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The Effects of a Money-financed Fiscal Stimulus in a Small Open Economy with Fiscal Theory of Price Level*.

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Abstract

We investigate the effectiveness of the *MF* fiscal stimulus in a small open economy in which the fiscal theory of price level (FTPL) is premised, different from Okano and Eguchi (Forthcoming, *IMFER*). In normal times, we find that the effectiveness of the *MF* fiscal stimulus decreases as openness increases. The negative relationship between inflation and the real money balance derived from the FTPL, hampers the effect, strengthening the *MF* fiscal stimulus in a small open economy. This result contradicts that of Okano and Eguchi (forthcoming, *IMFER*). However, in a liquidity trap, an adverse demand shock induces a huge money injection, overwhelming the negative relationship. Thus, in a small open economy, the effectiveness of *MF* fiscal stimulus is larger than in a closed economy. This finding is consistent with that of Okano and Eguchi (forthcoming, *IMFER*).

Keywords: Fiscal Stimulus, Money Financing, Small Open Economy, Zero Lower Bound
JEL Classification: E31, E32, E52, E62, F41

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1 Introduction

Following Gali[8], who analyzed the effectiveness of a money-financed (*MF*) fiscal stimulus in a closed economy, Okano and Eguchi[14] developed a small open economy model without iterated government budget constraints and showed that the *MF* fiscal stimulus was more effective in boosting output and inflation in a small open economy than in a closed economy, irrespective of whether in normal times or in a liquidity trap. We focus on the fiscal theory of price level (FTPL), which is derived by iterating government budget constraints and is not premised on Okano and Eguchi[14] and investigate the effectiveness of the *MF* fiscal stimulus in a small open economy in a liquidity trap. In conclusion, overall, the effectiveness of the *MF* fiscal stimulus is lower in FTPL than in non-FTPLs, irrespective of whether it is a small open economy or a closed economy. Premising the FTPL in a small open economy, the effectiveness of the *MF* fiscal stimulus is stronger than that in a closed economy in a liquidity trap. Okano and Eguchi[14] findings are still applicable even if FTPL is premised. However, in normal times, the *MF*fiscal stimulus is less effective in a small open economy than in a closed economy. Okano and Eguchi[14] findings were reversed during normal times.

We derive our own version of the central equation of the FTPL following Cochrane[6], which implies that the ratio of nominal consolidated government debt to the price level corresponding to the consumer price index (CPI) depends on the expected present value of the consolidated government's fiscal surpluses. Consolidated government debt comprises government debt, interest payments, and money. The consolidated government's fiscal surplus consists of the primary surplus and seigniorage, which originates from households and involves compensation for providing liquidity from the central bank. Thus, not only outstanding government debt but also money balance affects the price level, namely, the CPI (level). The central equation of the FTPL following Cochrane[6] can be arranged as the second-order differential equation which implies that (i) the consolidated government's fiscal surplus depends on burden to redeem consolidated government's debt with interest payment, (ii) if the consolidated government's fiscal surplus is not sufficient to redeem it, the consolidated government "inflate away" it and (iii) if the consolidated government renews its debt including the real money balance, "inflate away" is not necessary and deflationary pressure applies occasionally.

Here, effect (iii) is important for understanding our results. As mentioned, Gali[8] and Okano and Eguchi[14] show that a huge money injection bolsters CPI inflation, therefore the *MF* fiscal stimulus has a strong effect even in a liquidity trap. However, once the FTPL is introduced, its effectiveness is hampered because money injection applies pressure to decrease the CPI inflation. Money injection applies pressure to increase the current real money balance. This increase in the current real money balance is the renewal of consolidated government debt, which mitigates the burden of redeeming the consolidated government debt and applies pressure to decrease CPI inflation. In other words, the pressure to increase CPI inflation resulting from money injections is canceled through the FTPL. Therefore, the effectiveness of the *MF* fiscal stimulus in the FTPL is less than that in the non-FTPL, as analyzed by Gali[8] and Okano and Eguchi[14].

The FTPL also plays an important role in changing the effectiveness of *MF* fiscal stimulus in a small open economy. In normal times, we find that the effectiveness of the *MF* fiscal stimulus in the FTPL is weaker than in non-FTPL, as mentioned. An increase in government expenditure under an *MF* fiscal stimulus applies pressure to increase CPI inflation by increasing domestic inflation. An increase in CPI inflation resulting from an increase in government expenditure decreases the burden of redeeming consolidated government debt, thereby decreasing the current money balance. In a small open economy, the ratio of domestic inflation to CPI inflation is less than one so that the decrease in the current real money balance is less than that in a closed economy. Here, in a small open economy, current real money balance, which is renewal of consolidated government's debt, is relatively higher in a small open economy than a closed economy. As a result, the need to "inflate way" the debt is diminished in a small open economy. Thus, the increase in CPI inflation is smaller in a small open economy than in a closed economy. An increase in the CPI inflation depreciates the nominal exchange rate and applies pressure to worsen the terms of trade (the relative import price to the domestic price, TOT). However, as mentioned, the increase in CPI inflation is small, and the nominal exchange rate does not sufficiently depreciate to worsen TOT. An improvement in TOT corresponds to a relative increase in the price of domestic goods compared to import goods. Consequently, output decreases because consumers tend to prefer the cheaper imported goods over domestic products. Thus, as TOT improves, the increase in output in a small open economy is less than that in a closed economy. This result contradicts that of Okano and Eguchi[14].

We then calculate fiscal multipliers and find that they decrease as openness increases. Our results on the relationship between fiscal multipliers and openness under the *MF* fiscal stimulus in normal times are opposite to those of Okano and Eguchi[14] who find that fiscal multipliers increase as openness increases. Interestingly, in a fully opened small open economy in which openness is one, the fiscal multiplier under the *MF* fiscal stimulus is identical to that under the debt-financed (*DF*) fiscal stimulus with CPI inflation targeting (CIT). In a fully opened small open economy, the ratio of domestic inflation to CPI inflation is zero, meaning CPI inflation remainds unchanged and is zero. The consolidated government has no incentive to change its current real money balance following the FTPL. Thus, our central equation of the FTPL works as if the CIT is in a fully opened, small open economy.

However, once an adverse demand shock occurs, which plays an important role in generating a liquidity trap, the effectiveness of the *MF* fiscal stimulus is stronger in a small open economy. This result is applicable regardless of whether it is in normal times or in a liquidity trap and is consistent with Okano and Eguchi[14]. An adverse demand shock directly decreases the CPI inflation through the consumption Euler equation. In a small open economy, the CPI inflation is less sticky because it includes import inflation, which has no stickiness. Thus, the decrease in CPI inflation is more severe in a small open economy. Owing to the FTPL, a more severe decrease in CPI inflation requires huge money injections to increase the real money balance, which renewsal of consolidated government debt. Because of the greater increase in the real money balance resulting from higher money growth in a small open economy, the decrease in the nominal interest rate is larger than that in a closed economy. The larger decline in the nominal interest rate leads to a greater reduction in the consumption real interest rate, therefore resulting in a cumulative output in a small open economy that is larger than that in a closed economy. Then, we calculate fiscal multipliers and find that they increase as openness increases. As openness increases, the real money balance increases so that multipliers positively relates to openness. This result is contrary on the result in normal times without adverse demand shock although this result is consistent with Okano and Eguchi[14].

The result of a liquidity trap is almost the same as during normal times with arising an adverse demand shock. In the liquidity trap, the ZLB constraint is introduced and is prevents negative nominal interest rates. As mentioned, in a small open economy, an adverse demand shock decreases CPI inflation, and this decrease in the CPI inflation is larger than that in a small open economy.

