EK-125xii

Very Preliminary. Do NOT Quote.

A Role Model of Monetary and Fiscal Policy Rules in an Economy with Sovereign Risk: Evidence from Indonesia *

OKANO, Eiji† — Nagoya City University

Masataka Eguchi — Komazawa University

This Draft: Jun., 2019

Abstract

We developed a small open economy model following Gali and Monacelli[?] with the FTSR advocated by Uribe[21]. The model is estimated through Bayesian technique. Welfare costs function is derived through approximating utility function and any linear terms are eliminated appropriately. We solve the minimization problem and compare results under the optimal monetary and fiscal policy, namely, counter factual simulation with data. We find that the optimal monetary and fiscal policy is not necessarily the policy minimize volatility on inflation but that policy stabilizes the default rate. In addition, if Indonesia adopted optimal monetary and fiscal policy, welfare costs could be slushed approximately 85.6%.

Keywords: Sovereign Risk; Optimal Monetary Policy; Fiscal Theory of the Price Level

JEL Classification: E52; E60; F41; F47

†Corresponding Author. Graduate School of Economics, Nagoya City University, 1, Aza Yamanobata, Mizuho-cho, Mizuho-ku, Nagoya, 467-8501, Japan. Phone and Fax: +81-52-8725721; E-mail: eiji_okano@econ.nagoya-cu.ac.jp

1 Introduction

Uribe[21] argues that if the central bank's policy is to peg the price level, government surrenders its ability to inflate away the real value of nominal public liabilities, and so public debt default is inevitable. Alternatively, if the central bank's policy is to peg the nominal interest rate, government preserves its ability to suppress public debt default, but it no longer stabilizes the price level. This argument may be consistent with readers' intuition. Through a series of recent default scares stemming from the Greek debt crisis, the stabilizing of inflation and the suppressing of the default trade-off (SI–SD trade-off) observed by Uribe[21] appears to be increasingly emphasized, especially in the Euro area.

Okano and Inagaki[20] revisits Uribe[21]'s fiscal theory of sovereign risk (FTSR) from the viewpoint of a dynamic stochastic general equilibrium (DSGE) model with nominal rigidities as per Woodford (2003) and they find that there is not necessarily an SI–SD trade-off, and even if there is, it is not as severe as that suggested by Uribe[21]. That is, they develop a class of DSGE models with nominal rigidities and find that inflation stabilization is not inconsistent with default suppression. Default risk could be mitigated through stabilizing inflation, and this result differs markedly from that in Uribe[21]. they insist that policy authorities can then solve practically the SI–SD trade-off by adopting an optimal monetary and fiscal (optimal monetary and fiscal) policy where both the nominal interest and tax rates are available as policy instruments to minimize welfare costs through mostly stabilizing inflation. This is their most important policy contribution.

In this paper, we double-check the work in Okano and Inagaki[20] by developing a class of DSGE models with nominal rigidities. In addition, we change their closed economy setting to small open economy setting to estimate. We estimate our small open economy model with Uribe[21]'s Fiscal Theory of Sovereign Risk through Bayesian technique. Although Okano and Inagaki[20] does not derive welfare costs function appropriately, we derive it surely, that is, any linear terms which generates welfare reversal advocated by Kim and Kim[14] are eliminated in our welfare costs function. Interestingly, there appear a quadratic term of difference between the government debt coupon rate and the nominal interest rate for safety assets and a quadratic term of the interest rate spread for risky assets. We derive the FONCs for policy authorities such as the central bank and the government. We resort dynamic simulation and we find that the optimal monetary and fiscal policy is not necessarily inflation stabilization policy. This result is not consistent with Okano and Inagaki[20]. Rather, this is consistent with Uribe[21].

At this point, we review Uribe[21]'s FTSR. By iterating the government budget constraint forward and imposing an appropriate transversality condition, Uribe[21] shows that the default rate depends on the ratio of the net present value of the real fiscal surplus to real government debt with interest payment. That is, the default rate depends on government solvency. Thus, a decrease in the fiscal surplus, which is exogenous in his setting, decreases government solvency. Facing such a case, if the central bank stabilizes inflation, the burden of government debt redemption cannot be mitigated, and the default rate increases. If the central bank gives up trying to stabilize inflation, the burden of government debt redemption can be mitigated by inflation, which decreases real government debt, and the default is mitigated. This is Uribe[21]'s FTSR as hinted at by 'fiscal theory of price level' in Cochrane[7], Leeper[15] and Woodford[23], and the FTSR shows that there is indeed an SI-SD trade-off.

Okano and Inagaki[20] derive quite different policy implication from Uribe[**?**] by developing a class of DSGE model, that is, endogenizing production and they show that inflation stabilization policy is not necessarily consistent with suppressing default. How then does endogenized production derive quite different results? First, recall that the fiscal surplus is the difference between tax revenue and government expenditure, and suppose that a tax, which is one of the policy instruments in the optimal monetary and fiscal policy in their analysis, is levied on output and government expenditure is exogenous. Here, the most important thing is that the fiscal surplus not only acutely involves the default rate but also involves inflation through the output gap. That is, stabilizing the fiscal surplus stabilizes not only the default rate but also both inflation and the output gap. Thus, both inflation and default can be stabilized or suppressed.

We do not necessarily deny Okano and Inagaki[20] although we have to pay attention that they focus on inflation stabilization while we focus on optimal monetary policy under appropriate welfare costs function which stems from second-order approximated utility function without any linear terms. Under our estimation results, the coefficients not of inflation but of a quadratic term of difference between the government debt coupon rate and the nominal interest rate for safety assets and a quadratic term of the interest rate spread for risky assets is the highest. That is, policy authorities extremely consider those quadratic terms to minimize and they have to neglect stabilizing inflation. Thus, there is the SI-SD trade-off under the optimal monetary and fiscal policy.

Finally, we discuss the relationship between our work and previous work addressing sovereign risk or crises in the field of macroeconomics. First, Arellano[1] develops a model in which the default probability depends on some stochastic process and shows that default is more likely in recessions. He succeeds in matching his model with Argentinian data, and his assumption concerning the default mechanism is subsequently applied by Mendoza and Yue[17] and Corsetti et al.[9]. In their analysis, Mendoza and Yue[17] attempt to explain the negative relationship between output and default observed in the data. That is, they clarify the reason why deep recession often accompanies sovereign default. Corsetti et al.[9] develop a model including financial intermediaries showing that sovereign risk may give rise to indeterminacy and imply that fiscal retrenchment via government spending cuts can help to curtail the risk of macroeconomic instability and, in extreme cases, even stimulate economic activity. Their model stems from Curdia and Woodford[10] and is inclusive of the zero lower bound of nominal interest rates.

Subsequently, Corsetti and Dedola[8] develop a model for a sovereign debt crisis driven by either self-fulfilling expectations or weak fundamentals and analyze the mechanism through which either conventional or unconventional monetary policy can rule out the former. Their finding that swapping government debt for monetary liabilities can preclude self-fulfilling debt crises one of several unconventional monetary policies. Elsewhere, and similar to our analysis, Bacchetta, Perazzi and von Wincoop[2] develop a class of DSGE models and analyze both conventional and unconventional monetary policy. They find that the central bank cannot credibly avoid a self-fulfilling debt crisis.

Here, we make it clear where our analysis differs from this earlier body of work. With the exception of Corsetti and Dedola[8] and Bacchetta, Perazzi and von Wincoop[2], the main concerns in these analyses are how sovereign default affects the macroeconomic dynamics, especially the output dynamics, whereas we focus on how the optimal monetary and fiscal policy affects default. Although Corsetti and Dedola[8] and Bacchetta, Perazzi and von Wincoop[2] analyze monetary policy, they do not consider fiscal policy or how to use it as a stabilization or welfare cost-minimization tool. Thus, our purposes are not identical, and we can say that we propose monetary and fiscal policies to both stabilize inflation and suppress default risk, whereas they propose monetary policy only to suppress default risk.[1]

Repeatedly, we emphasize that while previous work in the area obtains important implications, none of these examines optimal monetary and fiscal policy. While Okano and Inagaki[20] certainly discusses this trade-off, we emphasize that there is a trade-off under the 'genuine' optimal monetary and fiscal policy. Needless to say, neither Uribe[21] nor Corsetti et al.[9] nor Mendoza and Yue[17] derives this result. Examining the 'genuine' optimal monetary and fiscal policy in this paper is novel.

The remainder of the paper is organized as follows. Section 2 develops the model. Section 3 estimates the model through Bayesian technique. Section 4 derives welfare costs function. Section 5 solves the linear-quadratic (LQ) problem. Section 6 calibrates the model. Section 7 analyzes welfare costs. Section 8 (to be added) derives optimal monetary and fiscal policy rules replicate optimal allocation brought about by optimal monetary policy. Section 9 concludes the paper. The appendices provide some additional analysis with Appendix A examining the steady state.

2 The Model

We derive a model basically based on De Paoli[?], Okano and Inagaki[?] and Gali and Monacelli[?]. The currency union consists of the small open economy (SOE) and the rest of the world (ROW), which together organize a monetary union. The households on the interval $j \in [0, \tilde{\alpha})$ belong to the SOE while those on the interval $m \in [\tilde{\alpha}, 1]$ belong to the ROW. Generic good $h \in [0, \tilde{\alpha})$ is produced in the SOE and generic good $f \in [0, \tilde{\alpha})$ is produced in the ROW. We assume that there is a default risk in the SOE. There is a seminar work on a currency union, Gali and Monaclelli[?] but we adopt De Paoli[?]'s setting to expand Okano and Inagaki[?]'s closed economy model. The reason why is that we compare a currency union setting with a flexible exchange rate regime setting. De Paoli[?]'s flexible exchange rate model is cmpatible with currency union model. Thus, we adopt De Paoli[?].

