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Abstract

This paper focuses on temperature options pricing using the marginal substitution value approach proposed by Davis (in Quantitative Finance, 2001). We first extend a utility function dealt with in Davis's work to the functions of "HARA" type which is well-known as a general class of utility functions in theoretical economics. The extension allows us to analysis a sensibility of "temperature options" price with respect to a risk aversion coefficient. In the framework, we next evaluate temperature option price contract on accumulated cooling degree days (CDDs) for a electric power producer. As an illustrative example, a time series model of daily air temperature at Nagoya city in Japan is estimated and on the basis of the model and our pricing formula, the temperature option prices for CDDs in summer are calculated numerically. The result verifies the temperature put option has a property as an insurance product for the purchaser.

1 Introduction

Weather derivatives are financial products whose underlying assets are given by some changes of weather conditions. Its history dates to 1997, Enron and Koch transacted first degree-day swap in the winter. From then on, against a background of intense competition and destabilization business environment among companies by global liberalization and deregulation, weather derivatives have been mainly developing as a risk reduction tool for companies which are exposed to weather risk.

"Temperature options" mainly treated in the present article are typical products of weather derivatives, whose underlying asset are some temperature indices. It is well-known that the "accumulated heating/cooling degree days (HDDs/CDDs) are commonly used as such underlying asset. These are calculated by accumulation of differences between a fixed basis temperature and actual temperatures for a certain fixed period; they are often used to measure degree of warmness in winter or coldness in summer. Since these indices highly correlate with profit of some companies - especially energy companies supplying electric power or gas, temperature options on HDDs and CDDs are transacted for risk reduction strategy in such companies.

However, one can not simply apply the "risk-neutral pricing method" (see Nielsen [8]) in complete market as typified by Black and Scholes to pricing of temperature options,

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since temperature movement is no tradable asset as mentioned above. Therefore, for this pricing problem, some attempts on the basis of the expected utility theory has been proposed (*e.g.* Cao and Wei [1]). The paper proposed by Davis in 2001 [3] is also one of such works included in this framework. The work is outlined as follows: First, temperature movement is simulated by AR(1) model. Underlying assets HDDs or CDDs, which are defined as accumulations of temperature index given by the temperature model, are described as a "geometric Brownian motion process" which is familiar to financial engineering. Next, by using the marginal substitution value approach in the expected utility theory, an explicit and simple pricing formula of the temperature derivatives on the above model for the investor with a logarithmic CRRA (Constant Relative Risk Averson) utility is derived. As the actual temperature data, the daily temperatures at Birmingham, England are adopted.

Thus, in Davis's work, a purchaser with a logarithmic CRRA utility is treated. On the other hand, a utility function which is applied to pricing is often taken in a more wide class of such functions. Moreover, in order to analysis a degree of risk averse of a purchaser of temperature options, we must extend the class of utility function dealt with in Davis's work to the more general one. Such an analysis must be important, since temperature options may be regarded as a sort of insurance products. Indeed, daily temperature data is only a physical index and is not tradable in market, the insurance value of the derivatives depends on a trading player. Moreover, for the company whose future profit correlates with temperature, the value is enormous. Consequently, the purchase of a temperature option needs to take account for a hedgeable temperature risk value at pricing them.

In consideration of the facts mentioned above, it seems to be natural that we generalize a class of utility functions in the framework of Davis's work and carry out a risk analysis of the purchaser; this is just a main purpose of the present article. That is, we will formally derive a general pricing formula of temperature derivatives by the extension of the class of the utility function dealt with in Davis's work to "HARA (Hyperbolic Absolute Risk Aversion)" ones, because the HARA class is a general class of the utility functions with a parameter corresponding to investor's risk aversion coefficient and hence we may investigate a sensibility of temperature options price with respect to the risk aversion coefficient.

This paper is outlined as follows: In Section 2, we first review Davis's works together with giving some notations of HDDs/CDDs and options on them. In his framework, an option pricing formula for energy companies with logarithm CRRA utility function is given, which is similar to Black-Scholes formula for call-options in complete financial market model. Under the same assumptions as those in Davis's work, we next will relax an assumption on the utility function and extend the option pricing formula to that for utility functions of HARA class. Then, it is difficult to find out the explicit formula in the case of the general utility functions belonging in HARA class, and hence, we derive the pricing formula in the suitable form to calculate the approximate value numerically with Monte Carlo simulation.