Owing to FTPL, an increase in the real money balance is necessary, and money growth is higher in a small open economy than in a closed economy. Although the increase in the real money balance in a small open economy is higher than that in a closed economy, the nominal interest rate is the same as that in a closed economy because of the ZLB constraint (in normal times, it pushes away to negative territory). However, a higher money injection in a small open economy bolsters the CPI (level), and the recovery in CPI inflation is faster in a small open economy than in a closed economy. CPI inflation in a small open economy is less sticky; thus, the recovery is fast. This faster recovery in CPI inflation reduces the real consumption interest rate, and this decrease is larger than that in a closed economy. Thus, the cumulative output in a small open economy is greater than that in a closed economy. The *MF* fiscal stimulus is more effective in a small open economy than in a closed economy. This result is consistent with those of Okano and Eguchi[14].

The remainder of this paper is organized as follows: Section 2 discusses related literature. Section 3 derives the model and defines the fiscal and monetary policies. Section 4 presents steady-state and equilibrium dynamics. Section 5 discusses the effects of fiscal stimuli in normal times. Section 6 discusses the effects of fiscal stimuli in a liquidity trap in which a zero lower bound is available. Finally, Section 7 concludes the study.

2 Related literature

Developed by Leeper[12], Woodford[17], and Sims[16] with one-period debt, the FTPL states that government budget constraints and surpluses determine price levels. Leeper[12] explored the interaction between monetary and fiscal policies using a stochastic maximizing model. He defined policy as "active" or "passive" depending on how it responds to government debt shocks and showed how different combinations of active and passive policies affect the existence and uniqueness of equilibria, the role of fiscal behavior in determining the effects of monetary shocks on prices, and the interpretation of Friedman's 1948 policy framework. Woodford[17] investigated how the price level is determined in different monetary regimes, even when the money supply is endogenous, based on the FTPL. Christiano and Fitzgerald[4] reviewed the FTPL and emphasized its implications for the feasibility of price stability. Cochrane[5] extended the fiscal theory to include long-term debt and analyzed the optimal policy. Cochrane[7] showed that the default risk can "inflate away" through inflating the government debt.

Similar to our study, few authors have focused on a consolidated government. Cochrane[6] showed that the price level is determined by the government debt valuation equation in a frictionless economy with money. Although we do not focus on monetary policy, Benigno[2] showed the inefficiencies and instability of the international monetary system by extending the FTPL and considering a consolidated government. We follow Cochrane[6] to derive a class of central equations of the FTPL because his analysis, similar to us considered not only consolidated government, but also the issuing, of non-interest-bearing debt, namely money.

Auerbach and Obstfeld[1] studied the effect of open market operations in raising inflation and output when the economy is at the ZLB owing to a temporary adverse shock. Buiter[3] analyzed the impact of a money-financed transfer to households (a "helicopter drop") in a relatively general setting, emphasizing the importance of "irredeemability" of money as the ultimate source of the expansionary effect on consumption of such a policy. Gali[8] expanded Auerbach and Obstfeld[1] and Buiter[3] proposed a theoretical framework of money-financed stimulus by comparing the effectiveness of money-financed stimulus and debt-financed stimulus. Okano and Eguchi[14] differed from Gali[8], who analyzed effectiveness in a small open economy by referring to Gali and Monacelli[11]. However, in both Gali[8] and Okano and Eguchi[14], FTPL was not considered in their models, whereas it was considered in this study.

3 The Model

The model consisted of policy and non-policy blocks. The FTPL equation was adopted for the policy block. The nonpolicy block is almost the same as that in Okano and Eguchi[14], whose model was developed following Gali and Monacelli[11]. We assume a representative household, sticky prices for domestic goods (i.e., Calvo pricing is applied for domestic goods), and flexible wages. A representative household lives in an infinitesimally small open economy with a complete international financial market. In a small open economy, the law of one price (LOOP) is applicable, and exports are elastic to changes in TOT, similar to Gali and Monacelli[11].

In the policy block, the consolidated government (consisting of fiscal and monetary authorities acting in a coordinated manner) finances expenditures and issues a riskless nominal one-period bond with a nominal interest rate and (non-interest-bearing) money. Similar to Okano and Eguchi[14] and Gali[8], taxation is a lump-sum fashion. The most important difference between Okano and Eguchi[14] is that the government budget constraint is iterated, and an appropriate transversality condition (TVC) is imposed, similar to Cochrane[6].

3.1 The Fiscal and Monetary Policy Framework

3.1.1 Government: Budget Constraints and Financing Regimes

We assume that the government (consisting of fiscal and monetary authorities acting in a coordinated manner) finances its expenditures through lump-sum taxes and issuing a riskless nominal one-period bond with a nominal interest rate and (non-interest-bearing) money. Therefore, the consolidated budget constraint is:

$$P_{H,t}G_t + B_{t-1}\left(1 + i_{t-1}\right) = P_t TR_t + B_t + \Delta M_t, \tag{1}$$

where $P_{H,t}$ denotes the domestic price index, $P_t \equiv P_{H,t}^{1-\nu} P_{F,t}^{\nu}$ denotes the CPI, $P_{F,t}$ denotes the import goods price in units of domestic currency, $\nu \in [0,1]$ denotes the openness of the small open economy, B_t denotes the nominal riskless one-period domestic government bond in units of domestic currency, i_t denotes the net nominal interest rate, TR_t denotes the lump-sum tax revenue, M_t denotes the (non-interest bearing) money, Δ is the difference operator, and G_t denotes the (real) government expenditure index. The definitions of $P_{H,t}$ and $P_{F,t}$ are shown precisely in Section 3.2.1.

Dividing both sides of Eq. (1) using the CPI yields:

$$\mathcal{S}_t^{-\nu} G_t + \mathcal{B}_{t-1}\mathcal{R}_{t-1} = TR_t + \mathcal{B}_t + \frac{\Delta M_t}{P_t}, \tag{2}$$

where $\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}}$ denotes the TOT, $\mathcal{B}_t \equiv \frac{B_t}{P_t}$ denotes real domestic government debt outstanding, and $\mathcal{R}_t \equiv (1 + i_t)\,\Pi_{t+1}^{-1}$ denotes the (ex-post) gross real interest rate. The following analysis focuses on the equilibrium near a steady state with zero inflation, no trend growth, and no government expenditure, taxes, or debt. The constancy of real balances requires $\Delta M = 0$, and hence zero seigniorage in the steady state. Note that the variables without time scripts are the steady-state values.

Multiplying both sides of Eq.(2) by $1 + i_t$, iterating forward j times, plugging Euler equation $U_{c,t}Z_t = \beta \mathcal{R}_t U_{c,t+1} Z_{t+1}$, taking the limit for $j \to \infty$, and imposing an appropriate TVC $\lim_{j\to\infty} \beta^{t+j+1} \mathcal{R}_{t+j} \left(\mathcal{B}_{t+j} + L_{t+j}\right) = 0$, one can write:

$$\begin{aligned} U_{c,t} Z_t \mathcal{R}_{t-1} \left(\mathcal{B}_{t-1} + L_{t-1}\right) \quad = \quad & \left\{ \sum_{h=0}^{\infty} \beta^h U_{c,t+h} Z_{t+h} SP_{t+h} \right. \\ & \left. + \sum_{h=0}^{\infty} \beta^{h-1} U_{c,t+h-1} Z_{t+h-1} \left(\frac{i_{t+h-1}}{1 + i_{t+h-1}} \right) L_{t+h-1} \right\} \Pi_t, \end{aligned} \tag{3}$$

where $U_{c,t}$ denotes the marginal utility of consumption, Z_t denotes the exogenous preference shifter, $L_t \equiv \frac{M_t}{P_t}$ denotes the real money balance, $\beta \equiv (1+\rho)^{-1}$ denotes the subjective discount factor, ρ denotes the time preference which is identical with steady state value of the net nominal interest rate, and $SP_t \equiv TR_t - \mathcal{S}_t^{-\nu} G_t$ denotes the (real) fiscal surplus. $\left(\frac{i_t}{1+i_t}\right) L_t$ is the opportunity cost of holding the real money balance deprived from households so that Eq.(3) shows that the consolidated government liability in terms of the marginal utility of consumption equals the sum of net present value of the sum of the fiscal surplus in terms of the marginal utility of consumption and the net present value of the sum of the opportunity cost of holding the real money balance.