2.1 Households

2.1.1 Preferences

A representative household's preferences are given by:

$$\mathcal{U}_H \equiv \mathrm{E}_0 \left(\sum_{t=0}^{\infty} \beta^t U_{H,t} \right) \ ; \mathcal{U}_F \equiv \mathrm{E}_0 \left(\sum_{t=0}^{\infty} \beta^t U_{F,t} \right), \tag{1}$$

where $U_{H,t} \equiv \ln C_t - \frac{1}{1+\varphi} N_{H,t}^{1+\varphi}$ and $U_{F,t} \equiv \ln C_t^* - \frac{1}{1+\varphi} N_{F,t}^{1+\varphi}$ denote the period utility in countries H and F, respectively, $N_{H,t} \equiv \frac{1}{\tilde{\alpha}} \int_{\tilde{\alpha}}^{1} N_t(h)\, dh$ and $N_{F,t} \equiv \frac{1}{\tilde{\alpha}} \int_0^{\tilde{\alpha}} N_t(f)\, df$ denote the hours of labor in countries H and F, respectively E_t is the expectation conditional on the information set at

[1] Furthermore, they do not focus on fiscal policy, and their model is unsuitable for analyzing fiscal policy regardless, whereas our model can analyze and evaluate the effect of fiscal policy. In terms of other differences, the government in Arellano[1] does not levy tax on any economic agents, while Mendoza and Yue[17], Corsetti et al.[9] and Bacchetta, Perazzi and von Wincoop[2] assume either lump-sum taxes or transfers. Thus, under their settings, it is not possible to analyze fiscal policy. In contrast, in our work, government changes the tax rate to minimize welfare costs, and fiscal policy can be analyzed specifically, and we can then easily observe the effects of the optimal monetary and fiscal policy on default. This is the advantage of our analysis over these existing studies from the viewpoint of model building.

period t, $\beta \in (0,1)$ is the subjective discount factor, C_t is the consumption index and φ is the inverse of the elasticity of labor supply.

The consumption index is defined as follows:

$$C_t \equiv \left[\upsilon^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + (1-\upsilon)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \; ; \; C_t^* \equiv \left[(\upsilon^*)^{\frac{1}{\eta}} \left(C_{H,t}^*\right)^{\frac{\eta-1}{\eta}} + (1-\upsilon^*)^{\frac{1}{\eta}} \left(C_{F,t}^*\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \quad (2)$$

where $C_{H,t} \equiv \left[\left(\frac{1}{\tilde{\alpha}}\right)^{\frac{1}{\varepsilon}} \int_0^{\tilde{\alpha}} C_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right]^{\frac{\varepsilon}{\varepsilon-1}}$ and $C_{F,t} \equiv \left[\left(\frac{1}{1-\tilde{\alpha}}\right)^{\frac{1}{\varepsilon}} \int_{\tilde{\alpha}}^1 C_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}}$ denote consumption subindexes of the continuum of differentiated goods in the SOE while $C_{H,t}^* \equiv \left[\left(\frac{1}{\tilde{\alpha}}\right)^{\frac{1}{\varepsilon}} \int_0^{\tilde{\alpha}} C_t^*(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right]^{\frac{\varepsilon}{\varepsilon-1}}$ and $C_{F,t}^* \equiv \left[\left(\frac{1}{1-\tilde{\alpha}}\right)^{\frac{1}{\varepsilon}} \int_{\tilde{\alpha}}^1 C_t^*(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}}$ denote those in the ROW, $C_t(h)$ and $C_t(f)$ are generic goods in the SOE while $C_t^*(h)$ and $C_t^*(f)$ are those in the ROW, respectively and $\eta > 0$ is the elasticity of substitution between goods produced in the SOE and the ROW. The parameter determining the SOE's households preference for the ROW's goods $(1-\upsilon)$ is a function of the relative size of the ROW $(1-\tilde{\alpha})$ and of the degree of openness α, more specifically, $(1-\upsilon) = (1-\tilde{\alpha})\alpha$. Similarly, $\upsilon^* = \tilde{\alpha}\alpha$ is applied.

By solving cost-minimization problems for households, we have the optimal allocation of expenditures as follows:

$$C_t(h) = \frac{1}{\tilde{\alpha}} \left(\frac{P_t(h)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} \; ; \; C_t(f) = \frac{1}{1-\tilde{\alpha}} \left(\frac{P_t(f)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t}, \quad (3)$$

$$C_t^*(h) = \frac{1}{\tilde{\alpha}} \left(\frac{P_t^*(h)}{P_{H,t}^*} \right)^{-\varepsilon} C_{H,t}^* \; ; \; C_t^*(f) = \frac{1}{1-\tilde{\alpha}} \left(\frac{P_t^*(f)}{P_{F,t}^*} \right)^{-\varepsilon} C_{F,t}^*, \quad (4)$$

where $P_{H,t} \equiv \left[\frac{1}{\tilde{\alpha}} \int_0^{\tilde{\alpha}} P_t(h)^{1-\varepsilon} dh \right]^{\frac{1}{1-\varepsilon}}$ and $P_{H,t}^* \equiv \left[\frac{1}{\tilde{\alpha}} \int_0^{\tilde{\alpha}} P_t^*(h)^{1-\varepsilon} dh \right]^{\frac{1}{1-\varepsilon}}$ denote the price of the SOE goods sold in the SOE and it sold in the ROW, respectively, $P_{F,t} \equiv \left[\frac{1}{1-\tilde{\alpha}} \int_{\tilde{\alpha}}^1 P_t(f)^{1-\varepsilon} df \right]^{\frac{1}{1-\varepsilon}}$ and $P_{F,t}^* \equiv \left[\frac{1}{1-\tilde{\alpha}} \int_{\tilde{\alpha}}^1 P_t^*(f)^{1-\varepsilon} df \right]^{\frac{1}{1-\varepsilon}}$ denote the price of the ROW goods sold in the SOE and it sold in the ROW, respectively, $P_t(h)$ and $P_t^*(h)$ denote the price of generic good h sold in the SOE and it sold in the ROW, respectively and $P_t(f)$ and $P_t^*(f)$ denote the price of generic good f sold in the SOE and it sold in the ROW. We assume the law of one price (LOOP) definitely, e.g., $P_t(h) = P_t^*(h)$ and $P_t(f) = P_t^*(f)$. Then, $P_{H,t} = P_{H,t}^*$ and $P_{F,t} = P_{F,t}^*$ are definiteli applicable. For short, we refer $P_{H,t}$ and $P_{F,t}^*$ as the producer price indeces (PPIs) in the SOE and the ROW, respectively.

The total demand for goods produced in countries H and F is given by:

$$C_{H,t} = \tilde{\alpha} \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \; ; \; C_{F,t} = (1-\tilde{\alpha}) \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t, \quad (5)$$

$$C_{H,t}^* = \alpha \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} C_t^* \; ; \; C_{F,t}^* = (1-\tilde{\alpha}) \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-\eta} C_t^*, \quad (6)$$

where:

$$P_t \equiv \left[\upsilon P_{H,t}^{1-\eta} + (1-\upsilon) P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}, \; ; \; P_t^* \equiv \left[\upsilon^* \left(P_{H,t}^*\right)^{1-\eta} + (1-\upsilon^*) \left(P_{F,t}^*\right)^{1-\eta} \right]^{\frac{1}{1-\eta}}, \quad (7)$$

denote the consumer price indeces (CPIs) in the SOE and the ROW, respectively. While the LOOP is aplicable, the home bias specification leads to deviations from purchasing power parity, that is, $P_t = P_t^*$ does not necessarily applied.

The sequence of budget constraints assumes the following form:

$$
\begin{aligned}
R^H_{t-1}B^n_{H,t-1}\left(1-\delta_t\right)+R^*_{t-1}B^{*n}_{F,t-1}\mathcal{E}_t+W_{H,t}N_{H,t}+PR_{H,t} &\geq P_tC_t+B^n_{H,t}+B^{*n}_{F,t}\mathcal{E}_t,\\
R^H_{t-1}\frac{B^n_{H,t-1}}{\mathcal{E}_t}\left(1-\delta_t\right)+R^*_{t-1}B^{*n}_{F,t-1}+W^*_{F,t}N_{F,t}+PR^*_{F,t} &\geq P^*_tC^*_t+\frac{B^n_{H,t}}{\mathcal{E}_t}+B^{*n}_{F,t},
\end{aligned}
\tag{8}
$$

where $R_t \equiv 1+r_t$ denotes the gross (risk-free) nominal interest rate in the SOE, r_t the net interest rate in the SOE, R^H_t the the gross nominal interest rate for risky assets, $R^*_t \equiv 1+r^*_t$ denotes the gross (risk-free) nominal interest rate in the ROW, r^*_t the net interest rate in the ROW, $B^n_{H,t}$ is nominal government debt issued by the SOE's government, $B^n_{F,t}$ is nominal government debt issued by the ROW's government, $W_{H,t}$ and $W_{F,t}$ are the nominal wage indeces in the SOE and the ROW, respectively (the definition is shown in sec. 2.2), $N_{H,t}$ and $N_{F,t}$ denote the total labor input in the SOE and the ROW, respevtively (the definition is shown in sec. 2.2), $PR_{H,t}$ and $PR_{F,t}$ denotes profits from the ownership of the firms in the SOE and the ROW, respectively, δ_t is the default rate, $sp_{H,t} \equiv \frac{SP_{H,t}}{SP_H}-1$ is the percentage deviation of the (real) fiscal surplus from its steady-state value, $SP_{H,t}$ denotes the (real) fiscal surplus (the definition is shown in sec. 2.3). Furthermore, we define V as the steady-state value of any variables V_t and v_t as the percentage deviation of V_t from its steady-state value. Thus, SP_H is the steady-state value of the fiscal surplus. The second term on the left-hand side (LHS) in Eq.(12) implies that the government may default on the share of δ_t and households cannot obtain $R^G_{t-1}B^n_{H,t-1}\delta_t$ if the government defaults.

