 ($T = 289$) and March 1991 to September 2026 ($T = 427$). "Shrunk" applies the benchmark's data-chosen shrinkage of the second-step weighting matrix; "one-step" uses the identity; "optimal" uses the unshrunk inverse long-run covariance matrix. Standard errors in parentheses are conditional on the first step; those of the one-step estimate are very large because the instrument coefficients are weakly identified under equal weighting. "GMM objective" uses each column's own weighting matrix and is not comparable across columns. "RMS Euler error" is the root-mean-square of $\mathbb{E}[MR_i] - 1$ over the 27 test assets, multiplied by 12×10^4, over the column's own sample or over September 2002 to September 2026. "Corr. with benchmark" is the correlation of the SDF with the benchmark SDF over September 2002 to September 2026; the equity premium is the annualised premium of the market return implied by the SDF, $-12\,\mathrm{Cov}(M, R_M)/\mathbb{E}[M]$, over the same months.

Optimisation. The zero-beta portfolio w_θ uses the covariance matrix of the residuals from regressions of the test-asset returns on the factors, shrunk with the nonlinear estimator of Ledoit and Wolf (2020). The GMM objective is minimised over $\theta = (\gamma, \delta)$ with the instrument coefficients γ expressed as coefficients on standardised instruments, so that all parameters have comparable scale; the estimates are reported in the original units. At each step, three starting values for δ (zero, the US estimates, and a market-only loading) are run under the same weighting matrix, and the second step adds the first-step solution as a fourth starting value; each run alternates a least-squares solver (Levenberg–Marquardt, applied to the square-root-weighted moment vector) with a Nelder–Mead search until the objective stops falling, and the run with the smallest objective is retained. For each of

the benchmark, equities-only and FF25 specifications, the four second-step runs reach the same objective to a relative precision of 10^{-11}.

Shrinkage of the weighting matrix. Let $g_t(\theta_1)$ be the $m = 372$ moments at the first-step estimate and Ω their Newey–West long-run covariance matrix (Bartlett kernel, six lags). With $\Lambda = \mathrm{diag}(\Omega)$ and $P = \Lambda^{-1/2}\Omega\Lambda^{-1/2}$, the second step uses $\Omega_a = \Lambda^{1/2}[(1-a)P+aI]\Lambda^{1/2}$ and $W = \Omega_a^{-1}$, computed through the Cholesky factor of $(1-a)P + aI$. The intensity a is the Ledoit and Wolf (2004) estimate for shrinking the contemporaneous sample covariance matrix of the standardised moments, $z_t = (g_t - \bar{g})/\hat{\sigma}$, towards a scaled identity, truncated to $[0, 1]$. That estimate is optimal for the sample covariance matrix of independent observations, not for a long-run covariance matrix, so we use it as a regularisation rule; Appendix D.1 reports the results for fixed values of a from 0.05 to 1.

Why two steps. Pal Mustafi and Preston (2026) describe their estimator as iterated to convergence. On the US data, iterating the weighting matrix does not converge: after ten iterations the parameters still change at the first decimal, and the factor loadings move away from the published values (δ_{mkt} from -2.08 to about -3.1, with the standard deviation of the SDF rising to about 0.16). The two-step estimates reproduce the published loadings and the SDF moments (Table B1). We therefore use two steps for both countries.

Nonparametric SDF. The nonparametric SDF solves the quadratic programme of Hansen and Jagannathan (1991) with a non-negativity constraint: minimise the sample variance of M subject to $T^{-1}\sum_t M_{t+1}R_{i,t+1} = 1$ for every test asset and $M_{t+1} \geq \varepsilon$, with $\varepsilon = 10^{-8}$. For Japan the constraint binds in several months.

Conversion of the US SDF. The US parametric and nonparametric SDFs are re-estimated over July 1926 to September 2026 from the same US test assets and converted into yen month by month with (4), using the month-end yen–dollar rate and Japanese and US CPI inflation; the US CPI for October 2025, which was not published, is interpolated log-linearly. The extended-sample estimates differ little from those in Table B1.

C Additional results

C.1 Bootstrap distributions

Table C1 reports the bootstrap distribution of the change in the risk premium for six- and twelve-month blocks and for the two treatments of the first GMM step. For each block length the resampled months are drawn once and used for both liabilities and both treatments of the first step; the twelve-month rows with re-estimation of the first

step use the first 499 of those samples. The bootstrap of the long-sample SDF reported in Appendix D.1 also uses circular blocks of six months, but draws them within three segments, March 1991 to August 2002, September 2002 to December 2021 and January 2022 to September 2026, so that the length of each segment is preserved, and $B = 499$ samples. In every sample the second GMM step is re-estimated, including the data-chosen shrinkage, and so is the first unless it is held at its full-sample estimate (the "fixed" rows of Table C1); the estimates start from the full-sample values, with each least-squares step run to a tolerance of 10^{-12} or at most 5,000 function evaluations. The evaluation limit binds in at most 2 of the samples of any of the single-country bootstraps of the Japanese SDF; dropping those samples moves no end point of a 90 or 95 per cent interval by more than 1.5 basis points for the premia, or by more than 0.004 for the contributions of the long end. The risk premia are computed on the last two segments of each sample, and the intervals are percentile intervals.

Table C1: Bootstrap distribution of the change in the risk premium

Liability	Block	First stage	B	SE	90% interval	95% interval	Share ≤ 0	p
Consolidated	6	re-estimated	999	44.1	[−16.2, 127.0]	[−27.7, 153.6]	0.119	0.21
Consolidated	6	fixed	999	35.0	[4.4, 115.9]	[0.2, 133.7]	0.024	0.10
Consolidated	12	re-estimated	499	39.4	[−11.4, 116.9]	[−22.3, 143.9]	0.086	0.17
Consolidated	12	fixed	999	32.9	[5.8, 108.2]	[1.1, 127.4]	0.023	0.09
JGBs only	6	re-estimated	999	63.4	[−35.8, 171.4]	[−57.8, 202.2]	0.162	0.30
JGBs only	6	fixed	999	49.2	[−6.3, 148.9]	[−14.4, 174.1]	0.072	0.18
JGBs only	12	re-estimated	499	56.3	[−31.5, 151.8]	[−48.9, 191.1]	0.140	0.26
JGBs only	12	fixed	999	46.0	[−3.4, 143.4]	[−13.2, 170.2]	0.069	0.17

Notes: Change in the annualised risk premium (basis points) between January 2022 to September 2026 and September 2002 to December 2021, for the benchmark Japanese parametric SDF. Each bootstrap sample draws circular blocks of six or twelve months ("Block") separately within the two sub-periods and re-estimates the SDF: "re-estimated" repeats the first-step optimisation, the residual-covariance shrinkage, the long-run covariance matrix and its shrinkage, and the second step; "fixed" keeps the first-step estimates of the full sample and repeats the rest. "Share ≤ 0" is the share of bootstrap samples with a non-positive change; p, a two-sided bootstrap p-value for a zero change, is the share of samples whose deviation from the bootstrap mean is at least as large in absolute value as the estimated change.