Eq.(3) can be rewritten as:

$$\begin{aligned} \frac{U_{c,t} Z_t \left(1 + i_{t-1}\right) \left(B_{t-1} + M_{t-1}\right)}{P_t} \quad = \quad & \sum_{h=0}^{\infty} \beta^h U_{c,t+h} Z_{t+h} SP_{t+h} \\ & + \sum_{h=0}^{\infty} \beta^{h-1} U_{c,t+h-1} Z_{t+h-1} \left(\frac{i_{t+h-1}}{1 + i_{t+h-1}} \right) L_{t+h-1}. \end{aligned} \tag{4}$$

According to Cochrane[6], FTPL recognizes that nominal debt, including the monetary base, is a residual claim to government primary surpluses. If the surplus is insufficient, the government must default on or inflate its debt. Therefore, we can determine the price level using the valuation equation for government debt as follows:

$$\frac{\text{Nominal Government Debt}}{\text{Price Level}} = \text{Expected Present Value of Primary Surpluses.} \tag{5}$$

Eq.(4) is analogous to Eq.(5); therefore, Eq.(3) succeeds the character of Cochrane[6]'s FTPL.

Eq.(3) can be rewritten as:

$$1 = \frac{\sum_{h=0}^{\infty} \beta^h U_{c,t+h} Z_{t+h} SP_{t+h} + \sum_{h=0}^{\infty} \beta^{h-1} U_{c,t+h-1} Z_{t+h-1} \left(\frac{i_{t+h-1}}{1+i_{t+h-1}} \right) L_{t+h-1}}{U_{c,t} Z_t \left(1 + i_{t-1}\right) \left(L_{t-1} + \mathcal{B}_{t-1}\right)} \Pi_t,$$

which implies that fiscal stimulus, such as an increase in government expenditure, which applies pressure to decrease fiscal surplus SP_t can increase the (gross) CPI inflation Π_t premising that changes in the marginal utility of consumption $U_{c,t}$ and exogenous preference shock Z_t are negligible. More importantly, the previous expression implies that the effects of the monetary-financed scheme on increasing inflation are limited. Whereas an increase in money growth applies pressure to increase inflation, an increase in money growth increases the expected present value of primary surplus through an increase in the opportunity cost for holding real money balance deprived from households $\left(\frac{i_t}{1+i_t}\right) L_t$ and applies pressure to decrease inflation. The pressure to increase inflation is canceled by the pressure to decrease it. Therefore, the effects of *MF* fiscal stimulus weakens.

Eq.(3) can be rewritten as the following second-order difference equation:

$$\begin{aligned} U_{c,t}Z_tSP_t + \beta^{-1}U_{c,t-1}Z_{t-1}\frac{i_{t-1}}{1+i_{t-1}}L_{t-1} &= U_{c,t}Z_t\left(1+i_{t-1}\right)\left(\mathcal{B}_{t-1}+L_{t-1}\right)\Pi_t^{-1} \\ &\quad - \beta U_{c,t+1}Z_{t+1}\left(1+i_t\right)\left(\mathcal{B}_t+L_t\right)\Pi_{t+1}^{-1}. \end{aligned}$$

In the previous expression, the LHS is consolidated government's revenue in terms of the marginal utility of consumption. The first term in the RHS is consolidated government's burden to redeem its debt with real interest payment and the second term is the renewal of consolidated government's debt with real interest payment (or the issuance of new debt with real interest payments). The previous expression can be log-linearized as:

$$\hat{i}_{t-1} + \frac{b\left(1-\beta\right)}{\chi\beta}\widehat{sp}_t = \frac{b+\chi}{\chi}\hat{i}_{t-1} + \frac{1}{\chi}\hat{b}_{t-1} + \frac{b\left(1-\beta\right)^2+\chi\beta^2}{\chi\beta}\hat{l}_{t-1} - \frac{\beta}{\chi}\hat{b}_t - \beta\hat{l}_t - \frac{b+\chi\beta}{\chi}\pi_t,$$

where we use households' intertemporal optimality condition, namely, Euler equation Eq.(18), to eliminate marginal rate for consumption before log-linearizing. The definitions of lower-case letters with time subscripts, which are logarithmic variables, are shown in Table 1. Variables without a time script are the steady state values of the corresponding variables with a time script. In the previous expression, the LHS is (logarithmic) revenue which consists of interest payment deprived from households and the fiscal surplus (The principal to produce interest payment $\hat{l}_t$ is canceled on both sides). The first to the third terms in the RHS is expenditure which consists of burden to redeem government debt with interest payment and the real money balance. The fourth and fifth terms represent the renewal of government debt (or newly issued government debt) and the renewal of the real money balance (or newly issued real money balance). The sixth term represents so-called inflation tax, which increases to reduce the burden of repaying the consolidated government's debt when the government's revenue or debt renewal is insufficient, and vice versa.

Plugging logarithmic definition of the fiscal surplus $\widehat{sp}_t = \widehat{tr}_t - \hat{g}_t$ into the LHS in the previous expression, we have:

$$\widehat{tr}_t = b\hat{i}_{t-1} + \hat{b}_{t-1} + \frac{b\left(1-\beta\right)^2+\chi\beta^2}{\beta}\hat{l}_{t-1} - \beta\hat{b}_t - \beta\chi\hat{l}_t - \left(b+\chi\beta\right)\pi_t + \hat{g}_t, \tag{6}$$

where $\chi \equiv \frac{L}{Y}$ and $b \equiv \frac{\mathcal{B}}{Y}$ are the steady state inverse velocity and target debt ratio, respectively. Here, the interest payment deprived taken from households is canceled on both sides and disappears from the LHS in Eq.(6). That is, seigniorage is less than the burden of paying interest on government debt. Eq.(6) is a fiscal policy rule that complies with the FTPL regime. Eq.(6) shows that the burden to redeem consolidated government's debt including interest payment and the real money balance and the government expenditure is not covered by lump-sum tax and newly issued debt including newly issued real money, the government "inflate away" as referred by Cochrane[7].

In addition, the fifth and sixth terms in the RHS imply that injecting money to boost CPI inflation is less effective in the FTPL. An increase in the money growth increases the real money balance; this increased real money balance helps alleviate the burden to redeem the consolidated government's debt. In such a case, "inflate away" is not necessary. Thus, the pressure to increase the CPI inflation from money injections weakens in the FTPL. Furthermore, the *MF* fiscal stimulus in the FTPL is less effective.

As in Gali[8] and Okano and Eguchi[14], in the non-FTPL regime, we assume the following simple tax rule throughout the analysis:

$$\widehat{tr}_t = \psi_b \hat{b}_{t-1}. \tag{7}$$

Eq.(7) shows that tax variations depend on $\psi_b \hat{b}_{t-1}$, which is endogenous and varies in response to deviations in the debt ratio from its long-run target, where ψ_b is a tax adjustment parameter. Note that $\psi_b > \rho$ guarantees that $\lim_{k\to\infty} \mathrm{E}_t (b_{t+k}) = 0$; that is, the debt ratio converges to its long-run target. Accordingly, the government's TVC $\lim_{k\to\infty} \Lambda_{t,t+k} \mathcal{B}_{t+k} = 0$ is satisfied for any price level path as long as the discount factor $\Lambda_{t,t+k}$ converges to zero as $k \to \infty$, which is the case in all the experiments considered below, where $\Lambda_{t,t+k} \equiv \prod_{j=0}^{k-1} \mathcal{R}_{t+j}^{-1}$ is the domestic discount factor. We assume the previous property, referred to as the Ricardian (or passive) fiscal policy (e.g., Leeper[12]), as in the standard specifications of the New Keynesian model, must be combined with an active monetary policy (implicitly assumed below) to guarantee a local unique equilibrium.

Log-linearizing Eq. (2), gives:

$$\hat{b}_t = \hat{g}_t + (1+\rho)\,\hat{b}_{t-1} + (1+\rho)\, b\hat{i}_{t-1} - (1+\rho)\, b\pi_t - \widehat{tr}_t - \chi \Delta m_t, \tag{8}$$

where Δm_t denotes the money growth.

