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September 9, 2001

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**Abstract**

The empirical evidence has showed that the empirical distributions of log return on stocks has usually more or less skewness and kurtosis when compared with normal distributions. In this article we attempt to verify that the Geometric Lévy Process's capability of capturing the empirical evidence. We also evaluate the option prices by MEMM and compare them with the results of Black-Scholes model.

**Key words:** skewness and kurtosis, incomplete market, geometric Lévy process, minimal entropy martingale measure, computational option pricing.

## 1 Introduction

The normal distribution has a long and illustrious history, and then has become the workhorse of the financial asset pricing literature. But as attractive as the normal distribution is, it is not consistent with all the feature of the empirical evidence, which has showed that the empirical distributions of daily log return on stocks has usually more or less skewness and kurtosis when compared with normal distributions. The empirical evidence suggests the presence of jump components in asset price process that are responsible for relatively large and sudden movements, but occur relatively infrequently[see Scott(1997) and Hilliard(1998)].

Early studies of asset returns attempted to capture this empirical evidence by modeling the distribution of continuously compounded returns as a member of the stable class, of which the normal is a special case. Lévy

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initiated a general investigation of stable distributions and provided a complete characterization of them through their log-characteristic function in 1924. Lévy also showed that the tail probabilities of stable distributions approximate those the Pareto distribution in 1925. For applications to financial asset returns, stable distribution were popular in the 1960's and early 1970's (see Mandelbrot(1963), Fama(1965), Samuelson(1967, 1976), Granger and Morgenstern(1970), Fama and Roll(1971), Blattberg and Gonedes(1974) Fielitz(1976), Hagerman(1978), simkowize and Beedles(1980), Tucker(1992) ), but they are less commonly used today. Campbell(1997) summarized that they have fallen out of favor partly because they make theoretical modeling so difficult; standard finance theory almost always requires finite second moments of returns, and often finite higher moments as well. Stable distributions also have some counterfactual implications.

Recently, Y.Miyahara(2001) has developed [Geometric Lévy Process & MEMM] pricing model for the incomplete market. This model exhibits several appealing economic properties, including a capability of describing all manner of new-information arrival and consistency with the efficient-markets hypothesis because the model exhibits certain martingale properities.

In this article, applying [Geometric Lévy Process & MEMM] pricing model, we attempt to verify the model's capability of capturing the empirical evidence by geometric Lévy process on 2 simple models, meanwhile, evaluate the options price by MEMM and compare them with Black-Scholes model.

This article is organized as follow. Section 2 explains the [Geometric Lévy Process & MEMM] pricing model briefly. Section 3 verifies the model's capability of capturing the empirical evidence by geometric Lévy process on 2 simple models. Section 4 evaluates the options price under MEMM by computer simulation, and compare them with Black-Scholes model. and section 5 offers a summary and conclusions.

## 2 Geometric Lévy Process Model

The empirical evidence has showed that the empirical distributions of daily log return on stock have usually more or less skewness and kurtosis when compared with normal distributions. It also suggests the presence of jump components in asset price process, which are responsible for relatively large and sudden movements but occur relatively infrequently. Hence we employ geometric Lévy processes with jump part to capture the above empirical evidence. In this section, we explain the Geometric Lévy process model according to Y. Miyahara(2001).

The price process of underlying asset $S(t)$ is of the form $S(t) = S_0 e^{X(t)}$, here $X(t)$ is a Lévy process given by

$$X(t) = \gamma_0 t + \sigma W(t) + Y(t), \qquad Y(0) = 0, \tag{1}$$

where $W(t)$ is a standard Brownian motion and $Y(t)$ is the jump part of $X(t)$. We assume that $Y(t)$ is a compound Poisson process, namely

$$Y(t) = \int_{[0,t]} \int_{(-\infty,\infty)/\{0\}} x N_p(dsdx) \tag{2}$$

where $N_p(dsdx)$ is a counting measure of a Poisson point process. We express the Lévy measure $\nu(dx)$ as

$$\nu(dx) = c\rho(dx), \qquad c > 0, \qquad \rho((-\infty,\infty)) = 1, \tag{3}$$

and we assume, for the simplicity, that

$$\rho(dx) \equiv \sum_{j=1}^{n} p_j \delta_{a_j}; \quad \sum_{j=1}^{n} p_j = 1; \quad p_j > 0. \tag{4}$$

We will investigate the following two simple cases in this article..

- **Model 1 (Gaussian + 1-jump model):**

Suppose that the Lévy process consists of a Brownian motion and a jump process, then the Lévy process is in the following form

$$X(t) \equiv \gamma_0 t + \sigma W(t) + Y(t), \tag{5}$$

and the Lévy measure $\nu(dx)$ of the jump part $Y(t)$ is

$$\nu(dx) = c\delta_a(dx), \qquad c > 0, \qquad a \neq 0.$$

- **Model 2 (2-jumps model)**

When the Lévy process is supposed to have no continuous part and to consist of 2-kinds of jumps. Then the Lévy process is in the following form

$$X(t) \equiv Y(t), \tag{6}$$

and the Lévy measure $\nu(dx)$ is

$$\nu(dx) = c(p_1\delta_{a_1}(dx) + p_2\delta_{a_2}(dx)),$$

where

$$c > 0, \quad p_1, p_2 > 0, \quad p_1 + p_2 = 1, \quad a_1, a_2 \neq 0, \quad a_1 \neq a_2.$$

# 3 Simulation of Log Return Distributions of $X(1)$

In this section, in order to verify our model's capability of capturing the empirical evidence, we simulate the log return distributions of $X(1)$.

### 3.1 Modeling

Both model 1 and model 2 in section 2 have four parameters $(\gamma_0, \sigma, a, c)$, or $(a_1, a_2, p_1, c)$ respectively, which characterize each log return distributions of $X(1)$. Our approach is to determine the parameters for modeling by the method of moments so that the model's moment would be equal to the sample's moment of the asset.