In Sections 3 and 4, by using the new formula together with computer simulation, we give an empirical example of pricing call and put temperature options on CDDs in an analogous way to that in Davis's paper. As the actual temperature data, then, the daily temperatures at Nagoya city are adopted. Here, we shortly introduce Nagoya city: Nagoya has a large population of almost two million. The city is geographically located roughly in the center of Japan which is regarded as the major urban and one of the foremost industrial area. For instance, the headquarters and production plants of Toyota Corp., one of the most major motor companies, are located close to Nagoya. Thus, Nagoya is a typical major city in Japan, and this is the reason why we use the temperature data as an example. In Section 3, we estimate a time series model of the daily air temperatures at Nagoya. Moreover, through this result we identify the geometric Brownian motion describing the underlying assets CDDs and examine the adequacy of the modeling. In Section 4, as an illustrative example, we calculate call and put option prices on the CDDs in summer for an electric power company with HARA utility in terms of our pricing formula and Nagoya temperature model. Then, we investigate a risk sensibility of temperature options with respect to a risk aversion coefficient which is contained in the utility functions of HARA class. In particular, the results on the put option will indicate that such a option has a property as an insurance product. Finally, in Section 5, we give some concluding remarks.

2 A Pricing Temperature Option Model and the Option Pricing Formula

In this section, we first review the pricing model of temperature derivatives proposed by Davis and his pricing formula, and in the latter half, we proceed to an extension of the framework.

We start with some definitions of weather indices. Let T_t be the average of the temperatures in degrees on date t. The daily number of "heating degree" HDD and "cooling degree" CDD on date t are defined as

$$\mathrm{HDD}_t = \max\{\hat{T} - T_t,\ 0\}, \tag{1}$$
$$\mathrm{CDD}_t = \max\{T_t - \hat{T},\ 0\}, \tag{2}$$

where $\hat{T}$ is the basis temperature which is determined beforehand. The accumulated heating degree days HDDs and the accumulated cooling degree days CDDs over an N day period ending at date t are defined as

$$X_t = \begin{cases} \sum_{i=1}^{N} \mathrm{HDD}_{t-i+1}, \\ \sum_{i=1}^{N} \mathrm{CDD}_{t-i+1}, \end{cases} \tag{3}$$

In the followings, we use these HDDs and CDDs are taken as the underlying indices on which the temperature options are written. Moreover, throughout this paper, we pick up the options of European type as a typical temperature derivative. We denote them by

$$B(X_T) = \begin{cases} A\max\{X_T - K,\ 0\} & \text{if } B \text{ is call option,} \\ A\max\{K - X_T,\ 0\} & \text{if } B \text{ is put option,} \end{cases} \tag{4}$$

where K is the strike index and A is the nominal pay-off rate.

Now, we proceed to the Davis's pricing model. As mentioned in Section 1, temperature derivatives are written on non-tradable assets, and hence those can not be priced by using the replicating portfolio. In consideration of this fact, Davis [3] approached to this problem by using marginal substitution value which was proposed as a pricing method for the derivatives at incomplete markets (Davis [2]).

The outline of the pricing method is as follows: An investor with concave utility function U and starting initial cash endowment x forms a dynamic portfolio whose cash value at time t is given by $H_\eta^\pi(t)$ on trading strategy $\pi \in \tau$, where τ denotes the set of admissible trading strategies. By maximizing the value function $V(x) = \sup_{\pi\in\tau} E[U(X_x^\pi(T))]$, the fair price $\hat{p}$ of the European option $B(X_T)$ was derived as follows:

Basic Pricing Formula (Davis [2]):

$$\hat{p} = \frac{E[U'(H_\eta^*(T))B(X_T)]}{V'(\eta)}, \tag{5}$$

where $V'(\eta) = \frac{d}{d\eta} E[U(H_\eta^*(T))]$, and $H_\eta^*(T)$ denotes the optimal portfolio value. On the basis of this formula, Davis [3] gave a fair price of a temperature option on HDDs or CDDs for an energy company in the framework of his financial model under the following assumptions:

Assumption 1: The underlying index X_t and the power spot price S_t are governed by the following geometric Brownian motions

$$\begin{aligned} dX_t &= \nu X_t dt + \gamma X_t dw_1(t), && (6)\\ dS_t &= \mu S_t dt + \sigma S_t dw_2(t), && (7) \end{aligned}$$

where $dw_1(t)$ and $dw_2(t)$ are standard Brownian motions with correlation $E[dw_1 dw_2] = \rho dt$.

Assumption 2: The profit of the company Y_t is formed as linear

$$Y_t = \alpha X_t S_t, \tag{8}$$

where α is an constant. We note that from the assumptions 1 and 2, Y_t is also governed by a geometric Brownian motion

$$dY_t = \theta Y dt + \xi Y dw_t, \tag{9}$$

where

$$\begin{aligned} \theta &= \nu + \mu + \rho\gamma\sigma,\\ \xi &= \sqrt{\gamma^2 + \sigma^2 + 2\rho\gamma\sigma}. \end{aligned}$$

Assumption 3: The optimal portfolio value H_η^* is equal to Y_t. Namely

$$\begin{aligned} H_\eta^*(T) &= Y_T, && (10)\\ \eta &= Y_0. && (11) \end{aligned}$$

For an energy producer, this assumption is natural, because he simply produces up to the level of current demand and sells at market price.

