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Sovereign Risk in the Euro Zone and Monetary Policy

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Sovereign Risk in the Euro Zone and Monetary Policy*

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Abstract

We analyze monetary policy in a currency union with sovereign risk using a three-country model including a two-country currency union and introduce an ad hoc assumption that one of the two countries is exposed to sovereign risk. In our model, if expected fiscal revenue is less than current public debt, one country can default whereas the other country cannot default. In a canonical setting that analyzes optimal monetary policy, the optimal monetary policy is consistent with the Taylor rule. However, the Taylor rule is not optimal and persistently causes defaults in our model. In contrast, an interest rate peg, namely government bonds swaps, terminates defaults after one period and is optimal in our model.

Keywords: Sovereign Risk; European Crisis; Fixed-interest-rate Rule; Fiscal Theory of Price Level

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1 Introduction

After the Lehman Crisis in Sep. 2008, inflation concerns appeared accompanied by a slow economic recovery in the euro zone, including Germany, through a depreciation in the euro. However, the IMF and the member countries of the European Monetary Union (EMU) faced the necessity of providing financial support to Greece in Mar. 2010 because of the Greek fiscal crisis in late 2009. To prevent contagion of this crisis to other member countries in the EMU, the European Stability Mechanism (ESM) was established at that time. However, unfortunately this crisis extended to other member countries in the EMU. Ireland faced a financial crisis stemming from the Greek crisis in autumn 2010 and it was decided that Ireland would be supported financially in Nov. 2010. In 2011, Portugal faced a fiscal crisis and financial support for Portugal was provided in Apr. 2011. In addition, fiscal crisis concerns also developed over Spain, Italy and other member countries in the EMU and these concerns increased government bond yields in these countries. To reverse increases in government bond yields, the European Central Bank (ECB) intervened and supported the government bond market in those countries. At present, some euro zone countries are facing inflation worries while other countries are facing sovereign risk concerns, and therefore conducting monetary policy in the euro zone in the presence of both types of problems is difficult because of the inconsistency between stabilizing inflation and reducing sovereign risk. The ECB, as the only central bank in the EMU, conducts a common monetary policy in the EMU that cannot target multiple objectives.

The member countries in the EMU have debated these conflicting objectives. The member countries held an urgent high-level meeting in Brussels in Jul. 2011, and developed a policy response to concern over the financial uncertainty stemming from the Greek crisis. The policy includes expansion of the European Financial System Facility (EFSF), purchase of Greek government bonds and a 20% debt waiver on these bonds. In addition, the private financial sector agreed to roll over their 7.5-year government bonds for 15-year or 30-year government bonds. Because of concern about domestic public sentiment, Angela Merkel, the German Chancellor, furiously opposed the policy, in particular the issuing of euro zone bonds and increasing the EFSF's funding. How can the euro zone best cope with this problem? Our research agenda is to identify policies that address not only the problems in the European economy but also those in the world economy.

Our paper has policy implications for the current EU situation including the support of government bond markets in Portugal, Spain and Greece by the EFSF or ECB, even though such support may cause loosening of the fiscal discipline in these countries. However, loosening of the fiscal discipline can be presumed by selling operations if these countries subsequently face inflation concerns. In short, combining buying operations for government bonds issued by countries where the nominal interest rate is increasing and selling operations for those issued by countries facing inflation concerns, namely government bond swaps, could dissolve the trade-off between inflation concerns and sovereign risk in a currency union, especially the euro zone.

A notable feature of our model is that a country in a currency union may go into default. Such a country issues government bonds with sovereign risk. This feature makes it possible to examine monetary policy in the current euro zone context because it includes government issuance of bonds with sovereign risk. Woodford[13] shows the importance of the relationship between the first and the second moves in a game in conducting monetary policy. Our results verify the results of Woodford[13]. In an economy with sovereign risk, Ricardian equivalence does not exist and fiscal policy is an active policy, namely the first move policy. In such an economy, the Taylor rule, which is an active policy, namely the first move policy, should not be adopted, and instead a fixed-interest-rate rule which is a passive policy, namely the second move policy, should be adopted. The Taylor rule does not provide a unique solution in a rational expectations equilibrium (REE), whereas a fixed-interest-rate rule always guarantees a unique solution in an REE in such an economy.

In many recent papers analyzing optimal monetary policy, including Woodford[15], Ricardian equivalence is implicitly assumed for simplicity. It is shown that allocations brought about by optimal monetary policy can be replicated approximately by introduction of the Taylor rule. Discussions regarding optimal monetary policy under this assumption are expanded to the situation of a currency union by Beetsma and Jensen[1], Benigno[4] and Ferrero[6]. However, needless to say, these authors discuss optimal monetary policy in a currency union by assuming Ricardian equivalence.[1]

Uribe[12] presents important policy implications for this problem, although his model is simplistic. His model features the fiscal theory of the price level (FTPL), which is advocated by Cochrane[5], Sims[11] and Woodford[13]. Uribe[12] analyzes monetary policy in an economy with sovereign risk and shows that the Taylor rule prolongs the default period, whereas a monetary policy rule that keeps the interest rate on risky assets equal to the interest rate on nonrisky assets and pegs them at their steady-state value, namely the fixed-interest-rate rule results in an immediate convergence to default. However, exogenous production, namely an endowment economy, is assumed in his model and the marginal utility of consumption is constant over time. Because of this, the GDP gap and dynamics in inflation, which are focused on in the dynamic stochastic general equilibrium (DSGE) literature, cannot be discussed in detail. In addition, his model has no foreign sector, and thus interaction with foreign economies cannot be discussed. Hence, discussion on monetary policy in an economy with sovereign risk is only the first step.

We introduce Uribe[12]'s idea into our three-country model, which is based on Gali and Monacelli[7]'s small open economy model, and we analyze monetary policy in a currency union with sovereign risk. As mentioned, we develop a policy implication that the fixed-interest-rate rule results in an immediate convergence to default as shown by Uribe[12]. In addition, we show that fixed-interest-rate rule is a sufficient condition for uniqueness of the REE solution. Interestingly, we also show that volatilities in inflation and the foreign GDP gap under the fixed-interest-rate rule are smaller than those under the Taylor rule.

The paper is organized as follows. Section 2 describes the trade-off faced by ECB between inflation concerns and sovereign risk in countries undergoing a fiscal crisis. Section 3 derives our DSGE model with sovereign risk. Section 4 presents the macroeconomic dynamics numerically. Section 5 discusses the solution uniqueness. Section 6 concludes.

[1]Ricardian equivalence is not necessarily satisfied in Ferrero[6]. However, his log-linearized model satisfies Ricardian equivalence around the steady state.

2 Inflation Concerns and Sovereign Risk

The ECB defines its role as maintaining the purchasing power of the euro and price stability in the euro zone, and conducts monetary policy to keep inflation at 2% or less in the short term and around 2% on average over the midterm. DSGE models imply that inflation is stabilized by increasing the nominal interest rate. An increase in the nominal interest rate involves an increase in the natural interest rate and prevents an increase in the GDP gap. That is, the nominal interest rate removes inflationary pressure and stabilizes the inflation rate. In fact, as shown in Fig. 11, the policy interest rate, namely the nominal interest rate, increases when inflation increases and vice versa. After the Lehman Crisis in Sep. 2008, the policy interest rate was suddenly reduced because of a liquidity shortage in the euro zone, although it had been raised in Apr. 2011. This increase in the policy interest rate resulted from a 12-month inflation figure of 2.6% announced in Mar. 2011, which was the highest figure in the previous 29 months. Following on from this, a new 12-month inflation rate of 2.6 % was announced in Jun. 2011, and the policy rate was increased in Jul. 2011.

In contrast, there are several countries, including Greece, where a fiscal crisis emerged. Although the announced Greek fiscal deficit was 3.6% of GDP, it was later revealed that the actual deficit was 13.6%. Greek government bonds were sold in large quantities. As shown in Fig. 22, long-term interest rates in Greece increased significantly. The yield on Greek 10-year government bonds increased to 17.83% in Jun. 2011 and the spread in yields between Greek government bonds and German federal bonds widened to 14.81%. Following the discovery of Greece's huge fiscal deficit, other European countries' fiscal deficits received the attention of investors. For example, Irish and Portuguese fiscal deficits were 14.7% and 8% of GDP, and long-term interest rates in those countries increased suddenly when the yield on Greek government bonds increase relative to the yield on German federal bonds. Furthermore, the government bond yields for those countries were increasing relative to the yield on German federal bonds, such that the yield on Portuguese 10-year government bonds reached 8.5% in Apr. 2011 and the yield on Irish 10-year government bonds reached 8.5% in Jul. 2011. In such a situation, an increase in the policy interest rate increases funding costs in those countries and may increase the possibility of default. Hence, monetary easing is necessary to avoid default on government bonds. Monetary easing involves lowering the policy interest rate to stabilize inflation in the euro zone. At present, the ECB faces a trade-off between a restrictive monetary policy to stabilize inflation and a monetary easing policy to avoid default on government bonds.

3 The Model

We derive a three-country model including a currency union based on Gali and Monacelli[7]'s small economy model. The currency union consists of countries H and F, which organize a monetary union. We assume that there is a default risk in country F and the mechanism of default follows Uribe[12]. The households on the interval $[0, \alpha)$ belong to country H while those on the interval $[\alpha, 1]$ belong to country F. Country A is outside the currency union where the households line up on the interval $[1, 2]$.