Suppose that $\hat{\mu}(z)$ is the characteristic function for $X(1)$, and set

$$\frac{d^k\hat{\mu}}{dz^k}(0) = (i)^k m_k, \qquad m_k \in R, \quad and \quad k = 1, 2, 3, 4, \tag{7}$$

where $i = \sqrt{-1}$. Then we can obtain four equations on parameters, so the values of the parameters may be obtained from these equations.

The following notations are devised for the convenience.

$$b_1 = m_1, \qquad b_2 = m_2 - m_1^2$$
$$b_3 = m_3 + 2m_1^3 - 3m_1m_2, \qquad b_4 = m_4 - 6m_1^4 + 12m_1^2m_2 - 4m_1m_3 - 3m_2^2.$$

#### 3.1.1 On Model 1

The characteristic function of $X(1)$ is

$$\hat{\mu}(z) = e^{iz\gamma - \frac{1}{2}z^2\sigma^2 + c(e^{iza}-1)}. \tag{8}$$

Then, from (7) we obtain the following equations [1]

$$\begin{aligned}
\gamma_0 + ca &= m_1 \\
(\gamma_0 + ca)^2 + \sigma^2 + ca^2 &= m_2 \\
(\gamma_0 + ca)^3 + 3(\gamma_0 + ca)(\sigma^2 + ca^2) + ca^3 &= m_3 \\
(\gamma_0 + ca)^4 + 6(\gamma_0 + ca)^2(\sigma^2 + ca^2) + 3(\sigma^2 + ca^2)^2 + 4(\gamma_0 + ca)ca^3 + ca^4 &= m_4
\end{aligned}$$

This equations are changed to

$$\begin{aligned}
\gamma_0 + ca &= m_1 = b_1 \\
\sigma^2 + ca^2 &= m_2 - m_1^2 = b_2 \\
ca^3 &= m_3 + 2m_1^3 - m_1m_2 = b_3 \\
ca^4 &= m_4 - 6m_1^4 + 12m_1^2m_2 - 4m_1m_3 - 3m_2^2 = b_4
\end{aligned}$$

and we can derive

$$\gamma_0 = b_1 - \frac{b_3^3}{b_4^2}, \tag{9}$$

$$\sigma = \sqrt{b_2 - \frac{b_3^2}{b_4}}, \tag{10}$$

[1] $\hat{\mu}(0) = 1$, $\frac{d\hat{\mu}}{dz}(0) = i(\gamma + ca)$, $\frac{d^2\hat{\mu}}{dz^2}(0) = -(\sigma^2 + ca^2)$, $\frac{d^3\hat{\mu}}{dz^3}(0) = -ica^3$, $\frac{d^4\hat{\mu}}{dz^4}(0) = ca^4$

$$a = \frac{b_4}{b_3}, \tag{11}$$

$$c = \frac{b_3^4}{b_4^3}, \tag{12}$$

#### 3.1.2 On Model 2

The characteristic function for $X(1)$ may be

$$\hat{\mu}(z) = e^{c(p_1 e^{iza_1} + p_2 e^{iza_2} - 1)}. \tag{13}$$

Similar to model 1, we can obtain the equations [2]

$$\begin{aligned}
c(p_1 a_1 + p_2 a_2) &= m_1 \\
c^2(p_1 a_1 + p_2 a_2)^2 + c(p_1 a_1^2 + p_2 a_2^2) &= m_2 \\
c^3(p_1 a_1 + p_2 a_2)^3 + 3c^2(p_1 a_1 + p_2 a_2)(p_1 a_1^2 + p_2 a_2^2) + c(p_1 a_1^3 + p_2 a_2^3) &= m_3 \\
c^4(p_1 a_1 + p_2 a_2)^4 + 6c^3(p_1 a_1 + p_2 a_2)^2(p_1 a_1^2 + p_2 a_2^2) + 3c^2(p_1 a_1^2 + p_2 a_2^2)^2 & \\
+4c^2(p_1 a_1 + p_2 a_2)(p_1 a_1^3 + p_2 a_2^3) + c(p_1 a_1^4 + p_2 a_2^4) &= m_4
\end{aligned}$$

or

$$\begin{aligned}
c(p_1 a_1 + p_2 a_2) &= m_1 = b_1 \\
c(p_1 a_1^2 + p_2 a_2^2) &= m_2 - m_1^2 = b_2 \\
c(p_1 a_1^3 + p_2 a_2^3) &= m_3 + 2m_1^3 - m_1 m_2 = b_3 \\
c(p_1 a_1^4 + p_2 a_2^4) &= m_4 - 6m_1^4 + 12m_1^2 m_2 - 4m_1 m_3 - 3m_2^2 = b_4
\end{aligned}$$

Since $p_1 + p_2 = 1$, from above equations, we can derive

$$a_1 = \frac{-B + \sqrt{B^2 - 4AC}}{2A}, \tag{14}$$

$$a_2 = \frac{b_3 - b_2 a_1}{b_2 - b_1 a_1} = \frac{-B - \sqrt{B^2 - 4AC}}{2A}, \tag{15}$$

$$p_1 = \frac{(b_2 - b_1 a_2) a_2}{(a_2 - a_1)[b_2 - b_1(a_1 + a_2)]} \tag{16}$$

$$p_2 = \frac{(b_2 - b_1 a_1) a_1}{(a_1 - a_2)[b_2 - b_1(a_1 + a_2)]} \tag{17}$$

$$c = -\frac{b_2 - b_1(a_1 + a_2)}{a_1 a_2}. \tag{18}$$

where

$$A = b_2^2 - b_1 b_3, \qquad B = b_1 b_4 - b_2 b_3, \qquad C = b_3^2 - b_2 b_4.$$

[2] $\hat{\mu}(0) = 1, \quad \frac{d^k \hat{\mu}}{dz^k}(0) = i^k c(p_1 a_1^k + p_2 a_2^k)$

### 3.1.3 Restriction to the parameter

According to the properties of geometric Lévy process, when using the method of moments to determine the parameters, there are several condition restrictions to the parameters on both model 1 and model 2.