Assumption 4: Utility is of logarithmic CRRA type,

$$U(Y_t) = \log Y_t. \tag{12}$$

Then, the fair pricing formula (5) to the following form:

$$\hat{p} = E\left[\frac{Y_0}{Y_T} B(X_T)\right]. \tag{13}$$

Moreover, using Gisanov's Theorem, we can rewrite (13) for the option (4) as a formula similar to the Black-Scholes one: The price $\hat{p}$ is provided by

$$\hat{p} = \begin{cases} e^{-qT}\{X_0\Phi(d_1) - Ke^{-(r-q)T}\Phi(d_2)\} & \text{if } B \text{ is call option,} \\ e^{-qT}\{-X_0\Phi(-d_1) + Ke^{-(r-q)T}\Phi(-d_2)\} & \text{if } B \text{ is put option,} \end{cases} \tag{14}$$

where

$$\begin{aligned} r &= \mu + \nu - \gamma^2 - \sigma^2 - \rho\sigma\gamma, \\ q &= \mu - \sigma^2, \\ d_1 &= \frac{log(\frac{X_0}{K}) + (r - q + \frac{\gamma^2}{2})T}{\gamma\sqrt{T}}, \\ d_2 &= d_1 - \gamma\sqrt{T}, \\ \Phi(x) &= \int_{-\infty}^{x} \left(\frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} \right) dy. \end{aligned}$$

This is the final form of the pricing formula proved by Davis [3].

Now, we proceed to develop the above Davis's formula. That is, we relax the assumption 4 and find out a pricing formula for an energy company with a utility function of HARA class under the assumptions 1-3. As mentioned in Section 1, it is difficult to derive option prices explicitly for HARA utility. On account of this, we give our option pricing formula in the suitable form for a numerical simulation. In consideration of the assumption 3, we first rewrite (5) as

$$\hat{p} = \frac{E\left[\frac{d}{dY_T} U(Y_T) B(X_T)\right]}{\frac{d}{dY_0} E[U(Y_T)]}. \tag{15}$$

Then, we additionally assume that the following equation holds;

$$\frac{d}{dy} E[U(Y_T)] = E\left[\frac{d}{dy} U(Y_T)\right], \tag{16}$$

where $y = Y_0$. Since the solution Y_T of (9) with initial value y is given by $Y_T = y\exp\{(\theta - 1/2\xi^2)T\} + \xi w_T$, the denominator in (15) is rewritten as

$$\begin{aligned} \frac{d}{dy} E[U(Y_T)] &= E\left[\frac{dU}{dY_T} \frac{dY_T}{dy}\right] \\ &= E\left[\frac{dU}{dY_T} \left(\frac{Y_T}{y}\right)\right]. \end{aligned} \tag{17}$$

Substituting (17) into (15), we obtain the following expression as the fair price $\hat{p}$:

$$\hat{p} = \frac{E\left[\frac{dU}{dY_T} B(X_T)\right]}{E\left[\frac{dU}{dY_T} \left(\frac{Y_T}{y}\right)\right]}. \tag{18}$$

This is just a key formula we want. Here we note that this final result has only a "formal" meaning as a formula to carry out numerical simulations since it is not verified that (16) holds exactly.

By using the formula (18), the price $\hat{p}$ for a utility functions of HARA class is derived: A utility function of HARA class is defined as

$$U(Y) = \frac{1}{b-1}(a+bY)^{1-\frac{1}{b}} \quad b \neq 0, 1 \tag{19}$$

on a domain $\{Y : a + bY > 0\}$ for given constants a and b. Note that this function becomes a logarithmic utility function

$$U(Y) \quad = \quad \log(a+Y) \tag{20}$$

at $b = 1$. In the case of $a = 0$ and $b = 1$, it reduces to the log CRRA function treated in Davis [3]. This function also becomes an exponential utility function

$$U(Y) = -e^{-\frac{Y}{a}} \tag{21}$$

if $b = 0$ and $a > 0$ (see Ingersoll [7]). Here we note if $a \geq 0$, it is easy to see that (16) holds exactly. Substituting (19)-(21) to (18), respectively, we find out the price $\hat{p}$ for these utility functions. The results are shown in Table 1.

Utility form	Option price
$U(Y) = \frac{1}{b-1}(a+bY)^{1-\frac{1}{b}}$	$\frac{E[(a+bY_T)^{-\frac{1}{b}}B(X_T)]}{E\left[(a+bY_T)^{-\frac{1}{b}}\left(\frac{Y_T}{y}\right)\right]}$
$U(Y) = \log(a+Y)$	$\frac{E\left[\frac{1}{a+Y_T}B(X_T)\right]}{E\left[\frac{1}{a+Y_T}\left(\frac{Y_T}{y}\right)\right]}$
$U(Y) = -e^{-\frac{Y}{a}}$	$\frac{E\left[e^{-\frac{1}{a}Y_T}B(X_T)\right]}{E\left[e^{-\frac{1}{a}Y_T}\left(\frac{Y_T}{y}\right)\right]}$

Table 1: Option price for utility functions of HARA class.