3.1.2 Experiments

Below, we analyze two stylized fiscal interventions that take the form of an exogenous increase in government expenditure using the basic New Keynesian model with a small open economy setting as a reference framework. The intervention is announced in period zero and implemented from that period onward, similar to Gali[8] and Okano and Eguchi[14]. For concreteness, we assume that:

$$\hat{g}_t = \delta^t > 0,$$

for $t = 0, 1, 2, ...$, where $\delta \in [0, 1)$ measures the persistence of an exogenous fiscal stimulus. We normalized the size of the stimulus to correspond to 1% of the steady state output in period zero.

We analyze the effects of fiscal interventions under the *MF*. We define this regime as one in which seigniorage is adjusted in every period to keep real debt $\mathcal{B}_t$ constant. By substituting $\hat{b}_t = 0$ into Eq. (8), we obtain

$$\Delta m_t = \frac{1}{\chi}\left[\hat{g}_t - \widehat{tr}_t + (1+\rho)\, b \left(\hat{i}_{t-1} - \pi_t\right)\right], \tag{9}$$

for $t = 0, 1, 2, ...$ The previous assumptions, combined with Eq. (8) imply that under the *MF* regime, the government does not need to adjust taxes because of an increase in government expenditure, either in the short or long run, relative to their initial level. A monetary policy must give up control of the nominal interest rate and instead adjust the money supply to meet the government's financing needs.

Under the *DF* scheme, the fiscal authority issues debt to finance the fiscal stimulus, eventually adjusting the tax path to attain the long-run debt target $\mathcal{B}$. We assume that the monetary authority pursues an independent price stability mandate. For concreteness, we assume that, if feasible, it conducts a policy such that

$$\pi_{H,t} = 0, \tag{10}$$

$$\pi_t = 0, \tag{11}$$

for all t. The domestic inflation targeting (DIT) Eq. (10) or CIT Eq. (11) is applicable to the *DF* scheme. The money supply, and therefore, seigniorage, then adjusts endogenously to bring about the interest rate required to stabilize prices, as well as the regime generally assumed in the New Keynesian literature on the effects of fiscal policy.

3.2 Non-policy Block

3.2.1 Households

The small open economy has a representative household with a continuum of members indexed by $j \in [0,1]$.

The household utility function is:

$$\sum_{t=0}^{\infty} \beta^t \left[\mathcal{U}\left(C_t, L_t, N_t; Z_t\right)\right], \tag{12}$$

where, $C_t \equiv \frac{1}{(1-\nu)^{1-\nu}\nu^{\nu}} C_{H,t}^{1-\nu} C_{F,t}^{\nu}$ denotes the consumption index, $C_{H,t} \equiv \left[\int_0^1 C_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$ is the index of domestic goods consumption, $C_{F,t} \equiv \left[\int_0^1 C_{F,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$ is the quantity of a composite foreign good consumed, $\epsilon > 0$ is the elasticity of substitution between goods, $\nu \in [0,1]$ is the measure of openness, $N_t \equiv \int_0^1 N_t(j)\, dj$ is the hours of labor, Z_t is the exogenous preference shifter, $\beta \equiv \frac{1}{1+\rho} \in (0,1)$ denotes the subjective discount factor, and ρ is the rate of time preference with $P_{H,t} \equiv \left[\int_0^1 P_{H,t}(j)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$ and $P_{F,t} \equiv \left[\int_0^1 P_{F,t}(j)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$.

Period utility is

$$\mathcal{U}\left(C_t, L_t, N_t; Z_t\right) \equiv \left[U\left(C_t, L_t\right) - V\left(N_t\right)\right] Z_t,$$

with $V(\cdot)$ increasing and convex and $U(\cdot)$ increasing and concave.

Optimal allocation of any given expenditure within each category of goods yields the following demand function:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} C_{H,t} \quad ; \quad C_{F,t}(j) = \left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{-\epsilon} C_{F,t}, \tag{13}$$

for all j. The optimal allocation of expenditures between domestic and foreign goods impliesthat

$$C_{H,t} = (1-\nu)\,\mathcal{S}_t^{\nu} C_t \quad ; \quad C_{F,t} = \nu \mathcal{S}_t^{-(1-\nu)} C_t. \tag{14}$$

The sequence of budget constraints is as follows:

$$\begin{aligned} P_t C_t + B_{H,t} + \mathcal{E}_t B_{H,t}^* + M_t \quad &= \quad B_{H,t-1}\left(1 + i_{t-1}\right) + \mathcal{E}_t B_{H,t-1}^*\left(1 + I_{t-1}^*\right) + M_{t-1} + W_t N_t \\ &\quad - P_t TR_t + D_t, \end{aligned}$$

where $B_{H,t}$ denotes the nominal riskless one-period domestic government bond in domestic currency units held by domestic households, $B_{H,t}^*$ is the nominal riskless one-period foreign government bond in units of foreign currency held by domestic households, i_t is the domestic nominal interest rate, i_t^* is the foreign nominal interest rate, $\mathcal{E}_t$ is the nominal exchange rate (the price of foreign currency in units of the domestic currency), W_t is the nominal wage, and D_t is the nominal dividend. Note that $B_t = B_{H,t} + B_{F,t}$, where $B_{F,t}$ denotes nominal riskless one-period domestic government bonds in units of the domestic currency held by foreign households.[1]

Dividing both sides of the previous expression by the CPI P_t yields:

$$\begin{aligned} C_t + \mathcal{B}_{H,t} + \mathcal{Q}_t \mathcal{B}^*_{H,t} + L_t &= \Pi_t^{-1} \mathcal{B}_{H,t-1} \left(1 + i_{t-1}\right) + \left(\Pi_t^*\right)^{-1} \mathcal{Q}_t \mathcal{B}^*_{H,t-1} \left(1 + i^*_{t-1}\right) \\ &\quad + \Pi_t^{-1} L_{t-1} + \frac{W_t}{P_t} N_t - TR_t + \frac{D_t}{P_t}, \end{aligned} \tag{15}$$

where $\mathcal{B}_{H,t} \equiv \frac{B_{H,t}}{P_t}$ denotes real domestic government debt held by domestic households, $\mathcal{B}^*_t \equiv \frac{B^*_{H,t}}{P^*_t}$ is real foreign government debt held by domestic households, $\mathcal{Q}_t \equiv \frac{\mathcal{E}_t P^*_t}{P_t}$ is the real exchange rate (the ratio of the CPI expressed in domestic currency), $\Pi^*_t \equiv \frac{P^*_t}{P^*_{t-1}}$ denotes the (gross) foreign inflation, P^*_t is the foreign price index, and $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is (gross) CPI inflation.

Assuming complete international financial markets, the equilibrium price (in units of the domestic currency) of a riskless bond denominated in a foreign currency is $\left(1 + i^*_t\right)^{-1} = Q_{t,t+1} \mathcal{E}_{t+1}$, where $Q_{t,t+1}$ denotes the price of a one-period discount bond paying off one unit of the domestic currency. We combine the previous pricing equation with the domestic bond-pricing equation, $\left(1 + i_t\right)^{-1} = Q_{t,t+1}$ to obtain a version of the uncovered interest parity (UIP) condition:

$$Q_{t,t+1} \left[\left(1 + i_t\right) - \left(1 + i^*_t\right) \left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right) \right] = 0. \tag{16}$$

We define $\mathcal{A}_t \equiv \left[\left(1 + i_{t-1}\right) \mathcal{B}_{H,t-1} + \mathcal{Q}_{t-1} \mathcal{B}^*_{H,t-1} \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \left(1 + i^*_{t-1}\right) + L_{t-1} \right] \Pi_t^{-1}$ as the representative household's real financial wealth at the beginning of period t. Therefore, we can rewrite Eq. (15).

$$C_t + \frac{1}{1 + i_t} \mathcal{A}_{t+1} \Pi_{t+1} + L_t \left(1 - \frac{1}{1 + i_t} \right) = \mathcal{A}_t + \frac{W_t}{P_t} N_t - TR_t + \frac{D_t}{P_t}, \tag{17}$$

where we assume the standard solvency constraint $\lim_{k \to \infty} \Lambda_{t,t+k} \mathcal{A}_{t+k} \geq 0$, thereby ruling out the Ponzi scheme.

