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Optimal Monetary Policy and Nominal Exchange Rate Volatility under Local Currency Pricing

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Optimal Monetary Policy and Nominal Exchange Rate Volatility under Local Currency Pricing*

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Abstract

We analyze the fluctuation in inflation and nominal exchange rate under optimal monetary policy with local currency pricing, by developing a two-country model belonging to DSGE with local currency pricing and comparing fluctuations under local currency pricing with fluctuations under producer currency pricing. Although preceding DSGE literatures assuming producer currency pricing show that stabilizing domestic inflation is optimal from the view point of minimizing welfare costs, we show that completely stabilizing consumer price index inflation is optimal from that view point. In addition, we show that completely stabilizing consumer price index inflation is equivalent with completely stabilizing nominal exchange rate.

Keywords: Local Currency Pricing, DSGE, Optimal monetary policy, Fixed Exchange Rate Regime, CPI Inflation

JEL Classification: E52; E62; F41

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1 Introduction

While new open economy macroeconomics (NOEM) try to analyze optimal exchange rate regime dynamic stochastic general equilibrium (DSGE) literatures discuss optimal monetary policy. Roughly speaking, NOEM has the policy implication that flexible exchange rate regime is optimal from the view point of maximizing social welfare which is equivalent to minimizing welfare costs, if firms set their prices following producer currency pricing (PCP) while fixed exchange rate regime is optimal if firms set their prices following local currency pricing (LCP). Some DSGE literatures assuming an open economy have policy implication that stabilizing producer price index (PPI) inflation which is equivalent to domestic or GDP inflation is optimal from that view point. Some DSGE literatures assuming an open economy assuming PCP and those do not have attention to price setting behavior except for few papers. NOEM and DSGE literatures have not still reconciled in policy implications in an open economy and there is enough room to discuss optimal monetary policy under the LCP.

To analyze the sort of inflation rate which should be stabilized under the LCP and reconcile policy implications derived by NOEM and DSGE literatures, we develop a two-country economy model under the LCP belonging to DSGE and study the fluctuation not only in inflation but also in nominal exchange rate. Although some DSGE literatures assuming an open economy under the PCP show that stabilizing PPI inflation is optimal from the view point of minimizing welfare costs, we show that completely stabilizing consumer price index (CPI) inflation is optimal from that view point. In addition, we show that completely stabilizing CPI inflation is equivalent with completely stabilizing nominal exchange rate under the LCP.

Now, we review some preceding papers to show the importance of our aim to study the sort of inflation rate which should be stabilized under the LCP and reconcile policy implications derived by NOEM and DSGE literatures in this paper. By developing not only PCP but also LCP model following NOEM, Deverereux and Engel[3] discuss the optimal exchange rate regime from the view point of welfare maximization and show that fixed exchange rate regime is desirable under the LCP although floating exchange rate regime is desirable under the PCP. Their finding is not trivial but important because conventional papers show that optimal monetary policy in an open economy requires exchange rate flexibility. However, because of inwardness of NOEM, they cannot show effects on price stability with fixed exchange rate regime under the LCP. Hence they do not provide what kind of inflation rate should be stabilized following Woodford's[10] motivation.

Gali and Monacelli[5] show that optimal monetary policy in a small open economy is consistent with domestic price inflation targeting. Although they do not mention explicitly, they assume the PCP. In addition, they compare three policy regimes, PPI inflation based and CPI inflation based Taylor rules and fixed exchange rate regime and show that PPI inflation based Taylor rule brings the closest macroeconomic volatility from macroeconomic volatility brought about by optimal monetary policy among those three regimes.[1] Their policy implication is also important because their policy implication implies that

[1]Correctly, Gali and Monacelli[5] dub not PPI inflation based Taylor rule but domestic inflation based Taylor rule. However, the definition of their domestic inflation is consistent with our definition of the PPI inflation.

outcome of optimal monetary policy is not fundamentally different from the one of the closed economy. While they do not highlight the firms' price setting behavior, Gali and Monacelli[5] imply that PPI inflation targeting is optimal under the PCP. In addition, they comply Woodford's[10] motivation.

Somehow, some DSGE literatures do not focus on the firms' price setting behavior and those assume the PCP. There are few DSGE literatures focus on the firms' price setting behavior. Based on Gali and Monacelli[5]'s model, Monacelli[7] introduces exporters whose price setting behavior can be regarded as the LCP and analyze monetary policy in a low-pass through environment. He can show that outcome of monetary policy is quite different not only from canonical papers but also Gali and Monacelli[5] who implies that stabilization in the PPI inflation achieves stabilization in output gap simultaneously. Because of low of one price (LOOP) gap, the analysis of monetary policy of an open economy is fundamentally different from the one of a closed economy. While he focuses on important point, he cannot comply Woodford's[10] motivation. He does not reply what kind of inflation rate should be stabilized under such a low-pass through environment stemming from LCP and cannot reconcile policy implications derived by NOEM and DSGE literatures while he shows importance of commitment on the monetary policy. Another few DSGE author is Okano[9] who shows that CPI inflation targeting stabilizes output to changes in demand shock by utilizing a two-country economy model under the LCP. Although his paper is insightful, he failures to show clear policy implication on CPI inflation targeting to changes in productivity shock and he does not derive microfounded loss function which stems from second-order Taylor expanded utility function alike with Monacelli[7]. Hence, it cannot be said that Okano complies Woodford's[10] motivation. In addition, he cannot derive clear policy implication on fluctuations in the nominal exchange rate under the CPI inflation targeting under the LCP and cannot reconcile with Deverereux and Engel[3]'s policy implication. It is quite obvious that analyzing optimal monetary policy following DSGE under the LCP to comply Woodford's[10] motivation and reconcile Deverereux and Engel[3]'s policy implication.

As mentioned above, our aims in this paper are finding the sort of inflation rate which should be stabilized under the LCP and reconciling policy implications derived by NOEM with DSGE literatures. To achieve our aims, we develop both the LCP and the PCP model which assume a two-country. We derive well microfounded loss function under both the LCP and the PCP, stemming from second-order Taylor expanded utility function following Woodford[11] and Gali[4]. We assume that central banks in two countries solve optimization problem under both the LCP and the PCP and impulse response functions (IRFs) are calculated. We calculate IRFs under the special case in which the relative risk aversion and the elasticity of substitution between goods produced in both two countries are unity and under the general case in which the relative risk aversion and the elasticity of substitution between goods produced in both two countries are 3 and 4.5, respectively.[2] Note that those elasticity settings in the special case is consistent with Gali and Monacelli's[5] setting and those elasticity settings in the general case is consistent with Benigno and Benigno's[1]. To compare with the result on Gali and Monacelli[5] and to discuss optimal

[2]The relative risk aversion and the elasticity of substitution between goods produced in both two countries are often dubbed the intertemporal elasticity of substitution and the intratemporal elasticity of substitution, respectively.

monetary policy on general parameterization, we analyze both two cases. Because we are interested in macroeconomic volatility which affects on the welfare costs based on second-order approximated utility function and are interested in nominal exchange rate volatility under the PCP and the LCP, we calculate macroeconomic volatility including the nominal exchange rate varying the relative risk aversion and the elasticity of substitution between goods produced in both two countries. Finally, we calculate welfare costs varying the relative risk aversion and the elasticity of substitution between goods produced in both two countries.

Now, we mention our results as follows. First of all, we show that optimal monetary policy under the LCP brings no fluctuations not in the PPI inflation rate but in the CPI inflation rate. Roughly speaking, optimal monetary policy under the LCP is the CPI inflation targeting. This result is quite different from the result on Gali and Monacelli[5]. Our result is confirmed by IRFs, volatility on the CPI inflation and loss function stemming from second-order approximated utility function. Interestingly, the quadratic terms of CPI inflation rate appear our loss function and replace the quadratic terms of PPI inflation under the LCP, although the quadratic terms of PPI inflation appear in our loss function under the PCP alike with Gali and Monacelli[5] and Benigno and Benigno[1]. Next, we can reconcile with Devereux and Engel[3] because there are no fluctuations on nominal exchange rate under the LCP. Roughly speaking, optimal monetary policy under the LCP is consistent with fixed exchange rate regime and that is shown by Devereux and Engel[3]. In addition, this result is not depending on the relative risk aversion and the elasticity of substitution between goods produced in both two countries. That there are no fluctuation on nominal exchange rate is consistent regardless of those preferences. Because Devereux and Engel[3] analyzes under some restriction which consistent with unitary elasticity of substitution between goods produced in both two countries in our model, we can support their results and can generalize their policy implication. Summarizing our result, optimal monetary policy under the LCP is not only consistent with CPI inflation targeting but also consistent with fixed exchange rate. Details on our results are discussed on the rest of this paper.

The rest of this paper is organized as follows. Section 2 derives two models, the LCP and the PCP model. Section 3 analyzes optimal monetary policy by deriving welfare costs, FONCs for central bank with commitment and calibration. Section 4 analyzes effect on macroeconomic volatility and welfare costs of varying relative risk aversion and the elasticity of substitution between goods produced in two countries. Section 5 concludes this paper. An appendix shows analysis on international monetary policy cooperation between two countries, which is omitted in the text because we highlight fluctuations in inflation and nominal exchange rate.

2 The Model

We construct a two-country model belonging to the class of DSGE models with nominal rigidities and imperfect competition, basically following Gali and Monacelli[5] and Monacelli[7]. We alter Gali and Monacelli[5]'s small open economy model to two-country economy model following Obstfeld and Rogoff[8] although we assume all goods are tradables. The union-wide economy consists of two countries, countries H and F. Country H produces an array of differentiated goods indexed by the interval $h \in [0,1]$, while country F produces an array of differentiated goods indexed by $f \in [1,2]$. In addition, we derive two models, one of them is assumed the LCP and another one is assumed the PCP.

Note that we take a definition $v_t \equiv \ln\left(\frac{V_t}{V}\right)$ if there are no provisions where V_t denotes an arbitrary variable and V denotes steady state value of V_t.

2.1 LCP Model

Under the LCP, LOOP is not necessarily applied because firms can choose prices to sell goods in countries H and F separately. Thus, $P_t(h) = \mathcal{E}_t P_t^*(h)$ and $P_t(f) = \mathcal{E}_t P_t^*(f)$ hence $P_{H,t} = \mathcal{E}_t P_{H,t}^*$ and $P_{F,t} = \mathcal{E}_t P_{F,t}^*$ do not necessarily hold where $P_t(h)$ and $P_t(f)$ denote the price of a generic good produced in country H in terms of country H's currency, $P_{H,t} \equiv \left[\int_0^1 P_t(h)^{1-\varepsilon} dh\right]^{\frac{1}{1-\varepsilon}}$ and $P_{F,t} \equiv \left[\int_1^2 P_t(f)^{1-\varepsilon} dh\right]^{\frac{1}{1-\varepsilon}}$ denote indices of the price of generic goods produced in countries H and F, respectively, $\mathcal{E}_t$ denotes nominal exchange rate.[3] Note that quantities and prices particular to country F are denoted by asterisks while quantities and prices without asterisks are those in country H.