Recentred moment conditions. With more moment conditions than parameters, the sample moments are not zero at the full-sample estimate, and in the samples above the SDF is re-estimated on the same moment conditions as in the data. Following Hall and Horowitz (1996), we also examine a version of the bootstrap that minimises the GMM objective using the moments of each sample minus their full-sample mean at the full-sample estimate. The instruments, factor means and zero-beta portfolio are recomputed in each sample, as in the version without recentring. We use the comparison to assess the sensitivity of the intervals to recentring, not as a bias correction. Table C2 reports

both versions with the same resampled months, $B = 999$ samples and percentile intervals, for the benchmark premia and for the difference between the Japanese and US changes (Appendix D.4). For Japan the two versions agree: the interval for the premium before 2022 lies below zero, and the interval for the change includes zero. For the difference between the Japanese and US changes they do not: its interval includes zero without recentring and excludes it with recentring.

Table C2: Bootstrap intervals with and without recentred moment conditions

	Estimate	90 per cent interval	
		Moments as estimated	Moments recentred
Japan, to 2021	−30	[−50, −1]	[−53, −10]
Japan, from 2022	+22	[−27, 88]	[−28, 79]
Japan, change	+51	[−16, 127]	[−6, 116]
Japan minus United States, change[a]	−136	[−253, 43]	[−290, −14]

Notes: Annualised risk premia of consolidated liabilities (basis points) under the benchmark Japanese parametric SDF, over September 2002 to December 2021 ("to 2021") and January 2022 to September 2026 ("from 2022"), and their change. Six-month circular blocks, with the first GMM step re-estimated (the first row of Table C1). "Moments as estimated" re-estimates the SDF in each sample on the moment conditions of the data; "Moments recentred" subtracts their full-sample value at the full-sample estimate. Both use the same resampled months and percentile intervals.

[a] Change in the Japanese premium minus change in the US premium, from September 2002 to December 2021 to the common period January 2022 to June 2025, from the joint bootstrap of Appendix D.4, which re-estimates both SDFs in each sample.

C.2 Pricing wedge

Pal Mustafi and Preston (2026) check that the SDF estimated from equities also prices government debt, in the sense that $\mathbb{E}[MR^g] = 1$ is not rejected over the full sample. For Japan the full-sample wedge $\mathbb{E}[MR^g] - 1$ for the consolidated liability is +29 basis points per year ($t = 0.02$); relative to the one-month bill, which is one of the test assets, $\mathbb{E}[M(R^g - R^f)]$ is +58 basis points ($t = 1.6$; +39, $t = 0.8$ for JGBs alone). We do not reject the full-sample pricing restriction under this SDF. The raw wedge is estimated very imprecisely, with a standard error of about 1,460 basis points per year, and the relative wedge with a standard error of about 36, so no average convenience yield or market segmentation is detectable with this test. Table C3 reports the relative wedge by sub-period. Within sub-periods the wedge is dominated by realised excess returns, which it tracks almost one for one, so a large sub-period wedge (such as −218 basis points from 2022, or +252 in 2013–2016) mostly reflects realised returns and does not by itself

identify an ex ante mispricing. We therefore do not use sub-period wedges to identify a convenience yield or its change.

Table C3: Pricing wedge relative to the one-month bill

SDF	Liability	Period	Relative wedge (bp)	t	Realised excess return (bp)
Japanese parametric	Consolidated	Full sample	+58	1.61	+32
		2002:09–2012:12	+129	3.31	+95
		2013:01–2016:08	+252	2.46	+223
		2016:09–2021:12	+33	0.73	+11
		2022:01–2026:09	−218	−3.64	−226
	JGBs only	Full sample	+39	0.76	+7
		2002:09–2012:12	+134	3.27	+97
		2013:01–2016:08	+321	2.26	+278
		2016:09–2021:12	+50	0.69	+15
		2022:01–2026:09	−395	−3.75	−410
US parametric, in yen	Consolidated	Full sample	+36	1.04	+32
		2002:09–2012:12	+103	2.48	+95
		2013:01–2016:08	+230	2.77	+223
		2016:09–2021:12	+16	0.37	+11
		2022:01–2026:09	−236	−4.20	−226
	JGBs only	Full sample	+10	0.20	+7
		2002:09–2012:12	+105	2.43	+97
		2013:01–2016:08	+289	2.49	+278
		2016:09–2021:12	+23	0.33	+15
		2022:01–2026:09	−428	−4.30	−410

Notes: The relative wedge is $12\,\mathbb{E}[M_{t+1}(R^g_{t+1} - R^f_{t+1})]$ over the period, in basis points per year, with t-statistics based on Newey–West (six lags) standard errors that treat the SDF as given. The last column is the annualised mean realised return on the liability in excess of the one-month bill. The sub-periods break at the start of 2013, the year in which quantitative and qualitative easing began, of yield-curve control (September 2016) and of the post-pandemic inflation (2022).

C.3 Covariances by remaining maturity

Table C4 reports period averages of the 36-month rolling covariance between the benchmark SDF and the real return on the coupon JGBs held outside the Bank in five buckets of remaining maturity, which panel A of Figure C1 plots; panel B plots the Bank's share of each bucket. After 2022 the bucket covariances are all close to zero, between −0.4 and +1.3.

Table C5 adds up the buckets with their weights, in levels, for August 2005 to December 2012, 2013–2021 and 2022–2026; Table 2 in the text reports the shares, together with the weights, volatilities and correlations, for the period from September 2002 to December 2021 and for 2013–2021, so only the 2013–2021 columns cover the same months. Let $w_{b,t}$ be the share of bucket b in the market value of coupon JGBs held outside the Bank at the end of month $t-1$ and $r_{b,t}$ the bucket's real return; the outside coupon portfolio's return is

$\sum_b w_{b,t} r_{b,t}$, so that $\mathrm{Cov}(M, \sum_b w_{b,t} r_{b,t}) = \sum_b \mathrm{Cov}(M, w_{b,t} r_{b,t})$ exactly within each period. The table reports each bucket's term and its average weight.

