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of
Utility Indifference
Net Present Value Method

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ABSTRACT

The net present value (NPV) method is a well-known and established standard approach for assessing project value, but in this method, the risk to investors caused by uncertainty about future cash inflow is not taken into consideration entirely. From this point of view, Miyahara has recently proposed a risk assessment method for generation investment using the net present value based on utility indifference pricing, which is called the "utility indifference net present value (UNPV) method." If one applies the UNPV method to an actual project value assessment, however, a vital problem on the utility function of the investor remains to be solved; it is generally difficult to identify this function. In this paper, a simplified procedure of the UNPV method is proposed, which is expected to circumvent the problem. The new procedure is formulated through a simple regression equation which is described in terms of some moments of randomized NPV: The regression equation is derived by approximating the utility function and estimating a probit model on the sign of UNPV under the assumption that the investor pays attention only to the execution of the project. Then, the sign of the equation determines whether or not one should execute the project. The additional remarks and conjectures on the proposed method are also given.

1. INTRODUCTION

The net present value (NPV) method is a well-known and established standard approach for assessing project value [1-3]. With the NPV method, however, the risk to investors caused by uncertainty about future cash inflow is not taken into consideration entirely; indeed, in the framework, the present value of a project is calculated empirically on the basis of the discounted value of future cash inflow with the interest rate [4]. One possible effective approach to compensate for this defect is the project assessment method, which is based on "utility indifference pricing" in the expected utility theory [5]. In this approach, the net present value of future cash inflow is evaluated by adopting a sort of equivalence concept to an expected utility function that assigns a value to the risk for the investor.

From this point of view, Miyahara has recently proposed a risk assessment method for generation investment using the net present value based on utility indifference pricing [4]. The method is called the "utility indifference net present value (UNPV) method." In order to verify whether or not the new method is useful for a project assessment, Miyauchi *et.al.* have calculated the proposed UNPV assuming investment in a crude oil or gas thermal power plant [6-8]. For example, in [6], they have compared the evaluation of the power plant project worth by the new method with that by the conventional NPV method for an investor having a utility function of the risk avoidance type. The result has indicated that the proposed UNPV is smaller than the conventional NPV, thereby showing that the UNPV method in fact evaluates the risks properly.

If one applies the UNPV method to an actual project value assessment, however, a vital problem on the utility function of the investor remains to be solved; it is generally difficult to identify such a function, and hence it is also hard for the investor to calculate the UNPV exactly. Therefore, it becomes important and useful to formulate a simplified procedure for the method so as to avoid such a problem on the utility function through a sort of approximation for the original UNPV. In light of this, the author has recently sketched such a simplified procedure in an elementary case [8], although he has not provided a generalized formulation of the method yet.

The main purpose of this paper is to formulate a simplified UNPV method in a generalized form and to present additionally some important remarks and conjectures related to the new procedure. The method is governed through a simple regression equation which is described in terms of some moments of randomized NPV: The equation is formulated by approximating the utility function and estimating a probit model on the sign of UNPV under the assumption that the investor pays attention only to the execution of the project. In the simplified procedure, the sign of the new equation determines whether or not the investor executes the project. We will also comment on comparisons of the result of original UNPV method with that of our simplified method.

This paper is organized as follows :

In Section 2, a short review of the UNPV method as proposed by Miyahara [4] is given. In Section 3, we address a simplified UNPV method. For this purpose, we first apply the Taylor expansion to the utility function in UNPV, and derive an approximation equation of UNPV which is described in terms of the moments of the randomized NPV. Introducing binary index variables on execution of the project to the equation, we formulate a probit model, and by estimating the parameters in the model, we finally obtain a simple regression equation in which the sign decides whether or not the project should be executed. In Section 4, we briefly comment on our comparisons of the result of the original UNPV with that of our reduced UNPV under exponential utility. Following this, an important result of a quite recent computer simulation provided by Hirata, Miyauchi *et. al.* is introduced [8]. Finally, in Section 5, some concluding remarks are given.

2. UTILITY INDIFFERENCE NET PRESENT VALUE (UNPV) METHOD

We now start with a review of the UNPV method as proposed by Miyahara [4].