2.1.1 Households

The preferences of the representative household in country H are given by:

$$\mathcal{U} \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U_t, \tag{1}$$

where $U_t \equiv \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\varphi} N_t^{1+\varphi}$ denotes the period utility, E_t denotes the expectation, conditional on the information set at period t, $\beta \in (0,1)$ denotes the subjective discount factor, C_t denotes consumption, $N_t \equiv \int_0^1 N_t(h)\, dh$ denotes hours of work, σ denotes the relative risk aversion and φ denotes the inverse of the labor supply elasticity. The preferences of the representative household in country F is defined analogously.

More precisely, private consumption is a composite index defined by:

$$C_t \equiv \left[\left(\frac{1}{2}\right)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \left(\frac{1}{2}\right)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

where $C_{H,t} \equiv \left[\int_0^1 C_t(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}$ and $C_{F,t} \equiv \left[\int_1^2 C_t(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}}$ denote Dixit–Stiglitz-type indices of consumption across the home goods and foreign goods, respectively, and $\eta > 0$ denotes the elasticity of substitution between tradables and nontradables. Note that C_t^* is defined analogously to Eq.(2).

Total consumption expenditures by households in country H are given by $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_t C_t$. A sequence of budget constraints in country H is given by:

$$B_t + W_t N_t - T_t \geq P_t C_t + \mathrm{E}_t\left(Q_{t,t+1} B_{t+1}\right), \tag{3}$$

[3] By citing Betts and Devereux[2], Mark[6] clearly explanes the LCP.

where $Q_{t,t+1}$ denotes the stochastic discount factor, B_t denotes the nominal payoff of the bond portfolio purchased by households, W_t denotes the nominal wage, and T_t denotes lump-sum taxes. The budget constraint in country F is given analogously. Furthermore:

$$P_t \equiv \left(\frac{1}{2} P_{H,t}^{1-\eta} + \frac{1}{2} P_{F,t}^{1-\eta} \right)^{\frac{1}{1-\eta}}, \tag{4}$$

denotes the consumption price index (CPI). P_t^* is defined analogously to this equality. By log-linearizing this equality yields $p_t = \frac{1}{2} p_{H,t} + p_{F,t}$, which implies as follows:

$$\pi_t = \frac{1}{2} \pi_{H,t} + \frac{1}{2} \pi_{F,t}, \tag{5}$$

where $\pi_t \equiv p_t - p_{t-1}$ denotes CPI inflation with $\pi_{H,t} = p_{H,t} - p_{H,t-1}$ and $\pi_{F,t} = p_{F,t} - p_{F,t-1}$.

The optimal allocation of any given expenditure within each category of goods implies the demand functions, as follows:

$$\begin{aligned} C_t(h) &= \left(\frac{P_t(h)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} \quad ; \quad C_t(f) = \left(\frac{P_t(f)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t} \\ C_t^*(h) &= \left(\frac{P_t^*(h)}{P_{H,t}^*} \right)^{-\varepsilon} C_{H,t}^* \quad ; \quad C_t^*(f) = \left(\frac{P_t^*(f)}{P_{F,t}^*} \right)^{-\varepsilon} C_{F,t}^*. \end{aligned} \tag{6}$$

The optimal allocation of expenditures between domestic and foreign goods is given by:

$$\begin{aligned} C_{H,t} &= \frac{1}{2} \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \quad ; \quad C_{F,t} = \frac{1}{2} \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t, \\ C_{H,t}^* &= \frac{1}{2} \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} C_t^* \quad ; \quad C_{F,t}^* = \frac{1}{2} \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-\eta} C_t^*. \end{aligned} \tag{7}$$

The representative household maximizes Eq.(1) subject to Eq.(3). The optimality conditions are given by:

$$R_t \beta \mathrm{E}_t \left(\frac{C_{t+1}^{-\sigma} P_t}{C_t^{-\sigma} P_{t+1}} \right) = 1, \tag{8}$$

which is a conventional Euler equation and

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t}, \tag{9}$$

which is a standard intratemporal optimality condition where $R_t \equiv 1 + r_t$ satisfying $R_t^{-1} = \mathrm{E}_t Q_{t,t+1}$ denotes the gross nominal return on a riskless one-period discount bond paying off one unit of the common currency (in short, the gross nominal interest rate), and r_t denotes the net nominal interest rate. Eq.(8) is an intertemporal optimality condition, namely the Euler equation, and Eq.(9) is an intratemporal optimality condition. Optimality conditions in country F are given analogously.

Log-linearizing Eq.(8), we obtain:

$$c_t = \mathrm{E}_t c_{t+1} - \frac{1}{\sigma}\hat{r}_t + \frac{1}{\sigma}\mathrm{E}_t \pi_{t+1} \tag{10}$$

with $\hat{r}_t \equiv \ln\left(\frac{R_t}{R}\right)$.

There is relationship on the gross nominal interest rate between countries H and F which is uncovered interest rate parity (UIP) as follows:

$$R_t = R_t^* \mathrm{E}_t\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right)$$

with $R_t^* \equiv 1 + r_t^*$. Log-linearizing the UIP, we have the familiar expression as follows:

$$\mathrm{E}_t\left(\Delta e_{t+1}\right) = \hat{r}_t - \hat{r}_t^*,$$

with $\Delta v_t \equiv v_t - v_{t-1}$ and $e_t \equiv \ln\left(\frac{\mathcal{E}_t}{\mathcal{E}}\right)$.

Combining Eq.(8) and the UIP and iterating with an initial condition, we have the following optimal risk-sharing condition:

$$C_t^\sigma = \vartheta\left(C_t^*\right)^\sigma \mathcal{Q}_t,$$

with $\mathcal{Q}_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t}$ denoting the real exchange rate and ϑ denoting a constant depending on the initial value. Log-linearizing this equality, we have:

$$c_t = c_t^* + \frac{1}{\sigma} q_t. \tag{11}$$

2.1.2 Market Clearing

The market for tradables and for nontradables in country H clears when domestic demand equals domestic supply, as follows:

$$Y_t(h) = C_t(h) + C_t^*(h), \tag{12}$$

where $Y_t(h)$ denotes the output of good h, which is market clearing condition. Market clearing condition in country F is analogously. Plugging Eq.(7) into Eq.(12) yields:

$$Y_t(h) = \frac{1}{2}\left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \frac{1}{2}\left(\frac{P_t^*(h)}{P_{H,t}^*}\right)^{-\varepsilon}\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} C_t^*. \tag{13}$$

Let $Y_t \equiv \left[\int_0^1 Y_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh\right]^{\frac{\varepsilon}{\varepsilon-1}}$ represent index for aggregate output in country H. Under the LCP, we obtain:

$$\begin{aligned} Y_t &= \frac{1}{2}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \frac{1}{2}\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} C_t^*, \\ &= \frac{1}{2}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t\left[1 + \left(\frac{P_{H,t}}{P_t}\right)^{\eta}\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} \mathcal{Q}_t^{-\frac{1}{\sigma}}\right], \end{aligned} \tag{14}$$

by combining Eqs.(13), Dixit–Stiglitz aggregators for output and prices, where we take Eq.(11) in the second lines in Eq.(14).

We define the terms of trade (TOT) as follows:

$$\mathcal{S}_t \equiv \frac{P_{F,t}}{\mathcal{E}_t P_{H,t}^*}, \tag{15}$$

where $\mathcal{S}_t$ is foreign TOT. The numerator is export price of goods produced in country F in terms of country H's currency and the denominator is export price of goods produced in country F in terms of country H's currency. Log-linearizing Eq.(15), we have:

$$s_t = p_{F,t} - e_t - p_{H,t}^*. \tag{16}$$

Plugging Eq.(21) into log-linearized Eq.(14), we have:

$$y_t = c_t + \frac{\eta}{2} s_t + \frac{1}{2}\left(\eta - \frac{1}{\sigma}\right) q_t,$$

which is log-linearized market clearing in country H under the LCP. Although there is a difference between this equality and Eq.(42) because logarithmic real exchange rate q_t appears in this equality. However, this equality boils down to Eq.(42) because the PPP is applied which implies that $q_t = 0$ although we assume the LCP. We discuss about the PPP under the LCP in section 2.3.

Combining Eq.(42) and its counterpart in country F, we have:

$$s_t = \frac{1}{\eta}\left(y_t - y_t^*\right) - q_t,$$

which clarifies relationship between the TOT and relative output under the LCP. As mentioned, $q_t = 0$ is applied although we assume the LCP. Hence, this equality boils down to Eq.(41).

2.1.3 Firms

Each producer uses a linear technology to produce a differentiated good as follows:

$$Y_t(h) = A_t N_t(h), \tag{17}$$

where A_t denotes stochastic productivity in country H. Firms in country F have a technology analogously to firms in country H.

Using Dixit–Stiglitz aggregators, Eq.(17) can be rewritten as:

$$N_t = \frac{Y_t D_t}{A_t}, \tag{18}$$

with $D_t \equiv \int_0^1 \frac{Y_t(h)}{Y_t} dh$. Because d_t is $o\left(\| \xi \|^2\right)$, a first order approximation of this equality is given by:

$$y_t = a_t + n_t, \tag{19}$$

which is consistent with Gali and Monacelli's[5] log-linearized production function.