3.1 Households

A representative household's preferences are given by:

$$\mathcal{U}_H \equiv \mathrm{E}_0 \left(\sum_{t=0}^{\infty} \beta^t U_{H,t} \right) \; ; \mathcal{U}_F \equiv \mathrm{E}_0 \left(\sum_{t=0}^{\infty} \beta^t U_{F,t} \right) \; ; \; \mathcal{U}^* \equiv \mathrm{E}_0 \left(\sum_{t=0}^{\infty} \beta^t U_t^* \right) \tag{1}$$

where $U_{H,t} \equiv \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\varphi} N_{H,t}^{1+\varphi}$, $U_{F,t} \equiv \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\varphi} N_{F,t}^{1+\varphi}$ and $U_t^* \equiv \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\varphi} (N_{F,t})^{1+\varphi}$ denotes the period utility in countries H, F and A, respectively, E_t denotes the expectation conditional on the information set at period t, $\beta \in (0,1)$ denotes the subjective discount factor, C_t denotes the consumption index, $N_{H,t} \equiv \int_0^{\alpha} N_{H,t}(h)\, dh$, $N_{F,t} \equiv \int_{\alpha}^{1} N_{H,t}(f)\, df$ and $N_t^* \equiv \int_1^2 N_t^*(a)\, da$ denote hours of labor in countries H, F and A, respectively, and φ denotes the inverse of the elasticity of labor supply. Note that variables in country A are accompanied by an asterisk while those in the currency union are not. The consumption index is defined as follows:

$$C_t \equiv 2 C_{E,t}^{\frac{1}{2}} C_{A,t}^{\frac{1}{2}}, \tag{2}$$

where $C_{E,t} \equiv \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} C_{H,t}^{\alpha} C_{F,t}^{1-\alpha}$ denotes consumption in the currency union, $C_{H,t} \equiv \left[\left(\frac{1}{\alpha} \right)^{\frac{1}{\varepsilon}} \int_0^{\alpha} C_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right]^{\frac{\varepsilon}{\varepsilon-1}}$, $C_{F,t} \equiv \left[\left(\frac{1}{1-\alpha} \right)^{\frac{1}{\varepsilon}} \int_{\alpha}^{1} C_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}}$ and $C_{A,t} \equiv \left[\int_1^2 C_t(a)^{\frac{\varepsilon-1}{\varepsilon}} da \right]^{\frac{\varepsilon}{\varepsilon-1}}$ denotes consumption subindexes of the continuum of differentiated goods produced respectively in countries H, F and A, $C_t(h)$, $C_t(f)$ and $C_t(a)$ denote generic goods produced in countries H, F and A, respectively, $\sigma > 1$ denotes the degree of relative risk aversion and $\varepsilon > 1$ denotes the elasticity of substitution across goods. Consumption in country A is defined analogously to Eq.(2).

Total consumption expenditures are given by $P_{H,t}C_{H,t}+P_{F,t}C_{F,t}+P_{A,t}C_{A,t} = P_t C_t$, where $P_{H,t} \equiv \left[\left(\frac{1}{\alpha} \right) \int_0^{\alpha} P_t(h)^{1-\varepsilon} dh \right]^{\frac{1}{1-\varepsilon}}$, $P_{F,t} \equiv \left[\left(\frac{1}{1-\alpha} \right) \int_{\alpha}^{1} P_t(f)^{1-\varepsilon} df \right]^{\frac{1}{1-\varepsilon}}$ and $P_{A,t} \equiv \left[\int_1^2 P_t(a)^{1-\varepsilon} da \right]^{\frac{1}{1-\varepsilon}}$ denote the producer price indexes (PPIs) in countries H and F, respectively. In addition:

$$P_t \equiv P_{H,t}^{\alpha} P_{F,t}^{1-\alpha}, \tag{3}$$

denotes the consumer price index (CPI) and:

$$P_{E,t} \equiv P_{H,t}^{\alpha} P_{F,t}^{1-\alpha} \tag{4}$$

denotes the price index of goods produced in the currency union. The CPI in country A is defined analogously to Eq.(3).

By log-linearizing (3) and taking a first differential, we have:

$$\pi_t = \frac{1}{2}\pi_{E,t} + \frac{1}{2}\pi_{A,t}, \tag{5}$$

where $\pi_t \equiv p_t - p_{t-1}$ denotes CPI inflation and $\pi_{E,t} \equiv p_{E,t} - p_{E,t-1}$ and $\pi_{A,t} \equiv p_{A,t} - p_{A,t-1}$ denote PPI inflation in a currency union and country A, respectively. Note that we define $v_t \equiv \frac{dV_t}{V}$, which is the percentage deviation from an arbitrary variable's steady-state value where V_t is an arbitrary variable and V is the arbitrary variable's steady-state value. Similar to Eq.(5), we have:

$$\pi_{E,t} = \alpha \pi_{H,t} + (1-\alpha)\,\pi_{F,t} \tag{6}$$

by log-linearizing Eq.(4), where $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$ and $\pi_{F,t} \equiv p_{F,t} - p_{F,t-1}$ denote PPI inflation in countries H and F, respectively.

By solving cost minimization problems for households, we have the optimal allocation of expenditures as follows:

$$C_t(h) = \frac{1}{\alpha}\left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}\ ;\ C_t(f) = \frac{1}{1-\alpha}\left(\frac{P_t(f)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t}. \tag{7}$$

Hence, total demand for goods produced in countries H and F is given by:

$$C_{H,t} = \alpha\left(\frac{P_{H,t}}{P_t}\right)^{-1} C_t\ ;\ C_{F,t} = (1-\alpha)\left(\frac{P_{F,t}}{P_t}\right)^{-1} C_t. \tag{8}$$

In addition, total demand for goods produced in a currency union and country A is given by:

$$C_{E,t} = \frac{1}{2}\left(\frac{P_{E,t}}{P_t}\right)^{-1} C_t\ ;\ C_{A,t} = \frac{1}{2}\left(\frac{P_{A,t}}{P_t}\right)^{-1} C_t. \tag{9}$$

Similar to Eq.(7), we have:

$$C_t(a) = \left(\frac{P_t(a)}{P_{A,t}}\right)^{-\varepsilon} C_{A,t}, \tag{10}$$

by solving the cost minimization problem for households in country A.

By aggregating households' budget constraints, we have:

$$\begin{aligned} D_{H,t} + W_{H,t}N_{H,t} + \Gamma_{H,t} &\geq P_tC_t + \mathrm{E}_t\left(Q_{t,t+1}D_{H,t+1}\right), \\ D_{F,t} + W_tN_{F,t} + \Gamma_t &\geq P_tC_t + \mathrm{E}_t\left(Q_{t,t+1}D_{F,t+1}\right), \end{aligned} \tag{11}$$

where $\mathrm{E}_t(Q_{t,t+1})$ denotes the stochastic discount factor, $D_{H,t}$ and $D_{F,t}$ denote the nominal payoff in period t of the portfolio held at the end of period $t-1$ in countries H and F, respectively, $W_{H,t}$ and $W_{F,t}$ denote the nominal wage in countries H and F, respectively, and $\Gamma_{H,t}$ and $\Gamma_{F,t}$ denote profits from ownership of the firms in countries H and F, respectively. The first and second equalities are the aggregated budget constraints in countries H and F, respectively.

Similar to Eq.(11), the household budget constraint in country A is given by:

$$D_t^* + W_t^*N_{F,t} + \Gamma_t^* \geq P_tC_t + \mathrm{E}_t\left(Q_{t,t+1}D_{t+1}^*\right). \tag{12}$$

The representative household in a currency union maximizes Eq.(1) subject to Eq.(11). The optimality conditions are given by:

$$\beta \mathrm{E}_t\left(\frac{C_{t+1}^{-\sigma}P_t}{C_t^{-\sigma}P_{t+1}}\right) = \frac{1}{R_t^f}, \tag{13}$$

$$C_t^{\sigma}N_{H,t}^{\varphi} = \frac{W_t}{P_t}\ ;\ C_t^{\sigma}N_{F,t}^{\varphi} = \frac{W_t^*}{P_t}. \tag{14}$$

Eq.(13) is the intertemporal optimality condition and Eq.(14) is the intratemporal optimality condition, where $R_t^f \equiv 1 + r_t^f$ denotes the gross risk-free nominal interest rate that satisfies $\frac{1}{R_t^f} = \mathrm{E}_t\left(Q_{t,t+1}\right)$ and r_t^f denotes the net interest rate.

Log-linearizing Eq.(13) yields:

$$c_t = \mathrm{E}_t\left(c_{t+1}\right) - \frac{1}{\sigma}\hat{r}_t^f + \frac{1}{\sigma}\mathrm{E}_t\left(\pi_{t+1}\right), \tag{15}$$

with $\hat{r}_t^f = \frac{dR_t^f}{R}$.

The representative household in country A maximizes Eq.(1) subject to Eq.(12). The optimality conditions are given by:

$$\beta \mathrm{E}_t\left(\frac{\left(C_{t+1}^*\right)^{-\sigma} P_t^*}{\left(C_t^*\right)^{-\sigma} P_{t+1}^*}\right) = \frac{1}{R_t^*}, \tag{16}$$

$$\left(C_t^*\right)^{\sigma}\left(N_t^*\right)^{\varphi} = \frac{W_t^*}{P_t^*}. \tag{17}$$

Eq.(16) is the intertemporal optimality condition and Eq.(17) is the intratemporal optimality condition, where $R_t^* \equiv 1 + r_t^*$ denotes the gross nominal interest rate in country A that satisfies $\frac{1}{R_t^*} = \mathrm{E}_t\left(Q_{t,t+1}^*\right)$. Log-linearizing Eq.(16) yields:

$$c_t^* = \mathrm{E}_t\left(c_{t+1}^*\right) - \frac{1}{\sigma}\hat{r}_t^* + \frac{1}{\sigma}\mathrm{E}_t\left(\pi_{t+1}^*\right), \tag{18}$$

with $\hat{r}_t^* = \frac{dR_t^*}{R}$ and $\pi_t^* \equiv p_t^* - p_{t-1}^*$.

Combining Eqs.(13) and (16) with an appropriate initial condition yields:

$$C_t = C_t^*, \tag{19}$$

which implies that marginal utility of consumption is no different between the currency union and country A.

3.2 Firms

A typical firm in each country produces a differentiated good with a linear technology represented by the production function:

$$Y_t(h) = A_{H,t} N_{H,t}(h), \quad ; \quad Y_t(f) = A_{F,t} N_{F,t}(f),$$

where $Y_{H,t}(h)$ and $Y_{F,t}(f)$ denote the output of a generic good in countries H and F, respectively, and $A_{H,t}$ and $A_{F,t}$ denote the productivity in countries H and F, respectively. Firms in country A have a technology similar to firms in the currency union. That is, $Y_t(a) = A_t^* N_t^*(a)$ holds.