Then, as mentioned, we see that each result has a simple form that allows easy calculation of $\hat{p}$ by Monte Carlo simulation. Furthermore, a risk aversion coefficient of the HARA utility functions is a monotonically decreasing function of a for **a** fixed b employed. The fact allows us a sensibility analysis of an option price with respect to a risk aversion coefficient.

3 An Air Temperature Model and the Estimation Problem

In this section, we estimate a time series model to describe daily air temperatures by using the empirical data of Nagoya in a similar manner to that in Davis [3]. As in the preceding section, the indices HDDs and CDDs are modeled by a geometric Brownian motion, and hence the distributions are assumed to be log-normal. In consideration of this, as an illustrative example, we also derive a distribution of CDDs derived from the numerical results by the temperature model and examine whether the obtained distribution is close to being log-normal one or not.

We first give some additive notations and assumptions on daily temperatures T_t. Suppose that T_t decomposes to a trend component and a stochastic component as

$$T_t = \bar{T}_t + D_t, \tag{22}$$

where $\bar{T}_t$ denotes the long-term average temperature on date t (for instance, a data period is 16 years, $\bar{T}_t$ is an simple arithmetic average for 16 years on date t) and $D_t = T_t - \bar{T}_t$ denotes the deviation from $\bar{T}_t$.

In Davis [3], the stochastic component D_t was described by a low-order time series model AR (1). On the other hand, in this paper, we assume that it is governed by the following ARMA(p, q) model:

$$D_t = \sum_{i=1}^{p} \phi_i D_{t-i} + \sum_{i=0}^{q} \psi_i \epsilon_t, \tag{23}$$

where $\psi_0 = 1$, and ϵ_t are the standard gauss noises (see Hamilton [4]). Because as observed later, the shape of the autocorrelation function and the partial autocorrelation function of D_t with respect to the daily temperatures of Nagoya suggest that D_t should be described by a higher-order ARMA model.

Now, we proceed to estimate the orders and parameters of this model on the basis of the empirical temperature data. For this purpose, we use the daily air temperature data in Nagoya for the 16 years period 1990-2005 from Japan Meteorological Agency's website (http://www.jma.go.jp/jma/indexe.html). Fig.1 shows the shape of the autocorrelation function and the partial autocorrelation function of D_t with respect to the daily temperatures data, and the results implicate D_t is governed by ARMA model mentioned above.

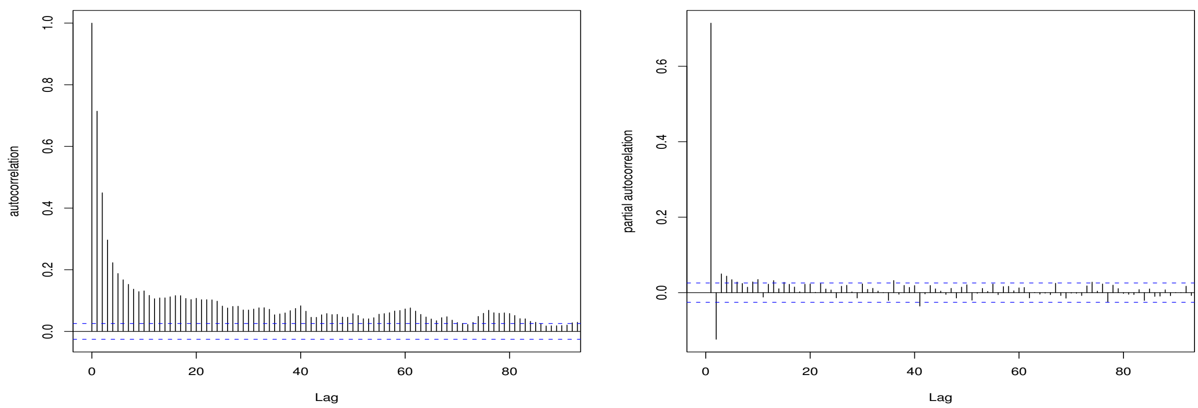

Fig. 1: The Shapes of autocorrelation and partial autocorrelation of the empirical data of D_t.

To choose the optimal ARMA order, we use the AIC model order selection criteria. Then, the estimation results are given as in the followings:

The Estimation results of the Nagoya temperature model:

- The optimal time series model over the 16 years is ARMA(6,4)

- The estimate values of the parameters:
  $\boldsymbol{\phi} = (0.183, 0.273, 0.462, 0.537, -0.652, 0.154)$,
  $\boldsymbol{\psi} = (0.616, 0.0609, -0.493, -0.917)$,
  $\mathrm{Var}(\epsilon) = 2.354$.

where $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$ are coefficients vector defined by $\boldsymbol{\phi} = (\phi_1, \cdots, \phi_p)$ and $\boldsymbol{\psi} = (\psi_1, \cdots, \psi_q)$, respectively. $\mathrm{Var}(\epsilon)$ is an estimate value of the variance of ϵ_t.