Households maximize Eq. (12), and subject to Eq. (17) and have the following optimality conditions:

$$U_{c,t} = \beta \left(1 + i_t\right) \Pi_{t+1}^{-1} U_{c,t+1} \frac{Z_{t+1}}{Z_t}, \tag{18}$$

$$\frac{W_t}{P_t} = \frac{V_{n,t}}{U_{c,t}}, \tag{19}$$

$$h \left(\frac{L_t}{C_t} \right) = \frac{i_t}{1 + i_t}, \tag{20}$$

with $h \left(\frac{L_t}{C_t} \right) \equiv \frac{U_{l,t}}{U_{c,t}}$. $h \left(\frac{L}{C} \right) \equiv \frac{U_l}{U_c}$ is a continuously decreasing function satisfying $h\left(\bar{\chi}\right) = 0$ for some $0 < \bar{\chi} < \infty$. This guarantees that the demand for the real money balance is bounded as the interest rate approaches zero, with a satiation point attained at $L = \bar{\chi} C$. Eqs. (18), (19), and (20) are the consumption Euler equation and the intertemporal optimality condition that determine the labor supply under the assumption of a competitive labor market and money demand schedule, respectively. These optimality conditions require the TVC $\lim_{k \to \infty} \Lambda_{t,t+k} \mathcal{A}_{t+k} = 0$.

[1]Similar to B_t, $B^*_t = B^*_{F,t} + B^*_{H,t}$ is applicable where B^*_t denotes the nominal riskless one-period foreign government bond in units of foreign currency and $B^*_{F,t}$ denotes the nominal riskless one-period foreign government bond in units of foreign currency held by foreign households.

3.2.2 International Risk-sharing Condition

Assuming a complete financial market, a condition analogous to Eq. (15) must hold for a representative household in a foreign country. By combining this condition with Eq. (18) with UIP and the definition of the real exchange rate, we obtain the international risk-sharing condition:

$$U_{c,t}^{-1} = \vartheta \left(U_{c,t}^{*}\right)^{-1} \mathcal{Q}_t \frac{Z_t}{Z_t^{*}}, \tag{21}$$

where $U_{c,t}^{*}$ denotes the counterpart of $U_{c,t}$ in the foreign country, Z_t^{*} denotes the foreign exogenous preference shifter, and ϑ is a constant that depends on initial conditions.

We assume LOOP; that is, $P_{F,t}(j) = \mathcal{E}_t P_{F,t}^{*}(j)$ for all j, where $P_{F,t}^{*}(j)$ denotes the price of foreign good j in units of foreign currency. By integrating all the goods, we obtain

$$P_{F,t} = \mathcal{E}_t P_{F,t}^{*}, \tag{22}$$

where $P_{F,t}^{*}$ denotes the foreign currency price of the foreign goods. Our treatment of the rest of the world as a (approximately) closed economy (with goods produced in a small open economy representing a negligible fraction of the world's consumption basket) implies that the foreign price index coincides with the foreign currency prices of foreign goods; that is, $P_t^{*} = P_{F,t}^{*}$.

By substituting the definition of the CPI for that of the real exchange rate, we have

$$\mathcal{Q}_t = \mathcal{S}_t^{1-\nu}, \tag{23}$$

This implies that the assumption of complete markets at the international level leads to a simple relationship between consumption at home and abroad, and TOT. By substituting Eq. (23) into Eq. (21), we obtain

$$U_{c,t}^{-1} = \vartheta \left(U_{c,t}^{*}\right)^{-1} \mathcal{S}_t^{1-\nu} \frac{Z_t}{Z_t^{*}}. \tag{24}$$

3.2.3 Domestic Producers

A typical domestic firm produces a differential good by using technology.

$$Y_t(j) = N_t(j)^{1-\alpha},$$

where $Y_t(j)$ is the output of generic good j and α denotes the index of decreasing returns to labor. The index for the aggregate domestic output is $Y_t \equiv \left[\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$. By integrating the previous expression, we obtain:

$$N_t^{1-\alpha} = Y_t \left[\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\frac{\epsilon}{1-\alpha}} dj\right]^{1-\alpha}, \tag{25}$$

where $\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\frac{\epsilon}{1-\alpha}} dj$ denotes price dispersion.

In each period, a subset of firms of measure $1-\theta$, with $\theta \in [0,1]$ being an index of price rigidities drawn randomly from the population, reoptimizes the price of their goods, subject to a sequence of isoelastic demand schedules for the latter. The remaining θ firms retain their prices. In other words, the firms are subject to Calvo pricing. Prices are set in the domestic currency, domestic and export markets share the same price, and LOOP also applies to exports.

The first-order necessary condition (FONC) for domestic producers is

$$\sum_{k=0}^{\infty}\theta^k\left[\Lambda_{t,t+k}\left(\frac{1}{P_{t+k}}\right)Y_{t+k|t}\left(\tilde{P}_{H,t}-\mathcal{M}MC^n_{t+k|t}\right)\right]=0, \tag{26}$$

where $\mathcal{M}\equiv\frac{\epsilon}{\epsilon-1}$ denotes the (desired) price markup, $Y_{t+k|t}\equiv Y_t\left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}}\right)^{-\epsilon}$ is output in period $t+k$ for a firm that last reset its price in period t, $\tilde{P}_{H,t}$ is the price set in period t by firms reoptimizing their price in that period, $MC^n_{t+k|t}$ is the nominal marginal cost in period $t+k$ for a firm that last reset its price in period t, and $MC^n_t\equiv W_t\left(\frac{N_t^\alpha}{1-\alpha}\right)$ is the nominal marginal cost.

3.2.4 Demand for Exports and Global Shocks

The demand for exports of domestic goods j is

$$EX_t(j)=\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon}EX_t, \tag{27}$$

where EX_t is the aggregate export index.

Following Gali and Monacelli[11], the aggregate exports are

$$EX_t=\nu\mathcal{S}_tY_t^*, \tag{28}$$

where Y_t^* denotes (per-capita) world output.

3.2.5 The Market-clearing Condition

The market-clearing condition is:

$$Y_t(j)=C_{H,t}(j)+EX_t(j)+G_t(j).$$

Plugging Eqs. (13), (14), (27), (28), and (21) into the previous expression, we obtain

$$Y_t=(1-\nu)\mathcal{S}_t^\nu C_t+\nu\mathcal{S}_tY_t^*+G_t, \tag{29}$$

where we use the optimal allocation of the output $Y_t(j)=\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon}Y_t$. We assume that $Y_t^*=C_t^*$, where C_t^* denotes the (per capita) world consumption.

4 Steady State and Equilibrium Dynamics

4.1 Steady State

The following analysis considers the equilibrium in the neighborhood of a steady state with zero inflation and zero government expenditure. Note that the steady state price markups must be at the desired level at zero inflation. By combining this result with Eqs.(19), (20), and (29), all evaluated at the steady state, one can derive the conditions, jointly determining the steady state output and real balances which are given by the system:

$$\begin{aligned}(1-\alpha)U_c\left(N^{1-\alpha},L\right)&=\frac{\varepsilon}{\varepsilon-1}V_nN^\alpha,\\ h\left(\frac{L}{N^{1-\alpha}}\right)&=\frac{\rho}{1+\rho},\\ \mathcal{S}&=1,\end{aligned}$$

which are assumed to have unique solutions.[2] The last condition above implies $\mathcal{Q} = 1$ which assures purchasing power parity (PPP) in the steady state. Thus, an increase in the CPI inflation applies pressure to depreciates the nominal exchange rate through an increase in the CPI (level) and vice versa.