Analogous to consumption indexes, we define $Y_{H,t} \equiv \left[\left(\frac{1}{\alpha}\right)\int_0^{\alpha} Y_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh\right]^{\frac{\varepsilon}{\varepsilon-1}}$ and $Y_{F,t} \equiv \left[\left(\frac{1}{1-\alpha}\right)\int_{\alpha}^{1} Y_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df\right]^{\frac{\varepsilon}{\varepsilon-1}}$. Combining these definitions and the PPI indexes, we have:

$$Y_t(h) = \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon} Y_{H,t} \; ; \; Y_t(f) = \left(\frac{P_t(f)}{P_{F,t}}\right)^{-\varepsilon} Y_{F,t}. \tag{20}$$

In addition, we define $Y_t^* \equiv \left[\int_1^2 Y_t^*(a)^{\frac{\varepsilon-1}{\varepsilon}} dh\right]^{\frac{\varepsilon}{\varepsilon-1}}$ and we have:

$$Y_t^*(a) = \left(\frac{P_t^*(a)}{P_{A,t}^*}\right)^{-\varepsilon} Y_t^*. \tag{21}$$

By combining the production technology in the currency union and Eq.(20), we have an aggregate production function related to aggregate employment as follows:

$$N_{H,t} = \frac{Y_{H,t} Z_{H,t}}{A_{H,t}} \ ; \ N_{F,t} = \frac{Y_{F,t} Z_{F,t}}{A_{F,t}}, \tag{22}$$

where $Z_{H,t} \equiv \int_0^\alpha \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon} dh$ and $Z_{F,t} \equiv \int_\alpha^1 \left(\frac{P_t(f)}{P_{F,t}}\right)^{-\varepsilon} df$ denote the price dispersions in countries H and F, respectively. By log-linearizing Eq(22), we have:

$$y_{H,t} = n_{H,t} - a_{H,t} \ ; \ y_{F,t} = n_{F,t} - a_{F,t}. \tag{23}$$

Notice that $Z_{H,t}$ and $Z_{F,t}$ disappear in Eq.(23) because of $o\left(\|\xi\|^2\right)$.

By combining the production technology in country A and Eq.(21), we have an aggregate production function related to aggregate employment as follows:

$$N_t^* = \frac{Y_t^* Z_t^*}{A_t^*}, \tag{24}$$

with $Z_t^* \equiv \int_1^2 \left(\frac{P_t^*(a)}{P_{A,t}^*}\right)^{-\varepsilon} da$ being the price dispersion in country A. Log-linearizing Eq.(24) yields:

$$y_t^* = n_t^* - a_t^*. \tag{25}$$

Each firm is a monopolistic producer of one of the differentiated goods. Each firm sets their prices $P_t(h)$, $P_t(f)$ and $P_t^*(a)$ taking as given P_t, P_t^*, $P_{H,t}$, $P_{F,t}$, $P_{A,t}^*$, C_t and C_t^*. We assume that firms set prices in a staggered fashion in the Calvo–Yun style, according to which each seller has the opportunity to change its price with a given probability $1-\theta$, where an individual firm's probability of re-optimizing in any given period is independent of the time elapsed since it last reset its price. When a firm has the opportunity to set a new price in period t, it does so in order to maximize the expected discounted value of its net profits. The FONC for firms are given by:

$$\begin{aligned}
\mathrm{E}_t\left[\sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \tilde{Y}_{H,t+k}\left(\tilde{P}_{H,t} - \frac{\varepsilon}{\varepsilon-1} P_{H,t+k} MC_{H,t+k}\right)\right] &= 0, \\
\mathrm{E}_t\left[\sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \tilde{Y}_{F,t+k}\left(\tilde{P}_{F,t} - \frac{\varepsilon}{\varepsilon-1} P_{F,t+k} MC_{F,t+k}\right)\right] &= 0, \\
\mathrm{E}_t\left[\sum_{k=0}^{\infty} \theta^k Q_{t,t+k}^* \tilde{Y}_{t+k}^*\left(\tilde{P}_{A,t}^* - \frac{\varepsilon}{\varepsilon-1} P_{A,t+k}^* MC_{t+k}^*\right)\right] &= 0, \qquad (26)
\end{aligned}$$

where $MC_{H,t} \equiv \frac{W_{H,t}}{(1-\tau_{H,t})P_{H,t}A_{H,t}}$, $MC_{F,t} \equiv \frac{W_{F,t}}{(1-\tau_{F,t})P_{F,t}A_{F,t}}$ and $MC_t^* \equiv \frac{W_t^*}{(1-\tau_t^*)P_{A,t}^*A_t^*}$ denote the real marginal costs in countries H, F and A, respectively, $\tilde{Y}_{H,t+k} \equiv \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} Y_{H,t+k}$, $\tilde{Y}_{F,t+k} \equiv \left(\frac{\tilde{P}_{F,t}}{P_{F,t+k}}\right)^{-\varepsilon} Y_{F,t+k}$ and $\tilde{Y}_{t+k}^* \equiv \left(\frac{\tilde{P}_t^*}{P_{t+k}^*}\right)^{-\varepsilon} Y_{t+k}^*$ denote demand for goods produced in countries H, F and A, respectively, when firms choose a new price, $\tilde{P}_{H,t}$, $\tilde{P}_{F,t}$ and $\tilde{P}_{A,t}^*$ denote newly set prices in countries H, F and A, respectively, and $\tau_{H,t}$, $\tau_{F,t}$ and $\tau_{A,t}$ denote the tax rate in countries H, F and A, respectively.

By log-linearizing Eq.(26), the inflation dynamics can be written as:

$$\begin{aligned} \pi_{H,t} &= \beta \mathrm{E}_t\left(\pi_{H,t+1}\right) + \kappa mc_{H,t}, \\ \pi_{F,t} &= \beta \mathrm{E}_t\left(\pi_{F,t+1}\right) + \kappa mc_{F,t}, \\ \pi_{A,t}^* &= \beta \mathrm{E}_t\left(\pi_{A,t+1}^*\right) + \kappa mc_t^*, \end{aligned} \tag{27}$$

with $\kappa \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$. Eq.(27) shows that not only an increase in current marginal cost but also an increase in future marginal cost increases current inflation because expected inflation appears on the RHS in Eq.(27).

Substituting Eq.(14) and Eq.(17) into the definition of the real marginal cost yields:

$$\begin{aligned} MC_{H,t} &= \mathcal{S}_{E,t}^{\frac{1-\alpha}{2}} \mathcal{S}_{H,t}^{\frac{1}{2}} \frac{C_t^\sigma N_{H,t}^\varphi}{\left(1-\tau_{H,t}\right) A_{H,t}}, \\ MC_{F,t} &= \mathcal{S}_t^{\frac{\alpha}{2}} \mathcal{S}_{F,t}^{\frac{1}{2}} \frac{C_t^\sigma N_{F,t}^\varphi}{\left(1-\tau_{F,t}\right) A_{F,t}}, \\ MC_t^* &= \mathcal{S}_{H,t}^{-\frac{\alpha}{2}} \mathcal{S}_{F,t}^{-\frac{1-\alpha}{2}} \frac{\left(C_t^*\right)^\sigma \left(N_t^*\right)^\varphi}{\left(1-\tau_t^*\right) A_t^*}, \end{aligned} \tag{28}$$

where $\mathcal{S}_{E,t} \equiv \frac{P_{F,t}}{P_{H,t}}$ denotes the terms of trade (TOT) in the currency union, $\mathcal{S}_{H,t} \equiv \frac{P_{A,t}}{P_{H,t}}$ denotes the TOT between countries H and A and $\mathcal{S}_{F,t} \equiv \frac{P_{A,t}}{P_{F,t}}$ denotes the TOT between countries F and A. Log-linearizing Eq.(28) yields:

$$\begin{aligned} mc_{H,t} &= \frac{1-\alpha}{2} s_{E,t} + \frac{1}{2} s_{H,t} + \sigma c_t + \varphi n_{H,t} - a_{H,t} + \frac{\tau}{1-\tau} \hat{\tau}_{H,t}, \\ mc_{F,t} &= -\frac{\alpha}{2} s_{E,t} + \frac{1}{2} s_{F,t} + \sigma c_t + \varphi n_{F,t} - a_{F,t} + \frac{\tau}{1-\tau} \hat{\tau}_{F,t}, \\ mc_t^* &= -\frac{\alpha}{2} s_{H,t} - \frac{1-\alpha}{2} s_{F,t} + \sigma c_t^* + \varphi n_{F,t} - a_t^* + \frac{\tau}{1-\tau} \hat{\tau}_t^*, \end{aligned} \tag{29}$$

with $\hat{\tau}_{H,t} \equiv \frac{d\tau_{H,t}}{\tau}$, $\hat{\tau}_{F,t} \equiv \frac{d\tau_{F,t}}{\tau}$ and $\hat{\tau}_t^* \equiv \frac{d\tau_t^*}{\tau}$.

3.3 Government

Similar to private consumption, government expenditure is a Dixit–Stiglitz aggregator defined by:

$$G_{H,t} \equiv \left[\left(\frac{1}{\alpha}\right) \int_0^\alpha G_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh\right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad ; \quad G_{F,t} \equiv \left[\left(\frac{1}{1-\alpha}\right) \int_\alpha^1 G_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $G_{H,t}$ and $G_{F,t}$ denote indexes of government expenditure in countries H and F, respectively. For simplicity, we assume that each government allocates a level of government expenditure only among domestic goods. Each government implies the following demands for the generic goods h and f:

$$G_t(h) = \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon} G_{H,t} \ ; \ G_t(f) = \left(\frac{P_t(f)}{P_{F,t}}\right)^{-\varepsilon} G_{F,t}. \tag{30}$$

Similar to that in a currency union, government expenditure in country A is a Dixit–Stiglitz aggregator defined by $G_t^* \equiv \left[\int_1^2 G_t^*(a)^{\frac{\varepsilon-1}{\varepsilon}} dh\right]^{\frac{\varepsilon}{\varepsilon-1}}$. Hence, we have:

$$G_t^*(a) = \left(\frac{P_t^*(a)}{P_{A,t}^*}\right)^{-\varepsilon} G_t^*. \tag{31}$$

The flow government budget constraints are given by:

$$\begin{aligned} P_t B_{H,t} &= R_{t-1}^f P_{t-1} B_{H,t-1} - \int_0^\alpha P_t(h)\left[\tau_{H,t} Y_t(h) - G_t(h)\right] dh, \\ P_t B_{F,t} &= R_{t-1} P_{t-1} B_{F,t-1}(1-\delta_t) - \int_\alpha^1 P_t(f)\left[\tau_{F,t} Y_t(f) - G_{F,t}(h)\right] dh, \end{aligned} \tag{32}$$

where $B_{H,t}$ and $B_{F,t}$ denote the per capita value of bond issuance of government bonds in real terms in countries H and F, respectively, $R_t \equiv 1 + r_t$ denotes the gross nominal interest rate for risky assets, r_t denotes the net nominal interest rate for risky assets and δ_t denotes the default rate. We assume that government bonds issued by country H's government are safety assets, although ones issued by country F's government are risky assets. Hence, δ_t and R_t appear in the second line in Eq.(32). The first and the second equalities in Eq.(32) are the government budget constraints in countries H and F, respectively.