It is assumed that the time series of cash flow $X = \{X_n, n = 1, 2, \cdots, N\}$ is obtained from the project every year in future. In order to randomize the NPV method, we regard the time series of cash flow as the series of random variables. Then, we define the random present value (RPV) obtained from one trial as:

$$RPV(X) = \left\{ \sum_{n=1}^{N} \left(X_n / (1+r)^n \right) \right\}, \qquad (1)$$

where N is the designed depreciation years and r is a suitable discount rate. The ordinary present value PV is given by the expectation of RPV:

$$PV(X) = E[RPV(X)], \qquad (2)$$

where $E[\bullet]$ denotes the expectation. Let I be the present cost of the project. In the conventional net present value method, the net present value NPV is calculated as follows:

$$NPV(X) = PV(X) - I\,, \tag{3}$$

and it is decided to execute the project if *NPV*>0 [1,2]. In a way analogous to that in the NPV method, we define the random net present value *RNPV* as

$$RNPV(X) = RPV(X) - I\,. \tag{4}$$

To evaluate the uncertain return for the project, which is given as the *RNPV*, we apply the utility indifference pricing to it [4-6]. In the framework of the expected utility theory, the uncertain return R is evaluated by the equation:

$$E[u(-v + R + x_0)] = u(x_0)\,, \tag{5}$$

where $u(x)$ is the utility function with $u(0) = 0$, and x_0 is an initial wealth. Then, the value of the return R as the "utility indifference price" is defined by the value of v. This means that the expected return is equal to the utility of the initial wealth if the value v is paid for the right to obtain the uncertain return R, and in this context, R and v are balanced.

Now, we substitute *RNPV* given by Eq. (4) for R in Eq. (5) to obtain:

$$E[u(-v + RNPV(X) + x_0)] = u(x_0)\,. \tag{6}$$

In the following, we call the value v satisfying equation (6) the "utility net indifference present value (*UNPV*)." In our framework, it is decided to execute the project if *UNPV*>0, instead of *NPV*>0. Note that if the utility function is given as $u(x) = x$, *UNPV* coincides with *NPV*.

Remark 2.1: As mentioned above, utility indifference pricing (6) to random net present values (*RNPV*) is a key concept to formulate the UNPV method. In terms of stochastic dynamical theory, the equation is regarded as a sort of invariance law in a sense of expectation for stochastic systems described by *RNPV*. This fact may allow us to formulate the UNPV method in a general framework of conserved quantities and symmetries in stochastic dynamical systems. On stochastic dynamical systems and the numerical methods with invariance character, for example, see References [9] and [10].

In the UNPV method, therefore, how to choose the utility function of the investor is of utmost importance. For example, Miyauchi *et. al.* [6-8] assumed the utility function $u(x)$ of investors in power generation plants to be the risk aversion type, that is,

$$u(x) = 1 - \exp(-bx), \tag{7}$$

where b is a positive constant, since most such investors seek to avoid risk. Even in this case, however, it is difficult to identify the parameter b from the historical results on project assessment, and this is the weak point of the UNPV method as mentioned in Section 1. In the following section, we will formulate a simplified UNPV method through some approximations for the original UNPV so that an investor can determine whether or not to execute the project even if he does not identify the utility function.

3. SIMPLIFICATION OF UNPV METHOD

In what follows, for simplicity, we set the initial wealth x_0 in (6) as 0. Then, Eq. (6) turns into

$$E[u(-v + RNPV(X))] = 0. \tag{8}$$

Then, we set Y and Z as

$$Y = -v + Z = -v + RNPV(X), \tag{9}$$

and we rewrite Eq. (8) in terms of Y and Z as:

$$E[u(Y)] = E[u(-n + Z)] = 0. \tag{10}$$

The first key of our simplification is a Taylor approximation of the expectation of utility function by the moments of *RNPV*. Suppose that the utility function $u(x)$ has such a proper smoothness that there exists the m-th order Taylor expansion of utility function $u(Y)$ on a neighbourhood of $E[Y]$. Then the finite expansion is given by

$$u(Y)=u(E[Y])+u'(E[Y])(Y-E[Y]])+\frac{1}{2}u''(E[Y])(Y-E[Y])^2$$
$$+\frac{1}{6}u^{(3)}(E[Y])(Y-E[Y])^3+\cdots+\frac{1}{m!}u^{(m)}(E[Y])(Y-E[Y])^m+O(|(Y-E[Y])^m|).$$