- On model 1
  the parameters $\sigma, a, c$ must satisfy the following conditions.

$$\sigma > 0, \quad a \neq 0, \quad c > 0.$$

- On model 2
  the parameters $p_1$ and $c$ must satisfy the following conditions,

$$0 < p_1 < 1, \quad c > 0,$$

  and for the solvability of the equations, it should hold that

$$B^2 - 4AC > 0$$

We will see the following example which explains the restrictions.

**Example 1** when $m_1 = 0.0, m_2 = 1.0$, both model 1 and model 2 have unique parameter sets in real value on same restriction such that if and only if $m_4 > m_3^2 + 3$.

When we choose the third moment and fourth moment for simulation, we must obey the above restriction.

## 3.2 Simulation results

The normalized third moment and fourth moment are represent of skewness, kurtosis respectively. The normal distribution has skewness equal to zero, as do all other symmetric distribution. The normal distribution has kurtosis equal to 3, but fat-tailed distribution with extra probability mass in the tail areas have higher kurtosis.

### 3.2.1 On model 1

When the sample's moments $\{m_k, k = 1, 2, 3, 4\}$ are given, then we can obtain the parameter sets $(a, c, \gamma, \sigma)$ from above equations respectively. The moments used in the model are summarized in table 1.

**Table 1:value of parameters for Model 1**

| | $m_1 = 0.0,$ | | | $m_2 = 1.0$ | | |
|---|---|---|---|---|---|---|
| | $m_4 = 4.0$ | | | $m_4 = 5.0$ | | |
| parameter | $m_3 = 0.3$ | $m_3 = 0.6$ | $m_3 = 0.9$ | $m_3 = 0.3$ | $m_3 = 0.6$ | $m_3 = 0.9$ |
| $\gamma_0$ | -0.027 | -0.216 | -0.729 | -0.007 | -0.054 | -0.182 |
| $\sigma$ | 0.954 | 0.800 | 0.436 | 0.977 | 0.906 | 0.771 |
| a | 3.333 | 1.667 | 1.111 | 6.667 | 3.333 | 2.222 |
| c | 0.008 | 0.130 | 0.656 | 0.001 | 0.016 | 0.082 |

Suppose that the $j$th sample path of $n+1$ prices $X_0^j$, $X_1^j$,..., $X_n^j$ equally spaced at intervals of length $h$ over the fixed time span $[0,T]$, so that $X_k^j \equiv X^j(kh), k = 0,1,\ldots,n,\ j = 1,2,\ldots,m$ and $T = nh$. Let random variable $\epsilon_k^j$ have a standard normal distribution and $\eta_k^j$ have an unite distribution, both random variables are supposed to have a independent and identical distribution (IID) for $\{k = 0,1,\ldots,n,\ j = 1,2,\ldots,m\}$. Then we can design the following algorithm for simulations.

- **algorithm 1.** simulation for Model 1:

  1. obtain random variable $\tau$ from the Poisson distribution:
  $$\tau = -\frac{\log(1-\eta_k^j)}{ch}.$$
  2. iterate the following calculation for $i = k+1, k+2, \ldots, k+\tau$.
  $$X_i^j = X_{i-1}^j + \gamma_0 h + \sigma\epsilon_i^j\sqrt{h}.$$
  3. $X_{k+\tau+1}^j = X_{k+\tau}^j + a$.
  4. iterate from step 1 to step 3 until $k = n$.
  5. iterate step 4 for $j = 1,2,\ldots,m$.

Then we are able to simulate for $X(1)$ by the algorithm 1 when $X_0^j$ is given. In order to present the experiment distributions, denote that $X_{max} = max\{X_n^j, \forall j, j = 1,2,\ldots,m\}$ and $X_{min} = min\{X_n^j, \forall j, j = 1,2,\ldots,m\}$. Then we translate the following counting function $f_i(\Delta x)$ into density function of $X(1)$.

$$f_i(\Delta x) \equiv \sum_{i=1}^{H} \delta_{(i\Delta x)}, \qquad H = cell(\frac{X_{max}-X_{min}}{\Delta x}), \tag{19}$$

where

$$\delta_{(i\Delta x)} = \begin{cases} 1, & if X_{min} - i\Delta x \leq X_n^j < X_{min} + i\Delta x, \quad j = 1,2,\ldots,m \\ 0, & otherwise. \end{cases}$$

Therefore the experiment distributions of $X(1)$ may be computed, the simulation results are shown in figure 1 and figure 2. When the third moment and fourth moment change, the value of parameters would become difference. Then the distributions reflect the difference and have significant skewness and kurtosis when compared with normal distributions. Two points are noteworthy from the figure 1 and figure 2.

- when $m_4 = 4.0$, the distributions are left skewed. The larger the value of $m_3$ is, the larger the degree of left skewness is.
- when $m_4 = 5.0$, the distributions have less skewness than $m_4 = 4.0$, but highly peaked, heavy tailed. The larger the value of $m_3$ is. the larger the degree of peakedness, tailedness is.

### 3.2.2 On model 2

When the sample's moments $\{m_k, k = 1,2,3,4\}$ are given, then we can obtain the parameter sets $(a_1, a_2, p_1, p_2, c)$ from the above equations respectively.