Substituting Eq.(30) and Eq.(21) into Eq.(32)yields:

$$\begin{aligned} P_t B_{H,t} &= R_{t-1}^f P_{t-1} B_{H,t-1} - P_{H,t}\left(\tau_{H,t} Y_{H,t} - G_{H,t}\right), \\ P_t B_{F,t} &= R_{t-1} P_{t-1} B_{F,t-1}(1-\delta_t) - P_{F,t}\left(\tau_{F,t} Y_{F,t} - G_{F,t}\right). \end{aligned} \tag{33}$$

The first and the second equalities in Eq.(33) are the government budget constraints in countries H and F, respectively. Dividing both sides of Eq.(33) by P_{t-1} yields:

$$\begin{aligned} \Pi_t B_{H,t} &= R_{t-1}^f B_{H,t-1} - \Pi_t \mathcal{S}_t^{-(1-\alpha)}\left(\tau_{H,t} Y_{H,t} - G_{H,t}\right), \\ \Pi_t B_{F,t} &= R_{t-1} B_{F,t-1}(1-\delta_t) - \Pi_t \mathcal{S}_t^{\alpha}\left(\tau_{F,t} Y_{F,t} - G_{F,t}\right). \end{aligned} \tag{34}$$

By log-linearizing Eq.(34), we have:

$$\begin{aligned} b_{H,t} = \ & \frac{1}{\beta} b_{H,t-1} + \frac{1}{\beta}\hat{r}_{t-1}^f - \frac{1}{\beta}\pi_{H,t} + \frac{(1-\alpha)(1+\beta)}{2\beta} s_{E,t} - \frac{\alpha-(1-\beta)}{2\beta} s_{H,t} \\ & - \frac{1-\alpha}{2\beta} s_{F,t} + \frac{1-\alpha}{\beta} s_{E,t-1} + \frac{\alpha}{2\beta} s_{H,t-1} + \frac{1-\alpha}{2\beta} s_{F,t-1} - \frac{\tau}{\varsigma_B} y_{H,t} \\ & + \frac{\varsigma_G}{\varsigma_B} g_{H,t} - \frac{\tau}{\varsigma_B}\tau_{H,t} \end{aligned}$$

$$
\begin{aligned}
b_{F,t} \quad = \quad & \frac{1}{\beta} b_{F,t-1} + \frac{1}{\beta} \hat{r}_{t-1} - \frac{\delta}{\beta (1-\delta)} \hat{\delta}_t - \frac{1}{\beta} \pi_{F,t} + \frac{\alpha (1+\beta)}{2\beta} s_{E,t} \\
& - \frac{1 - \alpha - (1-\beta)}{2\beta} s_{F,t} - \frac{\alpha}{2\beta} s_{H,t} - \frac{\alpha}{\beta} s_{E,t-1} + \frac{1-\alpha}{2\beta} s_{F,t-1} + \frac{\alpha}{2\beta} s_{H,t-1} \\
& - \frac{\tau}{\varsigma_B} y_{F,t} + \frac{\varsigma_G}{\varsigma_B} g_{F,t} - \frac{\tau}{\varsigma_B} \tau_{F,t} \qquad (35)
\end{aligned}
$$

with $\hat{\delta}_t \equiv \frac{d\delta_t}{\delta}$ where $\varsigma_G \equiv \frac{G}{Y}$ denotes the steady-state ratio of government expenditure to GDP and $\varsigma_B \equiv \frac{B}{Y}$ denotes the steady-state share of government debt to GDP.

Similar to Eq.(33), the government budget constraint in country A is given by:

$$
P_t^* B_t^* = R_{t-1}^* P_{t-1}^* B_{t-1}^* - P_{A,t}^* \left(\tau_t^* Y_t^* - G_t^* \right),
$$

which can be rewritten similar to Eq.(34) as follows:

$$
\Pi_t^* B_t^* = R_{t-1}^* B_{t-1}^* - \Pi_t^* \mathcal{S}_{H,t}^{\frac{\alpha}{2}} \mathcal{S}_{F,t}^{\frac{1-\alpha}{2}} \left(\tau_t^* Y_t^* - G_t^* \right). \qquad (36)
$$

Log-linearizing Eq.(36) yields the following:

$$
\begin{aligned}
b_t^* \quad = \quad & \frac{1}{\beta} b_{t-1}^* + \frac{1}{\beta} \hat{r}_{t-1}^* - \frac{1}{\beta} \pi_{A,t}^* + \frac{\alpha}{2} s_{H,t} + \frac{1-\alpha}{2} s_{F,t} - \frac{\alpha}{2\beta} s_{H,t-1} \\
& - \frac{1-\alpha}{2\beta} s_{F,t-1} - \frac{\tau}{\varsigma_B} y_t^* + \frac{\varsigma_G}{\varsigma_B} g_t^* - \frac{\tau}{\varsigma_B} \tau_t^*. \qquad (37)
\end{aligned}
$$

∘

Next, we discuss the government budget constraint in country F where there is a possibility of default and the default dynamics follow Uribe[12]. The appropriate transversality condition for government bonds issued by country F's government is given by:

$$
\lim_{j \to \infty} \beta^{t+j+1} \mathrm{E}_t \left[R_{t+j} \left(1 - \delta_{t+j+1}\right) \frac{P_{t+j} B_{F,t+j}}{P_{t+j+1}} \right] = 0. \qquad (38)
$$

Optimizing households must be indifferent between holding government bonds and holding state contingent claims. This implies the following intertemporal optimality condition:

$$
\beta \mathrm{E}_t \left(\frac{C_{t+1}^{-\sigma} P_t}{C_t^{-\sigma} P_{t+1}} \right) = \frac{1}{R_t \mathrm{E}_t \left(1 - \delta_{t+1}\right)}. \qquad (39)
$$

In Eq.(39), $\mathrm{E}_t \left(1 - \delta_{t+1}\right) R_t$ replaces R_t^f in Eq.(13). By log-linearizing Eq.(39), we have:

$$
c_t = \mathrm{E}_t \left(c_{t+1} \right) - \frac{1}{\sigma} \mathrm{E}_t \left(\pi_{t+1} \right) + \frac{\delta}{\sigma (1-\delta)} \mathrm{E}_t \left(\hat{\delta}_{t+1} \right), \qquad (40)
$$

with $\hat{\delta}_t \equiv \frac{d\delta_t}{\delta}$. By subtracting Eq.(40) from Eq.(15), we have:

$$
\hat{r}_t = \hat{r}_t^f + \frac{\delta}{1-\delta} \mathrm{E}_t \left(\hat{\delta}_{t+1} \right). \qquad (41)
$$

Eq.(41) shows that the interest rate on risky assets is equal to the interest rate on nonrisky assets with an expected default rate. That is, the interest rate on risky assets includes a risk premium.

Multiplying $R_t(1-\delta_{t+1})$ by both sides of Eq.(34) and iterating by substituting j times forward yields:

$$\begin{aligned} R_{t+j} B^n_{F,t+j}(1-\delta_{t+j+1}) &= \left[\prod_{k=0}^{j}(1-\delta_{t+k+1})\right] R_{t-1} B_{t-1}(1-\delta_t) \\ &\quad - \sum_{k=0}^{j}\left[\prod_{h=k}^{j} R_{t+h}(1-\delta_{t+h+1})\right] P_{F,t+k}\left(\tau_{F,t+k} Y_{F,t+k}\right. \\ &\quad \left. - G_{F,t+k}\right), \end{aligned}$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ denotes gross CPI inflation in a currency union. By dividing both sides of this equality P_{t+j+1}, we have:

$$\begin{aligned} R_{t+j}\frac{B_{t+j}}{P_{t+j+1}}(1-\delta_{t+j+1}) &= \left[\prod_{k=0}^{j} R_{t+k}(1-\delta_{t+k+1})\Pi^{-1}_{t+j+1}\right] R_{t-1}\frac{B_{F,t-1}}{P_t}(1-\delta_t) \\ &\quad + \sum_{k=0}^{j}\left[\prod_{h=k}^{j} R_{t+h}(1-\delta_{t+h+1})\Pi^{-1}_{t+h+1}\right] SP_{F,t+k}, \end{aligned}$$

where $SP_{F,t} \equiv \frac{P_{F,t}(\tau_{F,t}Y_{F,t}-G_{F,t})}{P_t}$ denotes the nominal fiscal surplus in terms of the CPI. Using the conditional expectation operator on both sides of this equality, substituting Eq.(39) into this equality and repeatedly applying the expected value operator, we have:

$$\begin{aligned} \mathrm{E}_t\left[R_{t+j}\frac{B^n_{F,t+j}}{P_{t+j+1}}(1-\delta_{t+j+1})\right] &= \beta^{-(j+1)}\frac{C_t^{-\sigma}}{\mathrm{E}_t\left(C_{t+1}^{-\sigma}\right)} R_{t-1}\frac{B^n_{F,t-1}}{P_t}(1-\delta_t) \\ &\quad - \mathrm{E}_t\left(\frac{1}{C^{-\sigma}_{t+j+1}}\sum_{k=0}^{j}\beta^{k-j-1}C^{-\sigma}_{t+k}SP_{F,t+k}\right). \end{aligned}$$

By multiplying both sides of this equality by β^j, taking the limit of $j \to \infty$ and substituting Eq.(38), we have:

$$C_t^{-\sigma} R_{t-1}\frac{B^n_{F,t-1}}{P_t}(1-\delta_t) = \sum_{k=0}^{\infty}\beta^k \mathrm{E}_t\left(C^{-\sigma}_{t+k}SP_{F,t+k}\right).$$

This equality is the central equation of FTPL where we assume $\delta_t = 0$, as pointed out by Uribe[12]. While Uribe[12] assumes an endowment economy implying that the marginal utility of consumption is constant over time, we introduce production activity explicitly. Introducing production activity implies that the marginal utility of consumption is not necessarily constant. Hence, in contrast to the equality derived by Uribe[12], the marginal utility of consumption appears on both sides of this equality. By rearranging this equality, we have the default rate in equilibrium as follows:

$$\delta_t = 1 - \frac{\sum_{k=0}^{\infty}\beta^k \mathrm{E}_t\left(C^{-\sigma}_{t+k}SP_{F,t+k}\right)}{C_t^{-\sigma}R_{t-1}\frac{B_{F,t-1}}{P_t}}. \tag{42}$$