Taking the expectation of the above equation, we find a finite expansion of the expectation of utility function of the left-hand side of Eq. (10), as follows:

$$\begin{aligned}E[u(Y)]&=u(E[Y])+u'(E[Y])E[Y-E[Y]]+\frac{1}{2}u''(E[Y])E[(Y-E[Y])^2]\\&+\frac{1}{6}u^{(3)}(E[Y])E[(Y-E[Y])^3]+\cdots+\frac{1}{n!}u^{(m)}(E[Y])E[(Y-E[Y])^m]+O(|E[(Y-E[Y])^m]|)\\&=u(E[Y])+\frac{1}{2}u''(E[Y])E[(Y-E[Y])^2]+\frac{1}{6}u^{(3)}(E[Y])E[(Y-E[Y])^3]\\&+\cdots+\frac{1}{m!}u^{(m)}(E[Y])E[(Y-E[Y])^m]+O(E[|(Y-E[Y])^m|]).\end{aligned}\tag{11}$$

Equation (11) and Eq. (9) together indicate that the expectation of utility function is approximated by the moments of *RNPV* (=*Z*) as:

$$\begin{aligned}&E[u(Y)]\approx u(E[Y])+\frac{1}{2}u''(E[Y])E[(Y-E[Y])^2]\\&+\frac{1}{6}u^{(3)}(E[Y])E[(Y-E[Y])^3]+\cdots+\frac{1}{n!}u^{(m)}(E[Y])E[(Y-E[Y])^m]\\&=u(-n+E[Z])+\frac{1}{2}u''(-n+E[Z])V(Z)+\frac{1}{6}u^{(3)}(-n+E[Z])M_3(Z)+\cdots+\frac{1}{m!}u^{(m)}(-n+E[Z])M_m(Z),\end{aligned}\tag{12}$$

where $E[Z]$, $V(Z)$ and $M_k(Z)=E[(Z-E[Z])^k]$ $(k=3,\cdots,m)$ are the mean, the variance and the k-th central moments of the random net present value $Z=RNPV$, respectively. The last equation together with (10) implies

$$u(-n+E[Z])+\frac{1}{2}u''(-n+E[Z])V(Z)+\frac{1}{6}u^{(3)}(-n+E[Z])M_3(Z)+\cdots+\frac{1}{m!}u^{(m)}(-n+E[Z])M_m(Z)\approx 0.\tag{13}$$

Suppose that the left-hand side of (13) is a C^1 function of $n, E[Z], V(Z), M_3(Z), \cdots, and\ M_m(Z)$ at $(n_0, E[Z]_0, V(Z)_0, M_3(Z)_0, \cdots, M_m(Z)_0)$. Using the theorem on implicit function, we obtain a C^1 -function g as

$$n = g(E[Z], V(Z), M_3(Z), \cdots, M_m(Z)) \quad , \tag{14}$$

on a neighbourhood of $(E[Z]_0, V(Z)_0, M_3(Z)_0, \cdots, M_m(Z)_0)$. Then we further approximate g as a linear function $E[Z], V(Z), M_3(Z), \cdots, and\ M_m(Z)$, and this is the second key of our simplification:

$$\begin{aligned} n &= b_1 E[Z] + b_2 V(Z) + b_3 M_3(Z) + \cdots + b_m M_m(Z) + O(E[Z], V[Z], M_3(Z), \cdots, M_m(Z)) \\ &\approx b_1 E[Z] + b_2 V(Z) + b_3 M_3(Z) + \cdots + b_m M_m(Z). \end{aligned} \tag{15}$$

Remark that a constant term in Eq. (15) must be set by 0, since if Z=0, Eq. (10) with $u(0)$=0 implies that $n = 0$.