4.2 Equilibrium Dynamics

We approximate the equilibrium around the steady state in which inflation is zero, as follows (ignoring the ZLB constraint at this point):

International Risk-sharing Condition

$$\hat{\xi}_t = -(1-\nu)\, s_t + \hat{\xi}_t^* - \zeta_t, \tag{30}$$

Market-clearing Condition

$$\hat{y}_t = \nu\,(2-\nu)\, s_t + (1-\nu)\,\hat{c}_t + \nu \hat{y}_t^* + \hat{g}_t, \tag{31}$$

Consumption Euler Equation

$$\hat{\xi}_t = \hat{\xi}_{t+1} + \hat{i}_t - \pi_{t+1} - \hat{\rho}_t, \tag{32}$$

Marginal Utility of Consumption

$$\hat{\xi}_t = -\sigma \hat{c}_t + \upsilon \hat{l}_t, \tag{33}$$

First-order Necessary Condition for Firms (New Keynesian Phillips Curve, NKPC)

$$\pi_{H,t} = \beta \pi_{H,t+1} - \kappa \hat{\mu}_t, \tag{34}$$

Price Markup Gap

$$\hat{\mu}_t = \hat{\xi}_t - \frac{\alpha+\varphi}{1-\alpha}\hat{y}_t - \nu s_t, \tag{35}$$

Money Demand Schedule

$$\hat{l}_t = \hat{c}_t - \eta \hat{i}_t, \tag{36}$$

Logarithmic First Differential of the Definition of the Real Money Balance

$$\hat{l}_{t-1} = \hat{l}_t + \pi_t - \Delta m_t, \tag{37}$$

Consolidated Government Budget Constraint

$$\hat{b}_t = (1+\rho)\,\hat{b}_{t-1} + (1+\rho)\, b\hat{i}_{t-1} - (1+\rho)\, b\pi_t + \hat{g}_t - \widehat{tr}_t - \chi \Delta m_t, \tag{38}$$

Combination of the Logarithmic First Differential of the Definition of the CPI and TOT

$$\pi_t = \pi_{H,t} + \nu\,(s_t - s_{t-1})\,, \tag{39}$$

[2]Following Gali and Monacelli[9], we found that TOT is determined uniquely.

Definition of the Trade Balance

$$\widehat{nx}_t = \hat{y}_t - \nu s_t - \hat{c}_t - \hat{g}_t, \tag{40}$$

Definition of the TOT

$$s_t = e_t + p_t^* - p_{H,t}, \tag{41}$$

Definition of Domestic Inflation

$$\pi_{H,t} = p_{H,t} - p_{H,t-1}, \tag{42}$$

Definition of Import Inflation

$$\pi_{F,t} = p_{F,t} - p_{F,t-1}, \tag{43}$$

Combination of the (Logarithmic) Definition of the TOT with the (Logarithmic) Definition of Domestic and Import Inflation

$$\pi_{F,t} = s_t - s_{t-1} + \pi_{H,t}, \tag{44}$$

with $\kappa \equiv \frac{(1-\theta\beta)(1-\theta)\Theta}{\theta}$, $\Theta \equiv \frac{1-\alpha}{(1-\alpha)+\alpha\epsilon}$, $\varphi \equiv \frac{V_{nn}N}{V_n}$, $\upsilon \equiv \frac{U_{cl}L}{U_c}$, and $\sigma \equiv -\frac{U_{cc}C}{U_c}$, where $\beta \equiv \frac{1}{1+\rho} \in (0,1)$ denotes the subjective discount factor, ρ denotes the rate of time preference, $\mu \equiv \log\mathcal{M}$ denotes the constant (desired) price markup, $\eta \equiv \frac{\epsilon_{lc}}{\rho}$, $\epsilon_{lc} \equiv \frac{1}{\sigma_l+\upsilon}$ and $\sigma_l \equiv \frac{U_{ll}L}{U_l}$ denotes the elasticity of substitution between consumption and real balances, $\chi \equiv \frac{L}{Y}$ is the inverse income velocity of money, $b \equiv \frac{\mathcal{B}}{Y}$ denotes the steady state share of government debt to output, and Δ is the difference operator. We assume $Z_t^* = 1$ and $Z_{t+1} = Z_t^{\varrho}$ with $\varrho = 0$. Therefore, $\hat{\rho}_t = \log Z_t = \zeta_t$ where $\hat{\rho}_t$ and ζ_t denote the demand and risk-sharing shocks, respectively. The presentation of the model and notation closely parallels those of Okano and Eguchi[14] and Gali[8]. Table 1 presents the notation for the variables.

Eqs. (30) to (36), (38), and (40) to (43) are derived by log-linearizing the international risk-sharing condition, market-clearing condition, Euler equation, the marginal utility of consumption, FONC for firms, the definition of the opportunity cost, money demand schedule, consolidated government budget constraint, definition of the trade balance, the definition of the TOT, the definition of domestic inflation, and the definition of import inflation.

Although Eqs. (41) to (44) do not play essential roles in determining the dynamic paths, it is necessary to calculate the nominal exchange rate and import inflation. We use the logarithmic definition of LOOP $p_{F,t} = e_t + p_{F,t}^*$ to derive Eq. (41). Plugging this into Eq. (41), Eq. (41) becomes $s_t = p_{F,t} - p_{H,t}$.

Our log-linearized model inherits the features of the small open economy of Gali and Monacelli[10], whose model consists of not only the New Keynesian IS curve and NKPC but also the international risk-sharing condition. In addition, the market-clearing conditions and average markup include the TOT. Both consumption and output are affected by changes in the TOT. Therefore, in contrast to Gali[8], not only the real consumption interest rate but also the TOT is involved in monetary–fiscal policy interactions.

4.3 Calibration

Our parametrization is identical to that of Okano and Eguchi[14] (Table 2).[3] Both our implied assumptions of perfect substitution between domestic and imported goods and our benchmark pa-

[3]For the relative risk aversion and the openness, Okano and Eguchi[14] follow Monacelli[13]. Except for those two, they adopt Gali[8].

rameterization of relative risk aversion to attain balanced trade, is similar to Okano and Eguchi[14]; that is, $\widehat{nx}_t = 0$ for all t as long as the demand shock $\hat{\rho}_t$ does not affect the economy.

5 Effects of the Fiscal Stimulus in Normal Times

We show the responses to an increase in government expenditure in normal times, when the ZLB is not applicable. Responses in FTPL in a closed and small open economy and in Non-FTPL in a small open economy are shown (openness $\nu = 0.4$ in a small open economy).

5.1 *MF* Fiscal Stimulus

Fig. 1 shows the dynamic effects of an increase in government expenditure under the *MF* fiscal stimulus in normal times. In Figs. 1, 4 and 6, the red line with circles, blue line with diamonds, and magenta line with pluses are responses in the FTPL in a small open economy, in the FTPL in a closed economy, and in the non-FTPL in a small open economy, respectively (In Figs. 1, 4 and 6, abbreviations SOE and CE stand for a small open economy and a closed economy, respectively). Under the *MF* fiscal stimulus, output increases and the increase in it in the non-FTPL is larger than that in the FTPL in a small open economy (Panel 1, Fig. 1). An increase in the government expenditure applies pressure to increase domestic inflation. This increase in domestic inflation works as so-called inflation tax. Given the revenue of the inflation tax which mitigates burden to redeem consolidated government's debt, the fiscal surplus with inflation tax increases (Panel 10, Fig. 1). Although the government expenditure increases, the fiscal surplus with inflation tax is sufficient for the money growth to decrease (Panel 6, Fig. 1). This decrease in the money growth, decreases the real money balance while it increases the nominal interest rate (Panel 5, Fig. 1).

In the non-FTPL in a small open economy, as mentioned by Okano and Eguchi[14], an increase in the domestic inflation applies pressure to depreciate the nominal exchange rate because of the PPP in the steady state, as mentioned in Section 4.1 (Panel 12, Fig. 1). This depreciation increases the CPI inflation through an increase in the import inflation (Panel 3, Fig. 1). Although the nominal interest rate increases, an increase in the CPI inflation decreases the real consumption interest rate (Panel 2, Fig. 1). In the non-FTPL, fiscal policy rule under the *MF* scheme is given by:

$$\widehat{tr}_t = 0, \tag{45}$$

which implies that there is no additional tax revenue because of $\hat{b}_t = 0$ for all t and the money growth increases after period 1 to finance government expenditure. Thus, the real consumption interest rate declines further through significant decrease in the nominal interest rate. Then, consumption increases. An increase in the CPI inflation depreciates the nominal exchange rate through the PPP in the steady state and the TOT worsens. So, the output increases vigorously. The above is review of Okano and Eguchi[14].