**Table 2:value of parameters for Model 2**

| | $m_1 = 0.0,$ | | | $m_2 = 1.0$ | | |
|---|---|---|---|---|---|---|
| | $m_4 = 4.0$ | | | $m_4 = 5.0$ | | |
| parameter | $m_3 = 0.3$ | $m_3 = 0.6$ | $m_3 = 0.9$ | $m_3 = 0.3$ | $m_3 = 0.6$ | $m_3 = 0.9$ |
| $a_1$ | 1.116 | 1.154 | 1.076 | 1.540 | 1.615 | 1.630 |
| $a_2$ | -0.816 | -0.554 | -0.176 | -1.240 | -1.015 | -0.730 |
| $p_1$ | 0.422 | 0.324 | 0.14 | 0.446 | 0.386 | 0.309 |
| $p_2$ | 0.578 | 0.676 | 0.86 | 0.554 | 0.614 | 0.691 |
| c | 1.099 | 1.562 | 5.263 | 0.524 | 0.610 | 0.840 |

Similar to mode1 1, we can design the following algorithm for simulation.

- **algorithm 2.** simulation for Model 2:
  1. obtain random variable $\tau$ from the Poisson distribution:
  $$\tau = -\frac{\log(1-\eta_k^j)}{ch}.$$
  2. iterate the following calculation for $i = k+1, k+2, \ldots, k+\tau$.
  $$X_i^j = X_{i-1}^j$$
  3. if $p_1 > \eta_{k+\tau+1}^j$
  $$X_{k+\tau+1}^j = X_{k+\tau}^j + a_1.$$
  else (or $p_2 > \eta_{k+\tau+1}^j$)
  $$X_{k+\tau+1}^j = X_{k+\tau}^j + a_2.$$
  4. iterate from step 1 to step 3 until $k = n$.
  5. iterate step 4 for $j = 1, 2, \ldots, m$.

Similar to model 1, we are able to simulate for the distributions of $X(1)$ by the algorithm 2. the simulation results are shown in figure 3 and figure 4. The distributions reflect the two jumps, have more significant skewness and kurtosis when compared with normal distributions. The distributions show also more skewness, peakedness and tailedness than model 1. Two points are noteworthy from the figure 3 and figure 4.

- when $m_4 = 4.0$, the distribution is left skewed, unlike model 1, the degree of left skewness is the biggest on $m_3 = 0.6$. Rank of peakedness is $m_3 = 0.6$, $m_3 = 0.3$, $m_3 = 0.9$ respectively.
- when $m_4 = 5.0$, the distribution is left skewed almost as well as when $m_4 = 4.0$, but the distribution have more peakedness, and tailedness when compared with $m_4 = 4.0$. the degree of left skewness is the biggest on $m_3 = 0.9$. Rank of peakedness is $m_3 = 0.9$, $m_3 = 0.3$, $m_3 = 0.6$ respectively.

# 4 [GLP & MEMM] model vs B-S model

The theory of option pricing in a complete market is now well understood. For example, recall that in the Black-Scholes model, the price process of underlying asset is assumed to generate by geometric Brownian motion, Under arbitrage-free assumption, the option prices are uniquely specified in term of expectations with respect to unique equivalent martingale measure by martingale pricing technique. However, when the price process of underlying asset is assumed to generate by geometric Lévy process with jump process, it is not possible to replicate the payoff of the option prices by a portfolio from the underlying asset and a bond. The market is incomplete due to the jump, and then there are more than one equivalent martingale measures. we select minimal entropy martingale measures(MEMM) to evaluate the European call option by martingale pricing technique.

## 4.1 [GLP & MEMM] pricing model

According to Miyahara(1999)'s proposition, when employed the geometric Lévy process to describe the price processes of underlying asset like model 1 and model 2, there exist a MEMM of the price process $S(t)$. Under the MEMM $P^*$, The Lévy process for both model 1 and model 2 may be described by following form.

- **On Model 1**

The Lévy process may be followed by

$$\bar{X}(t) = (\gamma_0 + \beta\sigma^2)t + \sigma W(t) + \bar{Y}(t), \tag{20}$$

and Lévy measure may be

$$\nu^*(dx) = c^*\delta_a(dx), \qquad c^* = ce^{\beta(e^a-1)},$$

where $\beta$ is the unique solution to following equation

$$\gamma_0 + (\frac{1}{2} + \beta)\sigma^2 + c(e^a - 1)e^{\beta(e^a-1)} = 0. \tag{21}$$

- **On Model 2**

The Lévy process may be generated by

$$\bar{X}(t) = X(0) + \bar{Y}(t), \tag{22}$$

and Lévy measure may be

$$\nu^*(dx) = c^*(p_1^*\delta_{a_1}(dx) + p_2^*\delta_{a_2}(dx)),$$

where

$$p_1^* = \frac{p_1 e^{\beta e^{a_1 - 1}}}{p_1 e^{\beta e^{a_1-1}} + p_2 e^{\beta e^{a_2-1}}}, \quad p_2^* = \frac{p_2 e^{\beta e^{a_2 - 1}}}{p_1 e^{\beta e^{a_1-1}} + p_2 e^{\beta e^{a_2-1}}}, \quad c^* = c(p_1 e^{\beta e^{a_1-1}} + p_2 e^{\beta e^{a_2-1}}),$$

and $\beta$ is the unique solution to following equation

$$p_1(e^{a_1 - 1})e^{\beta(e^{a_1}-1)} + p_2(e^{a_2-1})e^{\beta(e^{a_2}-1)} = 0. \tag{23}$$

## 4.2 Simulation result

We have known that the Lévy measure $\nu^*(dx)$ may be obtain under the MEMM $P^*$ through the $\beta$. Therefore, using the numerical bisection method, we are able to get an approximation value of $\beta$. Then we use the Monte Carlo simulation to calculate the approximation of this option price under the MEMM $P^*$.

### 4.2.1 On model 1

We are able to get an approximation value of $\beta$ by the numerical bisection method, and then obtain the Lévy measure $\nu^*(dx)$. the parameters of $\nu^*(dx)$ are summarized in table 3 when the sample's moments $\{m_k, k = 1, 2, 3, 4\}$ are given.