Next, we proceed to generate numerical samples of the index X_t given by (6) through a simulation of the air temperature model mentioned above and examine the distribution. For HDDs on the model, the similar result is also obtained, and hence, for simplicity, we only demonstrate the CDDs.

Let X_T be the CDDs at time $t = T$, which is accumulated over $N = 92$ days and set the beginning of the period as July, 1. Moreover, the basis temperature $\hat{T}$ is set 65 degree F. We generate 5000 number of samples of X_T on the basis of the daily air temperature time series model (23) for 92 days, and compare the distribution of $\log X_T$ and normal distribution. It shows that the distribution of $\log X_T$ is closely agreed with the normal distribution; indeed, the distribution statistically pass the normal test.

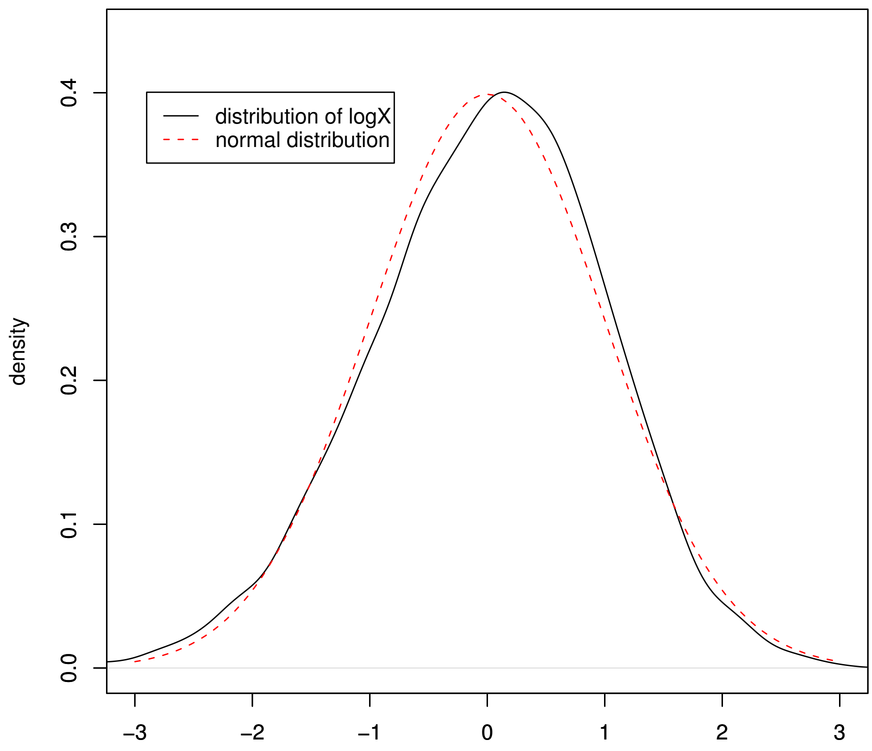

Fig. 2: A Comparison of the density distribution between $\log X_T$ and normal one.

These facts indicate that it is relevant that the distribution of CDDs is assumed to be log-normal, so that in view of statistical distributions, it may be natural to describe X_t as the geometric Brownian motion model (6).

Remark 1: Fig.3 and Fig.4 show the autocorrelation and partial autocorrelation of the empirical data X_t and those of the simulated X_t, respectively.

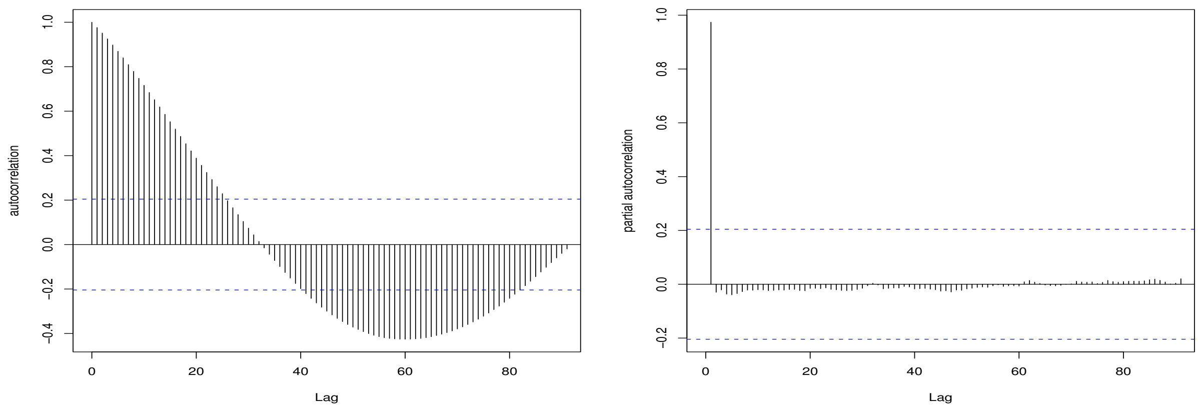

Fig. 3: The Shapes of autocorrelation and partial autocorrelation of the empirical data X_t.