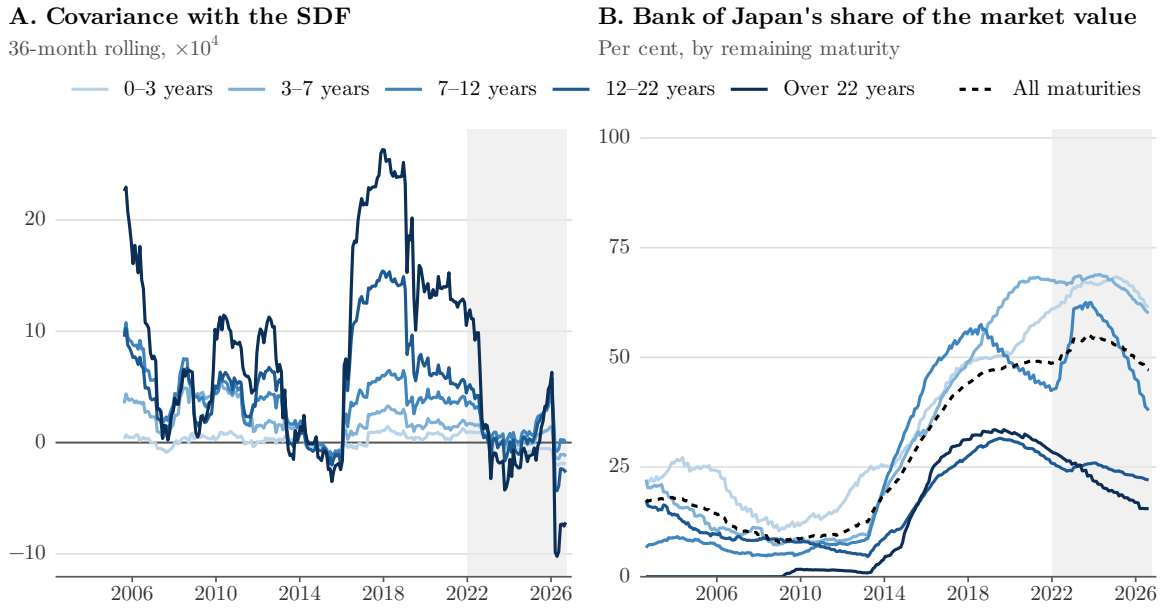


Figure C1: Covariance with the SDF and the Bank of Japan's share, by remaining maturity

Notes: Panel A: 36-month rolling covariance ($\times 10^4$) between the benchmark SDF and the real holding-period return on the coupon JGBs held outside the Bank in five buckets of remaining maturity; these are covariances of the bucket returns, not the weighted contributions of Table 2. Panel B: the Bank of Japan's share of the market value of the coupon JGBs in each bucket, and in all buckets together (dashed). The grey background marks the period from January 2022.

Table C4: Rolling covariances by remaining maturity, coupon JGBs held outside the Bank

Period	0–3 years	3–7 years	7–12 years	12–22 years	Over 22 years
2005:08–2012:12	+0.25	+3.10	+4.93	+4.70	+8.00
2013:01–2021:12	+0.28	+1.33	+3.10	+6.35	+11.48
2022:01–2026:09	−0.38	+0.29	+1.34	+0.68	+0.09

Notes: Period averages of the 36-month rolling covariance ($\times 10^4$) between the benchmark Japanese parametric SDF and the market-value-weighted real holding-period return on coupon JGBs held outside the Bank with the stated remaining maturity at the end of the previous month.

Table C5: Contribution of each maturity bucket to the covariance of the coupon JGB portfolio held outside the Bank

	Contribution to the covariance			Average weight (%)		
Remaining maturity	2005–12	2013–21	2022–26	2005–12	2013–21	2022–26
0–3 years	+0.14	+0.09	−0.32	31	26	20
3–7 years	+0.89	+0.28	−0.18	29	22	17
7–12 years	+0.98	+0.41	−0.03	22	16	19
12–22 years	+0.70	+1.41	−0.43	14	24	27
Over 22 years	+0.28	+1.48	−0.80	4	12	16
Total	+2.99	+3.67	−1.77	100	100	100

Notes: $\mathrm{Cov}(M, w_{b,t} r_{b,t})$ ($\times 10^4$) between the benchmark Japanese parametric SDF and the weighted net real holding-period return $w_{b,t} r_{b,t}$ of each bucket, where $w_{b,t}$ is the bucket's share of the market value of coupon JGBs held outside the Bank at the end of the previous month; the rows sum to the covariance of the SDF with the outside coupon portfolio's real return. "Average weight" is the period mean of $w_{b,t}$ in per cent. Periods: August 2005 to December 2012, January 2013 to December 2021, January 2022 to September 2026; the first starts in August 2005 to match the rolling covariances of Table C4 and differs from the period 2002–2021 used in Table 2.

Table C6 repeats the calculation for the 16 SDFs and specifications listed in its notes and reports the share of the covariance contributed by bonds beyond twelve years. Two of them are estimated on the months to December 2021 only, so that the location of the pre-2022 insurance value does not draw on asset prices from 2022 on; over those months their correlations with the benchmark SDF are 0.83 and 0.72. The SDFs differ in the size of the covariance, but the share of the long end lies between 42 and 66 per cent over 2002–2021 and between 66 and 80 per cent over 2013–2021. With the shrinkage intensity fixed at values from 0.05 to 1, it is 79 per cent over 2013–2021 and 50 to 53 per cent over 2002–2021.

The cutoff of the long-maturity segment. Beyond ten years of remaining maturity the outside portfolio contains only bonds issued at fifteen years or more: twenty-, thirty- and forty-year bonds and, until they matured, the fifteen-year floating-rate notes. Table C7 moves the cutoff of the long segment from twelve years to ten, fifteen and twenty years. Over 2013–2021 the segment beyond ten years is 40 per cent of the market value of the outside coupon portfolio, 75 per cent of its duration exposure and 83 per cent of its covariance with the SDF; beyond fifteen years the figures are 28, 59 and 69 per cent, and beyond twenty years 14, 36 and 45 per cent. At every cutoff the segment's share of the covariance exceeds its share of the market value by a wide margin and is close to or above its share of the duration exposure, and its share of the outside price loss from 2022 is 81, 77, 67 and 43 per cent.