How about *MF* fiscal stimulus in the FTPL? First, we refer the case of a closed economy ($\nu = 0$). As mentioned, an increase in the government expenditure increases the domestic inflation and the fiscal surplus with inflation tax increases. Money injection is no longer necessary so that the money growth decreases, the nominal interest rate increases and the real money balance decreases (Panels 4, 6, 10 and 11, Fig. 1). The scenario remains the same as in the non-FTPL, as discussed earlier.

Different from the non-FTPL, fiscal policy rule under the *MF* is given by:

$$\widehat{tr}_t = b\hat{i}_{t-1} + \frac{b(1-\beta)^2 + \chi\beta^2}{\beta}\hat{l}_{t-1} - \beta\chi\hat{l}_t - (b + \chi\beta)\pi_t + \hat{g}_t, \tag{46}$$

which implies that the lump-sum tax varies and an increase in the government expenditure can be financed by an increase in the tax. Actually, the tax increases (Panel 9, Fig. 1). Then, relationship between the real money balance and CPI inflation is highlighted. That fiscal policy implies that the lower the real money balance, the higher the CPI inflation and vice versa. A decrease in current real money balance means that the consolidated government fails or gives up to issue newly real money and the burden to redeem consolidated government's debt increases. Then, "inflate away" is necessary. This is the FTPL. Now, to mitigate the burden to redeem consolidated government's debt, the CPI inflation increases (Panel 3, Fig. 1). Although the CPI inflation increases, this increase is not large enough for the real consumption interest rate to decline and the real consumption interest rate then increases (Panel 2, Fig. 1). Thus, the increase in the output is not substantial (Panel 1, Fig. 1).

In the FTPL, an increase in the CPI inflation in a small open economy is less than it in a closed economy (Panel 3, Fig. 1). Okano and Eguchi[14] show that an increase in the CPI inflation is higher than it in a closed economy under the *MF* fiscal stimulus. According to them, an increase in the domestic inflation applies pressure to depreciate the nominal exchange rate so that an increase in the CPI inflation which includes the import inflation is higher in a small open economy. Our result is thus different. The reason for this stems from the fiscal policy rule under the *MF* in the FTPL Eq.(46). Ignoring the lump-sum tax and lagged and exogenous variables, Eq.(46) can be rewritten as:

$$\begin{aligned} \hat{l}_t &= -\frac{b+\beta\chi}{\beta\chi}(1-\nu)\pi_{H,t} - \frac{b+\beta\chi}{\beta\chi}\nu\pi_{F,t}, \\ &\quad -\frac{b+\beta\chi}{\beta\chi}(1-\nu)\pi_{H,t} - \frac{b+\beta\chi}{\beta\chi}\nu\Delta e_t, \end{aligned} \tag{47}$$

where we use log-linearized definition of the CPI (level) $p_t = (1-\nu)p_{H,t} + \nu p_{F,t}$ and the definition of the LOOP $p_{F,t} = e_t$, to derive the first and the second lines, respectively. Eq.(47) implies that as the openness increases, a change in the domestic inflation less affects the real money balance. As mentioned, the higher the CPI inflation, the lower the real money balance because higher CPI inflation mitigates burden to redeem the consolidated government's debt and renewal of the consolidated government's debt which includes current real money balance is not necessary. In a small open economy, the ratio of the domestic inflation to the CPI inflation is less than it in a closed economy. Thus, in a small open economy, pressure to decrease the real money balance is less than it in a closed economy and a decrease in the real money balance is mitigated in a small open economy (Panel 11, Fig. 1). Because of less decrease in the real money balance, an increase in the CPI inflation in a small open economy is less than it in a closed economy. Less increase in the CPI inflation implies that the import price is less than the domestic price. Actually, less CPI inflation suppress depreciation in the nominal exchange rate (Panel 12, Fig. 1). Thus, the TOT improves, although it worsens in the non-FTPL (Panel 7, Fig. 1). Finally, the cumulative output in a small open economy is less than it in a closed economy, although the real consumption interest rate is less than it in a closed economy (Panels 1 and 2, Fig. 1).

Now, we focus on Eq.(47) again. The second term in the RHS in Eq.(47) implies that depreciation in the nominal exchange rate decreases the real money balance because depreciation in the nominal exchange rate increases import inflation. Due to less increase in the CPI inflation, a depreciation in the nominal exchange rate is suppressed in the FTPL (Panel 12, Fig. 1). This suppression in depreciation contributes to suppress a decrease in the real money balance, along with less ratio of the domestic inflation to the CPI inflation.

Our finding here consists two and those are caused by the FTPL. First is that the FTPL weakens overall effectiveness of the *MF* fiscal stimulus. The FTPL looks to cancel the effects to increase the CPI inflation resulting from money injection. Second the FTPL strengthen that adverse effect which weaken the effects on the *MF* fiscal stimulus in a small open economy.

5.2 *DF* Fiscal Stimulus

The *DF* fiscal stimulus is characterized by Eqs.(10) or (11), namely, the DIT or the CIT. NKPC Eq.(34) connects the price markup gap which consists of marginal utility of consumption, output and the TOT with the domestic inflation strongly so that difference on fiscal policy rules Eqs.(6) and (7) does not change dynamics between the FTPL and the non-FTPL (Except for fiscal variables). That is, dynamics in the FTPL is same as it in the non-FTPL, which is shown in Okano and Eguchi[14].[4] Irrespective of fiscal regime, an increase in the output in a closed economy is same as it in a closed economy in the DIT. The domestic price is constant and the TOT does not change in the DIT. Thus, the openness does not change results.

Under the CIT, an increase in the output in a small open economy is larger than it in a closed economy. In a closed economy, CPI inflation is identical with domestic inflation. Thus, to stabilize the CPI inflation, the nominal interest rate is strongly hiked to an increase in the government expenditure. In a small open economy, the CPI inflation is not identical with the domestic inflation and zero domestic inflation is not necessary because the CPI inflation is weighted average of the domestic and the import inflation. Whereas an increase in government expenditure applies pressure to increase domestic inflation, a harsh hike in the nominal interest rate is not necessary, because a decrease in import inflation resulting from an increase in the nominal interest rate cancels the increase in domestic inflation, and CPI inflation is zero. Given the smaller increase in the nominal interest rate, consumption is higher than that in a closed economy. As a result, the increase in output is higher in a small open economy than in a closed economy.

5.3 Sensitivity Analysis

We now discuss the sensitivity of some of these qualitative findings in terms of the effectiveness of fiscal policies. We focus on the parameter measuring the degree of openness ν, a feature of small open economies not present in closed economies, rather than on the degree of price stickiness θ and the persistence of the shock δ, as Gali[8] does. Focusing on openness is important for understanding how the assumption of a small open economy affects the effectiveness of a fiscal stimulus. Following Gali[8] and Okano and Eguchi[14], we define the cumulative output multiplier $(1-\delta)\sum_{t=0}^{\infty}\hat{y}_t$.

5.3.1 Fiscal Multipliers: FTPL vs. Non-FTPL

Figs. 2 and 3 depicts the cumulative output multipliers for an increase in government expenditure as a function of openness ν. The multipliers are on the vertical axis and the level of openness is on the horizontal axis. Fig. 2 compares the multipliers in the FTPL with it in the non-FTPL.

[4] See Okano and Eguchi[14] for details.