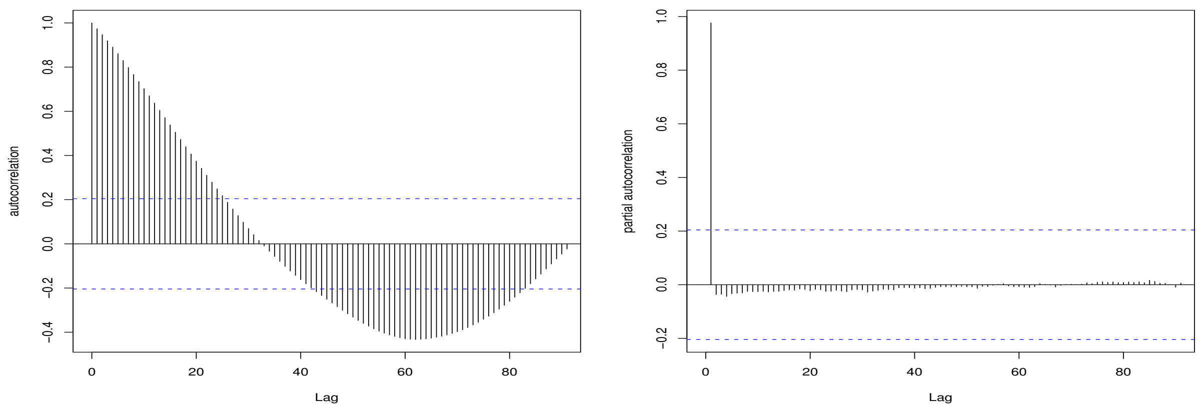

Fig. 4: The Shapes of autocorrelation and partial autocorrelation of the simulated X_t.

Here beginning of the accumulated term is 2005/7/1 and the accumulated days are 92 days. These results show that each sample autocorrelation pattern is very high and cyclic and implicate that X_t is not a Markov process. Hence, in view of stochastic processes, the X_t model by a geometric Brownian motion (6) may not always be appropriate. Nevertheless, if we treat an European type option whose value only depends on the probability distribution of underlying asset at time T, such a modeling is fully available.

The preceding result also gives an estimation method of unknown parameters of (6) which X_t satisfies. Finally, we will sum up it as some procedures.

Note that the distribution of $\log X_T$ on maturity T is given as

$$\log X_T \sim N\left(\log X_0 + \left(\nu - \frac{1}{2}\gamma^2\right)T, \gamma^2 T\right). \tag{24}$$

If we find out the mean m and the variance s^2 of $\log X_T$, the parameters (ν, γ) in (6) are obtained

$$\nu = \frac{-\log X_0 + m}{T} + \frac{\gamma^2}{2}, \quad \gamma = \sqrt{\frac{s^2}{T}}, \tag{25}$$

since

$$m = \log X_0 + \left(\nu - \frac{1}{2}\gamma^2\right) T, \quad s^2 = \gamma^2 T.$$

Therefore, if a time series model of air temperatures fitting empirical data is found out, unknown parameters (ν, γ) are estimated in the following procedures:
(Procedure 1) Simulate D_t by (23) together with the estimation results and generate the samples.
(Procedure 2) Substitute the samples of D_t into (22) and generate the samples of T_t.
(Procedure 3) Accumulate T_t over N days and compute X_T.
(Procedure 4) By Repeating (Pro.1)-(Pro.3), generate a large number of samples of X_T.
(Procedure 5) Calculate a distribution of simulated $\log X_T$ by (Pro.4), and estimate the mean m and the variance s^2 of $\log X_T$.
(Procedure 6) Using (25), determine ν and γ in (6).

In Table 2, we give some examples of (ν, γ) estimated by the above procedure.

N	ν	γ
31	0.0197	0.0232
62	0.0178	0.0130
92	0.0149	0.0105

Table 2: Estimation results of (ν, γ).

4 A pricing example on CDDs accumulated on Nagoya summer temperature data

Now, on the basis of the results in the preceding sections, we will calculate call and put option prices on the Nagoya temperature model for an electric power company, and examine the risk sensibility of them.

In the followings, we calculate the option prices in the setting below.

- Target securities: European call option and put option on CDDs.
- Maturity: 9/31.
- Basis temperature of the CDDs: 65 degrees F.
- Days over which the CDDs are accumulated: 92.
- Nominal pay-off rate: $A = 1$. Strike index: $K = 730$.