2.1.2 Intratemporal Optimality Conditions

The representative household in a currency union maximizes Eq.(1) subject to Eq.(12). The intratemporal optimality conditions are given by:

$$
C_tN^{\psi}_{H,t}=\frac{W_{H,t}}{P_t} \quad ; \quad C^*_tN^{\psi}_{F,t}=\frac{W^*_{F,t}}{P^*_t}, \tag{9}
$$

2.1.3 Intertemporal Optimality Conditions, Uncovered Interest Rate Parity and International Risk-sharing Condition

Another households' optimality conditions, namely, intertemporal optimality conditions are given by:

$$
\beta \mathrm{E}_t\left(\frac{P_tC_t}{P_{t+1}C_{t+1}}\right)=\frac{1}{R_t} \ ; \ \beta \mathrm{E}_t\left(\frac{P^*_tC^*_t}{P^*_{t+1}C^*_{t+1}}\right)=\frac{1}{R^*_t}. \tag{10}
$$

which is the standard intertemporal optimality condition. In the model, it is assumed that financial markets are complete and the uncovered interest rate which is given by:

$$
\frac{R_t}{R^*_t}=\mathrm{E}_t\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right) \tag{11}
$$

is applied. Eq.(11) implies that the difference between the nominal interest rate between the SOE and the ROW corresponds to changes in the expected nominal exchange rate and is taken to derive the LHS in Eq.(10).

There is another intertemporal optimality condition depicting the households' motivation to hold government debt with default risk. This is obtained by differentiating the Lagrangian by government nominal debt and is given by:

$$
\beta \mathrm{E}_t\left(\frac{P_tC_t}{P_{t+1}C_{t+1}}\right)=\frac{1}{R^H_t\mathrm{E}_t\left(1-\delta_{t+1}\right)}, \ ; \ \beta \mathrm{E}_t\left(\frac{P^*_tC^*_t}{P^*_{t+1}C^*_{t+1}}\right)=\frac{1}{R^H_t\mathrm{E}_t\left(1-\delta_{t+1}\right)}\frac{R_t}{R^*_t}. \tag{12}
$$

Similar to the LHS in Eq.(10), Eq.(11) is taken to derive the RHS in Eq.(12).

Combining Eqs.(10) and (12), we have:

$$R_t = R_t^H \mathrm{E}_t \left(1 - \delta_{t+1}\right), \tag{13}$$

which shows that the marginal rate of substitution for consumption is the same for households holding either the SOE's government debt $B_{H,t}$ the ROW's government debt B_t because both R_t and $R_t^H \mathrm{E}_t \left(1 - \delta_{t+1}\right)$ equal the marginal rate of substitution $\beta \mathrm{E}_t \left(\frac{P_t C_t}{P_{t+1} C_{t+1}}\right)$. That is, the consumption schedule is the same whether households the risky asset $B_{H,t}$ or the safety asset $B_{F,t}$.

2.1.4 International Risk Sharing Condition

Combining equalities in Eq.(10) and iterating it with initial condition yields:

$$C_t = Q_t C_t^*, \tag{14}$$

which is the international risk sharing condition where $Q_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t}$ denotes the CPI disparity between the SOE and the ROW corresponds to to the real exchange rate under flexible exchange rate regime.

2.1.5 Market-Clearing Conditions

Up to now, our model's features do not include market-clearing conditions, which in our analysis are quite similar to that in Gali and Monacelli[13] and Ferrero[11].

The market-clearing conditions in a currency union are given by:

$$\begin{aligned} Y_t(h) &= \tilde{\alpha}\left[C_t(h) + G_t(h)\right] + (1-\tilde{\alpha})\,\tilde{C}_t^*(h), \\ Y_t(f) &= \tilde{\alpha} C_t(f) + (1-\tilde{\alpha})\left[C_t^*(f) + G_t^*(f)\right], \end{aligned} \tag{15}$$

where $C_{H,t}^*(h)$ and $C_{F,t}^*(f)$ denote foreign demand for the SOE's goods and the ROW's goods, respectively.

We define $Y_{H,t} \equiv \left[\left(\frac{1}{\tilde{\alpha}}\right)\int_0^{\tilde{\alpha}} Y_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh\right]^{\frac{\varepsilon}{\varepsilon-1}}$ and $Y_{F,t} \equiv \left[\left(\frac{1}{1-\tilde{\alpha}}\right)\int_{\tilde{\alpha}}^1 Y_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df\right]^{\frac{\varepsilon}{\varepsilon-1}}$ which are the total demand for output in the SOE and the ROW, respectively where $\varepsilon > 1$ is the elasticity of substitution across goods. Combining these definitions and the PPI indexes, we have:

$$Y_t(h) = \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon} Y_{H,t}\ ;\ Y_t(f) = \left(\frac{P_t(f)}{P_{F,t}}\right)^{-\varepsilon} Y_{F,t}. \tag{16}$$

Similarly, government expenditure is a Dixit–Stiglitz aggregator defined by $G_{H,t} \equiv \left[\left(\frac{1}{\tilde{\alpha}}\right)^{\frac{1}{\varepsilon}}\int_0^{\tilde{\alpha}} G_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh\right]^{\frac{\varepsilon}{\varepsilon-1}}$ and $G_{F,t}^* \equiv \left[\left(\frac{1}{1-\tilde{\alpha}}\right)^{\frac{1}{\varepsilon}}\int_{\tilde{\alpha}}^1 G_t^*(f)^{\frac{\varepsilon-1}{\varepsilon}} df\right]^{\frac{\varepsilon}{\varepsilon-1}}$, where $G_{H,t}$ and $G_{F,t}^*$ denote the indexes of government expenditure in the SOE and the ROW, respectively. For simplicity, we assume that each government allocates a level of government expenditure only among domestic goods. Each government implies the following demands for the generic goods h and f:

$$G_t(h) = \frac{1}{\tilde{\alpha}}\left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon} G_{H,t}\ ;\ G_t^*(f) = \frac{1}{1-\tilde{\alpha}}\left(\frac{P_t^*(f)}{P_{F,t}^*}\right)^{-\varepsilon} G_{F,t}, \tag{17}$$

where $P_t(h)$ and $P_t(f)$ denote the price of generic good h and f, respectively, By plugging Eqs.(3) to (6), (14), (16) and (17) into Eq.(15), we have:

$$\begin{aligned} Y_{H,t} &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[\upsilon + \frac{\upsilon^*(1-\tilde{\alpha})}{\tilde{\alpha}}Q_t^{\eta-1}\right]C_t + G_{H,t}, \\ Y_{F,t} &= \left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta}\left[\frac{(1-\upsilon)\tilde{\alpha}}{1-\tilde{\alpha}}Q_t^{-(\eta-1)} + (1-\upsilon^*)\right]C_t^* + G_{F,t}^*. \end{aligned} \tag{18}$$

Finally, to portray our small open economy we use the de.nition of υv and υ^* and take the limit for $\tilde{\alpha} \to 0$. Consequently, Eq.(18) can be rewritten as:

$$\begin{aligned} Y_{H,t} &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[1 + \alpha\left(Q_t^{\eta-1} - 1\right)\right]C_t + G_{H,t}, \\ Y_{F,t} &= C_t^* + G_{F,t}^* \end{aligned} \tag{19}$$

Eq.(19) show how external changes in consumption affect the small open economy, but the reverse is not true. Moreover, movements in the real exchange rate do not modify the ROW's demand. Hereafter, we just focus on the SOE.

when taking the limit $\tilde{\alpha} \to 0$, Eq.(7) is rewritten as follows:

$$P_t \equiv \left[(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}\ ,\ ;\ P_t^* \equiv P_{F,t}^*, \tag{20}$$

because of $\upsilon = 1 - \alpha$ and $\upsilon^* = 0$ being applicable.

2.2 Firms

This subsection depicting the production, price setting and marginal cost and features of the firms is quite similar to Gali and Monacelli[13], although here tax is levied on firm sales and is not constant.[2]

A typical firm in each country produces a differentiated good with a linear technology represented by the production function:

$$Y_t(h) = A_{H,t}N_{H,t}(h),$$

where $A_{H,t}$ denotes the (total factor of) productivity.

By combining the production technology in the currency union and Eq.(16), we have an aggregate production function relating to aggregate employment as follows:

$$N_{H,t} = \frac{Y_{H,t}Z_{H,t}}{A_{H,t}}, \tag{21}$$

where $Z_{H,t} \equiv \frac{1}{\tilde{\alpha}}\int_0^{\tilde{\alpha}}\left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon}dh$ denotes the price dispersion.

Each firm is a monopolistic producer of one of the differentiated goods. Each firm sets their prices $P_t(h)$ taking as given P_t, $P_{H,t}$ and C_t. We assume that firms set prices in a staggered fashion in the Calvo–Yun style, according to which each seller has the opportunity to change its price with a given probability $1-\theta$, where an individual firm's probability of reoptimizing in any given period is independent of the time elapsed since it last reset its price. When a firm has the

[2]Unlike our setting, Gali and Monacelli[13] assume that constant employment subsidies and monopolistic power completely disappear.

opportunity to set a new price in period t, it does so in order to maximize the expected discounted value of its net profits. The FONCs for firms are given by:

$$\tilde{P}_{H,t} = \frac{\frac{\varepsilon}{\varepsilon-1}\sum_{k=0}^{\infty}(\beta\theta)^k \mathrm{E}_t\left[(P_{t+k}C_{t+k})^{-1}\tilde{Y}_{t+k}P_{H,t+k}MC_{H,t+k}\right]}{\sum_{k=0}^{\infty}(\beta\theta)^k \mathrm{E}_t\left[(P_{t+k}C_{t+k})^{-1}\tilde{Y}_{t+k}\right]} \tag{22}$$

with $\tilde{Y}_{H,t+k} \equiv \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} Y_{H,t+k}$ where $MC_{H,t} \equiv \frac{W_{H,t}}{(1-\tau_{H,t})P_{H,t}A_{H,t}}$ denotes the real marginal cost, $\tilde{P}_{H,t}$ denotes the newly set prices.

Plugging Eq.(13) into the definition of the marginal cost yields:

$$MC_{H,t} = \frac{1}{(1-\tau_{H,t})A_{H,t}}C_t N_{H,t}^{\psi}\left(\frac{P_{H,t}}{P_t}\right)^{-1}. \tag{23}$$

2.3 Government

2.3.1 Flow Government Budget Constraint

Government plays an important role in the model because fiscal policy is an important policy tool used to minimize welfare costs.