Equation (15) corresponds to an approximation equation of UNPV, and the sign of n determines whether or not the project should be executed. In order to apply the equation to estimate a new project, we need to determine the coefficients in Eq. (15) from the historical data on assessment of projects. Let $(n_i, E[Z]_i, V(Z)_i, M_3(Z)_i, \cdots, M_m(Z)_i)(i = 1,2,\cdots,n)$ be a series of historical data of $(n, E[Z], V(Z), M_3(Z), \cdots, M_m(Z))$. If we observed them perfectly, we would estimate the coefficients of Eq. (15) as the following regression model:

$$n_i = b_0 + b_1 E[Z]_i + b_2 V(Z)_i + b_3 M_3(Z)_i + \cdots + b_m M_m(Z)_i + e_i \tag{16}$$

where $\beta_0 + e_i$ is an error term and e_i is the random error whose distribution is Gaussian with the mean zero. However, in the actual projects, one may only know the sign of each $n_i (i = 1,2,\cdots,n)$ through the historical project data, since it is generally difficult to identify the utility function of the investor. Therefore, we here introduce the following binary index variables:

$$n_i^* = 1 \quad (if \quad n_i > 0) \quad n_i^* = 0 \quad (if \quad n_i \le 0) \quad (i = 1,2,\cdots,n) \quad . \tag{17}$$

As a result, our problem is reduced to estimate a so-called "probit" model ([11,12]) using Eq. (17) with Eq. (16) through

$(n^*_{i,} E[Z]_i, V(Z)_i, M_3(Z)_i, \cdots, M_m(Z)_i)(i = 1,2,\cdots,n)$, and this is the final key of our simplification of the UNPV method. Then, we find out that the conditional probability of $n_i^* = 1$ under the given values of $E[Z], V(Z), M_3(Z), \cdots, and\ M_m(Z)$ is calculated as

$$
\begin{aligned}
&P(n^*_i = 1 | E[Z]_i, V(Z)_i, M_3(Z)_i, \cdots, M_m(Z)_i) \\
&= P(-e_i < b_0 + b_1 E[Z]_i + b_2 V(Z)_i + b_3 M_3(Z)_i + \cdots + b_m M_m(Z)_i | E[Z]_i, V(Z)_i, M_3(Z)_i, \cdots, M_m(Z)_i) \\
&= F(b_0 + b_1 E[Z]_i + b_2 V(Z)_i + b_3 M_3(Z)_i + \cdots + b_m M_m(Z)_i),
\end{aligned} \tag{18}
$$

where F is the distribution function of $-e_i$, that is, the Gaussian distribution function. Note that one may assume the variance of $-e_i$ to be equal to 1, since the sign of n_i is invariant even if a positive constant is multiplied. That is, it is allowed to assume that F is the standard Gaussian distribution function as:

$$
F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2p}} e^{-\frac{1}{2}t^2} dt. \tag{19}
$$

In consideration of Eq. (18), we can estimates the coefficients in the right-hand side of Eq. (16) by the maximum likelihood method using the following likelihood function (or the log likelihood function):

$$
\begin{aligned}
L(b_0, b_1, b_2, b_3 \cdots, b_m) = &\prod\nolimits_{n^*_i=1} F(b_0 + b_1 E[Z]_i + b_2 V(Z)_i + b_3 M_3(Z)_i + \cdots + b_m M_m(Z)_i) \\
&\times \prod\nolimits_{n^*_i=0} \{1 - F(b_0 + b_1 E[Z]_i + b_2 V(Z)_i + b_3 M_3(Z)_i + \cdots + b_m M_m(Z)_i)),
\end{aligned}
$$

$$
\begin{aligned}
&\log L(b_0, b_1, b_2, b_3 \cdots, b_m) \\
&= \sum [n_i^* \log F(b_0 + b_1 E[Z]_i + b_2 V(Z)_i + b_3 M_3(Z)_i + \cdots + b_m M_m(Z)_i) \\
&\qquad + (1 - n_i^*) \log\{1 - F(b_0 + b_1 E[Z]_i + b_2 V(Z)_i + b_3 M_3(Z)_i + \cdots + b_m M_m(Z)_i)\}].
\end{aligned} \tag{20}
$$

The maximum likelihood estimators (MLE) $\hat{b}_0, \hat{b}_1, \hat{b}_2, \hat{b}_3, \cdots, and\ \hat{b}_m$ are given as a set of solutions to the following equations for Eq. (20):

$$
\frac{\partial \log L(b_0, b_1, \cdots b_m)}{\partial b_0} = 0, \frac{\partial \log L(b_0, b_1, \cdots b_m)}{\partial b_1} = 0, \cdots, \frac{\partial \log L(b_0, b_1, \cdots b_m)}{\partial b_m} = 0 .
$$