Similar to many DSGE literatures including Gali and Monacelli[5], we assume that firms set prices in Calvo–Yun-style price-setting behavior. Hence, a measure $1-\theta$ firms sets new prices each period, with an individual firm's probability of re-optimizing in any given period being independent of the time elapsed since it last set its prices. Each producer produces a single differentiated good and prices its good to reflect the elasticity of substitution across goods produced given the CPI. This is because each firm plays an active part in the monopolistically competitive market. In addition, we assume that firms have the ability to engage in price discrimination by setting a domestic price in terms of domestic currency for domestic sales that differs from the price that it sets for exports. This is the LCP behavior. Under the Calvo–Yun-style price-setting behavior and the LCP behavior in a monopolistically competitive market, the maximization problems which producers in country H face are as follows:

$$\max_{\tilde{P}_{H,t},\tilde{P}^*_{H,t}} \sum_{k=0}^{\infty} \theta^k \mathrm{E}_t \left\{ Q_{t,t+k} \left[\tilde{P}_{H,t} \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} C_{H,t+k} + \mathcal{E}_{t+k} \tilde{P}^*_{H,t} \left(\frac{\tilde{P}^*_{H,t}}{P^*_{H,t+k}} \right)^{-\varepsilon} C^*_{H,t+k} \right.\right.$$
$$\left.\left. - MC^n_{t+k} \left(\left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} C_{H,t+k} + \left(\frac{\tilde{P}^*_{H,t}}{P^*_{H,t+k}} \right)^{-\varepsilon} C^*_{H,t+k} \right) \right] \right\}, \quad (20)$$

where $\tilde{P}_{H,t}$ and $\tilde{P}^*_{H,t}$ are the prices chosen by firms when they obtain the chance to change prices associated with goods produced and sold in country H and goods produced in country H while sold in country F, respectively, $MC^n_t \equiv P_{P,t} MC_t$ denotes real marginal costs in country H, with $MC_t \equiv \frac{(1-\tau)W_t}{A_t P_{P,t}}$ and $P_{P,t}$ denotes producer price index (PPI) in country H, which are defined as follows:

$$P_{P,t} \equiv \frac{P_{H,t} C_{H,t} + \mathcal{E}_t P^*_{H,t} C^*_{H,t}}{C_{H,t} + C^*_{H,t}},$$

which can be rewritten as $P_{P,t} = P_{H,t}$ when the LOOP is applied. The PPI in country F is defined analogously. By log-linearizing this equality, we have $p_{P,t} = \frac{1}{2} p_{H,t} + \frac{1}{2}\left(e_t + p^*_{H,t}\right)$, which implies as follows:

$$\pi_{P,t} = \frac{1}{2}\pi_{H,t} + \frac{1}{2}\left(\Delta e_t + \pi^*_{H,t}\right), \quad (21)$$

where $\pi_{P,t}$ denotes the PPI inflation in country H and $\pi_{P_t} = \pi_{H,t}$ is applied when the LOOP is applied.

Note that the maximization problems which producers in country F face are analogously to Eq.(20). Because of nominal rigidities, Eq.(20) looks complicated. When there are no nominal rigidities, namely $\theta \to 0$, Eq.(20) problems boil down to:

$$\max_{P_{H,t},P^*_{H,t}} P_{H,t} C_{H,t} + \mathcal{E}_t P^*_{H,t} C^*_{H,t} - MC^n_t \left(C_{H,t} + C^*_{H,t} \right),$$

which implies that each firm sets its price in terms of local currency in which each firm's good is sold and pay costs to produce in terms of producer currency.

Under the LCP, we have multiple FONCs because firms can choose $\tilde{P}_{H,t}$ and $\tilde{P}^*_{H,t}$ separately. The FONCs for Eq.(20) are as follows:

$$\begin{aligned}
\mathrm{E}_t\left[\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}\left(\tilde{P}_{H,t}-\zeta MC^n_{t+k}\right)\left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon}C_{H,t+k}\right] &= 0,\\
\mathrm{E}_t\left[\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}\left(\tilde{P}^*_{H,t}\mathcal{E}_{t+k}-\zeta MC^n_{t+k}\right)\left(\frac{\tilde{P}^*_{H,t}}{P^*_{H,t+k}}\right)^{-\varepsilon}C^*_{H,t+k}\right] &= 0,
\end{aligned}$$

which can be log-linearized as follows:

$$\begin{aligned}
\tilde{p}_{H,t} &= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k\,\mathrm{E}_t\left(mc^n_{t+k}\right),\\
\tilde{p}^*_{H,t} &= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k\,\mathrm{E}_t\left(mc^n_{t+k}-e_{t+k}\right), \qquad (22)
\end{aligned}$$

with $\zeta \equiv \frac{\theta}{\theta-1}$ denoting a constant markup where we use the fact that $Q_{t,t+k} = \beta^k\left(\frac{C_{t+k}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+k}}$. Eq.(21) implies that firms set the price as a markup over a weighted average of expected future marginal costs. Especially, the first equality in Eq.(22) definitely corresponds to one derived by Gali and Monacelli[5]. The second equality in Eq.(22) is not a familiar expression although it implies that firms set the price as a markup over a weighted average of expected future nominal marginal costs. The second equality in Eq.(22) is the log-linearized FONC for firms which produce goods in country H and sell them in country F. Those firms set the price in terms of country F's currency as a markup over a weighted average of expected future nominal marginal costs in terms of country F's currency. We learn further the character of Eq.(22) after discuss some identities including the relative prices which is peculiar to LCP behavior.

Under the LCP, the LOOP is not necessarily applied because of Eqs.(20) and (22), which imply that firms set their price of goods in terms of local currency, namely the LCP. Because of that setting, there is the LOOP gap, which measures the degree of the pass-through. Now, we discuss the LOOP gap and the real exchange rate in our mode. Following Monacelli[7], we define the LOOP gap as follows:

$$\Psi_{H,t} \equiv \frac{\mathcal{E}_t P^*_{H,t}}{P_{H,t}}\ ;\ \Psi_{F,t} \equiv \frac{\mathcal{E}_t P^*_{F,t}}{P_{F,t}}$$

where $\Psi_{H,t}$ and $\Psi_{F,t}$ denote the LOOP gap for goods produced in countries H and F, respectively. When the LOOP is applied, we have $\Psi_{H,t} = \Psi_{F,t} = 1$.

Combining Eq.(7), the optimal risk-sharing condition and the definition of the TOT yields:

$$\Psi_{H,t} = \Psi^{-1}_{F,t}\mathcal{S}^{-1}_t\left(\frac{\mathcal{E}_t P^*_{F,t}}{P_{H,t}}\right)\mathcal{Q}_t^{-\frac{1}{\sigma\eta}},$$

which implies that the LOOP gap is a function of the TOT, the real exchange rate and the relative price of goods consumed domestically. Because

$\mathcal{S}_t^{-1}\left(\frac{\mathcal{E}_t P_{F,t}^*}{P_{H,t}}\right) = \Psi_{H,t}\Psi_{F,t}$, that equality can be rewritten as follows:

$$\mathcal{Q}_t = 1,$$

which implies that the PPP is applied although the LOOP is not applied.[4] Log-linearized version of this equality is given by:

$$q_t = 0. \tag{23}$$

In addition, plugging Eq.(23) into Eq.(11), we have $c_t = c_t^*$, which implies that the marginal utility of consumption between both countries are equal. In fact, households in both countries consume same goods although there is price discrimination. As mentioned, the LOOP is not necessarily applied although Eq.(23) implies that the PPP is definitely applied. This sounds inconsistent at glance. However, although a price of one goods violate the LOOP, the PPP is applied when another goods violate the LOOP inversely. In fact, plugging Eq.(23) into that equality, we have $\Psi_{H,t} = \Psi_{F,t}^{-1}$ and log-linearized version of this as follows:

$$\psi_{H,t} = -\psi_{F,t},$$

which implies that gains from price discrimination corresponds to loses from price discrimination.

Log-linearized marlet clearing conditions in countries H and F clarifies the relationship among nominal exchange rate, the price level and the TOT. Plugging log-linearized definition of the CPI into log-linearized marlet clearing conditions yields:

$$\begin{aligned} e_t &= p_t - p_t^* \\ &= p_{P,t} - p_{P,t}^* + s_t, \\ &= p_{P,t} - p_{P,t}^* + \frac{1}{\eta}\left(y_t - y_t^*\right) \end{aligned} \tag{24}$$

where we use Eq.(21) to derive the second line and Eq.(41) to derive the third line. Eq.(24) implies that output differential between both countries affects nominal exchange rate.

In turn, we discuss the character of Eq.(22), log-linearized FONCs for firms under the LCP. By taking the definition of the LOOP gap, Eq.(22) can be rewritten as follows:

$$\begin{aligned} \tilde{p}_{H,t} &= p_{H,t-1} + \sum_{k=0}^{\infty}(\beta\theta)^k \mathrm{E}_t\left(\pi_{H,t+k}\right) + \frac{1-\beta\theta}{2}\sum_{k=0}^{\infty}(\beta\theta)^k \mathrm{E}_t\left(\psi_{H,t+k}\right) \\ &\quad + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \mathrm{E}_t\left(mc_{t+k}\right), \\ \tilde{p}_{H,t}^* &= p_{H,t-1}^* + \sum_{k=0}^{\infty}(\beta\theta)^k \mathrm{E}_t\left(\pi_{H,t+k}^*\right) - \frac{1-\beta\theta}{2}\sum_{k=0}^{\infty}(\beta\theta)^k \mathrm{E}_t\left(\psi_{H,t+k}\right) \\ &\quad + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \mathrm{E}_t\left(mc_{t+k}\right), \end{aligned} \tag{25}$$

[4]This equality implies that the marginal utility of consumption in country H is definitely same as it in country F. Hence, the UIP can be derived by simply combining 8 and its counterpart in country F without assuming, although we describe that we assume the UIP in section 2.1.1.

where $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$ and $\pi^*_{H,t} \equiv p^*_{H,t} - p^*_{H,t-1}$ denotes inflation of goods both produced and sold in country H and inflation of goods produced in country H and sold in country F, respectively. As mentioned, firms set the price as a mark up over a weighted average of future marginal cost. In our LCP setting, those firms' sales are not measured by the PPI, because it is the weighted average of both price of goods selling in country H and in country F. However, real marginal cost is measured by the PPI, as shown in the definition of nominal marginal cost. That is, those firms obtain sales measured by $P_{H,t}$ and pay total costs measured by the $P_{P,t}$ and that gap is calculated by $p_{P,t} - p_{H,t} = \frac{1}{2}\psi_{H,t}$, which implies that that gap corresponds to the LOOP gap in country H. Although the firms selling goods in country H have no currency disparity in sales and payment, the LCP behavior generate the LOOP gap. Thus, a weighted average of expected future LOOP gap in country H appears in the first equality in Eq.(25).

The price setting behavior of the firms selling goods in country F generates the LOOP gap, alike with another firms which sell goods in country H. Those firms, namely exporters, obtain the sales of goods exported in terms of country F's currency and pay the total cost in terms of country H's currency. Their sales are measured by country H's currency. Hence, their sales in terms of country F' currency is multiplied by nominal exchange rate. They pay total costs which is measured by the PPI, alike with the firms selling goods in country H. The gap is calculated by $p_{P,t} - \left(p^*_{H,t} + e_t\right) = -\frac{1}{2}\psi_{H,t}$. Thus, a weighted average of expected future LOOP gap in country H appears in the second equality in Eq.(25) although the sign is contrary to the first equality. Similar mechanism works in firms in country F not only for selling goods domestically but also for exporters. Although our LCP setting is different from Monacelli[7], who assumes a small open economy and importers, our LCP setting clearly generates the LOOP gap and this setting affects the forms of New Keynesian Philips Curve (NKPC) and social welfare stemming from a second-order approximated utility function.