Substituting Eq.(17) into flow government budget constraints yields:

$$B_{H,t}^n = R_{t-1}^H B_{H,t-1}^n (1-\delta_t) - P_{H,t}(\tau_{H,t}Y_{H,t} - G_{H,t} - \zeta_{H,t}), \tag{24}$$

The first terms on the RHS correspond to the amount of redemption with the nominal interest payment. In particular, the first term on the RHS in the second equality shows that the SOE's government pays higher interest payments because of sovereign risk.

Dividing both sides of Eq.(24) by the consumer price index P_t yields:

$$B_{H,t} = R_{t-1}^H (1-\delta_t) B_{H,t-1}\Pi_t^{-1} - SP_{H,t}, \tag{25}$$

where $B_{H,t} \equiv \frac{B_{H,t}^n}{P_t}$ is the real government debts issued by the SOE's government and $SP_{H,t} = \frac{P_{H,t}}{P_t}(\tau_{H,t}Y_{H,t} - G_{H,t} - \zeta_{H,t})$ denotes the fiscal surplus in the SOE, $\tau_{H,t}$ denotes the tax rate in the SOE.

2.3.2 The FTSR

The appropriate transversality condition for the SOE's government debt is given by:

$$\lim_{j\to\infty}\beta^j R_{t+j}^H \frac{B_{H,t+j}^n}{P_{t+j+1}}(1-\delta_{t+j+1}) = 0.$$

Similar to Eq.(**??**), iterating the second equality in Eq.(24) forward and imposing the appropriate transversality condition for the SOE's government debt, we have:

$$1-\delta_t = \frac{\sum_{k=0}^{\infty}\beta^k \mathrm{E}_t\left(C_{t+k}^{-1}SP_{H,t+k}\right)}{C_t^{-1}R_{t-1}^H B_{H,t-1}\Pi_t^{-1}}, \tag{26}$$

where $R_t^R \equiv \frac{R_t^H}{R_t^G}$ denotes the (gross) premium difference between the government debt yield and its coupon rate. Eq.(26) and below imply that an increase in CPI or CPI inflation, does not necessarily occur if solvency is less than the government's debt. Not only inflation, but also default, mitigates the government's liability. Suppose that CPI is constant and there is no inflation. In this situation, if the solvency is about equal to the government's liability, the RHS is less than unity. On the other hand, the LHS is definitely less than unity through an increase in the default rate. In other words, if the government falls insolvent while CPI is strictly stable, default is inevitable. Uribe[21] pointed out that there is a trade-off between inflation stabilization and suppressing default by introducing default, namely sovereign risk, into the central equation of the FTPL. Similar to Uribe[21], at first glance Eq.(26) also implies that there is such a trade-off. Furthermore, he calibrates his model and compares it with the Taylor rule, which stabilizes inflation and the interest rate peg. Under the interest rate peg, the interest rate on risky assets corresponds to the risk-free asset interest rate pegged to the steady-state rate. This calibration further shows that default ceases just one period after the shock decreased the fiscal surplus, although default continues under the Taylor rule. This result implies that a Taylor rule to stabilize inflation includes the unwelcome possibility of magnifying sovereign risk, and this calls for an interest rate peg to counter default. Although Uribe[21] ignores the welfare perspective of these actions, his policy implications are persuasive. Paying attention to just Eq.(26), which is similar to Uribe's[21] model, we seem to obtain policy implications quite similar to those in Uribe[21].

Eq.(26) can be rewritten as a second-order differential equation as follows:

$$R_{t-1}^{H} B_{H,t-1} \Pi_t^{-1} \left(1-\delta_t\right) = SP_{H,t} + \beta \mathrm{E}_t \left[\frac{C_{t+1}^{-1}}{C_t^{-1}} R_t^{H} B_{H,t} \Pi_{t+1}^{-1} \left(1-\delta_{t+1}\right) \right]. \tag{27}$$

2.4 A Log-liear Representation of the Model

2.4.1 The Demand Block

Log-linearizing Eq.(7) yields:

$$\begin{aligned} p_t &= (1-\alpha)\, p_{H,t} + \alpha p_{F,t}, \\ p_t^* &= p_{F,t}^*. \end{aligned} \tag{28}$$

Note that Eq.(**??**) is applicable under $\tilde{\alpha} \to 0$.

By taking first differece on Eq.(28), we get:

$$\pi_t = (1-\alpha)\, \pi_{H,t} + \alpha \pi_{F,t}, \tag{29}$$

$$\pi_t^* = \pi_{F,t}^* \tag{30}$$

where $\pi \equiv \ln\left(\frac{P_t}{P_{t-1}}\right)$ denotes the CPI inflation in the SOE, $\pi_{H,t} \equiv \ln\left(\frac{P_{H,t}}{P_{H,t-1}}\right)$ denotes the GDP inflation in the SOE, $\pi_{F,t} \equiv \ln\left(\frac{P_{F,t}}{P_{F,t-1}}\right)$ denotes the GDP inflation in the ROW, $\pi_t^* \equiv \ln\left(\frac{P_t^*}{P_{t-1}^*}\right)$ denotes the CPI inflation in the ROW and $\pi_{F,t}^* \equiv \ln\left(\frac{P_{F,t}^*}{P_{F,t-1}^*}\right)$ denotes the GDP inflation in the ROW in terms of the SOE's currency.

As mentioned, $P_{F,t} = \mathcal{E}_t P_{F,t}^*$ is applicable due to the LOOP and is log-linearized as follows:

$$\pi_{F,t} = e_t - e_{t-1} + \pi_{F,t}^*, \tag{31}$$

with $e_t \equiv \ln\left(\frac{\mathcal{E}_t}{\mathcal{E}}\right)$, which implies that there is no difference between the GDP inflation in the ROW interms of the SOE's currency and in terms of the ROW's currency.

Log-linearizing Eq.(25) yields:

$$b_{H,t} = \frac{1}{\beta}\hat{r}_t^H - \frac{\delta}{\beta(1-\delta)}\hat{\delta}_t - \frac{1}{\beta}\pi_t + \frac{1}{\beta}b_{H,t} - \frac{1-\beta}{\beta}sp_{H,t}. \tag{32}$$

with $\hat{r}_t \equiv \ln\left(\frac{R_t}{R}\right)$, $\hat{\delta} \equiv \ln\left(\frac{\delta_t}{\delta}\right)$ and:

$$sp_{H,t} = -\alpha s_t + \frac{\beta\tau}{\sigma_B(1-\beta)}\hat{\tau}_{H,t} + \frac{\beta\tau}{\sigma_B(1-\beta)}y_{H,t} - \frac{\beta\sigma_G}{\sigma_B(1-\beta)}g_{H,t} - \frac{\beta}{\sigma_B(1-\beta)}\hat{\zeta}_{H,t}, \tag{33}$$

with $s_t \equiv p_{F,t} - p_{H,t}$, $s_t \equiv \ln\mathcal{S}_t$ and $\hat{\zeta}_{H,t} \equiv \frac{d\zeta_{H,t}}{Y}$. To derive Eq.(33), we use log-linearized definition of the TOT $s_t = p_{F,t} - p_{H,t}$. Notice that we define V as the steady state value of any variables V_t and $v_t \equiv \ln\left(\frac{V_t}{V}\right)$ as percentage deviation from its steady state value.

Log-linearizing Eq.(27) yields:

$$\begin{aligned} c_t = {} & \mathrm{E}_t(c_{t+1}) - \hat{r}_t^H + \mathrm{E}_t(\pi_{t+1}) + \frac{\delta}{1-\delta}\mathrm{E}_t\left(\hat{\delta}_{t+1}\right) - b_{H,t} + \frac{1}{\beta}\hat{r}_{t-1}^H - \frac{1}{\beta}\pi_t - \frac{\delta}{\beta(1-\delta)}\hat{\delta}_t \\ & + \frac{1}{\beta}b_{H,t-1} - \frac{1-\beta}{\beta}sp_{H,t}, \end{aligned} \tag{34}$$

with: $\hat{r}_t^H \equiv \ln\left(\frac{R_t^H}{R^H}\right)$.

Log-lienarizing Eq.(13) yields:

$$\hat{r}_t = \hat{r}_t^H - \frac{\delta}{1-\delta}\mathrm{E}_t\left(\hat{\delta}_{t+1}\right) \tag{35}$$

Log-linearizing Eq.(19) yields:

$$\begin{aligned} y_{H,t} &= \alpha\eta\sigma_C s_t + \alpha(\eta-1)\sigma_C q_t + \sigma_C c_t + \sigma_G g_{H,t}, \\ y_{F,t} &= \sigma_C c_t^* + \sigma_G g_{F,t}^*, \end{aligned} \tag{36}$$

By taking first difference on the log-linearized definition of the TOT, we get:

$$\pi_{F,t} = \pi_{H,t} + s_t - s_{t-1}, \tag{37}$$

which shows linkage between the GDP inflation and and the changes in the TOT.

Log-linearizing the definition of the CPI disparity yields:

$$q_t = (1-\alpha)s_t. \tag{38}$$

We assume that there is a mesurement error on the real exchange rate, that is $\tilde{q}_t = q_t + \xi_t$ where $\tilde{q}_t$ denotes the date on the real exchange rate published and ξ_t denotes the mesurement error the real exchange rate. Combining Eq.(38) and the previous expression yields:

$$\tilde{q}_t = (1-\alpha)s_t + \xi_t. \tag{39}$$

Log-linearizing Eq.(14) yields:

$$c_t = q_t + c_t^*,$$

which can be rewritten as follows:

$$c_t = (1-\alpha)s_t + c_t^*, \tag{40}$$

Plugging Eq.(38) into the first equality in Eq.(**??**) yields:

$$y_{H,t} = \alpha\sigma_C\omega_\eta s_t + \sigma_C c_t + \sigma_G g_{H,t} \tag{41}$$

with $\omega_\eta \equiv \eta + (\eta - 1)(1-\alpha)$. Eq.(41) is our version of national income identity.