As mentioned by Uribe[12], Eq.(42) shows that if the expected net present value of the primary balance in terms of the marginal utility of consumption is equal to the real balance of government debt in terms of the marginal utility of consumption at the beginning of a period, the default rate reaches zero.[2]

Eq.(42) can be rewritten as a second-order differential equation as follows:

$$R_{t-1}\Pi_t^{-1}B_{F,t-1}\left(1-\delta_t\right) = SP_{F,t} + \beta \mathrm{E}_t\left(\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\Pi_{t+1}^{-1}\right)R_t\left[1-\mathrm{E}_t\left(\delta_{t+1}\right)\right]B_{F,t}. \quad (43)$$

Eq.(43) shows that an increase in inflation decreases the default rate because inflation reduces the real debt burden for the government in the next period. By log-linearizing Eq.(43), we get:

$$\begin{aligned}\hat{\delta}_t \;=\; & -\frac{1-\beta}{\left(1-\delta\right)\delta}sp_{F,t} + \frac{1-\delta\beta\left(2-\delta\right)}{\delta}\hat{r}_{t-1} - \frac{1-\delta\beta\left(2-\delta\right)}{\delta}\pi_t + \frac{1-\delta\beta\left(2-\delta\right)}{\delta}b_{F,t-1} \\ & + \frac{\beta\sigma\left(1-\delta\right)}{\delta}\mathrm{E}_t\left(c_{t+1}\right) - \beta\sigma\left(1-\delta\right)c_t + \beta\left(1-\delta\right)\mathrm{E}_t\left(\pi_{t+1}\right) - \beta\left(1-\delta\right)\hat{r}_t \\ & - \beta\left(1-\delta\right)b_{F,t}. \end{aligned} \quad (44)$$

3.4 Market Clearing Conditions

The market clearing conditions in a currency union are given by:

$$\begin{aligned} Y_t\left(h\right) &= C_t\left(h\right) + C_t^*\left(h\right) + G_t\left(h\right), \\ Y_t\left(f\right) &= C_t\left(f\right) + C_t^*\left(f\right) + G_t\left(f\right), \end{aligned} \quad (45)$$

where $C_{H,t}^*\left(h\right)$ and $C_{F,t}^*\left(f\right)$ denote demands for good a produced in country A.

By combining Eqs.(7), (8), (7) and (15), we can rewrite Eq.(20) as follows:

$$\begin{aligned} Y_t\left(h\right) &= \left(\frac{P_t\left(h\right)}{P_{H,t}}\right)^{-\varepsilon}\left[\left(\frac{P_{H,t}}{P_t}\right)^{-1}C_t + G_{H,t}\right], \\ Y_t\left(f\right) &= \left(\frac{P_t\left(f\right)}{P_{F,t}}\right)^{-\varepsilon}\left[\left(\frac{P_{F,t}}{P_t}\right)^{-1}C_t + G_{F,t}\right]. \end{aligned} \quad (46)$$

Substituting Eq.(46) into Eq.(45) yields:

$$\begin{aligned} Y_{H,t} &= \left(\frac{P_{H,t}}{P_t}\right)^{-1}C_t + G_{H,t}, \\ Y_{F,t} &= \left(\frac{P_{F,t}}{P_t}\right)^{-1}C_t + G_{F,t}. \end{aligned} \quad (47)$$

Similar to Eq.(45), country A's market clearing condition is given by:

$$Y_t^*\left(a\right) = C_t\left(a\right) + C_t^*\left(a\right) + G_t^*\left(a\right),$$

[2]In other words, Eq.(42) shows that if the expected net present value of the primary balance in terms of the nominal marginal utility of consumption is equal to the real balance of government debt at the beginning of a period, the default rate reaches zero.

which can be rewritten as follows by substituting in the definition for government expenditure:

$$Y_t^* = \left(\frac{P_{A,t}^*}{P_t^*}\right)^{-1} C_t^* + G_t^*. \tag{48}$$

By substituting Eqs.(3) and (3) into Eq.(46), we have:

$$\begin{aligned} Y_{H,t} &= \mathcal{S}_{E,t}^{\frac{1-\alpha}{2}} \mathcal{S}_{H,t}^{\frac{1}{2}} C_t + G_{H,t}, \\ Y_{F,t} &= \mathcal{S}_{E,t}^{-\frac{\alpha}{2}} \mathcal{S}_{F,t}^{\frac{1}{2}} C_t + G_{F,t}, \end{aligned} \tag{49}$$

where $\frac{P_{H,t}}{P_t} = \mathcal{S}_{E,t}^{-\frac{1-\alpha}{2}} \mathcal{S}_{H,t}^{-\frac{1}{2}}$ and $\frac{P_{F,t}}{P_t} = \mathcal{S}_{E,t}^{\frac{\alpha}{2}} \mathcal{S}_{F,t}^{-\frac{1}{2}}$ are used to derive Eq.(49). By log-linearizing Eq.(49), we have:

$$\begin{aligned} y_{H,t} &= \frac{1-\alpha}{2} s_{E,t} + \frac{1}{2} s_{H,t} + c_t + \varsigma_G g_{H,t}, \\ y_{F,t} &= -\frac{\alpha}{2} s_{E,t} + \frac{1}{2} s_{F,t} + c_t + \varsigma_G g_{F,t}. \end{aligned} \tag{50}$$

Similar to Eq.(49), substituting an equality corresponding to Eq.(3) in country A and Eq.(3) into Eq.(48) yields:

$$Y_t^* = \mathcal{S}_{H,t}^{-\frac{\alpha}{2}} \mathcal{S}_{F,t}^{-\frac{1-\alpha}{2}} C_t^* + G_t^*, \tag{51}$$

where $\frac{P_{A,t}}{P_t} = \mathcal{S}_{H,t}^{\frac{\alpha}{2}} \mathcal{S}_{F,t}^{\frac{1-\alpha}{2}}$ is used to derive Eq.(51). By log-linearizing Eq.(51), we have:

$$y_t^* = -\frac{\alpha}{2} s_{H,t} - \frac{1-\alpha}{2} s_{F,t} + c_t^* + \varsigma_G g_t^*. \tag{52}$$

3.5 Natural Rate of Output and GDP Gap

In a flexible price equilibrium, the real marginal cost is constant and it corresponds to the inverse of the constant markup over time. Namely, $MC_{H,t} = MC_{F,t} = MC_t^* = \zeta^{-1}$. Substituting these conditions into Eq.(28) and log-linearizing yields:

$$\begin{aligned} \bar{y}_{H,t} &= \frac{(1-\alpha)(\sigma-1)}{2\varpi_1} s_{E,t} + \frac{\sigma-1}{2\varpi_1} s_{H,t} + \frac{\sigma\varsigma_G}{\varpi_1(1-\varsigma_G)} g_{H,t} - \frac{\tau}{(1-\tau)\varpi_1} \hat{\tau}_{H,t} \\ &\quad + \frac{1+\varphi}{\varpi_1} a_{H,t} \\ \bar{y}_{F,t} &= -\frac{\alpha(\sigma-1)}{2\varpi_1} s_{E,t} \frac{\sigma-1}{2\varpi_1} s_{F,t} + \frac{\sigma\varsigma_G}{(1-\varsigma_G)\varpi_1} g_{F,t} - \frac{\tau}{(1-\tau)\varpi_1} \hat{\tau}_{F,t} \\ &\quad + \frac{1+\varphi}{\varpi_1} a_{F,t} \\ \bar{y}_t^* &= -\frac{\alpha(\sigma-1)}{2\varpi_1} s_{H,t} - \frac{(1-\alpha)(\sigma-1)}{2\varpi_1} s_{F,t} + \frac{\sigma\varsigma_G}{(1-\varsigma_G)\varpi_1} g_t^* - \frac{\tau}{(1-\tau)\varpi_1} \hat{\tau}_t^* \\ &\quad + \frac{1+\varphi}{\varpi_1} a_t^*, \end{aligned} \tag{53}$$

where $\bar{y}_{H,t} \equiv y_{H,t} - x_{H,t}$、$\bar{y}_{F,t} \equiv y_{F,t} - x_{F,t}$ and $\bar{y}_t^* \equiv y_t^* - x_t^*$ denote the natural rate of output in countries H, F and A, respectively, and $x_{H,t}$, $x_{F,t}$ and x_t^* denote the GDP gap in countries H, F and A, respectively.

3.6 Monetary Policy

As mentioned above, Uribe[12] shows that the Taylor rule prolongs the default period, whereas a fixed interest rate reduces the default period to just one period in an economy subject to default. In our paper, we discuss whether Uribe[12]'s policy implication is applicable or not in our more complex model. To satisfy our objective, we define two policy rules, the Taylor rule and the fixed-interest-rate rule.

The Taylor rule is defined as follows:

$$\hat{r}_t^f = \phi \pi_t,$$

where ϕ denotes the reaction coefficient of the nominal interest rate to inflation. In contrast, we define the fixed interest rate as follows:

$$\hat{r}_t^f = \hat{r}_t = \frac{1}{\beta}.$$

The Taylor rule implies that the nominal interest rate is hiked following an increase in inflation and vice versa. $\phi > 1$ is the Taylor principle and the nominal interest rate is hiked by more than the increase in inflation if this condition holds. Inflation is immediately stabilized and the GDP gap is also stabilized via stabilizing inflation. Many papers introducing Calvo pricing into their DSGE models depict this mechanism. In contrast, the nominal interest rate is constant and corresponds to its steady-state value over time under the fixed-interest-rate rule. In addition, the nominal interest rate for nonrisky assets, which is the policy instrument, corresponds to the nominal interest rate for risky assets under the fixed-interest-rate rule. The fixed interest-rate rule is not complex and was adopted in the US. The nominal interest rate had been fixed from Apr. 1942 until the end of WWII in the US under the bond price support regime and it has been fixed since Feb. 1999 in Japan. A zero-interest-rate policy has been adopted in the US since Dec. 2006, which means that interest rates have been fixed.[3] Furthermore, the policy that equalizes the nominal interest rate for both nonrisky and risky assets corresponds to purchasing Exchange Traded Fund or Real Estate Investment Trust by the Bank of Japan.

The fixed-interest-rate rule is a policy that converges two interest rates with different levels of risk and pegs this interest rate to its steady-state value. As mentioned in Section 2, in Jun. 2011 the yield on Greek government bonds was 17.83%, while the yield on German federal bonds was 2.9%. If the steady-state value of the interest rate is 4 to 6%, Greek government bonds are purchased by the central bank while Germany federal bonds are sold by the central bank under the fixed-interest-rate rule. Hence, we designate the fixed-interest-rate rule as a government bond swap arrangement.