2.1.4 Marginal Cost and Natural Rate of Output

Plugging Eq.(9) into the definition of the marginal cost, we obtain as follows:

$$MC_t = (1-\tau)\frac{C_t^\sigma N_t^\varphi}{A_t}\left(\frac{P_{P,t}}{P_t}\right)^{-1}, \tag{26}$$

which is log-linearized as follows:

$$mc_t = \sigma c_t + \varphi n_t + \frac{1}{2}s_t - a_t, \tag{27}$$

which is consistent with Gali and Monacelli's[5] log-linearized marginal cost.

Under the flexible price equilibrium, $MC_t = \frac{1}{\zeta}$ implying that the real marginal cost is constant and corresponds to inverse of a constant markup is applied. Using this fact and combining Eqs.(14), (18) and Eq.(26), we have natural rate of output under the LCP in country H as follows:

$$\bar{Y}_t = \frac{1}{2}\left\{\frac{P_{P,t}}{P_t}\frac{\zeta^{-1}}{1-\tau}A_t^{1+\varphi}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta\sigma}\left[1+\left(\frac{P_{H,t}}{P_t}\right)^{\eta}\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta}\mathcal{Q}_t^{-\frac{1}{\sigma}}\right]^{\sigma}\right\}^{\frac{1}{\varphi+\sigma}} D_t^{-\frac{\varphi}{\varphi+\sigma}},$$

with $\bar{Y}_t$ denotes natural rate of output in country H, which implies that natural rate of output is a function not only of productivity but also of relative prices because of an open economy setting.

Before log-linearizing this equality, we define the output gap in country H x_t as the deviation of percentage deviation of output in country H y_t from its natural level $\bar{y}_t$. This relationship can be written as:

$$x_t \equiv y_t - \bar{y}_t, \tag{28}$$

which is definitely consistent with Gali and Monacelli[5]'s definition. The output gap in country F is defined analogously to Eq.(28)

Now, we log-linearize that equality. Log-linearized natural rate of output under the LCP is given by:

$$\bar{y}_t = \frac{\omega_1 \omega_2}{\omega_3} a_t - \frac{(\sigma\eta - 1)\,\omega_2}{\omega_3} a_t^*, \tag{29}$$

with $\omega_1 \equiv \eta\,(\sigma + 2\varphi) + 1$, $\omega_2 \equiv 2\eta\,(1 + \varphi)$ and $\omega_3 \equiv \omega_1^2 - (\sigma\eta - 1)^2$. While Gali and Monacelli[5] regard foreign output is exogenous because of their small open economy setting, foreign output, namely output in country F is endogenous in our two-country setting. Thus, productivity in country F replaces foreign output in Eq.(29).

We turn to discuss Eq.(27), percentage deviation of marginal cost from its steady state value. Plugging Eqs.(42), (41), (19) and (28) into Eq.(27) yields:

$$mc_t = \frac{\omega_1}{2\eta} x_t + \frac{\sigma\eta - 1}{2\eta} x_t^*, \tag{30}$$

which implies that real marginal cost in country H consists of output gap in both two countries.

2.1.5 The Demand and Supply Sides

Plugging Eqs.(21), (23), (42) and (41) into Eq.(10) yields New Keynesian IS Curve (NKIS) as follows:

$$x_t = \mathrm{E}_t\,(x_{t+1}) - \frac{2\eta}{\sigma_\alpha}\hat{r}_t + \frac{2\eta}{\sigma_\alpha}\mathrm{E}_t\,(\pi_{P,t+1}) + \frac{\sigma\eta - 1}{\sigma_\alpha}\mathrm{E}_t\left(\Delta x_{t+1}^*\right) + \frac{2\eta}{\sigma_\alpha}\bar{r}_t, \tag{31}$$

where $\bar{r}_t \equiv -\sigma_\alpha \frac{(1-\rho)(1+\varphi)\omega_4}{\omega_3} a_t - \sigma_\alpha \frac{(1-\rho)(\sigma\eta-1)(1+\varphi)\omega_5}{\omega_3} a_t^*$ denotes the natural rate of interest in country H with $\sigma_\alpha \equiv \sigma\eta + 1$, $\omega_4 \equiv \omega_1 - \frac{(\sigma\eta-1)^2}{\sigma_\alpha}$ and $\omega_5 \equiv \frac{\omega_1}{\sigma_\alpha} - 1$. The NKIS in country F, which is analogous to Eq.(31), can be derived by using Eqs.(41) and (23) and counterparts of Eqs.(10), (42) and(21).

Eq.(31) looks like ordinary NKIS in the DSGE literature at glance. Because of the LCP, Eq.(31) has some distinguished feature. Plugging Eq.(21) into Eq.(31), NKIS under LCP can be rewritten as follows:

$$\begin{aligned} x_t &= \mathrm{E}_t\,(x_{t+1}) - \frac{2\eta}{\sigma_\alpha}\hat{r}_t + \frac{\eta}{\sigma_\alpha}\mathrm{E}_t\,(\pi_{H,t+1}) + \frac{\eta}{\sigma_\alpha}\mathrm{E}_t\left(\pi_{H,t+1}^*\right) + \frac{\eta}{\sigma_\alpha}\mathrm{E}_t\,(\Delta e_{t+1}) \\ &\quad + \frac{\sigma\eta - 1}{\sigma_\alpha}\mathrm{E}_t\left(\Delta x_{t+1}^*\right) + \frac{2\eta}{\sigma_\alpha}\bar{r}_t \\ &= \mathrm{E}_t\,(x_{t+1}) - \frac{\eta}{\sigma_\alpha}\hat{r}_t - \frac{\eta}{\sigma_\alpha}\hat{r}_t^* + \frac{\eta}{\sigma_\alpha}\mathrm{E}_t\,(\pi_{H,t+1}) + \frac{\eta}{\sigma_\alpha}\mathrm{E}_t\left(\pi_{H,t+1}^*\right) \\ &\quad + \frac{\sigma\eta - 1}{\sigma_\alpha}\mathrm{E}_t\left(\Delta x_{t+1}^*\right) + \frac{2\eta}{\sigma_\alpha}\bar{r}_t, \end{aligned} \tag{32}$$

where we take log-linearized UIP to derive second line. As shown in the first line, changes in expected nominal exchange rate affects the NKIS. Second line shows that not only domestic nominal interest rate, but also foreign nominal interest rate appears the NKIS.

Plugging log-linearized Calvo's pricing rule and Eq.(30) to Eq.(25), we have equalities which determines the dynamics of inflation as follows:

$$\begin{aligned}\pi_{H,t} &= \beta \mathrm{E}_t\left(\pi_{H,t+1}\right) + \frac{\lambda}{2}\psi_{H,t} + \frac{\lambda\omega_1}{2\eta}x_t + \frac{\lambda\left(\sigma\eta-1\right)}{2\eta}x_t^*, \\ \pi_{H,t}^* &= \beta \mathrm{E}_t\left(\pi_{H,t+1}^*\right) - \frac{\lambda}{2}\psi_{H,t} + \frac{\lambda\omega_1}{2\eta}x_t + \frac{\lambda\left(\sigma\eta-1\right)}{2\eta}x_t^*, \qquad (33)\end{aligned}$$

with $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$. The first equality is inflation dynamics for goods sold domestically and the second equality is inflation dynamics for goods exported. Because Eq.(33) derived from Eq.(25), the FONCs for firms in country H, the third and the fourth terms in the RHS, which stem from real marginal cost in country H, are consistent between both equalities. The signs of the second terms in the RHS are inverse between both equalities. The reason is that the losses from price discrimination are compensated by the gains from price discrimination, and vice versa. Counter part of Eq.(33) are derived from counter part of Eq.(25).

Plugging Eq.(33) into Eq.(21), we have New Keynesian Philips Curve (NKPC) in country H as follows:

$$\pi_{P,t} = \beta \mathrm{E}_t\left(\pi_{P,t+1}\right) + \frac{\lambda\omega_1}{2\eta}x_t + \frac{\lambda\left(\sigma\eta-1\right)}{2\eta}x_t^* - \frac{\beta}{2}\mathrm{E}_t\left(\Delta e_{t+1}\right) + \frac{1}{2}\Delta e_t,$$

and plugging counter part of Eq.(33) into Eq.(21) yields counter part of this equality in country F. This NKPC is featured by appearance of changes in nominal exchange rate. Gali and Monacelli[5] mention that full stabilization of domestic prices coincides with full stabilization of output gap, namely $x_t = \pi_{H,t} = 0$ for all t. In our model, their domestic prices correspond to the PPI and they assume fully-exogenous foreign output which implies that the percentage deviation of marginal cost from its steady state value is not affected by the percentage deviation of foreign output from its steady state value. That is, they claim that full stabilization of PPI implies that output conforms its natural rate if we ignore foreign output gap or assume $\sigma\eta = 1$ in this equality. Even if we ignore foreign output gap or assume $\sigma\eta = 1$ in this equality, full stabilization of PPI does not necessarily imply that output conforms its natural rate because of changes in nominal exchange rate, as shown in the forth and the fifth terms in the RHS. Changes in nominal exchange rate as if work cost push shocks under the LCP. Thus, full stabilization of PPI no longer implies that output conforms its natural rate if we ignore foreign output gap or assume $\sigma\eta = 1$. Plugging Eqs.(24), (29) and (28) into that equality, we can eliminate changes in nominal exchange rate and obtain as follows:

$$\begin{aligned}\pi_{P,t} = {} & \beta \mathrm{E}_t\left(\pi_{P,t+1}\right) + \beta \mathrm{E}_t\left(\pi_{P,t+1}^*\right) - \frac{\beta}{\eta}\mathrm{E}_t\left(x_{t+1}\right) + \frac{\beta}{\eta}\mathrm{E}_t\left(x_{t+1}^*\right) + \kappa_{\varpi}x_t \\ & - \kappa_{\varsigma}x_t^* - \pi_{P,t}^* - \frac{1}{\eta}x_{t-1} + \frac{1}{\eta}x_{t-1}^* + \omega_6 a_t - \omega_6 a_{t-1} - \omega_6 a_t^* + \omega_6 a_{t-1}^* \qquad (34)\end{aligned}$$

with $\kappa_{\varpi} \equiv \frac{1+\beta+\lambda\omega_1}{\eta}$, $\kappa_{\varsigma} \equiv \frac{1+\beta-\lambda(\sigma\eta-1)}{\eta}$, $\omega_6 \equiv \frac{\omega_2\varpi_3(\sigma+\varphi)}{\omega_3}$ and $\varpi_3 \equiv 1+\beta\left(1-\rho\right)$. Exogenous shocks appear in Eq.(34) which shows that exogenous productivity affects PPI inflation.