Plugging Eqs.(38), (40) and the second equality in Eq.(36) into the first equality in Eq.(36) yields:

$$s_t = \frac{1}{\sigma_C\omega_\alpha}\left(y_{H,t} - y^*_{F,t}\right) - \frac{\sigma_G}{\sigma_C\omega_\alpha}\left(g_{H,t} - g^*_{F,t}\right),$$

with $\omega_\alpha \equiv \alpha\omega_\eta + (1-\alpha)$. Eq.(**??**) implies that the TOT depends on the output difference between the SOE and the ROW.

2.4.2 The Supply Block

Log-linearizing Eq.(21) yields:

$$n_{H,t} = y_{H,t} - a_{H,t}. \tag{42}$$

Notice that $Z_{H,t}$, $Z_{F,t}$, $Z^w_{H,t}$ and $Z^w_{F,t}$ disappear in Eq.(42) because these are $o\left(\|\xi\|^2\right)$.

By log-linearizing Eq.(22), we have:

$$\pi_{H,t} = \beta \mathrm{E}_t\left(\pi_{H,t+1}\right) + \kappa mc_{H,t}, \tag{43}$$

with $\kappa \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$ being the slop of New Keynesian Philips curve (NKPC). Eq.(43) is fundamental equality of our NKPC.

Log-linearizing the definition of the marginal cost yields:

$$mc_{H,t} = w^r_{H,t} + \frac{\tau}{1-\tau}\hat{\tau}_{H,t} - a_{H,t}, \tag{44}$$

where $w^r_{H,t} \equiv w_{H,t} - p_{H,t}$ denotes the real wage in country H, $\hat{\tau}_{H,t} \equiv \frac{d\tau_{H,t}}{\tau_H}$ denotes the percentage deviation of the tax rate from its steady-state value in country H, and We simply refer to the percentage deviation of the tax rate from its steady-state value $\hat{\tau}_{H,t}$ as the tax gap.

Log-linearizing Eq.(**??**) yields:

$$\tilde{w}_{H,t} = (1-\beta\theta_w)\sum_{k=0}^{\infty}(\beta\theta_w)^k \mathrm{E}_t\left(mrs_{H,t+k|t} + p_{t+k}\right), \tag{45}$$

Let define define the economy's average marginal rate of substition $MRS_{H,t+k} \equiv N^{\psi}_{H,t+k}C_{t+k}$ and $MRS_{F,t+k} \equiv N^{\psi}_{F,t+k}C_{t+k}$. Then, (log) marginal rate of substitution between consumption and hours in period $t+k$ for the household resetting the wage in period t $mrs_{t+k|t}$ can be rewritten as:

$$mrs_{H,t+k|t} = mrs_{H,t+k} - \varepsilon_w\psi\left(\tilde{w}_{H,t} - w_{H,t+k}\right), \tag{46}$$

with:

$$mrs_{H,t} = \psi n_{H,t} + c_t \tag{47}$$

Plugging this Eq.(46) Eq.(45) yields:

$$\tilde{w}_{H,t} = \beta\theta_w \mathrm{E}_t\left(\tilde{w}_{H,t+1}\right) + (1-\beta\theta_w)\, w_{H,t} - \frac{1-\beta\theta_w}{1+\varepsilon_w\psi}\mu^w_{H,t}, \tag{48}$$

with:

$$\mu^w_{H,t} \equiv w_{H,t} - p_t - mrs_{H,t},$$

which can be rewritten as:

$$\mu^w_{H,t} = w^r_{H,t} - \alpha s_t - mrs_{H,t}, \tag{49}$$

where $\mu^w_{H,t}$ denotes the (log) average wage murkup in country H and $w^r_{H,t} \equiv w_{H,t} - p_{H,t}$ denotes the (log) real wage in countries H and F, respectively.

Eq.(**??**) which can be log-linearized as:

$$w_{H,t} = \theta_w w_{H,t-1} + (1-\theta_w)\,\tilde{w}_{H,t}. \tag{50}$$

Combining Eqs.(48) and (50) yields the wage inflation equation as follows:

$$\pi^w_{H,t} = \beta \mathrm{E}_t\left(\pi^w_{H,t+1}\right) - \kappa_w \mu^w_{H,t}, \tag{51}$$

with $\kappa_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varepsilon_w\psi)}$ where $\pi^w_{H,t} \equiv w_{H,t} - w_{H,t-1}$ denotes the wage inflation in countries H and F, respectively.

There is an identity relating the changes in the real wage, the wage inflation and the domestic inflation as follows:

$$w^r_{H,t} - w^r_{H,t-1} = \pi^w_{H,t} - \pi_{H,t}. \tag{52}$$

2.4.3 No Sovereign Risk in the SOE

Log-linearizing Eq.(**??**) yields:

$$c_t = \mathrm{E}_t(c_{t+1}) - \hat{r}_t + \mathrm{E}_t(\pi_{t+1}) - b_{H,t} + \frac{1}{\beta}\hat{r}_{t-1} - \frac{1}{\beta}\pi_t + \frac{1}{\beta}b_{H,t-1} - \frac{1-\beta}{\beta}sp_{H,t}. \tag{53}$$

2.4.4 Flexible Excahnge Rate Regime

By log-linearizing Eq.(**??**), we have:

$$p_{F,t} = e_t + p^*_{F,t}, \tag{54}$$

with $e_t \equiv \text{in}\left(\frac{\mathcal{E}_t}{\mathcal{E}}\right)$. By taking first difference on Eq.(54), we have:

$$\pi_{F,t} = (e_t - e_{t-1}) + \pi^*_{F,t} \tag{55}$$

Log-linearizing Eq.(**??**) yields:

$$\hat{r}_t - \hat{r}^*_t = \mathrm{E}_t(e_t)\,e_t, \tag{56}$$

which implies that the difference on the nominal interest rate (for safety assets) between the SOE and the ROW corresponds to an expected changes in the nominal exchange rate.

2.5 Welfare Costs

Welfare costs function is derived mainly from second-order approximated utility function which is given by:

$$U_F = -\sum_{t=0}^{\infty} L_{F,t} + \text{t.i.p.} + \left(\|\xi\|^3\right)$$

Notice that we just discuss on country F ' s welfare costs for now. To avoid welfare reversal, linear terms are eliminated by exploiting second-ordered equalities which comprise the model.

Welfare costs function in country F is given by:

$$L_{F,t} \equiv \frac{\Lambda_n}{2}\hat{n}_{F,t}^2 + \frac{\Lambda_\pi}{2}\pi_{F,t}^2 + \frac{\Lambda_{\pi^w}}{2}\left(\pi_{F,t}^w\right)^2 + \frac{\Lambda_r}{2}\left(\hat{r}_t^R\right)^2$$

where $\hat{n}_{F,t} \equiv n_{F,t} - n_{F,t}^e$ denotes the welfare relevant employment gap and $n_{F,t}^e$ denotes the efficient employment level. Because of the default risk, the 4th term, quadratic term of the premium gap $\hat{r}_t^R = \frac{\delta}{1-\delta}\mathrm{E}_t\left(\delta_{t+1}\right) + \phi sp_{F,t}$ appears. Appearance of the term of the premium gap represents the cost of changing households ' portfolio. When the steady state value of interest spread is zero, namely $\phi = 0$, we have:

$$L_{F,t} \equiv \frac{\Lambda_n}{2}\hat{n}_{F,t}^2 + \frac{\Lambda_\pi}{2}\pi_{F,t}^2 + \frac{\Lambda_{\pi^w}}{2}\left(\pi_{F,t}^w\right)^2$$

where the quadratic term of the premium gap disappears. That is, the premium gap is the additional source of welfare costs.

2.5.1 Inflation Dynamics and Marginal Cost

By log-linearizing Eq.(**??**), we have:

$$\pi_{H,t} = \beta\mathrm{E}_t\left(\pi_{H,t+1}\right) + \kappa mc_t, \tag{57}$$

with $\kappa \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$ being the slop of New Keynesian Philips curve (NKPC). Eq.(57) is fundamental equality of our NKPC.

Substituting Eq.(**??**) into the definition of the real marginal cost yields:

$$MC_t = \frac{C_t N_t^\psi \mu_t^w}{\left(1-\tau_t\right)A_t}\frac{P_t}{P_{H,t}}. \tag{58}$$

Note that the marginal cost in the steady state, which is the inverse of a constant markup, is smaller than one, while the gross tax rate is definitely smaller than one. In such a case, the steady-state wedge between the marginal product of labor and the marginal utility of consumption is not unity. That is, monopolistic power remains because it is unable to be completely absorbed through taxation. As we discuss later, we need to derive our welfare criteria following Benigno and Woodford[5] because monopolistic power is no longer removed completely and the steady state is distorted.

Log-linearizing Eq.(58) yields:

$$mc_t = c_t + \psi n_t + \alpha s_t + \frac{\tau}{1-\tau}\hat{\tau}_t + \hat{\mu}_t^w - a_t, \tag{59}$$

where $\hat{\tau}_t \equiv \frac{d\tau_t}{\tau}$ denotes the percentage deviation of the tax rate from its steady-state value and $\hat{\mu}_t^w \equiv \frac{d\mu_t^w}{\mu}$ denotes the percentage deviation of the wage markup from its steady-state value. We simply refer to the percentage deviation of the tax rate from its steady-state value $\hat{\tau}_t$ and the percentage deviation of the wage markup from its steady-state value $\hat{\mu}_t^w$ as the tax gap and the wage markup gap, respectively.

2.6 Simple Rules

We assume that the central bank adopts following simple rules:

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + \varphi_\pi \pi_t + \varphi_y y_t + \epsilon_{r,t} \tag{60}$$

where $\epsilon_{r,t}$ is the monetary policy shock and Eq.(60) is a typical Taylor rule.