Finally, using the coefficients mentioned above, we define a "*simplified UNPV regression equation*" to determine the executing the project, as follows:

$$\hat{n} = \hat{b}_0 + \hat{b}_1 E[Z] + \hat{b}_2 V(Z) + \hat{b}_3 M_3(Z) + \cdots + \hat{b}_m M_m(Z). \tag{21}$$

As already mentioned, in Eq. (21), $E[Z]$, $V(Z)$ and $M_k(Z) = E[(Z - E[Z])^k]$ $(k = 3, \cdots, m)$ are the mean, the variance and the k-th central moments of the random net present value Z=*RNPV* given by Eq. (4), respectively. Then, in a way analogous to that in the usual probit model [11,12], if $\hat{n} > 0$ in Eq. (21) under a set of values $(E[Z], V(Z), M_3(Z), \cdots, M_m(Z))$ for a new project, one decides to execute the project, since the event $\hat{n} > 0$ indicates that the probability of the execution of the project is over 0.5. Indeed,

$$\begin{aligned} &P(n^*_i = 1 | E[Z]_i, V(Z)_i, M_3(Z)_i, \cdots, M_m(Z)_i) \\ &\quad = F(\hat{b}_0 + \hat{b}_1 E[Z] + \hat{b}_2 V(Z) + \hat{b}_3 M_3(Z) + \cdots + \hat{b}_m M_m(Z)) = F(\hat{n}), \end{aligned} \tag{22}$$

where F is given by Eq. (19), and hence $F(\hat{n}) > 0.5$. We call this procedure to execute a project through Eq. (21), as mentioned previously, a *"simplified UNPV method,"* and this is the result we want. Note that only the sign of $\hat{n}$ is essential; the value of $\hat{n}$ itself does not have meaning.

Remark 3.1: We can also formulate a simplified equation (21) on the basis of a "logit" model replacing the Gaussian distribution function F by the logistic distribution function

$$G(x) = \frac{e^x}{1 + e^x}$$

in Eq. (18) (*cf*. [11,12]).

Remark 3.2: Carrying out the higher order Taylor expansion in Eq. (15) under a proper smoothness of the function g in Eq. (14), one can formulate a simplified UNPV equation having a higher-order approximation in a way similar to that of the derivation of Eq. (21).

Remark 3.3: If $m=2$, the equation (21) is reduced to

$$\hat{n} = \hat{b}_0 + \hat{b}_1 E[Z] + \hat{b}_2 V(Z). \tag{23}$$

This is the most elementary case in our simplified UNPV regression equations under consideration of the risk of *RNPV*. In [8], the model (23) has been roughly sketched.

4. UNPV AND THE SIMPLIFIED UNPV FOR EXPONENTIAL UTILITY

We will now provide some remarks and conjectures on the accuracy of our simplified method.

It is important to examine the consistency of signs between the strict UNPV and the simplified UNPV. However, it seems to be difficult to carry out the examination analytically under a general utility function, and at present, the issue remains as a future problem. On the other hand, fortunately, quite recent works by Miyahara [13,14] suggest that the exponential utility function Eq. (7), $u(x) = 1 - e^{-bx}$, is the most proper for making risk value measures through utility indifference pricing theory. On account of this, in this section, we restrict the utility function of an investor to that of the exponential type. In this case, by inserting Eq. (7) into Eq. (8), we find out that the strict UNPV n is written in the following explicit form:

$$n = -\frac{1}{b}\log(E[e^{-bZ}]) , \tag{24}$$

where $Z=RNPV$ is defined by Eq. (4). Moreover, if the distribution of *RNPV* is Gaussian, with the mean m and the variance s, the above equation of UNPV is put into the following, more simple form:

$$n = m - \frac{1}{2}sb \ . \tag{25}$$

Indeed, using the Gaussian distribution of Z, we can calculate $E[e^{-bZ}]$ in Eq. (24) as:

$$\begin{aligned} E[e^{-bZ}] &= \frac{1}{\sqrt{2ps}}\int e^{-bz} e^{-\frac{1}{2s}(z-m)^2} dz \\ &= e^{-\frac{1}{2s}(2msb - s^2b^2)} \frac{1}{\sqrt{2ps}}\int e^{-\frac{1}{2s}(z+sb-m)^2} dz = e^{-mb+\frac{1}{2}sb^2} \end{aligned}$$

and therefore Eq. (24) turns into Eq. (25).