Monacelli[7] derives CPI based NKPC. Following Monacelli[7], we derive CPI based NKPC. Plugging the first equality in Eq.(33) and its counterpart in country F into Eq.(5) yields:

$$\pi_t = \beta \mathrm{E}_t\left(\pi_{t+1}\right) + \frac{\kappa_\alpha}{2} x_t + \frac{\kappa_\alpha}{2} x_t^* \tag{35}$$

with $\kappa_\alpha \equiv \lambda\left(\sigma+\varphi\right)$. As mentioned by Gali and Monacelli[5], κ_α is consistent with the slope coefficient of standard closed economy NKPC. A full stabilization not of PPI inflation but of CPI inflation implies that output conforms its natural rate when the nominal interest rate in both countries absorbs the effects from changes in productivity in NKISs. Gali and Monacelli[5] mention that a full stabilization of PPI inflation implies output conforms its natural level and there is no output gap, in their non-LCP setting under a small open economy, as mentioned. However, our CPI based NKPC Eq.(35) implies that a full stabilization of CPI inflation implies output conforms its natural level and there is no output gap, in our LCP setting under a two-country. This can be understood alternatively and intuitively by comparing Eqs.(5) and (21). To derive Eq.(34), we use Eq.(21) implying that the PPI inflation is affected by changes in nominal exchange rate while we use Eq.(5) to derive Eq.(35).

In addition, Eq.(35) contrasts CPI based NKPC in Monacelli[7]. In his LCP setting, imports purchase foreign goods at the costs in terms of foreign currency while they sell foreign goods at the costs in terms of domestic currency. Because importers maximize their profits, the LOOP gap appears in CPI based NKPC in Monacelli[7]. Our LCP setting is quite different from Monacelli's[7] setting. Goods markets are fully partitioned, there are no importers and each producer prices their goods in terms of consumer's currency. As mentioned in section 2.1.3, the LOOP gap does not appear in Eq.(35), different from Monacelli[7].

2.2 PCP Model

Under the PCP, the LOOP is applied which is given by $P_t\left(h\right) = \mathcal{E}_t P_t^*\left(h\right)$ and $P_t\left(f\right) = \mathcal{E}_t P_t^*\left(f\right)$ hence:

$$P_{H,t} = \mathcal{E}_t P_{H,t}^* \ ; \ P_{F,t} = \mathcal{E}_t P_{F,t}^* \tag{36}$$

and

$$p_{H,t} = e_t + p_{H,t}^* \ ; \ p_{F,t} = e_t + p_{F,t}^*, \tag{37}$$

are applied.

2.2.1 Households

The preference of the representative household, private consumption index, Consumption index, the optimal allocation of any given expenditure within each category of goods and the optimal allocation of expenditures between domestic and foreign goods are given by Eqs.(1), (2), (3), (4), (6) and (7), alike with the LCP model. Because households face same optimization problem, intertemporal and intratemporal optimality conditions are given by Eqs.(8) and (9). The UIP is applied in the PCP model, hence optimal risk-sharing condition is applied in PCP model. Log-linearized definition of the CPI, intertemporal optimality condition and the risk-sharing condition are also given by Eqs.(5), (10) and (11).

2.2.2 Market Clearing

Market clearing condition is given by Eq.(12) alike with the LCP model. Plugging Eqs.(6) and (7), we have Eq.(13). Because of LOOP, Eq.(13) can be rewritten as:

$$Y_t\left(h\right) = \left(\frac{P_t\left(h\right)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t.$$

by utilizing Eq.(36). Plugging Dixit-Stiglitz aggregator of output into this equality yields:

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t, \tag{38}$$

which is demand function consistent with Benigno and Benigno's[1].[5]

Definition of the TOT is given by Eq.(15). Plugging Eq.(36) into Eq.(15) yields:

$$\mathcal{S}_t = \frac{P_{F,t}}{P_{H,t}}, \tag{39}$$

which is applicable only to PCP model because Eq.(36) is not applicable to LCP model.

Log-linearizing Eq.(39), we have:

$$s_t = p_{F,t} - p_{H,t} \tag{40}$$

with $s_t \equiv \ln\mathcal{S}_t$. Eq.(40) is only if applicable to PCP model because Eq.(37) is not applied under the LCP, alike with Eq.(39).

Plugging Eq.(37) into this equality, we have:

$$s_t = \frac{1}{\eta}\left(y_t - y_t^*\right), \tag{41}$$

which clarifies relationship between the TOT and relative output under the PCP. Gali and Monacelli[5] and Benigno and Benigno[1] who assume the PCP derive same equality.

Log-linearizing Eq.(38) yields:

$$y_t = \frac{\eta}{2}s_t + c_t, \tag{42}$$

where we use Eq.(37). As mentioned, Eq.(42) is final form of log-linearized market clearing under the LCP in country H. The difference in price setting behavior between the LCP and the PCP does not affect market clearing.

[5] We do not assume government expenditure. Thus, government expenditure does not appear in those equalities although it appears in Benigno and Benigno[1].

2.2.3 Firms

Firms technology is given by Eq.(17) which can be rewritten as Eq.(18). Thus, log-linearized technology is given by Eq.(19) alike with LCP model.

We assume Calvo-Yun-style price setting behavior alike with LCP model. However, maximization problem which is faced by firms under the PCP is quite simple. Because of $\tilde{P}_{H,t} = \mathcal{E}_t \tilde{P}^*_{H,t}$ and Eq.(36), Eq.(20) can be rewritten as:

$$\max_{\tilde{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k \mathrm{E}_t \left\{ Q_{t,t+k} \left[\left(\tilde{P}_{H,t} - MC^n_{t+k} \right) \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \left(C_{H,t+k} + C^*_{H,t+k} \right) \right] \right\}, \quad (43)$$

which is familiar expression in literatures assuming Calvo pricing. Plugging Eq.(36) into the PPI definition, we have $P_{P,t} = P_{H,t}$ and plugging Eq.(37) into Eq.(21) yields:

$$\pi_{P,t} = \pi_{H,t}. \quad (44)$$

The FONC of Eq.(43) is given by:

$$\mathrm{E}_t \left[\sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left(\tilde{P}_{H,t} - \zeta MC^n_{t+k} \right) \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \left(C_{H,t+k} + C^*_{H,t+k} \right) \right] = 0,$$

which is familiar expression in literatures assuming the PCP. Log-linearizing this equality, we have:

$$\tilde{p}_{H,t} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \mathrm{E}_t \left(mc^n_{t+k} \right),$$

which corresponds to the first equality in Eq.(22). Terms related to the LOOP gap disappear because the LOOP is definitely applied in the PCP model. This equality can be rewritten as follows:

$$\tilde{p}_{H,t} = p_{H,t-1} + \sum_{k=0}^{\infty} (\beta\theta)^k \mathrm{E}_t \left(\pi_{H,t+k} \right) + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \mathrm{E}_t \left(mc_{t+k} \right), \quad (45)$$

which corresponds to one derived by Gali and Monacelli[5]. Because of the LOOP, LOOP gap disappears in Eq.(45), although LOOP gap appears in the first equality in Eq.(25).

2.2.4 Marginal Cost and Natural Rate of Output

Plugging Eq.(9) into the definition of the marginal cost, we obtain Eq.(26) and its log-linearized equality Eq.(27). However, the natural rate of output under the PCP is quite different from one under the LCP at glance. Combining not only Eqs.(14), (18) and Eq.(26) but also $P_{P,t} = P_{H,t}$, we have:

$$\bar{Y}_t = \left[\frac{\zeta^{-1}}{1 - \tau} A_t^{1+\varphi} \left(\frac{P_{H,t}}{P_t} \right)^{-(\sigma\eta - 1)} \right]^{\frac{1}{\sigma + \varphi}} D_t^{-\frac{\varphi}{\sigma + \varphi}},$$

which can be log-linearized as follows:

$$\bar{y}_t = \frac{\omega_1 \omega_2}{\omega_3} a_t - \frac{(\sigma\eta - 1)\,\omega_2}{\omega_3} a_t^*.$$

This equality is consistent with log-linearized natural rate of output under the LCP Eq.(29) although natural rate of output is quite different between the PCP and the LCP before log-linearizing. This implies that differences in price setting behavior do not affect the natural rate of output.

That natural rate of output under the PCP is consistent with one under the LCP implies that there is same relationship between marginal cost and output gap. In fact, plugging Eqs.(42), (41), (19) and (28) into Eq.(27) yields:

$$mc_t = \frac{\omega_1}{2\eta} x_t + \frac{\sigma\eta - 1}{2\eta} x_t^*,$$

which is consistent with Eq.(30). Difference between the PCP and the LCP models is price setting behavior. Because the marginal cost has no relationship with price setting behavior, Eq.(30) is applied under both the PCP and the LCP. Note that Gali and Monacelli[5] show that real marginal cost has relationship with just domestic output gap and their result is different from Eq.(30). This difference stems from our two-country setting. As mentioned, foreign output is not exogenous in our setting and productivity in country F appears in Eq.(29), while foreign output appears in their expression in terms of percentage deviation from its steady state value. In their setting, not foreign productivity but foreign output affects domestic natural rate of output. Foreign output gap no longer affects domestic output gap which stems from percentage deviation of domestic real marginal cost from its steady state value. Because percentage deviation of domestic real marginal cost from its steady state value corresponds to its deviation from its flexible price equilibrium value, foreign output gap disappears in Gali and Monacelli[5]. In fact we have $mc_t = \frac{\omega_1}{2\eta} x_t$ if we regard output in country F as exogenous.

2.2.5 The Demand and Supply Sides

Plugging Eqs.(21), (23), (41) and (42) into Eq.(10) yields NKIS as follows:

$$x_t = \mathrm{E}_t\left(x_{t+1}\right) - \frac{2\eta}{\sigma_\alpha}\hat{r}_t + \frac{2\eta}{\sigma_\alpha}\mathrm{E}_t\left(\pi_{P,t+1}\right) + \frac{\sigma\eta - 1}{\sigma_\alpha}\mathrm{E}_t\left(\Delta x_{t+1}^*\right) + \frac{2\eta}{\sigma_\alpha}\bar{r}_t, \tag{46}$$

which is consistent with NKIS under the LCP Eq.(31). While the LOOP is not applied in LCP model, the LOOP is applied in PCP model. Hence, NKISs are not completely same between both models although those are quite same at glance. Plugging Eq.(44) into Eq.(31), we have:

$$x_t = \mathrm{E}_t\left(x_{t+1}\right) - \frac{2\eta}{\sigma_\alpha}\hat{r}_t + \frac{2\eta}{\sigma_\alpha}\mathrm{E}_t\left(\pi_{H,t+1}\right) + \frac{\sigma\eta - 1}{\sigma_\alpha}\mathrm{E}_t\left(\Delta x_{t+1}^*\right) + \frac{2\eta}{\sigma_\alpha}\bar{r}_t,$$

which is applicable only if to PCP model and $\pi_{H,t}$ replaces $\pi_{P,t}$ in this equality. Because the LOOP is definitely applied in PCP model, neither changes in expected nominal exchange rate nor foreign nominal interest rate appear in NKIS under the PCP.