Table C6: Share of the covariance contributed by bonds beyond twelve years, by SDF

	2002:09–2021:12		2013:01–2021:12	
SDF	Covariance	>12 years (%)	Covariance	>12 years (%)
Japanese parametric (benchmark)	+3.63	53	+3.67	79
One-step GMM	+3.29	53	+3.42	77
FF25 (optimal W)	+3.50	62	+4.72	76
Equities only	+1.53	55	+1.67	77
Japanese nonparametric	+4.22	50	+3.89	74
US parametric, in yen	+1.28	49	+1.06	80
US nonparametric, in yen	+2.85	66	+4.59	69
Three instruments, 1991–2026	+2.58	63	+3.40	79
Three instruments, 1991–2026, one-step	+2.89	64	+3.89	80
Three instruments, 2002–2026	+3.93	57	+4.63	77
Benchmark specification, 2002–2021	+1.34	42	+0.93	73
Three instruments, 2002–2021	+1.18	45	+1.03	66
Shrinkage $a = 0.05$	+3.73	53	+3.78	79
Shrinkage $a = 0.3$	+3.32	52	+3.26	79
Shrinkage $a = 0.5$	+3.08	52	+2.92	79
Diagonal weighting ($a = 1$)	+3.13	50	+2.74	79

Notes: "Covariance" is $\sum_b \mathrm{Cov}(M, w_{b,t} r_{b,t})$ ($\times 10^4$) between the SDF and the real return on the coupon JGB portfolio held outside the Bank over the period; ">12 years" is the share of it contributed by the two buckets beyond twelve years, in per cent. The first seven SDFs are those of Table 1 and Appendix Table B2; the next three are the specifications with three instruments of Appendix D.1; the next two are estimated on September 2002 to December 2021 only, with the benchmark specification and with the three instruments; the last four are the benchmark specification with the shrinkage intensity a fixed at the stated value (Appendix D.1).

Inference on the contribution of the long end. The shares in Tables 2 and C6 are ratios of sample covariances, and the denominator, the covariance of the whole portfolio, is imprecisely estimated (Section 4.2). We therefore bootstrap the numerator in levels, in a bootstrap of the benchmark specification with six-month blocks that differs from the one of Table C1 only in drawing the blocks separately within three segments, September 2002 to December 2012, January 2013 to December 2021 and January 2022 to September 2026, so that every sample contains exactly the 108 months of 2013–2021 ($B = 999$). On each sample, with the SDF re-estimated, we compute the covariance of the SDF with the weighted returns of the two buckets beyond twelve years, $C_{>12} = \mathrm{Cov}(M, \sum_{b>12} w_{b,t} r_{b,t})$, and its excess over the contribution that their average weight $\bar{w}_{>12}$ would imply if every bucket contributed in proportion to its weight, $C_{>12} - \bar{w}_{>12} C$, where C is the covariance of the whole portfolio, both over the months of the sample drawn from the period. Over 2013–2021 $C_{>12}$ is 2.89 ($\times 10^4$) with a 90 per cent interval of $[0.06, 5.43]$ and a 95 per cent interval of $[-0.22, 6.21]$, non-positive in 4.6 per cent of the samples, and the excess is 1.59

with intervals of $[0.01, 2.92]$ and $[-0.17, 3.41]$, non-positive in 4.8 per cent. Over the period to 2021 the figures are 1.93 with intervals of $[0.17, 3.35]$ and $[-0.05, 3.67]$, non-positive in 3.2 per cent, and 1.01 with $[0.04, 1.87]$ and $[-0.05, 2.14]$, non-positive in 3.9 per cent. The median share of the long end across the samples is 79 per cent over 2013–2021, with a 90 per cent interval of $[57, 112]$. With the weights fixed at their period averages, which removes the co-movement of the weights with the SDF, the long end's share is 76 per cent over 2013–2021 and 54 per cent over the period to 2021, against 79 and 53 per cent with time-varying weights.

Table C7: The long-maturity segment under alternative cutoffs

	2002:09–2021:12			2013:01–2021:12			2022:01–2026:09
Remaining maturity	Weight	Dur. share	Contrib.	Weight	Dur. share	Contrib.	Price loss
Over 10 years	29	61	56	40	75	83	81
Over 12 years	26	57	53	36	70	79	77
Over 15 years	19	49	46	28	59	69	67
Over 20 years	8	26	26	14	36	45	43

Notes: Per cent. Coupon JGBs held outside the Bank with more than the stated remaining maturity at the end of the previous month. "Weight" is the period mean of the segment's share of the outside coupon portfolio's market value; "Dur. share" its duration share, defined as in Table 2; "Contrib." its share of the covariance between the benchmark SDF and the portfolio's real return, with time-varying weights, as in Table 2; "Price loss" its share of the outside holders' cumulative price loss from January 2022 to September 2026. The twelve-year row reproduces the "Over 12 years" row of Table 2.

D The post-2022 change in detail

This appendix reports the evidence summarised in Section 6: the sensitivity of the estimates to the specification and sample of the SDF, the decomposition of the change into nominal returns and inflation and its timing, rolling covariances, the comparison with the United States and the fiscal scale of the change.

D.1 Sensitivity to the SDF specification and sample

The benchmark SDF involves three choices specific to this application: the shrinkage of the second-step weighting matrix, the composition of the cross-section, and the estimation sample, which starts in 2002 because the instruments and the liability returns do. Table D1 varies all three. The first nine rows keep the 2002–2026 sample and the five instruments. Across them the pre-2022 premium on the consolidated liability is negative, between -30 and -12 basis points, and so is the premium on JGBs alone; the change from 2022 is

positive in all of them except the FF25 specification, between +17 and +63 basis points for the consolidated liability; with FF25 it is −2 for the consolidated liability and −48 for JGBs alone. Whether the post-2022 premium is itself positive depends on the cross-section: it is positive for every weighting matrix with the full set of 60 assets, +5 basis points with equities only and −29 with the FF25 cross-section.

Table D1: Sensitivity of the sub-period risk premia to the SDF specification and sample

	Consolidated			JGBs only		
Specification	To 2021	From 2022	Change	To 2021	From 2022	Change
Benchmark (a chosen by data)	−30 (8)	+22 (25)	+51 (26)	−37 (10)	+22 (36)	+59 (37)
Shrinkage $a = 0.05$	−30 (9)	+22 (26)	+52 (27)	−38 (11)	+22 (37)	+59 (38)
Shrinkage $a = 0.3$	−27 (8)	+24 (22)	+51 (23)	−33 (9)	+29 (32)	+62 (33)
Shrinkage $a = 0.5$	−25 (7)	+26 (20)	+52 (21)	−31 (8)	+34 (29)	+65 (30)
Diagonal weighting ($a = 1$)	−26 (7)	+37 (21)	+63 (22)	−31 (9)	+54 (30)	+85 (31)
One-step GMM ($W = I$)	−27 (8)	+13 (25)	+40 (26)	−33 (10)	+6 (36)	+40 (37)
Equities only	−12 (4)	+5 (12)	+17 (13)	−16 (5)	−1 (17)	+15 (18)
FF25 (optimal W)	−27 (12)	−29 (38)	−2 (39)	−36 (15)	−83 (54)	−48 (56)
CPI instruments lagged 1 month	−28 (8)	+23 (24)	+51 (25)	−35 (10)	+25 (35)	+60 (36)
FF25, market and call rate; three instruments; SDF estimation sample as shown						
SDF sample 2002–2026, shrunk W	−31 (11)	−3 (34)	+28 (36)	−40 (14)	−33 (49)	+7 (50)
SDF sample 1991–2026, shrunk W	−21 (7)	−18 (21)	+3 (22)	−26 (9)	−47 (31)	−20 (32)
SDF sample 1991–2026, one-step	−23 (8)	−23 (24)	0 (26)	−30 (10)	−58 (35)	−29 (37)