On fiscal policy, we assume a class of Bohn rule as following:

$$sp_t = \frac{\beta \varphi_b}{1-\beta} b_t. \tag{61}$$

2.7 Shock Processes

Shock processes are following as follows:

$$\begin{aligned} a_t &= \rho_a a_{t-1} + \epsilon_{a,t}, \\ y_t^* &= \rho_y y_{t-1}^* + \epsilon_{y,t}, \\ g_t &= \rho_g g_{t-1} + \epsilon_{g,t}, \\ \hat{\zeta}_t &= \rho_\zeta \hat{\zeta}_{t-1} + \epsilon_{\zeta,t}, \\ \hat{\mu}_t^w &= \epsilon_{\mu,t} \end{aligned} \tag{62}$$

3 Estimation

The model consists of Eqs.(**??**), (**??**), (**??**), (16), (16), (22), (**??**), (26), (18), (**??**), (**??**), (**??**), (57), (59), (60), (61) and (62). We adopt the data in Indonesia where is faced with sovereign debt crisis in 1998, 2000 and 2002. Sample periods of data is 1994 and 2012 and the data is quarterly.

To estimate, we adopt Bayesian technique and our estimation results are shown in Tab. 1. Fig. 1 shows Kalman smoother on the default rate and the government debt. Notice that those are percentage deviation from their steady state value and are consistent with the model. Fig. 2 shows the ratio of government debt to GDP calculated by the data. Fig. 3 shows the TFP inferred by the model and the data which has been brought to light.

As shown in te top panel of Fig. 1, the default rate is highly volatile from 1998 to 2000, when Indonesia faced Asian Financial Crisis and national financial collapse. By comparing the bottom panel of Fig. 1 with Fig. 2, the path of the government debt well captures the data, although Fig. 2 shows not the government debt itself but its ratio to GDP. Finally, the path of inferred TFP well captures the data, as shown by Fig. 3. Thus, it can be said that our estimation is successful.

4 Welfare Costs Function

The policy authorities, the central bank and the government minimize the policy authorities' welfare cost function over time. The period welfare cost function is derived from the welfare criterion. Basically, we derive the policy authorities' welfare criterion following Gali[12]. However, because of distorted steady state, where a linear term which generates welfare reversal, we have to eliminate such a linear term.[3] To eliminate such a linear term, we need to derive the second-order approximated aggregate supply equation which corresponds to Eq.(**??**) and the second-order

[3] The presence of linear terms generally leads to the incorrect evaluation of welfare simple example of this result is proposed by Kim and Kim[14].

approximated inter-temporal government solvency condition, which corresponds to Eq.(**??**). Thus, we follow not only Gali[12] but also Benigno and Woodford[5] and Benigno and Woodford[6] to derive the welfare criterion similar to Ferrero[11]. Notice that we do not impose $R_t^G = R_t^H$ when we derive the second-order approximated inter-temporal government solvency condition different from Okano and Inagaki[20].

Following Gali[12], second ordered approximated utility function is given by:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{U_t - U}{U_C C}\right) &= \sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{\Phi}{\sigma_C}y_t - \alpha\omega s_t\right) - \sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left[\frac{(1-\Phi)(1+\psi)}{\sigma_C 2}\left(y_t^2 - 2y_t a_t\right)\right.\\ &\quad \left. - \frac{(1-\Phi)\varepsilon}{\sigma_C 2\kappa}\pi_t^2\right] + \text{t.i.p.} + o\left(\|\xi\|^3\right), \end{aligned} \tag{63}$$

where t.i.p. denotes terms independent of policy, $o\left(\|\xi\|^3\right)$ are terms of order three or higher and $\Phi \equiv 1 - \frac{1-\tau}{\left(\frac{\varepsilon}{\varepsilon-1}\right)\mu^w}$ denotes the steady-state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor. In the RHS, there is linear terms $\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{\Phi}{\sigma_C}y_t - \alpha\omega s_t\right)$ generating "Welfere Reversal.[4]" To avoid welfare reversal, we have to eliminate the linear terms in the RHS in Eq.(**??**). Following Benigno and Woodford[5] and Benigno and Woodford[6], those linear terms can be rewritten as follows:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{\Phi}{\sigma_C}y_t - \alpha\omega s_t\right) &= -\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left[y_t\left(\tilde{\Omega}_1 y_t - 2\Omega_2 g_t - 2\tilde{\Omega}_3 a_t - 2\Omega_4\hat{\mu}_t^w - 2\Omega_5\hat{\zeta}_t - 2\alpha\Omega_y y_t^*\right)\right.\\ &\quad + \alpha s_t\left(\Omega_6 s_t - 2\Omega_7 g_t - 2\Omega_9 a_t - 2\Omega_{11}\hat{\mu}_t^w - 2\Omega_{12}\hat{\zeta}_t\right)\\ &\quad \left. + \frac{\Theta_6}{2}\left(\hat{r}_t^H - \hat{r}_t^G\right)^2\right] + \Upsilon_0 + \text{t.i.p.} + o\left(\|\xi\|^3\right),\end{aligned}$$

where $\tilde{\Omega}_1$, Ω_2, $\tilde{\Omega}_3$, Ω_4, Ω_5, Ω_6, Ω_7, Ω_y, Ω_9, Ω_{11} and Ω_{12} are complicated block of parameters, $\Upsilon_0 \equiv -\frac{\Theta_1}{1-\beta-\delta}\bar{\omega} + \frac{\Theta_2}{\kappa}\bar{\nu}$ denotes a transitory component and $\bar{\omega}$ and $\bar{\nu}$ are the second-order approximated solvency condition for government and the second-order approximated FONC for firms, respectively.

Plugging previous equality into Eq.(64) yields:

$$\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{U_t - U}{U_C C}\right) \simeq -\mathcal{L} + \Upsilon_0 + \text{t.i.p.} + o\left(\|\xi\|^3\right),$$

where:

$$\mathcal{L} \equiv \sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(L_t\right) \tag{64}$$

denotes the expected welfare costs,

$$L_t \equiv \frac{\Lambda_y}{2}\left(y_t - y_t^e\right)^2 + \frac{\Lambda_\pi}{2}\pi_{H,t}^2 + \frac{\alpha\Lambda_s}{2}\left(s_t - s_t^e\right)^2 + \frac{\Lambda_r}{2}\left(\hat{r}_t^H - \hat{r}_t^G\right)^2, \tag{65}$$

denotes period welfare costs with $\Lambda_y \equiv 2\Omega_1$, $\Lambda_\pi \equiv \frac{\varepsilon(1-\Phi)}{\sigma_C\kappa}$, $\Lambda_s \equiv 2\Omega_6$, $\Lambda_r \equiv \Theta_6$ and $\Omega_1 \equiv \frac{2\sigma_C\tilde{\Omega}_1 + (1+\psi)(1-\Phi)}{2\sigma_C}$ where $y_t^e \equiv \frac{\Omega_2}{\Omega_1}g_t + \frac{\Omega_3}{\Omega_1}a_t + \frac{\Omega_4}{\Omega_1}\hat{\mu}_t^w + \frac{\Omega_5}{\Omega_1}\hat{\zeta}_t + \frac{\Omega_y}{\Omega_1}y_t^*$ and $s_t^e \equiv \frac{\Omega_7}{\Omega_6}g_t + \frac{\Omega_9}{\Omega_6}a_t + \frac{\Omega_{11}}{\Omega_6}\hat{\mu}_t^w +$

[4] Tesar[**?**] has used the log-linearization method and derived a paradoxical result that the incomplete-markets economy produces a higher level of welfare than the complete-markets economy. Kim and Kim[14] point out that a reversal of welfare ordering implies that approximation errors due to the linearization.

$\frac{\Omega_{12}}{\Omega_6}\hat{\zeta}_t$ denote the efficient level of the output and the efficient level of the TOT, respectively. The first, the second and the third terms in the RHS in Eq.(65) show that the deviation of output from its efficient level, namely, the welfare relevant out put gap, the domestic inflation and the deviation of the TOT from its efficient level, namely, the welfare relevant TOT gap bring welfare costs to in this SOE.

5 The LQ Problem

5.1 FONCs for Policy Makers

The optimal stabilization policy can be determined following Lagrangean method. The policy makers choose the sequence $\{x_t, z_t, \pi_{H,t}, \pi_t, sp_t, \delta_t, \hat{r}_t, \hat{\tau}_t\}_{t=0}^{\infty}$ to minimize Eq.(61) subject to the infinite sequence of constraints Eqs.(**??**), (**??**), (16), (**??**), (42), (22), (**??**), (26), (18), (**??**), (**??**), (**??**), (57) and (59). The FONCs for policy authorities are shown in Appendix.

6 Numerical Analysis

6.1 Macroeconomic Dynamics

We run a series of dynamic simulations and adopt the following our estimation results shown in Tab. 1. We solve both cases, the constant tax rule and the optimal monetary and fiscal policy. Tab. 2 shows macroeconomic volatility under both cases. although both volatilities on welfare relevant output gap $y_t - y_t^e$ and GDP inflation are slightly higher than those under the constant tax rule, volatilities on the default rate is obviously smaller than it under the constant tax rule, under the optimal monetary and fiscal policy. There is a reason why the default rate is smaller under the optimal monetary and fiscal policy. Eq.(65), our period welfare cost function is distinguished because of the fourth term in the RHS. The fourth term in Eq.(65) is the quadratic term of the premium difference between the government debt yield $\hat{r}_t^H$ and its coupon rate $\hat{r}_t^G$, which is given by:

$$\begin{aligned}\hat{r}_t^H - \hat{r}_t^G &= \frac{\phi\varsigma\tau\sigma_B}{\tau}\mathrm{E}_t\left(\hat{\delta}_{t+1}\right) + \phi sp_t \\ &= -\frac{\phi\varsigma_\tau\sigma_B(\gamma-1)}{\tau+\phi\varsigma_\tau\sigma_B}sp_t + \frac{\phi\varsigma\sigma_B}{\tau+\phi\varsigma_\tau\sigma_B}b_t, \end{aligned} \tag{66}$$

which implies that the policy authorities has to minimize burden of government debt redemption with interest payment. To minimize burden of government debt redemption, the government has to slush government debt which improve fiscal surplus. As shown in Eq.(**??**), improving the fiscal surplus avoids default. Thus, the optimal monetary and fiscal policy makes the default rate stabilized. In fact, the volatility on the fiscal surplus under the optimal monetary and fiscal policy is smaller than it under the constant tax rule. Stabilized fiscal surplus stabilizes the default rate.