By rearranging Eq.(45), we have NKPC in country H under the PCP as follows:

$$\pi_{P,t} = \beta \mathrm{E}_t \left(\pi_{P,t+1}\right) + \frac{\lambda \omega_1}{2\eta} x_t + \frac{\lambda \left(\sigma\eta - 1\right)}{2\eta} x_t^*, \tag{47}$$

which is two-country version NKPC derived by Gali and Monacelli[5]. While foreign output gap appears in Eq.(47), that does not appear in the NKPC derived by Gali and Monacelli[5] who assume small open economy where foreign variables are exogenous. Because our model is a two-country model where the foreign variables are endogenous, foreign output gap appears in our NKPC under the PCP. In fact, if we regard output in country F as exogenous, we have:

$$\pi_{P,t} = \beta \mathrm{E}_t \left(\pi_{P,t+1}\right) + \frac{\lambda \omega_1}{2\eta} x_t,$$

which is quite similar to NKPC derived by Gali and Monacelli[5] and can be derived alternatively only if $\sigma\eta = 1$ in our two-country model under the PCP because foreign output gap disappears in such a case. Gali and Monacelli[5] mention that full stabilization of PPI implies that $x_t = \pi_{H,t} = 0$ which is plausible if output gap in country F disappears in Eq.(47). Because of two-country setting, foreign output gap does not disappears as long as we do not assume $\sigma\eta = 1$. Hence, full stabilization of PPI does not necessarily imply $x_t = \pi_{H,t} = 0$ in our two-country setting.

3 Optimal Monetary Policy under the LCP and the PCP

3.1 Welfare Costs

We assume central banks conduct optimal monetary policy. Central banks minimizes welfare costs. Welfare costs consist of the period loss function which is derived by the welfare criterion. Following Woodford[11] and Gali[4], we have second-order approximated utility function as follows:

$$\mathcal{W}_{LCP}^W = -\mathcal{L}_{LCP}^W + \text{t.i.p.} + o\left(\parallel \xi \parallel^3\right) \ ; \ \mathcal{W}_{PCP}^W = -\mathcal{L}_{PCP}^W + \text{t.i.p.} + o\left(\parallel \xi \parallel^3\right) \tag{48}$$

where $\mathcal{L}_{LCP}^W \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t L_{LCP,t}^W$ and $\mathcal{L}_{PCP}^W \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t L_{PCP,t}^W$ denote the loss function in LCP and PCP models, respectively, $\mathcal{W}_{LCP}^W = \frac{1}{2}\left(\mathcal{W}_{LCP} + \mathcal{W}_{LCP}^*\right)$ and $\mathcal{W}_{PCP}^W = \frac{1}{2}\left(\mathcal{W}_{PCP} + \mathcal{W}_{PCP}^*\right)$ denote average welfare criteria in LCP and PCP models, respectively, W_{LCP}^W and W_{PCP}^W denote welfare criteria in country H in LCP and PCP models, respectively with $\mathcal{W} \equiv \sum_{t=0}^{\infty} \mathrm{E}_0 \left(\mathcal{W}_t\right)$ and $\mathcal{W}_t \equiv \frac{U_t - U}{U_C C}$. Further:

$$L_{LCP,t}^W \equiv \frac{1}{2}\left[\frac{\varepsilon}{2\lambda}\pi_t^2 + \frac{\varepsilon}{2\lambda}\left(\pi_t^*\right)^2 + \left(\sigma + \varphi\right)\left(x_t^W\right)^2 + \frac{\left(1+\varphi\right)\eta^2}{4} z_t^2\right], \tag{49}$$

$$L_{PCP}^W \equiv \frac{1}{2}\left[\frac{\varepsilon}{2\lambda}\pi_{H,t}^2 + \frac{\varepsilon}{2\lambda}\left(\pi_{F,t}^*\right)^2 + \left(\sigma + \varphi\right)\left(x_t^W\right)^2 + \frac{\left(1+\varphi\right)\eta^2}{4} z_t^2\right], \tag{50}$$

are period loss function in countries H and F, respectively, , $z_t \equiv s_t - \bar{s}_t$ being the deviation of the TOT from its efficient level, $\bar{s}_t \equiv \frac{1+\varphi\eta}{\eta^2(1+\varphi)} \bar{y}_t^R$ being the efficient level of TOT. Note that we define $v_t^W \equiv \frac{1}{2}\left(v_t + v_t^*\right)$ and $v_t^R \equiv v_t - v_t^*$.

3.2 FONCs for Central Banks

We next briefly mention the FONCs for the central bank. We assume that the central bank in each country conducts optimal monetary policy with commitment cooperatively. Under the LCP, central banks minimizes Eq.(49) and the FONCs for them are given by:

$$
\begin{aligned}
\pi_t^W &= -\frac{1}{\varepsilon}\left(x_t^W - x_{t-1}^W\right), \\
z_t &= 0,
\end{aligned}
$$

where $\mu_{3,t}$ is a Lagrange multiplier associated with the average block of the NKPC. Because of commitment, lagged multiplies appear in the FONCs.

Under the PCP, central banks minimizes Eq.(50) and the FONCs for them are given by:

$$
\begin{aligned}
\pi_t^W &= -\frac{1}{\varepsilon}\left(x_t^W - x_{t-1}^W\right), \\
\pi_{P,t}^R &= -\frac{(1+\varphi)\,\eta^2}{\varepsilon\,(1+\eta\varphi)}\left(z_t - z_{t-1}\right).
\end{aligned}
$$

3.3 Calibration

We run a series of dynamic simulations and adopt the following benchmark parameterization. We set the price stickiness θ, the subjective discount factor β, the elasticity of substitution across goods ε and the inverse of the labor supply elasticity φ equal to 0.75, 11, 0.99, 3, respectively, which are consistent with quarterly time periods in the model.[6] We compare two cases. One of them is special case in which $\sigma = \eta = 1$ and another one is general case in which $\sigma = 3$ and $\eta = 4.5$. Note that $\sigma = \eta = 1$ is assumed by Gali and Monacelli[5] while $\sigma = 3$ and $\eta = 4.5$ are assumed by Benigno and Benigno[1]. We assume that productivity shifters are described according to the following AR(1) processes:

$$
a_t = \rho a_{t-1} + \xi_t \ ; \ a_t^* = \rho a_{t-1}^* + \xi_t^*,
$$

where ξ_t and ξ_t^* denote the i.i.d. shocks. We set ρ equal to 0.9. To examine the impulse response functions (IRFs), we consider one percent changes in the productivity shifter in country H a_t, and the productivity shifter in country F, a_t^*.

Impulse responses to one percent increase in productivity in country H in special case are shown in Figure 1 while macroeconomic volatilities are shown in 3rd and 4th columns in Table 1[7]. To eliminate the effect of changes in productivity, central banks decreases nominal interest rate under both the LCP and the PCP (Panels 7 and 8). Because of this, output gap in countries H and F are completely stabilized. Under the PCP, PPI inflation rate in countries H and F is completely stabilized (Panels 3 and 4). This result is consistent with Gali and Monacelli[5] who imply that PPI inflation targeting brings zero output

[6] $\theta = 0.75$ implies that average length of price contracts equal to 4.

[7] There are 2 eigenvalues larger than 1 in modulus for 2 forward-looking variables. Hence the rank condotion is verified

gap. This result can be understood by paying attention to Eq.(47). Plugging $\sigma = \eta = 1$ into Eq.(47) yields:

$$\pi_{P,t} = \beta \mathrm{E}_t \left(\pi_{P,t+1}\right) + \frac{\lambda \omega_1}{2\eta} x_t,$$

which is NKPC in the special case under the PCP. This NKPC implies that stabilizing PPI inflation brings stabilizing output gap simultaneously and is consistent with one derived by Gali and Monacelli[5] although the slope of our NKPC is slightly different from theirs because we assume a two-country economy.

Under the LCP, not PPI inflation but CPI inflation is stabilized and this result is quite different not only from Gali and Monacelli[5] but also other DSGE literatures assuming an open economy (Panels 5 and 6). This can be understood by paying attention to Eq.(35) which implies that CPI inflation becomes zero when output gap in countries H and F are stabilized. Hence, it may be said that CPI inflation targeting brings completely stabilizing output gap. Interestingly, nominal exchange rate is completely stabilized under the LCP which is consistent with Devereux and Engel[3] developing NOEM model, assuming the LCP and showing that fixed exchange rate is optimal regime from the viewpoint of maximizing welfare. This result stems from stabilizing CPI inflation rate. Perfect stabilization in CPI inflation is consistent with perfect stabilization in the CPI level.[8] In our model, PPP is always applied hence $e_t + p_t^* = p_t$ is applied. Perfect stabilization in CPI inflation implies $p_t = p_t^* = 0$ which is consistent with $e_t = 0$. Thus, under the LCP, there are neither changes in the CPI inflation nor nominal exchange rate.

Impulse responses to one percent increase in productivity in country H in the general case are shown in Figure 2.[9] In the general case under the PCP, inflation–output gap trade-offs are no longer dissolved simultaneously although Gali and Monacelli[5] show that that trade-offs dissolved simultaneously (Panels 1 to 4). Because our model is a two-country model, foreign output is endogenous while it is exogenous in Gali and Monacelli[5]'s small open economy model. In a small economy setting, foreign output gap disappears in the NKIS although that appears in the NKIS, as shown in Eq.(47). foreign output gap disappears in Eq.(47) only if $\sigma\eta = 1$. Hence, neither output gap nor PPI inflation stabilized simultaneously.

However, although output gap in countries H and F is not stabilized, CPI inflation is completely stabilized under the LCP. This can be understood by paying attention to Eq.(35). Average output gap is always stabilized not only under the PCP but also under the LCP (Rows 3 and 4 in Table 1). Hence, CPI inflation is stabilized because Eq.(35) can be rewritten as:

$$\pi_t = \beta \mathrm{E}_t \left(\pi_{t+1}\right) + \kappa_\alpha x_t^W.$$

In this NKPC, the slope is not affected by σ and η. Thus, result on volatility is not different between the special and the general cases. In addition, CPI inflation is completely stabilized, there is no fluctuation in the nominal exchange rate, alike with the special case (Panel 13). As mentioned, our result that there

[8]We assume zero inflation deterministic steady state.

[9]There are 2 eigenvalues larger than 1 in modulus for 2 forward-looking variables. Hence the rank condotion is verified

is no fluctuation in the nominal exchange rate under the LCP is consistent with the result of Devereux and Engel[3]. Devereux and Engel[3] assume Arminton Form of consumption which implies $\eta = 1$.[10] Now, we apply more general setting such as $\eta = 4$ while our result on fluctuations in nominal exchange rate is consistent with their result. This implies that Devereux and Engel[3]'s finding can be applied in general parameterization. We have further discussion on this topic in next section.