In Japan, the electricity market is not still developed sufficiently. In consideration of this, we assume the price of electricity S_t is a constant 1. Moreover, for simplicity, we set $X_0 = 186.8$ and $\alpha = 1$, respectively.

If the utility function of our procedure is of log CRRA type, the option price is determined explicitly by Davis's formula (14). On the other hand, in case of logarithm, exponential, and HARA type, the prices are evaluated by Table 1 with Monte Carlo simulations.

Then, The option pricing simulation algorithm is given as follows:
(Step 1) For given α, A, K, S, and X_0, we generate many numerical samples of X_T estimated by the procedures in Section 3.
(Step 2) Using the samples, we calculate the sample values of $B(X_T)$ and Y_T.
(Step 3) Finally, substituting the results of (Step 2) into each formula in Table 1 in Section 2, and calculating the sample mean, we obtain $\hat{p}$.

Table 3 shows the analytic solution of the European call and put option value under log CRRA utility by (14). Note that the parameters (ν, γ) used in this simulation are estimates showed in Table 2.

	call option	put option
price	6.99	8.08

Table 3: The option price in the case of log CRRA utility.

The simulation results for the European call and put options under logarithm, exponential, and HARA utility are shown in Fig.5$\sim$Fig.7. Here, to perform risk sensibility analysis later, in case of logarithm and exponential utility, we plot the call and put option prices with respect to a. And in case of HARA utility we plot them with respect to a for fixed $b = -1, -1/2$ and $-1/3$, respectively.

Now, we examine risk sensibility of the price values; this is just a merit of our pricing formula. For such analysis, it is often used ARA (Absolute Risk Aversion), which is well-known as the indicator of the degree of the response to risk, hence we also observe it. In our cases, they are expressed as $\mathrm{ARA}(Y) = 1/(a + Y)$, $\mathrm{ARA}(Y) = 1/a$, $\mathrm{ARA}(Y) = 1/(a + bY)$ for logarithm, exponential, and HARA, respectively, since it is defined as $\mathrm{ARA}(Y) = -U''(Y)/U'(Y)$. These results show that the value of ARA decreases with increasing the value of a. Fig.5$\sim$Fig.7 indicate that the values of the call option prices decrease with increasing the value of ARA on any utility function. In contrast with this, the values of the put option prices increase in keeping with the value of ARA on any utility function.

Finally, we proceed to investigate the relation between the put option prices and ARA. The results on the put options are consistent with the fact that an option purchaser has a taste for hedging temperature risk. Namely, an electric power company purchases the put option to hedge the risk of the battering of its profit by unusual cool summer. Hence, it is natural that the risk-hedging value, i.e. the put option value for the company increases in keeping with the value of ARA. Thus, the temperature options for an electric company may be regarded as a sort of insurance products.

Remark 2: As mentioned in Section 2, HARA utility function is defined on a domain $\{Y : a + bY > 0\}$. Therefore, in our numerical calculation, we only use the samples of Y_T satisfying this condition for a given a and b.

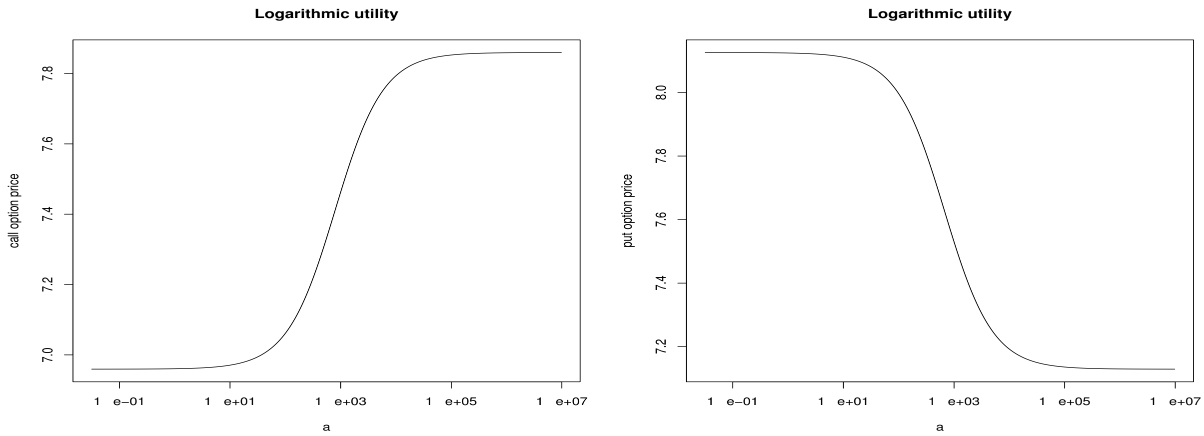

Fig. 5: The option prices for logarithm utility.