6.2 IRFs

Fig. 4 shows the impulse response function of the welfare relevant output gap, the GDP inflation and the default rate to one standard deviation change of lamp-sum transfer shock. To the shock, both the welfare relevant output gap and the GDP inflation under the constant tax rule are more stabilized than those under the optimal monetary and fiscal policy. However, the default rate under the optimal monetary and fiscal policy is more stabilized than it under the constant tax rule. These results are consistent with macroeconomic volatility shown in Tab. 1. That is, the optimal monetary and fiscal policy is not necessarily the policy stabilize both the inflation and the output gap because of default risk. In fact, the coefficients on the welfare relevant output gap, the GDP inflation, the efficient TOT gap and the difference of quadratic term of r_t^R and r_t^S, namely, Λ_y, Λ_π, Λ_s and Λ_r, are 2.2578, 0.6496, 1.5090 and 143.2793. That is, minimizing the difference of quadratic term of r_t^R and r_t^S is the most important policy agenda in an economy with sovereign risk.

7 Welfare Analysis

By taking $\beta \to 1$, welfare costs function Eq.(64) becomes as follows:

$$\mathcal{L} = \frac{\Lambda_y}{2} \mathrm{var}\left(y_t - y_t^e\right) + \frac{\Lambda_\pi}{2} \mathrm{var}\left(\pi_{H,t}\right) + \frac{\alpha \lambda_s}{2} \mathrm{var}\left(s_t - s_t^e\right) + \frac{\Lambda_r}{2} \mathrm{var}\left(\hat{r}_t^R - \hat{r}_t^S\right),$$

and we can calculate welfare costs under each regime following this criteria. Under the optimal monetary and fiscal policy, welfare costs are 47.4080 while welfare costs are 328.7759 under the constant tax rule. If Indonesia adopted optimal monetary and fiscal policy, welfare costs could be slushed approximately 85.6%.

8 Optimal Monetary and Fiscal Rule

to be added.

9 Conclusion

We developed a small open economy model following Gali and Monacelli[?] with the FTSR advocated by Uribe[21]. The model is estimated through Bayesian technique. Welfare costs function is derived through approximating utility function and any linear terms are eliminated appropriately. We solve the minimization problem and compare results under the optimal monetary and fiscal policy, namely, counter factual simulation with data. We find that the optimal monetary and fiscal policy is not necessarily the policy minimize volatility on inflation but that policy stabilizes the default rate. In addition, if Indonesia adopted optimal monetary and fiscal policy, welfare costs could be slushed approximately 85.6%.

Appendices

A Nonstochastic Steady State

We focus on equilibria where the state variables follow paths that are close to a deterministic stationary equilibrium, in which $\Pi_t = 1$ and $\frac{\tilde{P}_t}{P_t} = 1$. Because this steady state is nonstochastic, the productivity has unit values; i.e., $A_H = 1$. Following De Paoli[?], we assume $\mathcal{S} = 1$.

In this steady state, the gross nominal interest rate is equal to the inverse of the subjective discount factor, as follows:

$$R = R^* \beta^{-1},$$

which implies $\mathcal{E}_t$ is constant for all t.

Eq.(24) can be rewritten as:

$$\tilde{P}_t = \mathrm{E}_t \left(\frac{K_{H,t}}{P^{-1} F_{H,t}} \right) \tag{67}$$

with:

$$K_{H,t} \equiv \frac{\varepsilon}{\varepsilon - 1} \sum_{k=0}^{\infty} (\beta\theta)^k \left(P_{t+k} C_{t+k}\right)^{-1} \tilde{Y}_{t+k} MC_{t+k}^n \quad ; \quad F_{H,t} \equiv P_t \sum_{k=0}^{\infty} (\beta\theta)^k \left(P_{t+k} C_{t+k}\right)^{-1} \tilde{Y}_{t+k},$$

which implies that:

$$K_H = \frac{\frac{\varepsilon}{\varepsilon-1} Y MC^n}{(1-\alpha\beta)(PC)} \quad ; \quad F_H = \frac{PY}{(1-\alpha\beta)(PC)}.$$

These equalities imply that:

$$P = \frac{\varepsilon}{\varepsilon - 1} MC_H^n.$$

Thus, we have:

$$MC_H = \left(\frac{\varepsilon}{\varepsilon - 1} \right)^{-1}. \tag{68}$$

Furthermore, Eqs.(26) and (68) imply the following:

$$\begin{aligned} CN_H^{\psi} &= (1-\tau) \left(\frac{\varepsilon}{\varepsilon - 1} \right)^{-1}, \\ &= MC_H (1-\tau), \end{aligned} \tag{69}$$

where we use $MC_H = (1-\tau)^{-1} \frac{W}{P}$ and $CN^{\psi} = \frac{W}{P}$. Then, the previous expression can be:

$$MC_H = CN_H^{\psi} \left(\frac{1}{1-\tau} \right)$$

Eq.(70) implies the familiar expression:

$$(1-\tau) C^{-1} = \frac{\varepsilon}{\varepsilon - 1} N_H^{\psi}.$$

Note that because $\tau \in (0,1)$ and $\varepsilon > 1$, this steady state is distorted.

In the steady state, Eq.(**??**) implies:

$$SP_H = B_H \left[R^H (1-\delta) - 1 \right],$$

which can be rewritten as:

$$\frac{B_H}{SP_H} = \frac{\beta}{1-\beta}, \tag{70}$$

which implies $\frac{SP_H}{Y_H} = \frac{\sigma_B (1-\beta)}{\beta}$.

References

[1] Arellano, Cristina (2008), “Default Risk and Income Fluctuations in Emerging Economies,” *American Economic Review*, 98, 690–712.

[2] Bacchetta, Philippe, Elena Perazzi and Eric van Wincoop (2015), “Self-Fulfilling Debt Crises; Can Monetary policy Really Help?” Unpublished Manuscript.

[3] Benigno, Pierpaolo (2001), “Price Stability with Imperfect Financial Integration,” Unpublished Manuscript, New York University.

[4] Benigno, Pierpaolo (2009), “Price Stability with Imperfect Financial Integration,” *Journal of Money, Credit and Banking*, 41,121–149.

[5] Benigno, Pierpaolo and Michael Woodford (2005), “Inflation Stabilization and Welfare: The Case of a Distorted Steady State,” *Journal of the European Economics Association*, 3, 1185–1236.

[6] Benigno, Pierpaolo and Michael Woodford (2003), “Optimal Monetary and Fiscal Policy: A Linear-Quadratic Approach,” *NBER Macroeconomics Annual 2003*, Mark Gertler and Kenneth Rogoff (eds.), MIT Press.

[7] Cochrane, John H. (1998), “A Frictionless View of U.S. Inflation,” *NBER Macroeconomics Annual*, 13, 323–384.

[8] Corsetti, Giancarlo and Luca Dedola (2014), “The Mystery of the Printing Press: Monetary Policy and Self-fulfilling Debt Crises, ” Mimeo.

[9] Corsetti, Giancarlo, Keith Kuester, Andre Meier, and Gernot J. Mueller (2013), “Sovereign Risk, Fiscal Policy, and Macroeconomic Stability,” *Economic Journal*, F99-F132.

[10] Curdia, Vasco and Michael Woodford (2009), “Credit Frictions and Optimal Monetary Policy,” *BIS Working Papers*, #278, Bank for International Settlements.

[11] Ferrero, Andrea (2009), “Fiscal and Monetary Rules for a Currency Union,” *Journal of International Economics*, 77, 1–10.

[12] Gali, Jordi (2008), *Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework*, Princeton University Press, Princeton, NJ.

[13] Gali, Jordi and Tommaso Monacelli (2005), “Monetary Policy and Exchange Rate Volatility in a Small Open Economy,” *Review of Economic Studies*, 72, 707–734.

[14] Jinill Kim and Sunghyun Henry Kim (2003), “Spurious welfare reversals in international business cycle models,” *Journal of International Economics*, 60, 471–500.

[15] Leeper, Eric M. (1991), “Equilibria under ‘Active’ and ‘Passive’ Monetary and Fiscal Policyies, ” *Journal of Monetary Economics*, 27, 129–147.

[16] Lubik, Thomas A. and Frank Schorfheide (2007), “Do central banks respond to exchange rate movements? A structural investigation,” *Journal of Monetary Economics*, 54, 1069–1087.

[17] Mendoza, Enrique E. and Vivian Z. Yue (2012), "A General Equilibrium Model of Sovereign Default and Business Cycles," *Qurterly Journal of Economics*, 127, 889–946.

[18] Okano, Eiji (2014), "How Important is Fiscal Policy Cooperation in a Currency Union?" *Journal of Economic Dynamics and Control*, 38, 266–286.

[19] Okano, Eiji and Masashige Hamano (2016), "Inflation Stabilization and Default Risk in a Currency Union," *Macroeconomic Dynamics*, Forthcomming.

[20] Okano, Eiji and Kazuyuki Inagaki (2017), "Revisiting the Fiscal Theory of Sovereign Risk from a DSGE Viewpoint," Nagoya City University Society of Economics Discussion Papers in Economics, No. 611.

[21] Uribe, Martin (2006), "A Fiscal Theory of Sovereign Risk," *Journal of Monetary Economics* 53, 1857–1875.

[22] Woodford, Michael (2003), "Interest Rate and Prices: Foundations of a Theory of Monetary Policy," *Princeton University Press*, Princeton and Oxford.

[23] Woodford, Michael (1994), "Monetary Policy and Price Level Determinacy in a Cash-in-advance Economy," *Economic Journal*, 4, 345–380.