4 Macroeconomic Volatilities and Welfare Costs

In this section, we focus on macroeconomic volatilities and welfare costs under varying the relative risk aversion σ and the elasticity of substitution between goods produced in countries H and F η. There are many macroeconomic variables in our model and we focus on some important variables which is related to our loss functions Eqs.(49) and (50) and the nominal exchange rate.

Figure 3 shows effects on macroeconomic volatilities of varying the relative risk aversion $\sigma \in [1, 10]$ and the elasticity of substitution between goods produced in countries H and F $\eta \in [1, 10]$. Under the PCP, volatility of PPI inflation is definitely zero when $\eta = 1$ although the higher the η the higher the volatility (Panel 2). When $\eta = 1$, one of FONCs related to relative inflation for central banks under the PCP can be rewritten as:

$$\pi_{P,t}^{R} = -\frac{1}{\varepsilon}\left(x_t^R - x_{t-1}^R\right) \tag{51}$$

because $z_t = x_t^R$ is applied when $\eta = 1$. Along with another FONC related to average output gap for central banks under the PCP, those FONCs imply that stabilization in PPI inflation strictly consistent with stabilization in output gap. Hence, volatility of PPI inflation is definitely zero when $\eta = 1$. Equally, this implies that $\sigma = \eta = 1$ is not a sufficient condition to dissolve inflation–output gap trade-offs but just $\eta = 1$ is a sufficient condition to dissolve that trade-offs under the PCP. As implied by calibration in former section, volatility of CPI inflation is definitely zero regardless of σ and η under the LCP (Panel 3). This stems from NKIS under the LCP Eq.(35). Because of FONCs for central bank related to average inflation, there is no fluctuation in average output gap regardless of σ and η. This immediately implies that there is no fluctuation in CPI inflation countries H and F regardless of σ and η under the LCP.

Average output gap is completely stabilized under both the LCP and the PCP regardless of σ and η (Panels 5 and 6). This stems from the FONC for central banks related to average inflation which are common under both the LCP and the PCP and imply that average inflation and output gap are stabilized simultaneously. TOT deviation from efficient level is definitely zero regardless of σ and η under the LCP (Panel 7). This stems from the FONC for central banks under the LCP related to TOT deviation from efficient level. However, TOT deviation from efficient level is definitely zero under the PCP only if $\eta = 1$ (Panel 8). In that case, $z_t = x_t^R$ is applied and there is no fluctuation in output gap in countries H and F, as implied by Eq.(51). Both PPI inflation and output gap are completely stabilized when $\eta = 1$. Hence, TOT deviation from efficient

[10] In that case, Eq.(2) is rewritten by $C_t = 2C_{H,t}^{\frac{1}{2}}C_{F,t}^{\frac{1}{2}}$

level is definitely zero through complete stabilization in output gap in countries H and F. However, complete stabilization in TOT deviation from efficient level is no longer achieved when $\eta = 1$ is not applied.

Now, we discuss on volatility of the nominal exchange rate. As many literatures show, we show that optimal monetary policy is flexible exchange rate regime (Panel 10) under the PCP. On contrary, the nominal exchange rate is definitely zero regardless of σ and η under the LCP. Even if the LCP is assumed, the PPP is applied which implies that $s_t = p_t - p_t^*$. Because of optimal monetary policy, CPI inflation is definitely stabilized which is consistent with zero fluctuation in the CPI level under the LCP. Thus, the nominal exchange rate is definitely stabilized regardless of σ and η under the LCP. As mentioned in former section, our result is consistent with Devereux and Engel[3]'s result which shows that optimal monetary policy under the LCP is consistent with fixed exchange rate regime although they assume Armington form of consumption which corresponds to $\eta = 1$. Our model does not assume Armington form of consumption and there is no parametric restriction in η. Thus, our result implies that Devereux and Engel[3]'s policy implication is not applied in special parameterization but applied in general setting. In addition, we derive more important policy implication. Optimal monetary policy under the LCP stabilizes the CPI inflation definitely and coincides with complete stabilization in the nominal exchange rate. Furthermore, monetary policy which stabilizes the CPI inflation or the nominal exchange rate is optimal under the LCP. We do not analyze an explicit targeting rule or a regime such as CPI inflation targeting and fixed exchange rate regime. However, it can be said that CPI inflation targeting and fixed exchange rate regime are optimal and equivalent under the LCP although there is some room to discuss precisely.

Finally, we discuss effects on welfare costs varying the relative risk aversion σ and the elasticity of substitution between goods produced in countries H and F η. When we introduce $\beta \to 1$ in Eq.(48), we have welfare criteria as follows:

$$\begin{aligned}
\mathcal{L}_{LCP,t}^{W} &\equiv \frac{1}{2}\left[\frac{\varepsilon}{2\lambda}\mathrm{var}\left(\pi_t\right) + \frac{\varepsilon}{2\lambda}\mathrm{var}\left(\pi_t^*\right) + \left(\sigma+\varphi\right)\mathrm{var}\left(x_t^W\right) + \frac{\left(1+\varphi\right)\eta^2}{4}\mathrm{var}\left(z_t\right)\right], \\
\mathcal{L}_{PCP,t}^{W} &\equiv \frac{1}{2}\left[\frac{\varepsilon}{2\lambda}\mathrm{var}\left(\pi_{P,t}\right) + \frac{\varepsilon}{2\lambda}\mathrm{var}\left(\pi_{P,t}^*\right) + \left(\sigma+\varphi\right)\mathrm{var}\left(x_t^W\right) + \frac{\left(1+\varphi\right)\eta^2}{4}\mathrm{var}\left(z_t\right)\right],
\end{aligned}$$

and we utilize these equalities to calculate welfare costs.

Figure 4 depicts effects on welfare costs varying the relative risk aversion σ and the elasticity of substitution between goods produced in countries H and F η. We have already discussed some macroeconomic volatilities which comprise welfare costs. Thus, we can understand effects on welfare losses varying σ and η. Because there are no fluctuations in average output gap, CPI inflation in countries H and F and the TOT deviation from efficient level regardless of σ and η, there are no welfare losses regardless of σ and η under the LCP. However, there are no welfare costs only if $\eta = 1$ because there are no fluctuations in PPI inflation in countries H and F and the TOT deviation from efficient level under the PCP. Except for $\eta = 1$, there are some welfare costs because there are fluctuations in PPI inflation in countries H and F and the TOT deviation from efficient level under the PCP even if optimal monetary policy is conducted.

5 Conclusion

We analyze optimal monetary policy under the LCP model by comparing with the PCP model. We have two main findings as follows. We insist that optimal monetary policy under the LCP brings no fluctuations not in the PPI inflation rate but in the CPI inflation rate. Roughly speaking, optimal monetary policy under the LCP is the CPI inflation targeting. This result is quite different from the result on Gali and Monacelli[5]. We show that there are no fluctuations in the nominal exchange rate under the LCP. Roughly speaking, optimal monetary policy under the LCP is consistent with fixed exchange rate regime and that is shown by Deverereux and Engel[3]. We can reconcile with Deverereux and Engel[3] and we derive our policy implication complying Woodford's[10] motivation.

Our finding shed light on Mussa's puzzle which focuses on the fact that comovement of nominal exchange rate and real exchange rate along with Betts and Devereux[2] although they do not analyze optimal monetary policy. Because complete stabilization in CPI inflation rate coincides with complete stabilization in the nominal exchange rate under the LCP, one of answers to Mussa's puzzle may be optimal monetary under the LCP. Solving Mussa's puzzle along with the result on this paper is one of future research agenda.

Appendix

A Gains from International Monetary Cooperation

To be added.

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Table 1: Macroeconomic Volatility to One Percent Increase in Productivity

Variables	Pricing	Special ($\sigma = \eta = 1$)		General ($\sigma = 3$, $\eta = 4.5$)	
		a_t	a_t^*	a_t	a_t^*
x_t^W	LCP	0.0000	0.0000	0.0000	0.0000
	PCP	0.0000	0.0000	0.0000	0.0000
π_t^W	LCP	0.0000	0.0000	0.0000	0.0000
	PCP	0.0000	0.0000	0.0000	0.0000
x_t	LCP	0.0000	0.0000	0.0028	0.0028
	PCP	0.0000	0.0013	3.2172e-004	3.2172e-004
x_t^*	LCP	0.0000	0.0000	0.0028	0.0028
	PCP	0.0000	0.0000	3.2172e-004	3.2172e-004
$\pi_{P,t}$	LCP	0.0051	0.0051	0.0011	0.0011
	PCP	0.0000	0.0000	1.0486e-004	1.0486e-004
$\pi_{P,t}^*$	LCP	0.0051	0.0051	0.0011	0.0011
	PCP	0.0000	0.0000	1.0486e-004	1.0486e-004
π_t	LCP	0.0000	0.0000	0.0000	0.0000
	PCP	0.0051	0.0051	0.0012	0.0012
π_t^*	LCP	0.0000	0.0000	0.0000	0.0000
	PCP	0.0051	0.0051	0.0012	0.0012
$\hat{r}_t$	LCP	0.0011	0.0011	0.0023	0.0023
	PCP	0.0021	1.1460e-006	0.0024	0.0020
$\hat{r}_t^*$	LCP	0.0011	0.0011	0.0023	0.0023
	PCP	1.1460e-006	0.0021	0.0020	0.0024
y_t	LCP	0.0229	0.0000	0.0191	0.0038
	PCP	0.0229	0.0000	0.0217	0.0064
y_t^*	LCP	0.0000	0.0229	0.0038	0.0191
	PCP	0.0000	0.0229	0.0064	0.0217
s_t	LCP	0.0229	0.0229	0.0051	0.0051
	PCP	0.0229	0.0229	0.0063	0.0063
z_t	LCP	0.0000	0.0000	0.0000	0.0000
	PCP	0.0000	0.0000	0.0012	0.0012
e_t	LCP	0.0000	0.0000	0.0000	0.0000
	PCP	0.0229	0.0229	0.0057	0.0057

Figure 1: IRFs to Productivity in Country H in the Special Case ($\sigma = \eta = 1$)