Notes: Annualised risk premium in basis points before 2022 (September 2002 to December 2021), from 2022 (January 2022 to September 2026) and the change, for the Japanese parametric SDF estimated under alternative choices. Parentheses: fixed-SDF Newey–West standard errors. Shrinkage a is the weight on the identity in the correlation matrix of the second-step weighting matrix; the benchmark chooses it from the data ($a = 0.093$). "Equities only" drops the one-month bill from the 60 test assets; "FF25" uses the 25 portfolios, the market and the bill (27 assets) and the unshrunk optimal weighting matrix, whose long-run covariance matrix is positive definite in this specification. "CPI instruments lagged 1 month" dates the two CPI-based instruments one month before the return month instead of two. The last three rows use the 25 portfolios, the market and the overnight call rate as test assets and three instruments (the real call rate, trailing inflation and the ten-year yield minus the call rate), estimated on the sample shown; the covariances are measured from September 2002 in every row.

The last three rows use a smaller set of assets and instruments that is available from 1991, and vary the estimation sample of the SDF. The industry portfolios, the bill and two of the five instruments do not exist before 2002, but the 25 size-and-book-to-market portfolios, the market, the call rate and three instruments (the real call rate, trailing inflation and the term spread over the call rate) are available from 1991, so the SDF can be estimated on $T = 427$ months; the covariances are still measured from September 2002.

The matched comparison is between the first two of these rows, which use the same assets and instruments and the benchmark's shrinkage rule (the data-chosen intensities are small) and differ only in the estimation sample; a comparison with the benchmark itself would change the assets and instruments as well. With the 2002–2026 sample this specification tracks the benchmark (correlation 0.90) and gives a change of +28 basis points. With the sample from 1991 it keeps a pre-2022 safety premium (−21 basis points) but not the benchmark's weakening: the change is +3 (22) basis points for the consolidated liability and −20 for JGBs alone, and one-step GMM gives 0 (26) and −29.

The two SDFs estimated on September 2002 to December 2021 only (Appendix C.3) give pre-2022 premia of −11 and −9 basis points for the consolidated liability and −13 and −12 for JGBs alone: smaller than the benchmark, but of the same sign.[29]

The long-sample SDFs differ from the benchmark particularly in a few months of 2023, in which they read the co-movement of value stocks and JGBs differently (Appendix B); by contrast, February and March 2026, when JGBs rallied and then fell, push the covariance down in every domestic parametric specification of Table D1, whose SDFs were below their mean in February and above it in March.

The benchmark and the other parametric specifications estimated on the 2002–2026 sample, except FF25, show the premium weakening from 2022, whereas FF25, the Japanese nonparametric SDF and the SDFs estimated from 1991 give a change close to zero or negative. Among the domestic parametric specifications, a positive post-2022 premium on the consolidated liability appears only with the broader cross-sections of 59 or 60 assets. The size of the change also depends on the months of 2026 (Appendix D.2).

D.2 Nominal return, inflation, and timing

Write the net nominal return on the liability as $r^n_{t+1} = \tilde{R}^g_{t+1} - 1$ and net CPI inflation over the month as $\pi_{t+1} = \Pi_{t+1} - 1$. The net real holding-period return $r^g_{t+1} = \tilde{R}^g_{t+1}/\Pi_{t+1} - 1$ then satisfies, exactly,

$$r^g_{t+1} = \frac{r^n_{t+1}}{1+\pi_{t+1}} - \frac{\pi_{t+1}}{1+\pi_{t+1}}, \tag{7}$$

so that the covariance of the SDF with the real return, which is the covariance in (1) up to a constant, splits into a nominal-return component and an inflation component,

$$\mathrm{Cov}(M, r^g) = \mathrm{Cov}\big(M, \tfrac{r^n}{1+\pi}\big) - \mathrm{Cov}\big(M, \tfrac{\pi}{1+\pi}\big)\,. \tag{8}$$

[29] Appendix Table B3 also reports the long-sample estimate with the unshrunk optimal weighting matrix. It is excluded from these comparisons because it has a correlation of 0.50 with the benchmark, implies an equity premium of only 2.1 per cent per year over 2002–2026, against 8.9 for the benchmark, and fits the test assets considerably worse on an unweighted criterion (Appendix B); its change in the premium is −26 basis points. A bootstrap of the long-sample SDF with the benchmark's weighting (Appendix C.1) gives a standard error of 34 basis points for the change, against 44 for the benchmark, and a 90 per cent interval of $[-48, 63]$, which includes zero; the narrower interval reflects the smaller cross-section and instrument set as well as the longer sample.

The inflation component splits further into the change in the Statistics Bureau's index of all items less fresh food and energy, π^x, and the remainder $\pi - \pi^x$; the remainder is a residual of the decomposition, not the published contribution of fresh food and energy. The nominal component $r^n/(1+\pi)$ splits, within each period, into the fitted value and the residual of a projection on an approximation to the monthly return on a ten-year US Treasury (notes to Table D2). All of this is accounting: it tells us which co-movements changed, not why.