Tab.1 Estimation Resuls

Parameter	Prior			Posterior	
	Distribution	Mean	S.D	Mean	90% Interval
φ	Normal	2	0.5	2.0186	[1.2388 , 2.7505]
θ	Beta	1	0.25	0.0634	[0.0011 , 0.1191]
ν	Gamma	1	0.5	1.2092	[0.6579 , 01.7080]
η	Gamma	1	0.5	1.1886	[0.4823 , 1.9317]
α	Beta	0.3	0.2	0.3581	[0.2481 , 0.4612]
ϕ	Gamma	0.00001	0.2	0.0004	[0.0001 , 0.0007]
γ	Normal	1.145	0.2	0.1575	[0.9946 , 1.3145]
ϕ_π	Normal	1.5	0.5	2.3325	[2.0517 , 2.6363]
ϕ_y	Normal	0.5	0.5	1.0420	[0.6756 , 1.4247]
ρ_a	Beta	0.85	0.5	0.8511	[0.7712 , 0.9352]
ρ_y	Beta	0.85	0.5	0.6085	[0.4274 , 0.7714]
ρ_g	Beta	0.85	0.5	0.6641	[0.5400 , 0.7818]
ρ_ζ	Beta	0.85	0.5	0.9553	[0.8870 , 0.9998]
ρ_r	Beta	0.85	0.5	0.5827	[0.4456 , 0.7291]
σ_{ϵ_a}	Inv. Gamma	2	2	1.8460	[1.5925 , 2.0789]
σ_{ϵ_y}	Inv. Gamma	4	2	4.1755	[3.0949 , 5.2164]
σ_{ϵ_g}	Inv. Gamma	4	2	3.7893	[3.2796 , 4.2305]
σ_{ϵ_ζ}	Inv. Gamma	4	2	19.1987	[3.9686 , 41.5635]
σ_{ϵ_μ}	Inv. Gamma	2	2	2.4010	[1.7326 , 3.0213]
σ_{ϵ_r}	Inv. Gamma	7	2	6.3377	[5.3080 , 7.2818]
σ_{ϵ_q}	Inv. Gamma	10	2	14.6637	[12.8854 , 16.3621]
				Log data density	-1350.131

Tab. 2 Macroeconomic Volatility		
	Constant Tax Rule	Optimal Monetary and Fiscal Policy
Welfare Relevant Output Gap	3.1231	5.6541
GDP Inflation	2.0080	2.8882
CPI Inflation	2.5130	3.4000
Efficient TOT Gap	7.4379	6.2200
Difference bet. Gov. Debt Yield and Coupon	0.9382	0.0032
Interest Rate Spread	0.5021	0.0317
Fiscal Surplus	944.4973	870.4524
Default Rate (Not Percentage Deviation)	43.3754	13.1738

Figure 1: Kalman Smoother

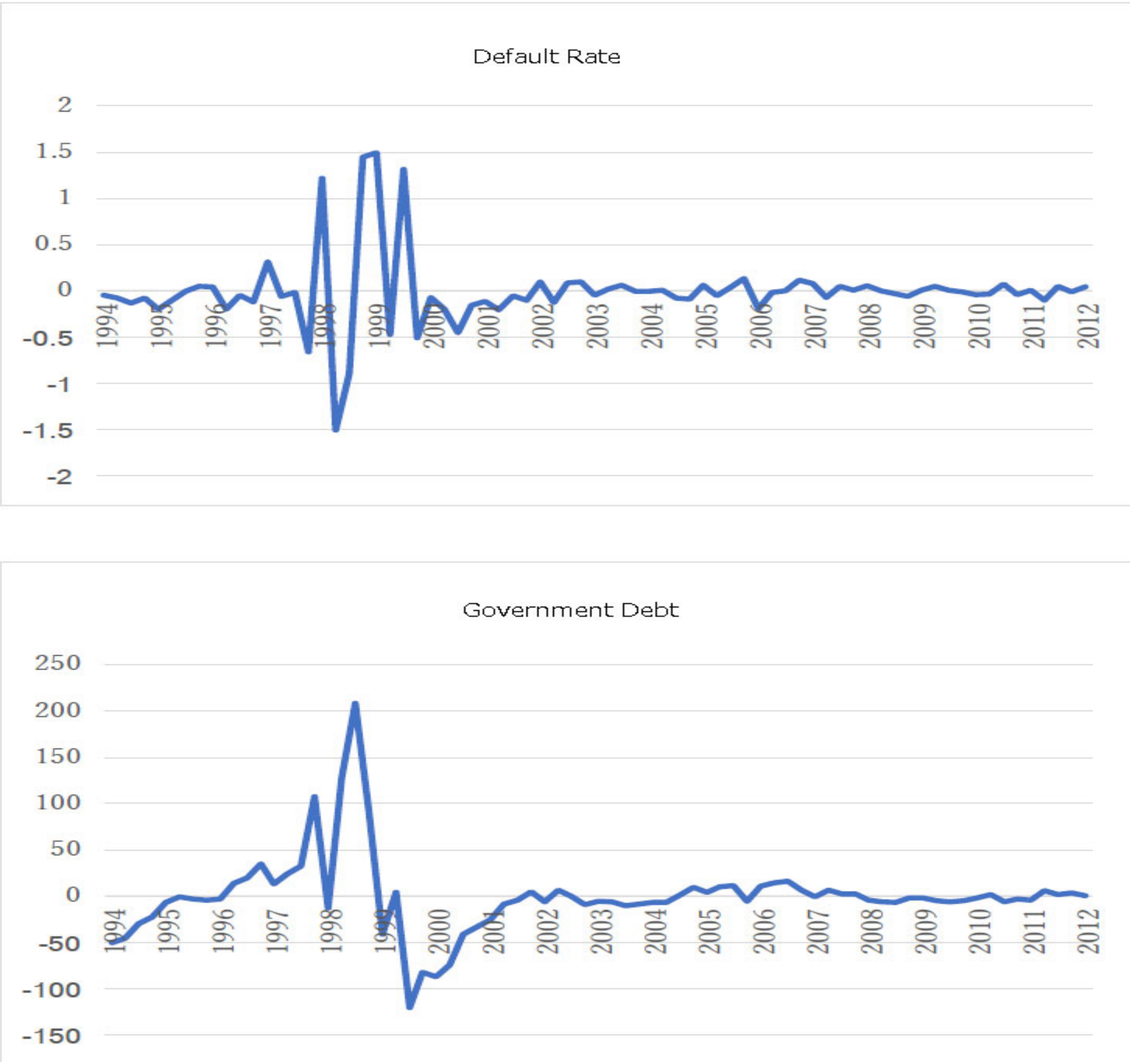

Figure 2: Ratio of Government Debt to GDP Calculated by Data

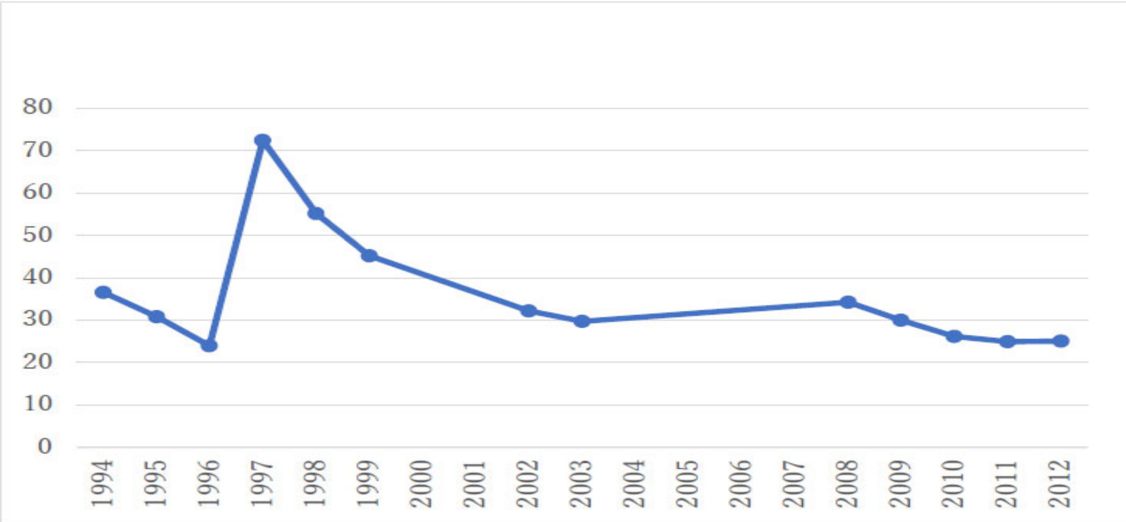

Figure 3: Productivity

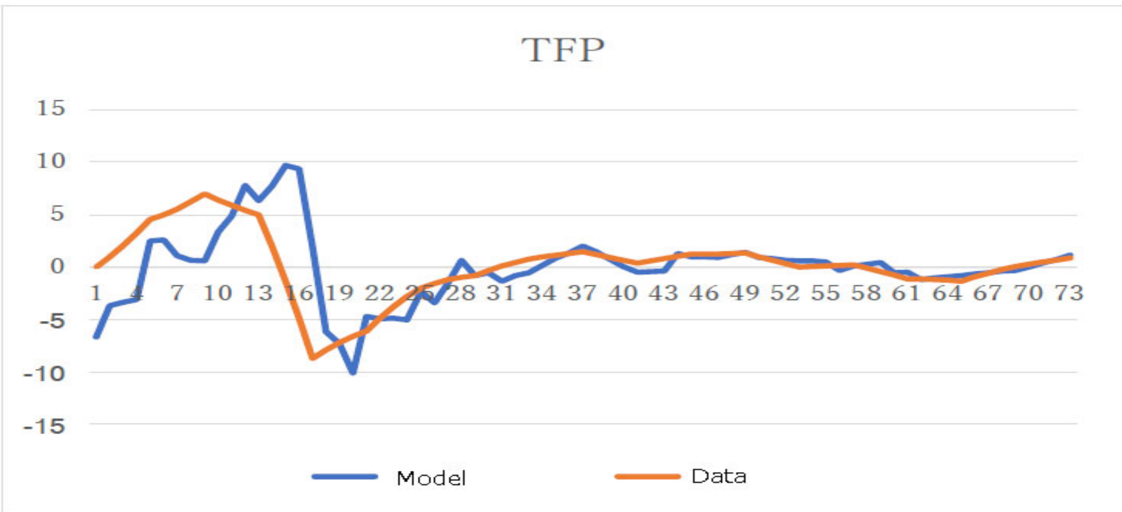

Figure 4: IRFs to Lump-sum Transfer Shock