Finally, we assume that the Taylor rule is always adopted in country A independently from the currency union's monetary policy as follows:

$$\hat{r}_t^* = \phi^* \pi_{A,t}^*.$$

[3] The zero-interest-rate policy was removed temporarily between Aug. 2000 and Mar. 2006.

4 Macroeconomic Dynamics

4.1 Parameterization

Following previous DSGE studies, we solve the model numerically because it is too difficult to solve the DSGE model analytically. Following Ferrero[6], in analyzing the monetary and fiscal policy mix in a currency union, we set the subjective discount factor, steady-state tax rate, steady-state government expenditure ratio to GDP, steady-state government debt ratio to GDP and degree of relative risk aversion to 0.99, 0.3, 0.276, 0.24 and 3, respectively. The elasticity of labor supply is set to 3 following Gali and Monacelli[7]. Following Beetsma and Jensen[1], who analyze the monetary and fiscal policy mix in a currency union similar to Ferrero[6], we set the degree of price stickiness to 0.75. Price stickiness is asymmetric in Beetsma and Jensen[1]'s two-country model. The degree of price stickiness is set equal to 0.75 in one of the countries in his two-country model. We set the reaction coefficient of the nominal interest rate to inflation equal to 1.5 not only in the currency union but also in country A. We assume that productivity, tax rate and government expenditure are exogenous AR(1) processes and their AR(1) coefficients are set equal to 0.9. The timing of the model is quarterly. Hence, we set the annual steady-state government debt to GDP ratio equal to 60%, which suggests a quarterly ratio of 24%. This is equal to the upper limit of the Stability and Growth Pact. Our setting on price stickiness implies that prices are constant for four quarters, namely one year.

4.2 Macroeconomic Dynamics

Although our model includes productivity, government expenditure and the tax rate in each country, we only focus on changes in these stemming from exogenous shocks in country F. Tab. 1 shows the macroeconomic volatilities for one-percent changes in each shock. Generally speaking, the nominal interest rate is hiked and inflation stabilized immediately in response to inflation pressure under the Taylor rule, and this scenario is common in DSGE models. In our model, however, this mechanism is not necessarily applied. For example, in response to an increase in government expenditure, government bond swaps make inflation more stable than under the Taylor rule. There is no notable difference between the volatilities of PPI inflation to an increase in the tax rate in country F under both rules. As for the GDP gap in country F, a fixed-interest-rate rule makes inflation more stable than under the Taylor rule for increases in government expenditure and the tax rate in country F. The volatility of the GDP gap in country H to an increase in government expenditure in country F under a fixed interest rate is smaller than under the Taylor rule. We do not derive the welfare criterion stemming from a second-order approximated utility function, and we cannot discuss the welfare associated with the introduction of both rules. In many DSGE studies, however, the welfare criteria stemming from a second-order approximated utility function includes variances of PPI inflation and the output gap, and these welfare criteria imply that stabilizing PPI inflation and the output gap minimizes welfare costs. Our results show that the Taylor rule is not the best way to stabilize PPI inflation and the output gap, and our results imply that the allocation under the Taylor rule is not close to the macroeconomic outcome under optimal monetary policy, which has been verified by Rotemberg and Woodford[10]. Hence, our results, which are derived using a DSGE model with sovereign risk, may bring Rotemberg and Woodford's[10] established theory into question. In addition, it is noteworthy that government bond swaps always stabilize PPI inflation and the GDP gap in country A, except for an increase in productivity in country F.

Figs. 1, 2 and 3 show impulse responses to a one-percent negative productivity shock, a one-percent positive shock in government expenditure and a one-percent negative tax rate shock in country F. Such changes immediately decrease tax revenue in country F. Following such changes in exogenous variables, government debt, not only in country F but also in countries H and A, increases infinitely under the Taylor rule. Government debt in each country, however, converges to the steady state in 20 to 23 quarters after changes in those exogenous variables under the fixed-interest-rate rule. The default rate is remarkable. Although the default rate increases to infinite changes in each of the exogenous variables under the Taylor rule, it converges in two quarters following the shocks under the fixed-interest-rate rule. This result is obvious because $\mathrm{E}_t\left(\hat{\delta}_{t+1}\right)=0$, which means the expected default rate is zero, and is applied under government bond swaps. The prices of government bonds in each country are the same, and the prices of risky and nonrisky assets are also the same under government bond swaps. This means that the central bank purchases government bonds in country F or risky assets when the prices of the bonds or prices of the assets decrease. In such a case, fundraising in country F is still easy and the country avoids a revenue shortfall when the threat of one emerges. This is the reason why the government debt converges in 20 to 23 quarters after the shocks. As shown by Uribe[12], who analyzes sovereign risk and price stability in a closed economy and shows that default is inevitable for price stability under the Taylor rule, where the fixed interest rate immediately converges to the default interest rate. Our results are consistent with Uribe's[12] results.

Finally, we consider the dynamics of the nominal exchange rate. For each shock, it depreciates by 0.08 to 1.28% in the first quarter under the Taylor rule, while it depreciates 0.05 to 0.7% under government bond swaps. The volatility under government bond swaps is smaller than the volatility under the Taylor rule for each shock.

5 Determinacy

Generally speaking, $\phi > 1$ is a necessary condition to guarantee uniqueness of an REE in a standard DSGE model.[4] That is, the reaction coefficient of the nominal interest rate to inflation must be larger than one for uniqueness of an REE. Woodford[**?**] shows that the Taylor rule stabilizes inflation as long as this condition and Ricardian equivalence are applied and the Taylor rule is a desirable policy rule from the viewpoint of price stability. This condition, which assures uniqueness of an REE, varies depending on the model assumptions, as shown in Benhabib, Schmitt-Grohe and Uribe[8]. We have to verify the

[4]Blanchard and Khan[2] show that the uniqueness of an REE is assured when the number of jumpers corresponds to the number of characteristic roots inside the unit circle. In a basic closed economy DSGE model or a DSGE model assuming Ricardian equivalence, Blanchard and Khan's[2] condition is guaranteed if $\phi > 1$. See Monacelli[9] for details.

uniqueness of an REE because our model has a distinguishing feature; it is a three-country model consisting of two countries in a currency union and one other country with sovereign risk. Of course, our solution under the Taylor rule and the fixed-interest-rate rule assure the uniqueness of an REE given our parameterization. Our parameterization follows notable preceding studies, although not all of the empirical papers have results that are consistent with our parameterization. In particular, the empirical result of price stickiness varies. Rotemberg and Woodford[10] report it to be 0.66, while Benigno and Lopez-Salid[3] assume that it is 0.875. Hence, in this section, we discuss values of the reaction coefficient of the nominal interest rate to inflation ϕ that guarantee uniqueness of an REE under varying degrees of price stickiness θ.

Fig. 4 shows values of the reaction coefficient of the nominal interest rate to inflation that guarantee uniqueness of an REE under varying degrees of price stickiness. The vertical axis is price stickiness and the horizontal axis is the reaction coefficient of the nominal interest rate to inflation. Uniqueness of an REE is assured in the area below the solid line. Note that we set the reaction coefficient of the nominal interest rate in country A to 1.5. As shown by Fig. 4, uniqueness depends on the degree of price stickiness, and uniqueness is guaranteed even if the reaction coefficient of the nominal interest rate to inflation is less than one. In addition, Fig. 4 shows that uniqueness is not guaranteed when price stickiness exceeds 0.8 in many cases. Although the empirical results are not consistent among previous studies, the Taylor rule may severely harm an economy if the degree of price stickiness exceeds 0.8 and the Taylor rule no longer contributes to stabilizing inflation. While the Taylor rule does not necessarily assure uniqueness of an REE, uniqueness is guaranteed if $\theta \in [0, 1]$ under government bond swaps.

Our result in this section supports Woodford's[14] implication. Following stable prices in WWII under a bond price support regime in the US and harsh inflation in the 1980s in Brazil under the Taylor rule, Woodford[14] advocates that the relationship between the first and the second moves in monetary and fiscal policy is important. The first and the second moves mean a combination of active monetary policy, namely the Taylor rule, and passive fiscal policy, namely Ricardian equivalence or a combination of passive monetary policy and active fiscal policy. There is a point in common between the US in WWII and Brazil in the 1980s in terms of huge government debt to finance war expenditure and public-works spending. Although it cannot be determined with certainty, if Ricardian equivalence is not applicable to the US in WWII and Brazil in the 1980s, fiscal policy in these countries was most probably active. In that case, monetary policy must be passive, namely a bond price support regime or the fixed-interest-rate rule that corresponds to government bonds swap. Woodford[14] points out that price stability is the role of the central bank, and the central bank adopted the Taylor rule in Brazil in the 1980s. Under the Taylor rule, people expected price stability and low inflation. This expectation did not decrease real government debt, and the revenue shortfall became chronic. Finally, Brazil issued public debt continuously, and eventually defaulted. This scenario was replicated in a previous section in our paper. If Ricardian equivalence is not applicable, the Taylor rule is perilous monetary policy.

Default is an ongoing problem, as illustrated by the recent financial crisis stemming from the Greek fiscal deficits in Mar. 2008. The yield of Greek government bonds reached 17.83%, an all-time high, and its spread with the yield on German federal bonds widened to 14.81percentage points. The default rate in our model is analogous to sovereign risk reflecting a risk premium. Internalizing default is a cause of the nonexistence of Ricardian equivalence.[5] Hence, our most practical policy implication for the current euro zone situation is that the ECB should purchase not only Greek government bonds but also Portuguese and Spanish government bonds and sell German federal bonds.

6 Conclusion

This paper developed a three-country model consisting of one country and a currency union with two countries, into which we introduce sovereign risk and analyze monetary policy. The Taylor rule is not a powerful enough policy to stabilize both inflation and the GDP gap in an economy with default. In addition, there is no uniqueness of an REE under the Taylor rule if price stickiness exceeds a certain level. In contrast, there is uniqueness of an REE under government bond swaps independent of price stickiness. Hence, the ECB should adopt government bond swaps, which is a policy involving the purchase of government bonds with sovereign risk and selling government bonds issued by countries with inflation concerns.