Hence, in the case of the exponential utility function, the comparison between these results and our simplified UNPV regression equations (21) and (23) may allow us to conjecture the following:

C1) When the distribution of *RNPV* is Gaussian, the sign of the simplified UNPV (23) in case of $m=2$ is almost consistent with that of the strict UNPV (25) for the given mean and variance of *RNPV* because of the similarity between Eq. (23) and Eq. (25).

C2) In contrast to C1), with increasing the difference of the distribution of *RNPV* from the Gaussian distribution, the necessity of a simplified UNPV equation having higher-order approximations (*i.e.* Eq. (21)) increases, if one wants to realize the higher accuracy of consistency of the signs between the strict UNPV (24) and our simplified equation.

C3) Replacing the right-hand side of Eq. (16) with that of Eq. (24) with an error term having a Gaussian distribution, one will expect to derive a more exact probit model and the simplified UNPV equation than the equation (21). Then, we need a maximum likelihood estimation for a *non-linear probit model* to determine the unknown parameter b in Eq. (24).

Among these conjectures, a pilot study on the issue C1) has been dealt with in a recent paper by Hirata *et.al* [8]. They examined the consistency of signs between the equations (23) and (25) through computer simulations in a way analogous to those of a generation investment model on a crude oil thermal power plant in [6]. In what follows, we will briefly review the paper [8].

The paper presents a project assessment for a simple gas thermal power plant as a basic study on the UNPV method. In the paper, it is assumed that only the daily price of the electric power in the market is described by a stochastic mean regression model. Through the daily price and the daily fuel cost, a series of daily profits are obtained, and the total profit of one year is given by a summation of the daily profits for one year. The random cash flow is represented as the annual profit that is the difference between the total profit of one year and the operating maintenance cost. The *RNPV* is calculated by such random cash flows for about twenty years, together with a suitable constant discount rate and the present cost through Eq. (1) and Eq. (4).

Remark 4.1: This model enables us to expect that the distribution of *RNPV* often becomes close to a Gaussian one. Because of the framework mentioned above, *RNPV* is provided by a summation of a large number of random variables, and thus the central limit theorem comes to work approximately on this type of *RNPV* (*cf.* [6]).

In the framework, a set of 47 fictional projects on the revenue of the generation unit are first prepared in order to produce the historical project data. Then, the sample mean and the sample variance of *RNPV* are calculated for each project by the Monte-Carlo simulation, and thus the strict UNPV is obtained for each project under an exponential utility (7) with a given parameter β. Next, on the basis of this data on *RNPV* and the sign of the strict UNPV, the simplified regression equation (23) is estimated. Finally, for each fictional project, the sign of the estimated equation is compared with that of the corresponding strict UNP. As a result, on all 47 projects, the signs perfectly coincided with each other [8]. On account of Remark 4.1, this fact may indicate the conjecture C1) is valid, and we will expect that our simplified method is fully relevant to determine whether or not to execute a project, instead of the strict UNPV.

5. CONCLUDING REMARKS

In this paper, we have focused on a simplification of a risk assessment method for projects based on utility indifference pricing as proposed by Miyahara. As for the issue on consistency between the strict UNPV and our simplified UNPV, we have already given it consideration in Section 4, together with some conjectures. Finally, we will comment on some other remarks which will become relevant in future works related to our simplified method and the original method.

• We can calculate the probability to execute a project by using a simplified UNPV regression equation (21) and the Gaussian distribution function F through Eq. (22). This result allows us to carry out an interval estimation and a statistical test on such a probability (*cf*.[11,12]), and therefore the risk sensitivity of the investor for the project may become clearer.

• As other project evaluation methods are based on the utility functions, for example, the contingent valuation method (CVM) is often used in the research field of infrastructure improvement [15], although risk assessment of the project is not always dealt with in the method. It may be interesting to compare the results of our UNPV method or the simplified method with those of CVM.

• Although this study has provided a fundamental procedure on a simplification of UNPV, many issues remain to be solved when applications are made to real projects. To acquire valid results from our method in real projects, we should compose strict models of price fluctuations and profit evaluations, and identify the parameters in the models.

Nevertheless, UNPV and the simplification method will be useful tools for evaluating project worth by considering changes of investment circumstances and risks.

We will come back to treat such topics in future works.

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