Table D2: Decomposition of the covariance between the SDF and the real return on government liabilities

Liability	Period	Total	Nominal	−Core (π^x)	−Remainder ($\pi - \pi^x$)	Fitted on UST	Residual
Consolidated	2002:09–2021:12	+2.49	+2.45	−0.20	+0.24	+0.73	+1.72
Consolidated	2022:01–2026:09	−1.74	−0.22	−0.78	−0.74	−0.05	−0.17
JGBs only	2002:09–2021:12	+3.08	+3.03	−0.20	+0.24	+0.84	+2.19
JGBs only	2022:01–2026:09	−1.78	−0.27	−0.78	−0.74	−0.09	−0.18

Notes: Sample covariance ($\times 10^4$) between the benchmark Japanese parametric SDF and the real holding-period return on government liabilities, and its exact decomposition (8). "−Core (π^x)" is $-\operatorname{Cov}(M, \pi^x/(1+\pi))$ with π^x the change in the Statistics Bureau's "all items, less fresh food and energy" index (which excludes fresh food, not all food); "−Remainder ($\pi - \pi^x$)" is the corresponding term for the residual $\pi - \pi^x$, which is not the published contribution of fresh food and energy to all-items inflation. "Fitted on UST" is the covariance of the SDF with the fitted value of a within-period regression of the nominal-return component $r^n/(1+\pi)$ on an approximation to the monthly return on a ten-year US Treasury (UST) built from the constant-maturity yield with a duration of 8.5; "Residual" is the covariance with the residual. The two inflation columns sum to the inflation component, the last two columns to the nominal component.

Table D2 reports the decomposition for the benchmark SDF, for the two sub-periods, for both definitions of the liability. Panel A of Figure D1 shows the same decomposition for the consolidated liability with the post-2022 period split at the end of 2023. Before 2022, the positive covariance of the consolidated liability (+2.49) is almost entirely a nominal-return covariance (+2.45): the estimated hedge came from nominal returns that were high in states the SDF prices highly, as the returns on long-term nominal bonds are when bad states come with lower yields. The two inflation components offset each other over this period. From 2022 the covariance is −1.74, of which −0.22 comes from the nominal return and −1.52 from inflation. The change between the periods, −4.23, therefore has two parts of unequal size: the nominal-return covariance that made the liability a hedge disappeared (−2.67, or 63 per cent of the change), and the inflation component, which had been close to zero, turned negative (−1.56, or 37 per cent).

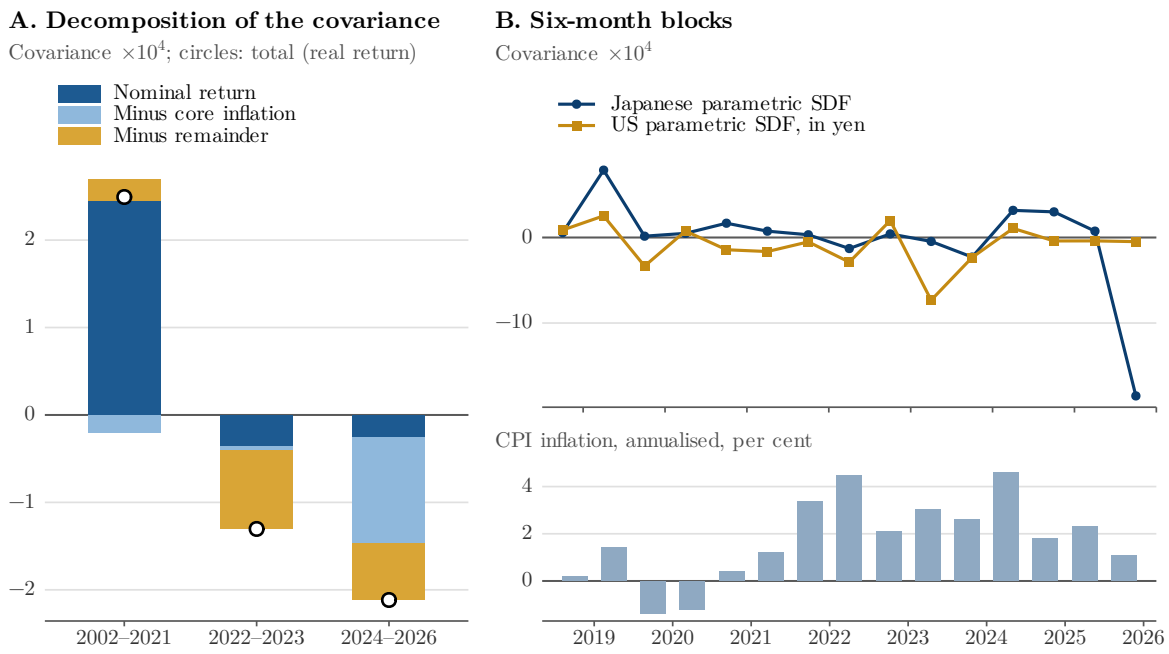


Figure D1: Decomposition and timing of the covariance, consolidated government liabilities

Notes: Panel A: exact decomposition (8) of the covariance ($\times 10^4$) between the benchmark SDF and the real return on consolidated government liabilities, for September 2002 to December 2021, January 2022 to December 2023, and January 2024 to September 2026; dots are the totals; the remainder $\pi - \pi^x$ is a residual, defined as in Table D2. Panel B: covariance in six-month blocks (January–June, July–December) for the benchmark SDF and the converted US SDF (upper part) and annualised CPI inflation in the block (lower part); July–September 2026 is not shown.

The level of the post-2022 covariance is a different matter: because the nominal component ends close to zero, 87 per cent of the negative covariance that remains is the co-movement of inflation with the SDF, inflation eroding the real value of the liability in months that the SDF prices highly. The converted US SDF reads the same period differently: its covariance with the consolidated liability, −1.36, is essentially a nominal-return covariance (−1.36). The share of inflation is thus a property of the Japanese SDF, not of the data alone. The inflation component is identical for the two definitions of the liability, because balances and JGBs are deflated by the same index; consolidation changes only the nominal component (−0.22 against −0.27), which accounts for the smaller covariances of the consolidated liability in these period estimates.

Panel B of Figure D1 plots the covariance in six-month blocks from 2019 together with annualised CPI inflation; July to September 2026 are too few months for a block and are not shown. The covariance of the consolidated liability with the benchmark SDF stays positive through the pandemic fiscal expansion of 2020 and 2021 and in the first half of 2022. Within these blocks it first turns negative in the second half of 2022 (−1.3), when annualised inflation reaches 4.5 per cent, and is negative again in the second half

of 2023 and the first half of 2024. It is positive again in the second half of 2024 and through 2025, and strongly negative in the first half of 2026 (-18.5), mainly because of February and March, when equities and JGBs rose and then fell together. The post-2022 estimate therefore combines the negative block covariances of the second half of 2022, the second half of 2023 and the first half of 2024 with the pronounced co-movement of bonds and equities in February and March 2026, separated by positive block covariances in the second half of 2024 and in 2025. Blocks of six months rest on few observations; they record where the covariance was concentrated, not a persistent regime.