Because of fiscal crisis concerns, the yields on long-term government bonds are beginning to increase not only in Portugal, Ireland, Italy, Greece and Spain, namely the PIIGS, but also in other euro zone countries, including France. As mentioned above, the ECB faces a trade-off on monetary policy amid an increase in long-term interest rates, although inflation is gradually increasing. Because of sudden increases in interest rates stemming from the fiscal crises, the ECB is now starting to purchase Italian and Spanish government bonds. In Italy and Spain, there is a notable increase in interest rates stemming from the risk premium associated with sovereign risk. The EFSF is now being used to purchase government debt in these countries. Because inflation concerns and sovereign risk coexist in the euro zone, introducing government bond swaps, which is a policy involving purchasing government bonds with sovereign risk and selling government bonds issued by countries with inflation concerns, is necessary. In the portfolio balance model, government bonds in two countries are imperfect substitutes when there is sovereign risk in one country. In such a case, government bond swaps make it possible for the interest rates in the two countries to move in opposite directions to each other. Our paper also showed that government bond swaps are more desirable than the Taylor rule.

Appendix

A Nonstochastic Steady State

In this section, we show the paths of state variables under a deterministic equilibrium that guarantees $\Pi_{H,t} = \Pi_{F,t} = \Pi^*_{A,t} = 1$ with $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$, $\Pi_{F,t} \equiv \frac{P_{F,t}}{P_{F,t-1}}$、$\Pi^*_{A,t} \equiv \frac{P^*_t}{P^*_{t-1}}$. In the equilibrium, we assume zero inflation.

[5] Ricardian equivalence is not applied in our model. As mentioned by Ferrero[6], our log-linearized model guarantees Ricardian equivalence around equilibrium.

In addition, $\tilde{\mathcal{X}}_H = \tilde{\mathcal{X}}_F = \tilde{\mathcal{X}}_A^* = 1$ is applied under this equilibrium with $\tilde{\mathcal{X}}_{H,t} \equiv \frac{\tilde{P}_{H,t}}{P_{H,t}}$, $\tilde{\mathcal{X}}_{F,t} \equiv \frac{\tilde{P}_{F,t}}{P_{F,t}}$ and $\tilde{\mathcal{X}}_{A,t}^* \equiv \frac{\tilde{P}_{A,t}^*}{P_{A,t}^*}$. Because this equilibrium is non-stochastic, $A_H = A_F = A^* = 1$. In addition, we assume $G_H = G_F = G^*$ and $\tau_H = \tau_F = \tau^*$. Following Ferrero[6], we assume $\mathcal{S}_{E,-1} = S_{H,-1} = S_{F,-1} = 1$.

The gross interest rate is given by:

$$R^f = R^* = \beta^{-1}.$$

Eqs.(13) and (24) imply:

$$R^f = R\left(1-\delta\right), \tag{54}$$

because we assume a positive default rate.

Eq.(17) can be rewritten as:

$$\begin{aligned} \tilde{P}_{H,t} &= \mathrm{E}_t\left(\frac{K_{H,t}}{P_{H,t}^{-1}F_{H,t}}\right) \\ \tilde{P}_{F,t} &= \mathrm{E}_t\left(\frac{K_{F,t}}{P_{F,t}^{-1}F_{F,t}}\right) \\ \tilde{P}_{A,t}^* &= \mathrm{E}_t\left(\frac{K_{A,t}^*}{\left(P_{A,t}^*\right)^{-1}F_{A,t}^*}\right), \end{aligned} \tag{55}$$

where:

$$\begin{aligned} K_{H,t} &\equiv \zeta\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}\tilde{Y}_{H,t+k}MC_{H,t+k}^n \quad ; \quad F_{H,t} \equiv P_{H,t}\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}^{-1}\tilde{Y}_{H,t+k} \\ K_{F,t} &\equiv \zeta\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}\tilde{Y}_{F,t+k}MC_{F,t+k}^n \quad ; \quad F_{F,t} \equiv P_{F,t}\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}\tilde{Y}_{F,t+k} \\ K_t^* &\equiv \zeta\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}^*\tilde{Y}_{\mathcal{N},t+k}^*MC_{t+k}^{n*} \quad ; \quad F_t^* \equiv P_{A,t}^*\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}^*\tilde{Y}_{t+k}^*. \end{aligned} \tag{56}$$

Eq.(56) implies the following:

$$\begin{aligned} K_H &= \frac{\zeta Y_H MC_H^n}{1-\theta\beta} \quad ; \quad F_H = \frac{P_H Y_H}{1-\theta\beta} \\ K_F &= \frac{\zeta Y_F MC_F^n}{1-\theta\beta} \quad ; \quad F_F = \frac{P_F Y_F}{1-\theta\beta} \\ K^* &= \frac{\zeta Y^* MC^{n*}}{1-\theta\beta} \quad ; \quad F^* = \frac{P_A^* Y^*}{1-\theta\beta}. \end{aligned}$$

Combining these equalities and Eq.(55) yields:

$$P_H = \zeta MC_H^n \ ; \ P_F = \zeta MC_F^n \ ; \ P_A^* = \zeta MC^{n*}. \tag{57}$$

Hence:

$$MC_H = MC_F = MC^* = \frac{1}{\zeta} \tag{58}$$

is applied. We define $MC \equiv \frac{1}{\zeta}$.

Eq.(48) implies the following:

$$\begin{aligned}
\frac{1-\tau}{\zeta} &= \mathcal{S}_E^{\frac{1-\alpha}{2}} \mathcal{S}_H^{\frac{1}{2}} C^{\sigma} Y_H^{\varphi} \\
\frac{1-\tau}{\zeta} &= \mathcal{S}_E^{-\frac{\alpha}{2}} \mathcal{S}_F^{\frac{1}{2}} C^{\sigma} Y_F^{\varphi} \\
\frac{1-\tau}{\zeta} &= \mathcal{S}_H^{-\frac{\alpha}{2}} \mathcal{S}_F^{-\frac{1-\alpha}{2}} C^{\sigma} \left(Y^*\right)^{\varphi}.
\end{aligned} \tag{59}$$

Substituting the initial condition for the TOT into Eq.(59) yields:

$$Y_H = Y_F = Y^* \equiv Y. \tag{60}$$

Eqs.(49) and (51) imply the following:

$$\begin{aligned}
Y_H &= \mathcal{S}_{H,t}^{\frac{1}{2}} \mathcal{S}_{E,t}^{\frac{1}{2}} C + G, \\
Y_F &= \mathcal{S}_{E,t}^{-\frac{\alpha}{2}} \mathcal{S}_{F,t}^{-\frac{1}{2}} C + G, \\
Y^* &= \mathcal{S}_H^{-\frac{\alpha}{2}} \mathcal{S}_F^{-\frac{1-\alpha}{2}} C + G.
\end{aligned} \tag{61}$$

Substituting the initial condition for the TOT and Eq.(60) into Eq.(61) yields:

$$Y = C + G. \tag{62}$$

Eq.(59) can be rewritten as:

$$(1-\tau) U_C (C) = \zeta U_N (N), \tag{63}$$

which is a familiar expression. Because of $\tau \in (0,1)$ and $\theta > 1$, this steady state is inefficient.

Eqs.(27) and (47) imply:

$$B \left(\frac{1-\beta}{\beta} \right) = \tau Y - G, \tag{64}$$

with $B \equiv B_H = B_F = B^*$.

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Table 1: Macroeconomic Volatility

Variables	Monetary policy	Shocks in country F		
		Productivity	Gov. expenditure	Tax Rate
GDP gap in country H	Taylor	0.0030	0.0020	0.0013
	GBS	0.0060	0.0013	0.0025
GDP gap in country F	Taylor	0.0031	0.0018	0.0067
	GBS	0.0055	0.0011	4.6771e-004
GDP gap in currency union	Taylor	0.0012	2.5775e-004	0.0035
	GBS	0.0057	0.0012	0.0012
GDP gap in country A	Taylor	0.0044	9.3797e-004	0.0020
	GBS	4.3432e-004	9.2430e-005	1.7856e-004
PPI inflation in country H	Taylor	0.0021	8.4394e-004	9.2156e-004
	GBS	0.0058	0.0012	0.0024
PPI inflation in country F	Taylor	0.0031	0.0012	0.0014
	GBS	0.0041	8.4492e-004	0.0018
Inflation in currency union	Taylor	0.0022	4.6207e-004	9.8619e-004
	GBS	0.0049	0.0010	0.0021
PPI inflation in country A	Taylor	3.3698e-004	0.0047	0.0101
	GBS	0.0051	7.1391e-005	1.3893e-004
Gov. debt in country H	Taylor	1.7935	0.3151	0.7955
	GBS	0.0048	0.0010	0.0020
Gov. debt in country F	Taylor	0.5820	0.2286	0.2988
	GBS	0.0035	7.4479e-004	0.0014
Gov. debt in country A	Taylor	0.0026	0.1495	0.3273
	GBS	0.0229	5.5011e-004	0.0011
Default rate	Taylor	3.7951	1.5005	1.9474
	GBS	0.0046	2.0480e-004	0.0018
Interest rate for Nonrisky assets	Taylor	0.0032	6.6163e-004	0.0014
	GBS	0.0000	0.0000	0.0000
Interest rate for risky assets	Taylor	4.0030	1.5825	2.0541
	GBS	0.0000	0.0000	0.0000
Interest rate in country A	Taylor	0.0338	0.0070	0.0150
	GBS	5.0105e-004	0.0229	2.0632e-004
Exchange rate	Taylor	0.1675	0.0018	0.0037
	GBS	8.3369e-004	5.2489e-004	0.0010