**Table 3:value of $\nu^*(dx)$ for Model 1**

| | $m_1 = 0.0,$ | | | $m_2 = 1.0$ | | |
|---|---|---|---|---|---|---|
| | $m_4 = 4.0$ | | | $m_4 = 5.0$ | | |
| parameter | $m_3 = 0.3$ | $m_3 = 0.6$ | $m_3 = 0.9$ | $m_3 = 0.3$ | $m_3 = 0.6$ | $m_3 = 0.9$ |
| $\beta$ | -0.470 | -0.353 | -0.321 | -0.493 | -0.434 | -0.294 |
| $\gamma_0^*$ | -0.455 | -0.442 | -0.790 | -0.477 | -0.410 | -0.357 |
| $\sigma$ | 0.954 | 0.800 | 0.436 | 0.977 | 0.906 | 0.771 |
| a | 3.333 | 1.667 | 1.111 | 6.667 | 3.333 | 2.222 |
| $c^*$ | 0.000 | 0.028 | 0.341 | 0.000 | 0.000 | 0.007 |

Under the MEMM $P^*$, European call option with strike price K and expiration at time T might be evaluated by

$$C(S(t), t; K) = E_{P^*}[Max(S(T) - K, 0)]. \tag{24}$$

similar to algorithm 1 by substituting $\nu^*(dx)$ for $\nu(dx)$, we use the Monte Carlo simulation to calculate the approximation of the European call option by

$$\bar{C} = \frac{1}{m}\sum_{i=l}^{H}(f_i(\Delta x)e^{\bar{X}_i} - K), \qquad \bar{X}_i = \bar{X}_{min} + i\Delta x \tag{25}$$

where

$$l = min\{i : cell(f_i(\Delta x)e^{X_n^i} - K > 0), \quad i = 1, 2, \ldots, H\}$$

The simulation result is showed in figure 3. The option prices reflect the skewness and kurtosis of distributions on our models , have significant diffidence when compared with Black-Scholes model. Two points are noteworthy:

1. The prices have some difference between the [Geometric Lévy Process & MEMM] pricing model and Black-Scholes model.
2. The prices also have some difference between the $m_3 > 0$ and $m_3 < 0$ for [Geometric Lévy Process & MEMM] pricing model.

### 4.2.2 On model 2

Similar to model 1, we can obtain the Lévy measure $\nu^*(dx)$ and calculate the option prices. the parameters of $\nu^*(dx)$ are summarized in table 4 when the sample's moments $\{m_k, k = 1, 2, 3, 4\}$ are given.

**Table 4:value of $\nu^*(dx)$ for Model 2**

| | $m_1 = 0.0,$ | $m_2 = 1.0$ | | | | |
|---|---|---|---|---|---|---|
| | $m_4 = 4.0$ | | | $m_4 = 5.0$ | | |
| parameter | $m_3 = 0.3$ | $m_3 = 0.6$ | $m_3 = 0.9$ | $m_3 = 0.3$ | $m_3 = 0.6$ | $m_3 = 0.9$ |
| $\beta$ | -0.379 | -0.345 | -0.321 | -0.325 | -0.296 | -0.274 |
| $a_1$ | 1.116 | 1.154 | 1.076 | 1.540 | 1.615 | 1.630 |
| $a_2$ | -0.816 | -0.554 | -0.176 | -1.240 | -1.015 | -0.730 |
| $p_1^*$ | 0.214 | 0.164 | 0.0.077 | 0.162 | 0.137 | 0.112 |
| $p_2^*$ | 0.786 | 0.836 | 0.923 | 0.838 | 0.863 | 0.888 |
| $c^*$ | 0.997 | 1.462 | 5.161 | 0.436 | 0.524 | 0.753 |

The simulation results are shown in figure 4. Two points are noteworthy:

1. The prices have some difference between the [Geometric Lévy Process & MEMM] pricing model and Black-Scholes model.
2. The prices also have some difference between the $m_3 > 0$ and $m_3 < 0$ for [Geometric Lévy Process & MEMM] pricing model.

## 5 Summary

To capture the empirical evidence, which show that the distributions of daily log return on stocks, has usually more or less skewness and kurtosis when compared with normal distributions, we apply the geometric Lévy process to define a price process of underlying asset, and provide the method of moments to determine the parameters so that the model's moment would be equal to the sample's moment of the asset. Our results of computer simulation show those two models are capable of capturing the skewness and kurtosis of distributions of daily log return on stocks eventhough they are simple models. They also show that the distributions are significant difference between model 1 and model 2 eventhough two models have same moments.

We select MEMM to evaluate European call option prices for the incomplete markets. the results of Monte Carlo simulation indicate that: (1)The prices are some difference between the [Geometric Lévy Process & MEMM] pricing model and Black-Scholes model. (2) The prices also are some difference between the $m_3 > 0$ and $m_3 < 0$ for [Geometric Lévy Process & MEMM] pricing model. [Geometric Lévy Process & MEMM] pricing model may be potentially useful for the empirical analysis.