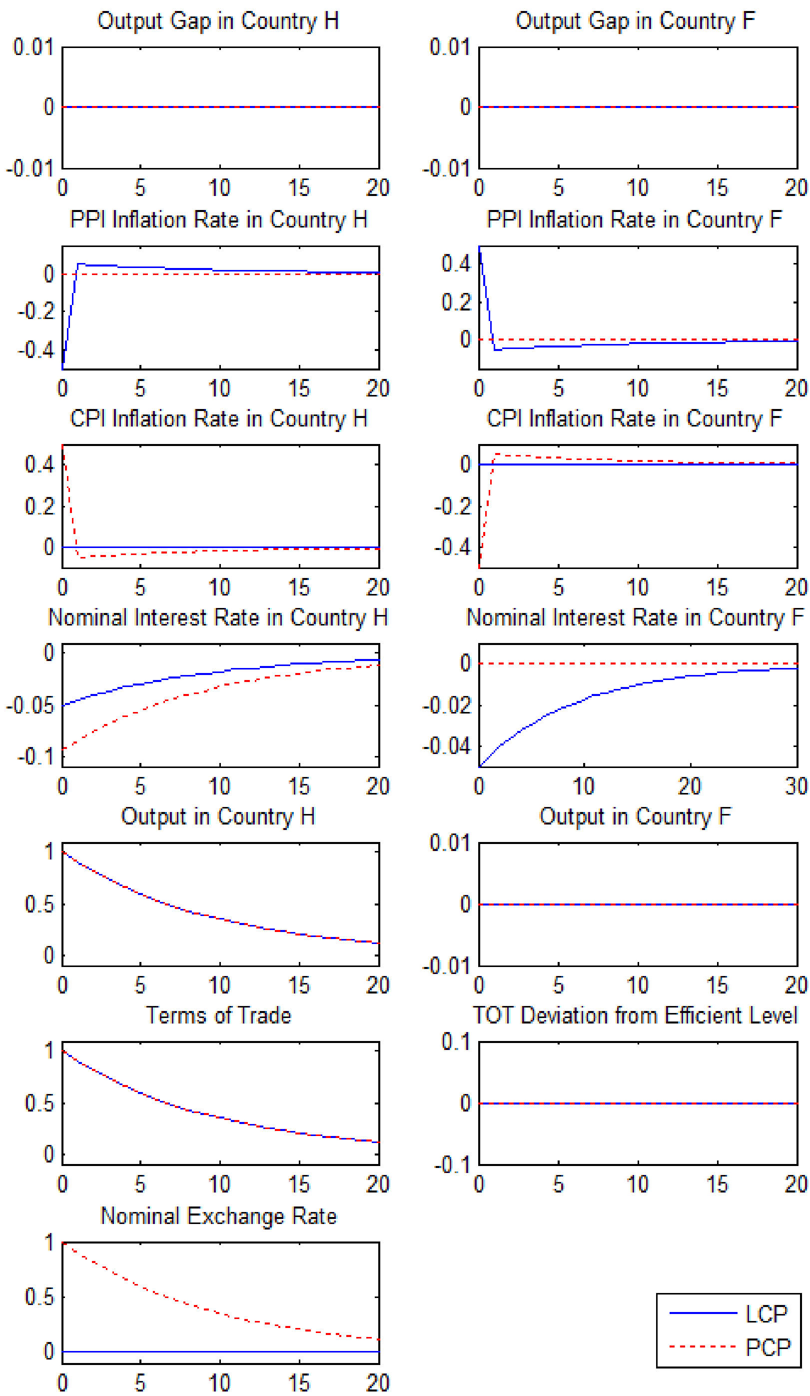

Figure 2: IRFs to Productivity in Country H in the General Case ($\sigma = 3$, $\eta = 4.5$)

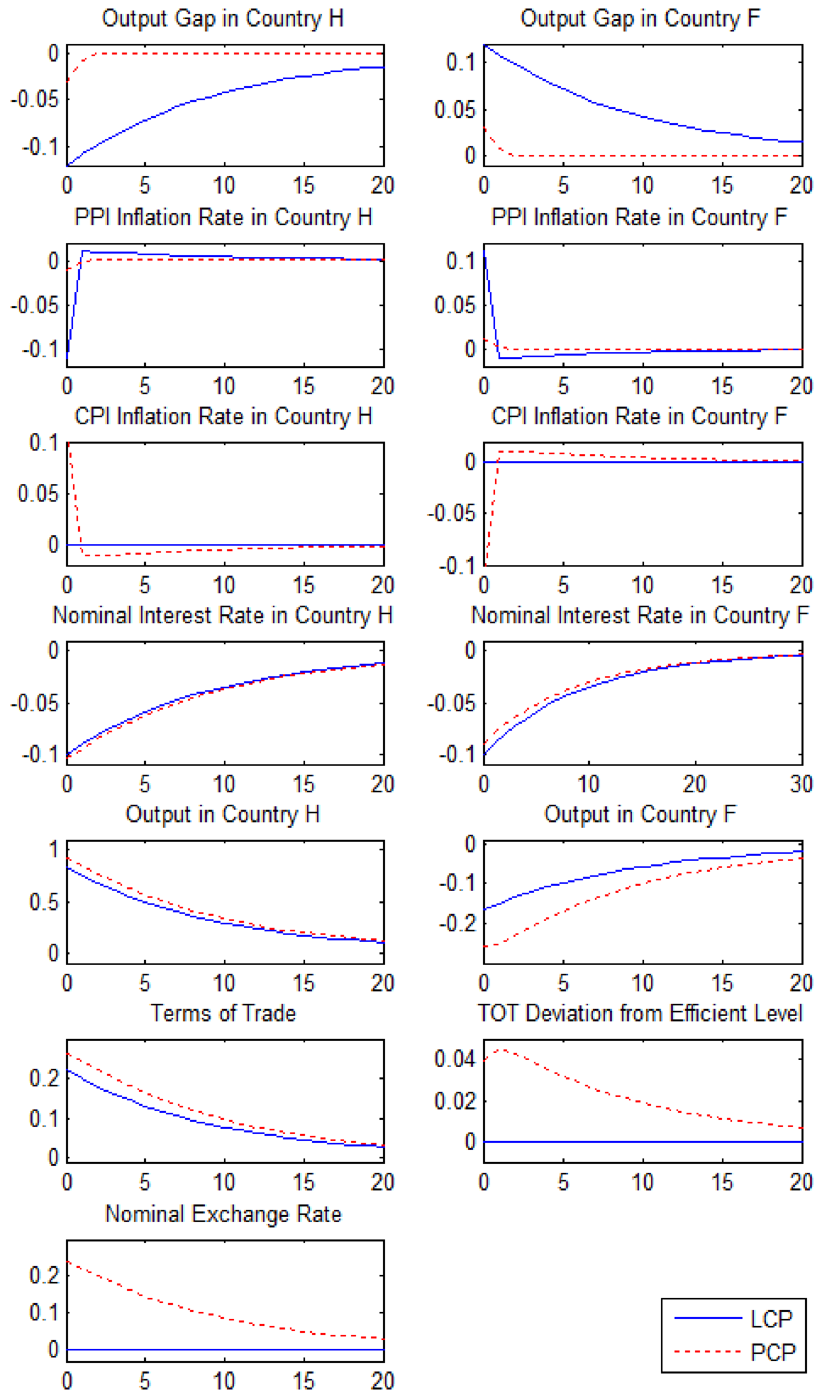

Figure 3: Effects on Macroeconomic Volatilities of Varying Relative Risk Aversion σ and Elasticity of Substitution between Goods Produced in Countries H and F η

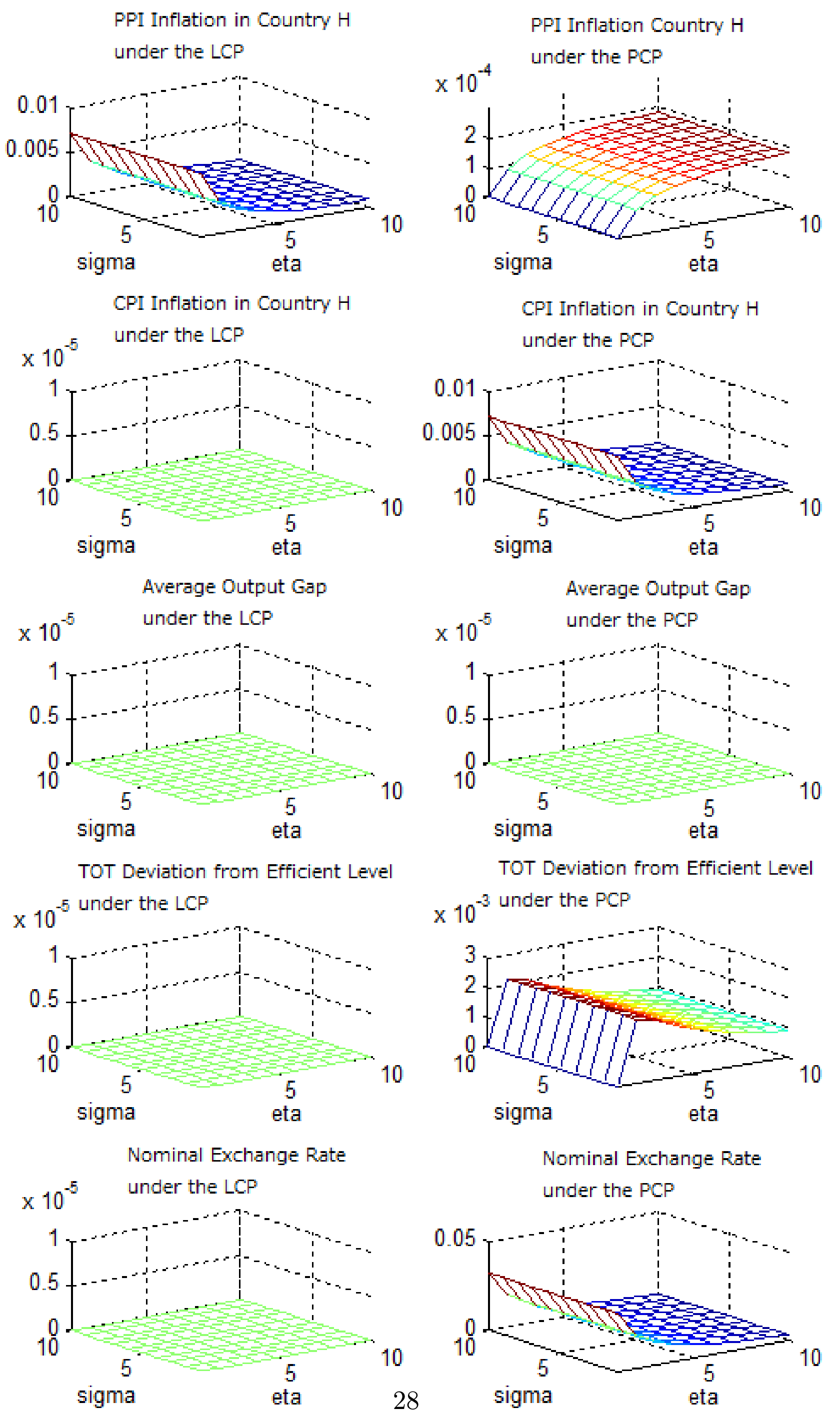

Figure 4: Effects on Welfare Costs of Varying Relative Risk Aversion σ and Elasticity of Substitution between Goods Produced in Countries H and F η