Figure 1: IRF to Negative Productivity Shock in Country F

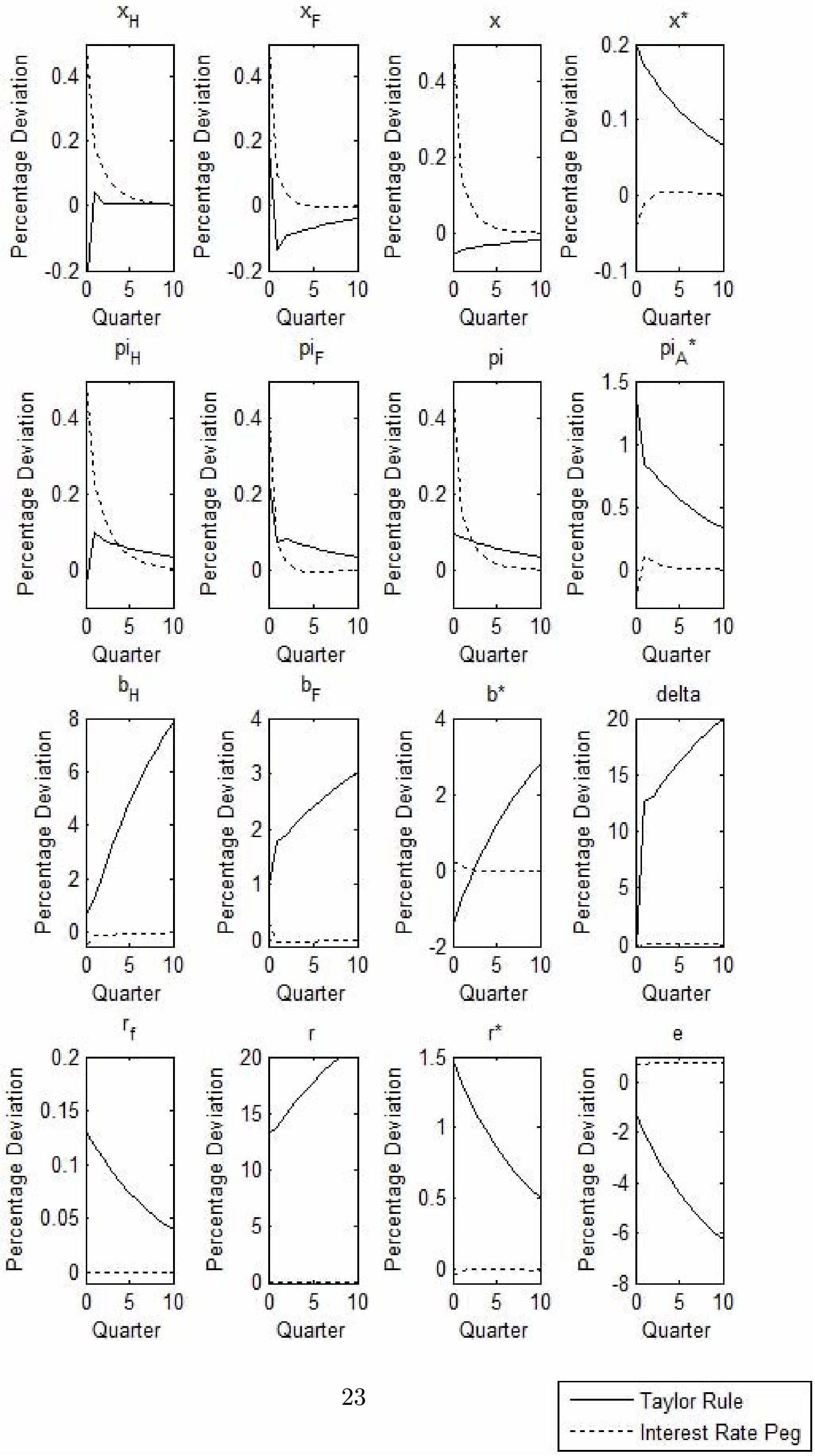

Figure 2: IRF to Positive Government Expenditure Shock in Country F

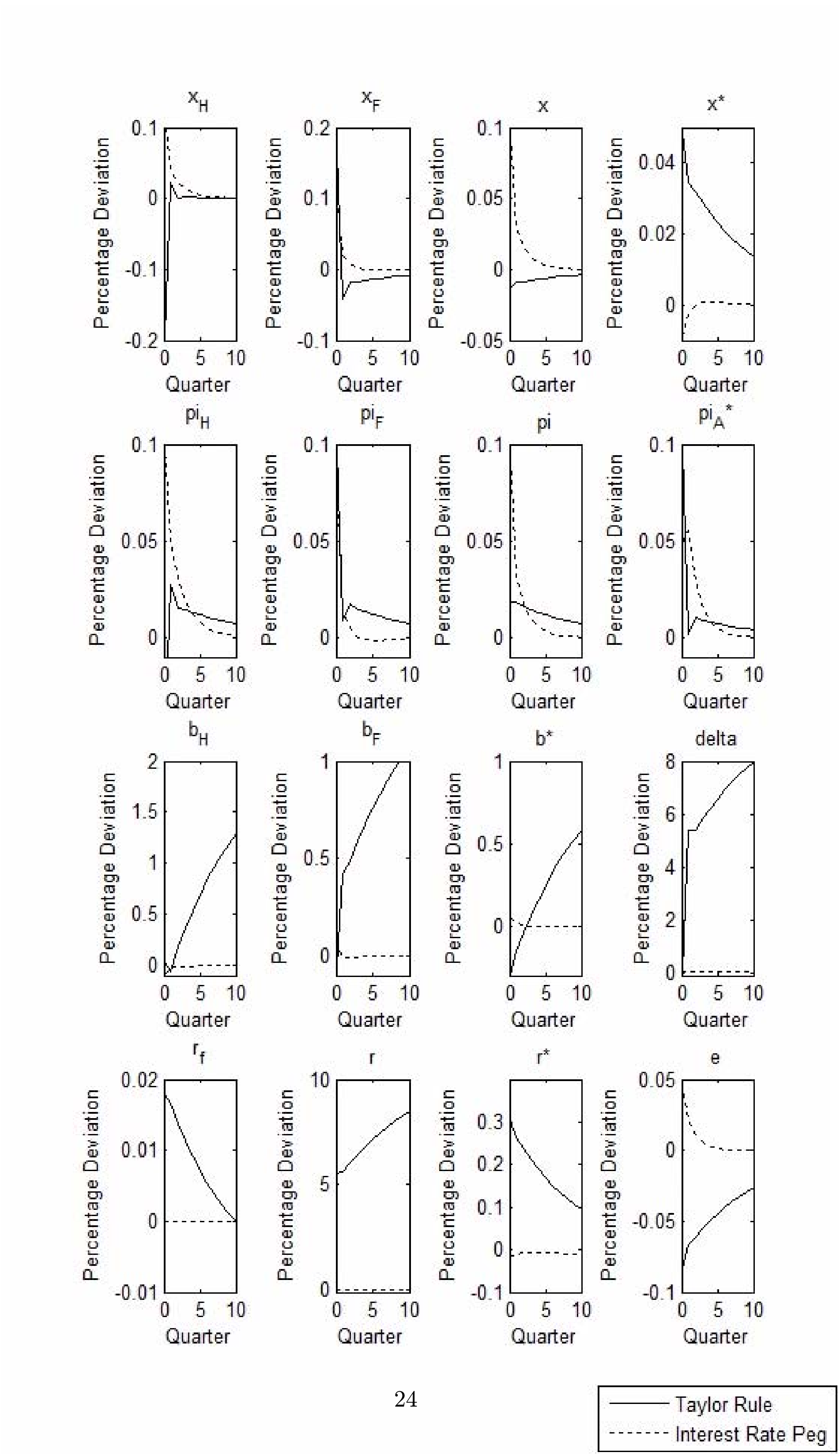

Figure 3: IRF to Negative Tax Rate Shock in Country F

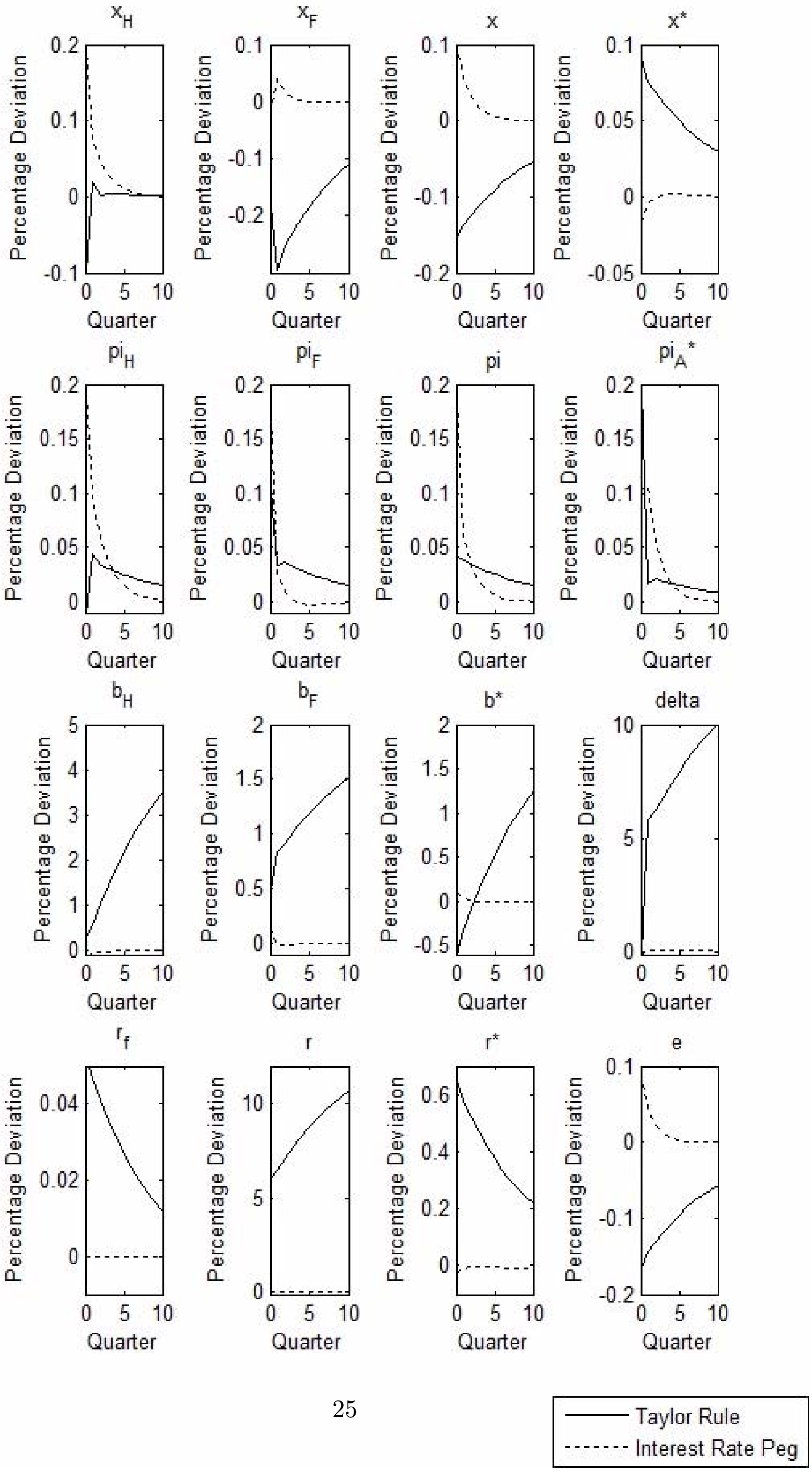

Figure 4: Determinacy and Price Stickiness