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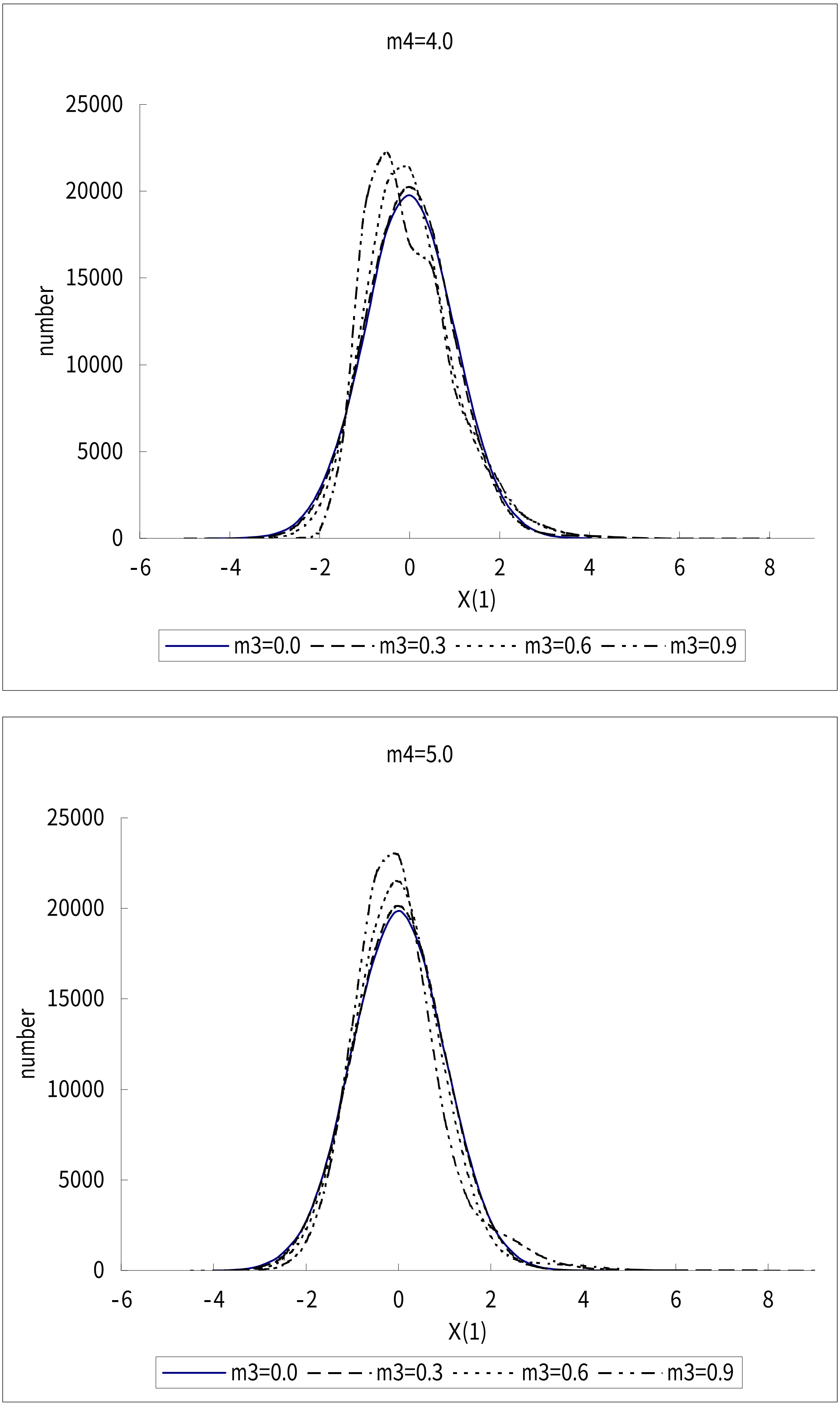
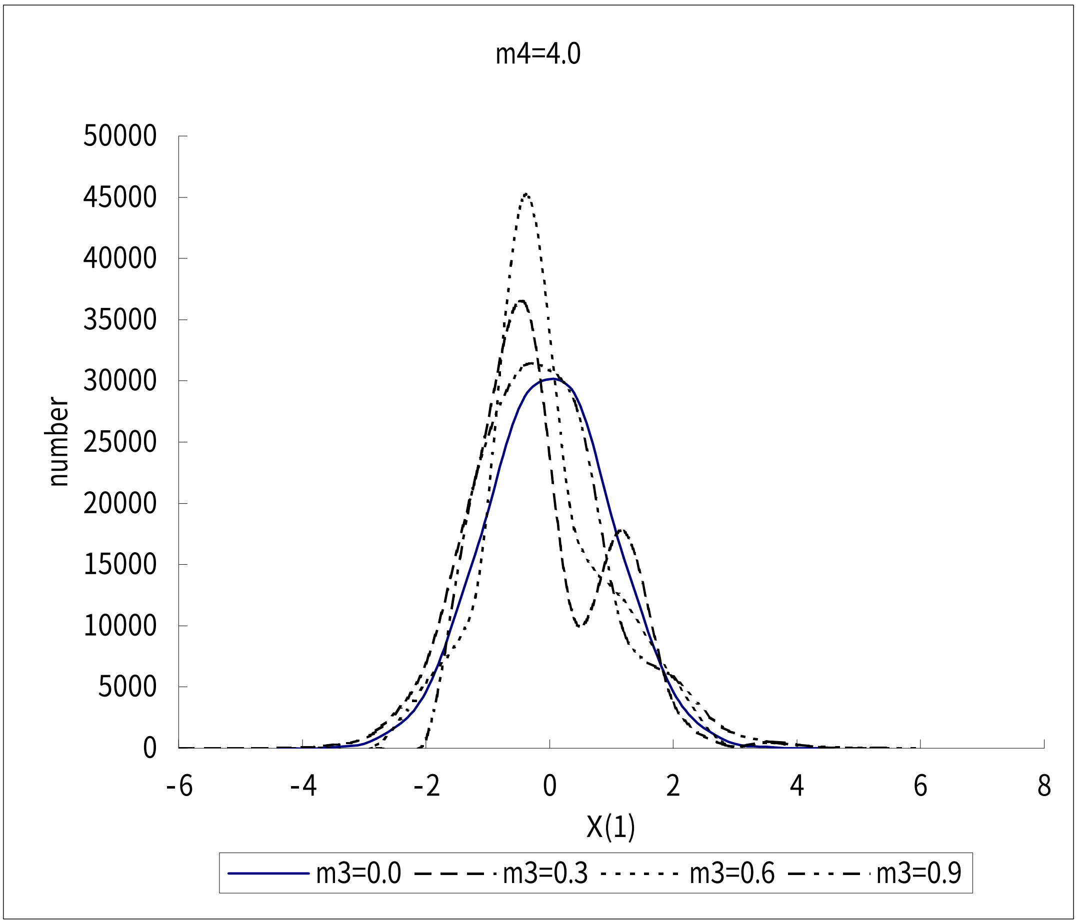



Fig.1. The distributions of X(1)
for Model 1 (m1=0 and m2=1).