D.3 Rolling covariances

Figure D2 plots backward-looking 36-month covariances for the four SDFs, with one-standard-error (68 per cent) bands for the consolidated liability that treat the SDF as given and the observations as independent, and Table D3 summarises them by period. The rolling estimates start in August 2005, when the first full window is available.

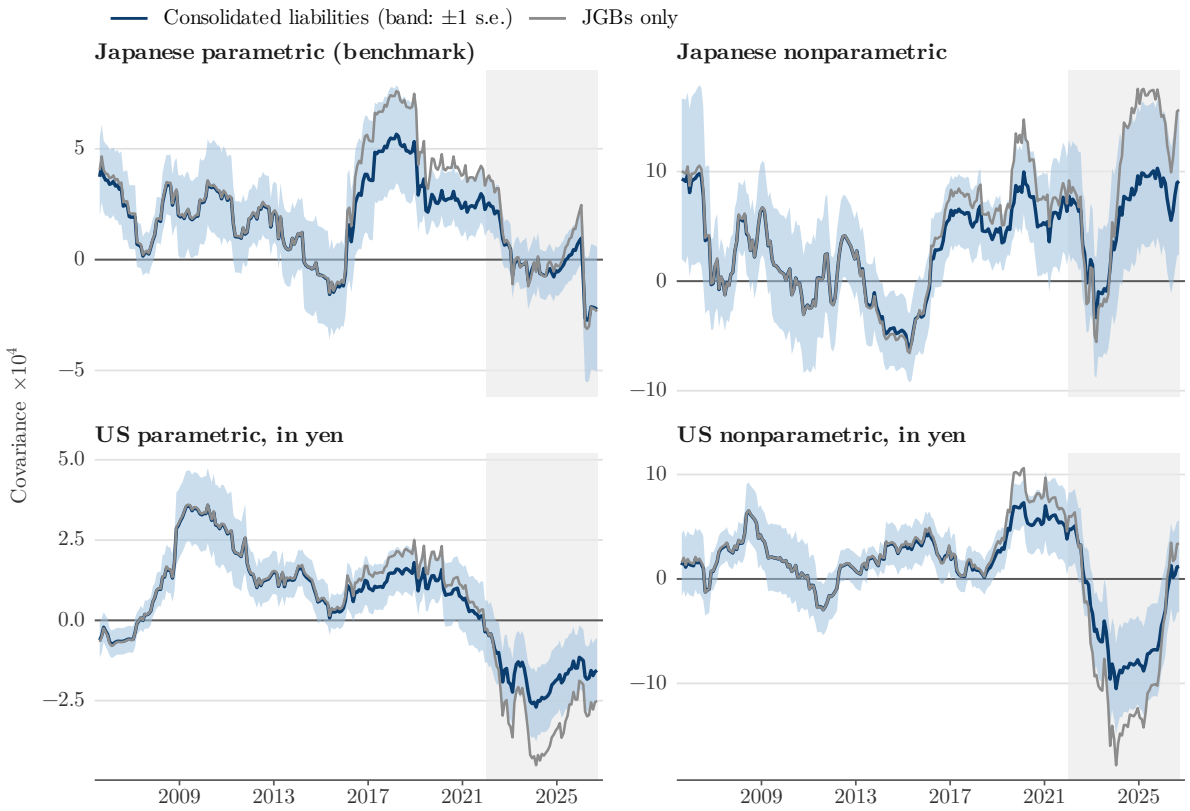


Figure D2: Rolling 36-month covariance between the SDF and the real return on government liabilities

Notes: Backward-looking 36-month sample covariance ($\times 10^4$) between each SDF and the real holding-period return on the consolidated government's liabilities (dark line) and on JGBs only (grey line). The blue bands are one-standard-error (68 per cent) bands for the consolidated liability that treat the estimated SDF as given and the observations within the window as independent. The grey background marks the period from January 2022. Positive values indicate that the liability pays off in states that the SDF prices highly.

Table D3: Rolling covariances by period

	Consolidated			JGBs only		
SDF	2005–12	2013–21	2022–26	2005–12	2013–21	2022–26
Japanese parametric	2.26 (0)	2.25 (0)	−0.07 (0)	2.36 (0)	3.29 (0)	0.26 (0)
US parametric, in yen	1.47 (16)	0.99 (0)	−1.69 (91)	1.49 (16)	1.30 (0)	−2.81 (89)
Japanese nonparametric	2.58 (0)	2.62 (18)	5.72 (0)	2.69 (0)	3.88 (17)	9.24 (0)
US nonparametric, in yen	1.43 (6)	3.20 (0)	−4.20 (60)	1.47 (6)	4.00 (0)	−7.08 (63)

Notes: Period averages of the 36-month rolling covariance ($\times 10^4$) between each SDF and the real return on government liabilities. In parentheses, the percentage of months in the period in which the upper edge of the one-standard-error band is below zero. Periods: August 2005 to December 2012, January 2013 to December 2021, January 2022 to September 2026.

For the benchmark SDF the covariance is positive in most of 2005 to 2021: its period average is 2.3 in 2005–2012 and 2.3 in 2013–2021, with 2014–2015 the main exception. From 2022 the covariance declines; it is mostly slightly negative from 2023 to mid-2025, positive in late 2025, and falls to its lowest values of the period from March 2026, after equities and JGBs rose and then fell together in February and March, with a 2022–2026 average of −0.07. From September 2022 the band includes zero in every month, although its upper end comes close to zero in the spring of 2026. The rolling estimate thus describes the hedge fading rather than turning into a clear source of risk. The converted US SDF moves further: its 2022–2026 average is −1.7 and the band lies below zero in 91 per cent of those months. Because the windows overlap, these shares of months are descriptive, not counts of independent tests.[30] The period covariance of Table D2 and the average of the rolling covariances use different centring and observation weights, so they need not agree (−1.74 against −0.07): each rolling estimate is centred on its own window means, the windows ending before late 2024 also contain months before 2022, and February and March 2026 enter only the last few windows.

D.4 The same years in the United States

The estimated insurance value of US Treasuries declined sharply over the same years, and Pal Mustafi and Preston (2026) find US government debt risky at the end of their sample. Table D4 applies the decomposition (8) to the US series of Hall and Sargent (2011, 2025) with the US SDF. The US series ends in June 2025, so the table reports Japan both over the common period, January 2022 to June 2025, and over the full post-2022 sample. Over the common period the change is far larger in the United States in the point estimates. The US covariance moves from +3.08 to −10.53 and the premium from −37 to +129 basis points, with 91 per cent of the post-2022 covariance in the nominal-return component: Treasury prices fell in the months that the SDF prices highly, as bond and equity prices fell together. In Japan the covariance over the same months is +0.02, a premium of 0 basis points (11): by mid-2025 the Japanese premium was close to zero. Including the observations after June 2025, and above all the first half of 2026 (Figure D1), moves the full post-2022 estimate to +22 basis points.