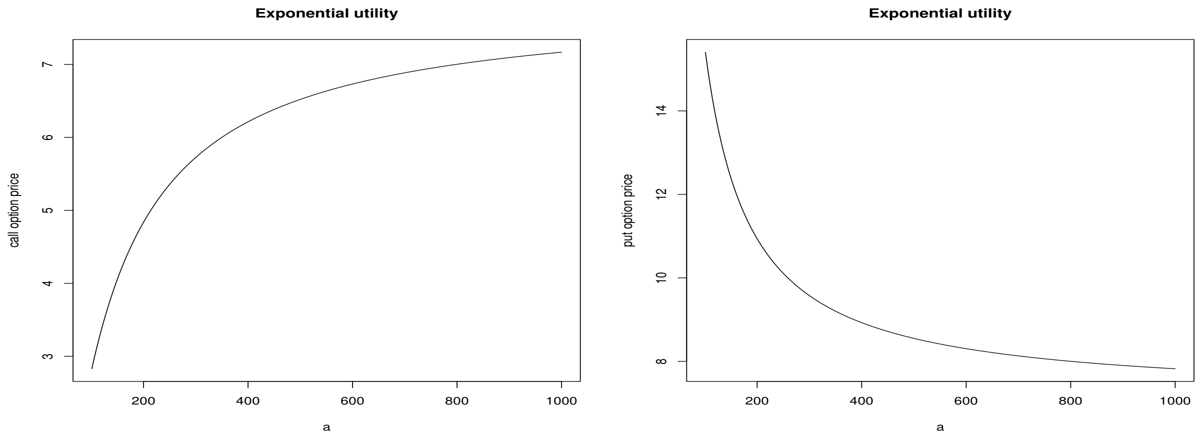

Fig. 6: The option prices for exponential utility.

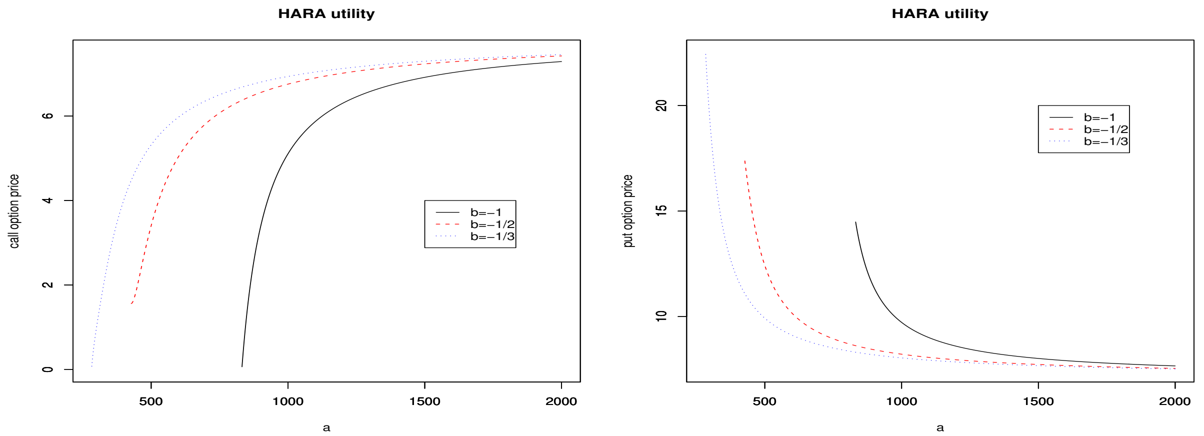

Fig. 7: The option prices for HARA utility.

5 Concluding Remarks

In this paper, we develop the Davis's formula for an energy company by extending a class of utility function treated in his work, and derive a pricing formula for HARA utility which is suitable to obtain the value numerically. Then, we estimate a time-series model to describe temperature changes in Nagoya and we calculate call and put option prices on CDDs by the pricing formula and Nagoya temperature model.

Furthermore, as a merit of our pricing formula, we examine the risk sensibility analysis for the calculated price on the basis of ARA, and observe that the movement of the value with respect to parameters is consistent with the movement of ARA; thus, we see that the price of the temperature option has the property as an insurance product.

We will finally refer to some concluding remarks: (1) As shown in Section 3, the assumption that the CDDs X_t is described as geometric Brownian motion may not be appropriate, since the empirical data shows the process is not Markovian, while the distribution of X_T at the terminal time T as log-normal seems natural. Therefore, for example, if one treats the options whose process depend on the "paths" of X_t, such an index should be formulated by better stochastic models. (2) Recently, as a similar approach to that in Davis's work, "utility indifference pricing method" is developed (*e.g.* Henderson [5,6]). The method allows us to price derivatives in an incomplete market by maximizing an investor's expected utility through dynamic trading. It may be natural that we apply this method to our problem on pricing temperature options and compare the results with those in the present paper. We will try such topics in future works.

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