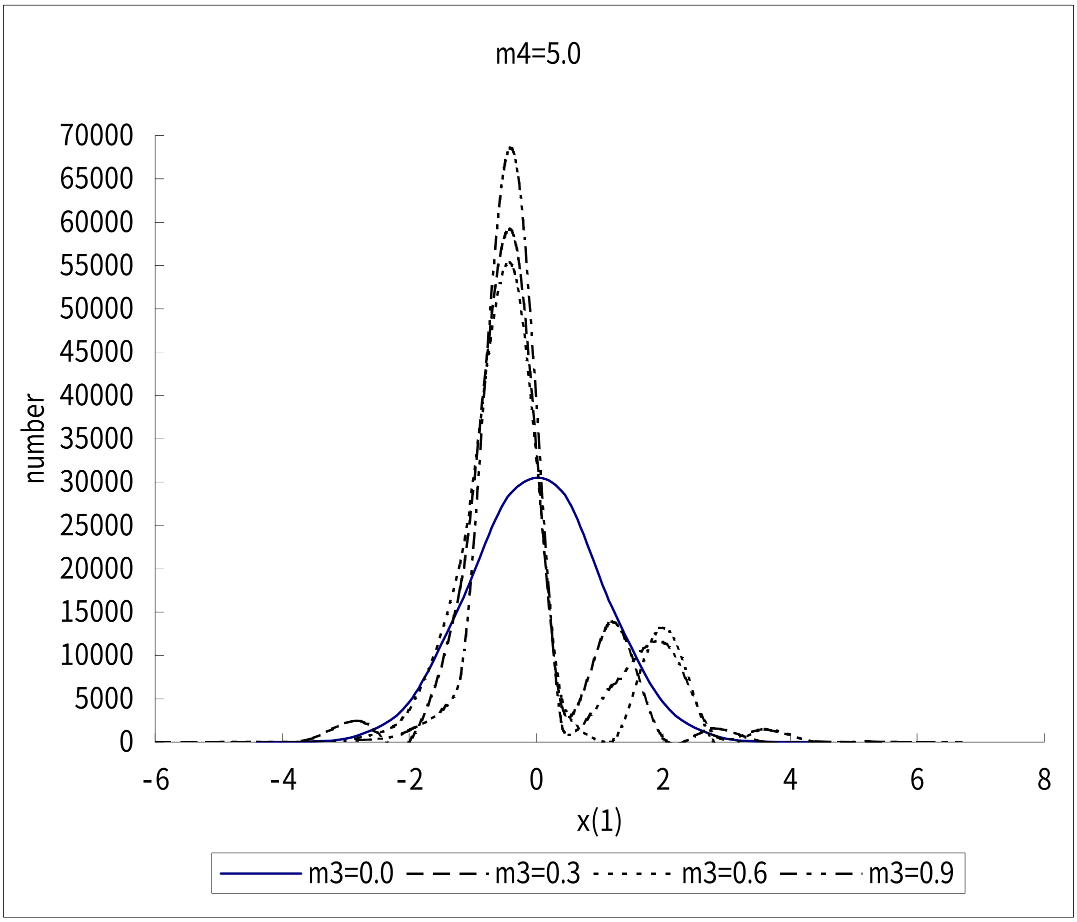




Fig.2. The distribution of X(1)
for Model 2 (m1=0 and m2=1).