[30] The Japanese nonparametric covariance rises after 2023, against the other three.

Table D4: Decomposition of the covariance: Japan and the United States

	Japan (consolidated)			United States (private)	
	2002:09–2021:12	2022:01–2025:06	2022:01–2026:09	2002:09–2021:12	2022:01–2025:06
Covariance, total	+2.49	+0.02	−1.74	+3.08	−10.53
nominal-return component	+2.45	+0.63	−0.22	+3.04	−9.58
inflation component	+0.05	−0.62	−1.52	+0.04	−0.95
Risk premium (bp per year)	−30 (8)	0 (11)	+22 (25)	−37 (10)	+129 (41)

Notes: Sample covariance ($\times 10^4$) between the SDF and the real holding-period return on government liabilities, and its exact decomposition (8) into the nominal-return and inflation components. Japan: consolidated liabilities and the benchmark Japanese parametric SDF (Table D2), over the period common to both countries and over the full post-2022 sample. United States: marketable Treasury debt held by private investors, from Hall and Sargent (2011, 2025), deflated by the US CPI, with the US parametric SDF re-estimated over July 1926 to September 2026 (Appendix B); the series ends in June 2025. Risk premia in basis points per year with fixed-SDF Newey–West standard errors in parentheses.

The comparison rests on two estimated SDFs. We re-estimate both on each of 999 joint block-bootstrap samples, which draw the same calendar months for the two countries from September 2002 on (six-month circular blocks within September 2002 to December 2021, January 2022 to June 2025 and July 2025 to September 2026, and within the earlier months for the United States alone). The difference between the two premia over the common period, −129 basis points, then has a 90 per cent interval of $[-226, 28]$, and the difference between their changes from before 2022, −136, one of $[-253, 43]$. Pricing the Japanese liability with the converted US SDF instead, so that one SDF prices both, gives a difference of −108 with an interval of $[-199, 15]$.

An asymptotic alternative leads to a different conclusion. Adding the estimation error of each SDF by the delta method, from the influence functions of the two-step GMM estimates,[31] raises the standard error of the difference between the changes only from 38 to 42 basis points, and its 90 per cent interval, $[-205, -67]$, excludes zero. The two approximations differ mainly for the United States, where the bootstrap re-estimates the SDF on resampled data from 1926 on. Which is more accurate here is an open question: asymptotic standard errors of two-step GMM estimates with several hundred moment conditions can be too small in finite samples, and the accuracy of the block bootstrap for such estimators with serially dependent data is not known. The bootstrap with recentred

[31]The calculation is a first-order linearisation of the GMM estimates, with the estimated weighting matrix and its shrinkage intensity held fixed. It combines the influence of the parameter estimates with the direct sampling variation of the covariance and of the mean of the SDF in the denominator of the premium, and the Newey–West long-run variance of their sum retains the cross-covariances.

moment conditions (Appendix C.1) also gives an interval that excludes zero, $[-290, -14]$. We keep the bootstrap without recentring as the reference, as elsewhere in the paper.

How much of the Japanese change is a transmission of the US repricing cannot be settled by this accounting, but the part of it that a linear projection on US Treasury returns captures is small under either SDF: -0.05 of a post-2022 covariance of -1.74 under the Japanese SDF, and -0.31 of -1.36 under the converted US SDF, whose nominal component is otherwise in the residual (-1.05). What the comparison does show is that the Japanese change was smaller than the US one in the point estimates, and composed differently; whether it reflects a change in the fiscal regime or the accommodation of an imported supply shock, the decomposition cannot say.

D.5 Fiscal scale of the change

With d_t the market value of the consolidated government's liabilities relative to GDP, Y_t real output and $\tilde{s}_t$ the primary surplus relative to GDP, the debt ratio evolves as (Pal Mustafi and Preston, 2026)

$$d_t = \frac{\tilde{R}_t^g / \Pi_t}{Y_t / Y_{t-1}} \, d_{t-1} - \tilde{s}_t. \tag{9}$$

To first order, keeping the ratio constant in expectation requires a primary surplus higher by $RP \times d$ a year, the conversion used in Section 4.2. Table D5 applies it to the period estimates, before and after 2022.

Table D5: Fiscal scale of the covariance-based premium on the covered liabilities

Period	Liability	Risk premium (bp)	Debt / GDP	Premium component (% GDP)
2002:09–2026:09	JGBs only	−32 (11)	1.11	−0.36
2002:09–2026:09	Consolidated	−25 (8)	1.50	−0.38
2002:09–2021:12	JGBs only	−37 (10)	1.14	−0.42
2002:09–2021:12	Consolidated	−30 (8)	1.42	−0.42
2022:01–2026:09	JGBs only	+22 (36)	1.01	+0.23
2022:01–2026:09	Consolidated	+22 (25)	1.84	+0.40
Change	JGBs only	+59 (37) [−36, 171]	1.11	+0.66 [−0.40, 1.91]
Change	Consolidated	+51 (26) [−16, 127]	1.50	+0.77 [−0.24, 1.91]

Notes: "Risk premium" is the annualised premium implied by the benchmark Japanese parametric SDF (Table 1), in basis points, with fixed-SDF Newey–West standard errors in parentheses and, for the change, the 90 per cent bootstrap interval in brackets. "Debt / GDP" is the period average of the market value of the liability held outside the Bank at the start of the month relative to nominal GDP; for the change it is the full-sample average, applied to both sub-periods. "Premium component" is $RP \times d$, the first-order fiscal scale of the premium on the covered liabilities (marketable JGBs held outside the Bank and current-account balances), in per cent of GDP per year, with the bootstrap interval of the change in brackets; a negative value corresponds to a safety premium. It is evaluated at period-average values of the premium and the debt ratio and is not a solution of the debt dynamics (Section 4.2).

For the change we apply the change in the premium to a common debt ratio, the full-sample average of 1.50 for the consolidated liability, so that the figure reflects the covariances and not the growth of debt. The change corresponds to 0.77 per cent of GDP per year for the consolidated liability and 0.66 per cent for JGBs alone. Its uncertainty is large. Conditional on the estimated SDF, the 90 per cent interval of the consolidated figure is $[0.13, 1.42]$ per cent of GDP. Once the SDF is re-estimated on each bootstrap sample, the interval is $[-0.24, 1.91]$ per cent of GDP for the consolidated liability and $[-0.40, 1.91]$ for JGBs alone; both contain zero.