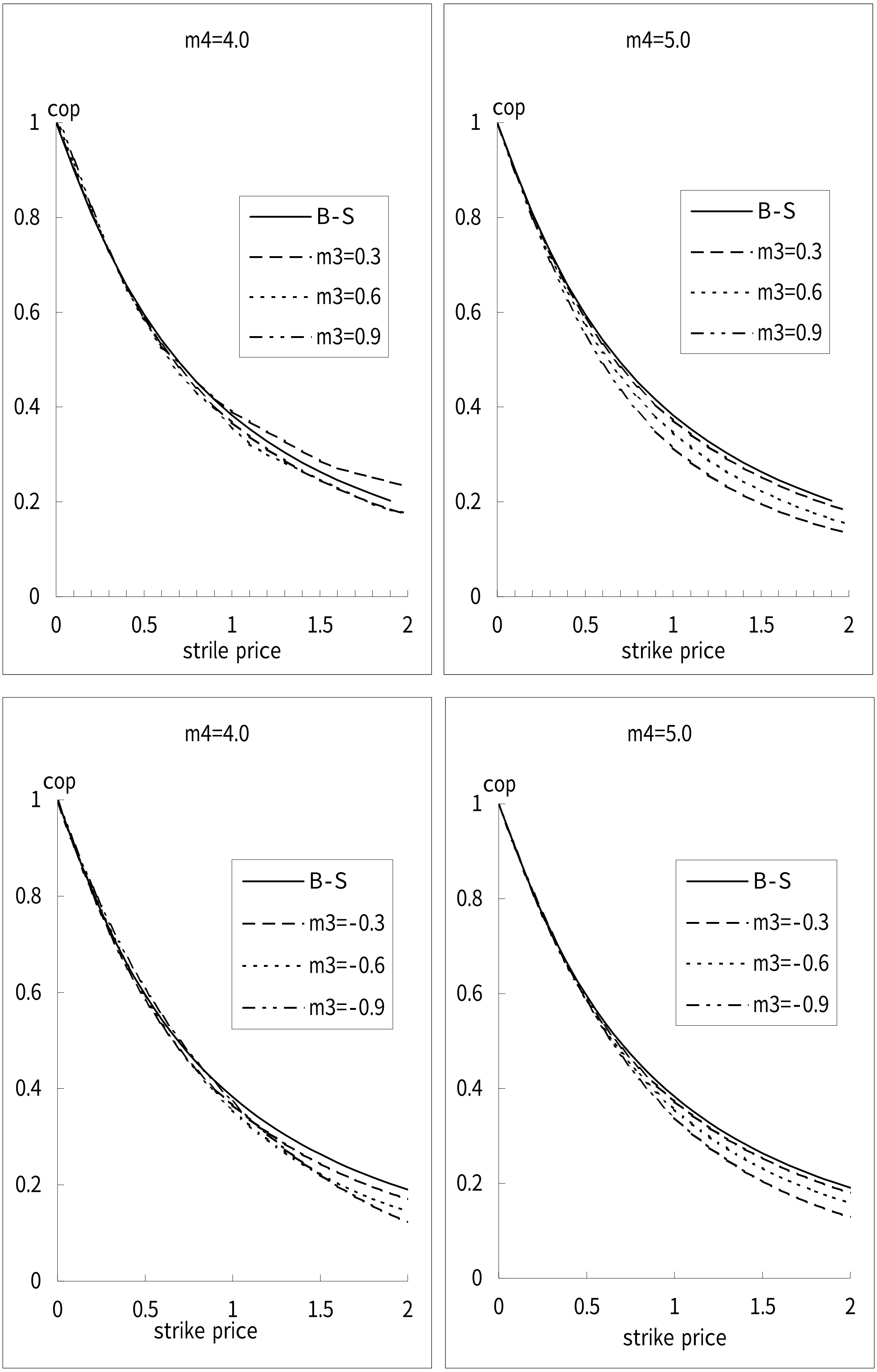



Fig.3. The European call option prices:
Black-Sholes Model vs Model 1+MEMM.

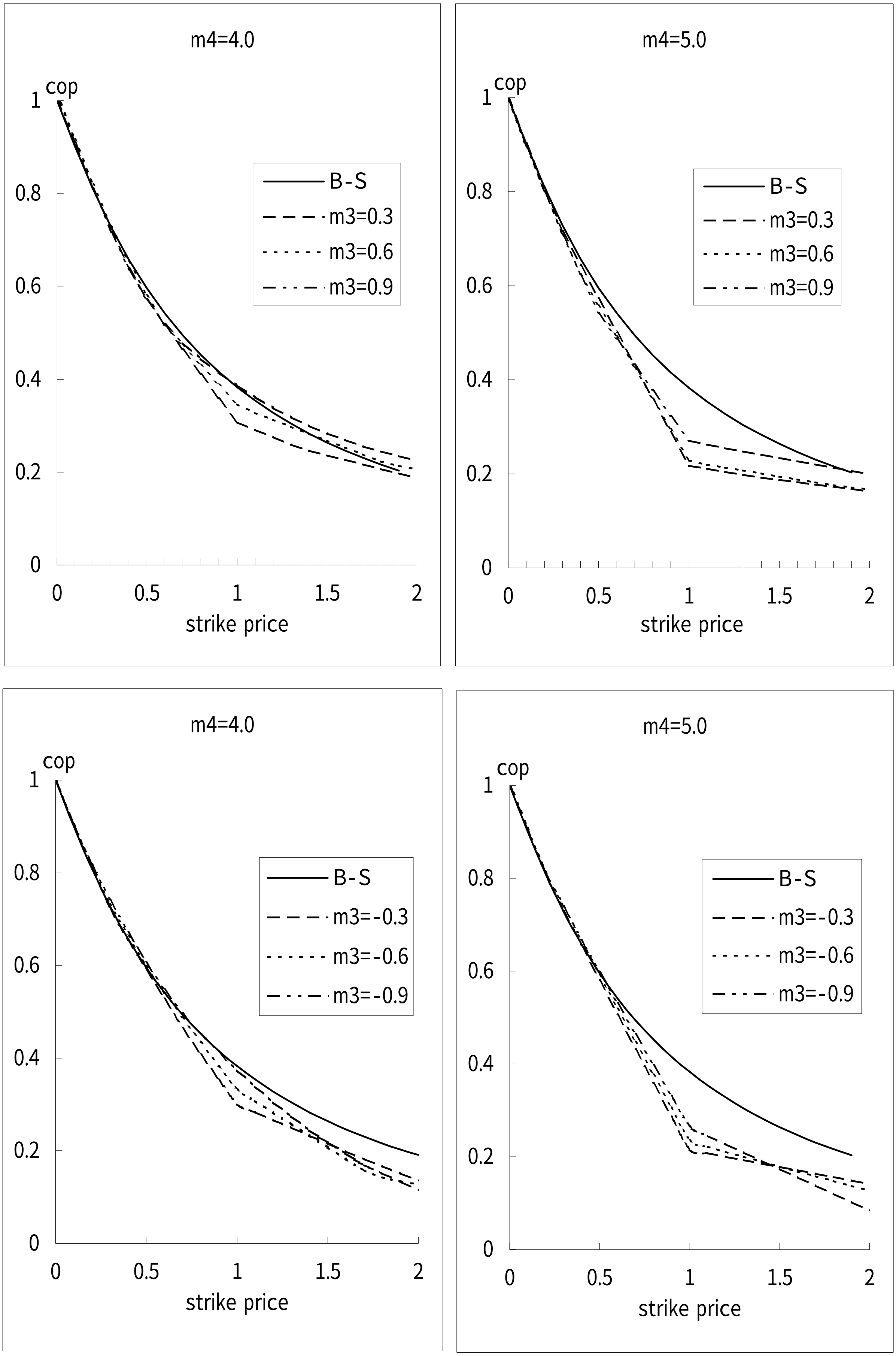



Fig.4. The European call option prices:
Black-Scholes Model vs Model 2+MEMM.