In Fig. 2, the red line with circles and magenta line with pluses are the multipliers in the FTPL and the non-FTPL, respectively. As Okano and Eguchi[14] show, the multipliers increase as the openness increases under the *MF* fiscal stimulus in the non-FTPL (Panel 2, Fig. 2). As the openness increases, the share of import inflation which has no stickiness increases in the CPI inflation. Thus, the higher the openness, the higher the sensitivity of the CPI inflation to increase in the money growth. An increase in the CPI inflation causes a decrease in the real consumption inflation and applies pressure to worsen the TOT and the output increases, finally. Thus, as the openness increases, the multipliers increase.

By contrast, in the FTPL, the multipliers decrease as the openness increases (Panel 1, Fig. 2). As mentioned in Section 5.1, As shown in Eq.(47), as the openness increases, changes in the domestic inflation less affects the real money balance. In addition, pressure to depreciate the nominal exchange rate weakens as the openness increases in the FTPL. As a decrease in the real money balance is mitigated, an increase in the CPI inflation is suppressed. Thus, the higher the openness, the lower the depreciation in the nominal exchange rate. In other words, the higher the openness, the more the improvement in the TOT. Thus, as the openness increases, the output decreases.

Under the *DF* fiscal stimulus, the multipliers in the FTPL are same as those in the non-FTPL. As mentioned in Section 5.2, the *DF* fiscal stimulus is characterized by Eqs.(10) or (11), namely, inflation targeting. NKPC Eq.(34) connects the price markup gap which consists of marginal utility of consumption, output and the TOT with the domestic inflation strongly so that difference on fiscal policy rules Eqs.(6) and (7) does not change dynamics between the FTPL and the non-FTPL (Except for fiscal variables).

5.3.2 Fiscal Multipliers: *MF* vs. *DF*

Fig. 3 compares the multipliers under the *MF* fiscal stimulus with those under the *DF* fiscal stimulus in the FTPL. In Fig. 3, the red line with circles is the multipliers under the *MF* fiscal stimulus, the blue line with diamonds is the multipliers under the *DF* fiscal stimulus with the DIT and the magenta line with plusses is the multipliers under the *DF* fiscal stimulus with the CIT. Those three lines are identical with the red lines with circles in Panels 1 to 3, Fig. 2, respectively.

There are two points in common with in Fig. 3. First is the left end in which the blue line with diamonds meets with the magenta line with plusses. Those lines are multipliers in the DIT and CIT and the openness is zero at the left end. When the openness is zero, the domestic inflation is identical with the CPI inflation. Thus, those two lines must meet at there. Second the right end in which the red line with circles and the magenta line with plusses. The red line with circles is the multiplier under the *MF* fiscal stimulus and the magenta line with plusses is the multipliers under the *DF* fiscal stimulus with the CIT. Thus, meeting both lines at the right end where the openness is one implies that the multipliers are same when the openness is one. In other words, effectiveness of the *MF* fiscal stimulus is identical with it under the *DF* fiscal stimulus with the CIT when a small open economy is fully opened. As previously mentioned, in the FTPL framework, as openness increases, the ratio of the domestic inflation to the CPI inflation decreases so that pressure to decrease the real money balance is mitigated as openness increases, in the FTPL. Thus, the higher the openness, the lower the CPI inflation. Lower CPI inflation suppresses depreciation in the nominal exchange rate. Thus, Eq.(47) implies that $\hat{l}_t = 0$ when the openness attains one. There is no reason to change the CPI inflation when $\hat{l}_t = 0$ so the CPI inflation is zero, namely,

$\pi_t = 0$. That is, *MF* fiscal stimulus is consistent with the CIT when the openness is zero. The multiplier under the *MF* fiscal stimulus is identical with it under the *DF* scheme with the CPI inflation if a small open economy is fully opened. In other words, the effectiveness of *MF* fiscal stimulus decreases and approximating *DF* fiscal stimulus with the CPI inflation targeting.

5.4 *MF* Fiscal Stimulus to an Adverse Demand Shock Coinciding with an Increase in the Government Expenditure

To understand the effectiveness of fiscal stimulus in a liquidity trap which is analyzed in Section 6, we show the effectiveness of the *MF* fiscal stimulus to an adverse demand shock which pushes the nominal interest rate away negative territory. If there is the ZLB constraint, the nominal interest rate sticks to the ZLB. Note that we continue to ignore the ZLB constraint at this point. Fig. 4 shows responses to an increase in the government expenditure under the *MF* fiscal stimulus in normal times with an adverse demand shock. While there is an increase in the government expenditure whose persistence and the size are 0.5 and 1%, respectively, an adverse demand shock $\hat{\rho}_t = -\gamma^t$ coexists following persistence $\gamma = 0.5$ with $\gamma \in [0, 1)$ and size of the shock is 1%.

To an increase in the adverse demand shock, although there is an adverse demand shock, the output increases in the non-FTPL (Panel 1, Fig. 4). However, the output in the FTPL decreases no matter whether a closed economy or a small open economy (Panel 1, Fig. 4). An adverse demand shock applies pressure to decrease the CPI inflation and this decrease causes revenue shortfall (Panel 10, Fig.4). To finance, money is injected and the real money balance increases (Panels 6 and 10, Fig. 4). As mentioned, in the FTPL, an increase in current real money balance reduces burden to redeem consolidated government's debt and applies pressure to decrease the CPI inflation. Thus, a decrease in the CPI inflation is larger in the FTPL (Panel 3, Fig. 4). As a result, a decrease in the real consumption interest rate is less in the FTPL and the cumulative output in the FTPL is remarkably less than it in the non-FTPL. (Panels 1 and 2, Fig. 4). In a small open economy in the FTPL, severe decrease in the CPI inflation appreciates the nominal exchange rate more and the TOT improves more (Panels 7 and 12, Fig. 4). This more improvement in the TOT contributes to less cumulative output in a small open economy in the FTPL.

The most important feature in Fig. 4 is that a decrease in the output in a small open economy is less than it in a closed economy in the FTPL. This result is opposite from the result in Fig. 1. There are two reasons why. One of them is lower real consumption interest rate (Panel 2, Fig. 4). An adverse demand shock directly attempt to decrease the CPI inflation through the consumption Euler equation Eq.(32). As mentioned, due to the fiscal policy rule under the *MF* in the FTPL Eq.(46), a decrease in the CPI inflation coincides with an increase in the real money balance (Consolidated government's debt cannot be "inflated away" so that renewal of consolidated government's debt is necessary). In a small open economy, a decrease in the CPI inflation applies pressure to appreciate the nominal exchange rate. This appreciation causes more severe decrease in the CPI inflation in a small open economy (Panel 3, Fig. 4). Due to the fiscal policy rule under the *MF* in the FTPL Eq.(46), more vigorous money injection is necessary in a small open economy (Panel 6, Fig. 4). Then, the real money balance increases and its increase is higher than it in a closed economy (Panel 11, Fig. 4). This higher real money balance contributes to decrease the nominal interest rate. A decrease in the nominal interest rate is higher than a closed economy (Panel 5, Fig. 4). As a result, the real consumption interest rate is less and consumption is more vigorous in a small open economy.

Another one is a decrease in the risk sharing shock ζ_t resulting from an adverse demand shock. An adverse demand shock stems from a decrease in the preference shifter Z_t. Thus, a decrease in the risk sharing shock coincides with an adverse demand shock. Combining Eqs.(30), (31), (33) and (40) yields $\widehat{nx}_t = -\nu\zeta_t$ which implies that the adverse demand shock makes the trade balance positive and the higher the openness, the higher the net export. In our benchmark parameterization, due to perfect substitution between the domestic and import goods, balanced trade is attained. However, once an adverse demand shock arises, the net export increases and the higher the openness, the higher the net export. Thus, due to two reasons, in a small open economy, the output is higher than it in a closed economy to an increase in the government expenditure coinciding with an adverse demand shock.

Finally, we discuss how the fiscal policy rule under the *MF* in the FTPL Eq.(46), which is summarized by Eq.(47), makes the effectiveness of the *MF* fiscal stimulus in a small open economy precisely. In normal times, without an adverse demand shock, the effectiveness of the *MF* fiscal stimulus is shown to be weaker in a small open economy. However, now we show that it is stronger in a small open economy in normal times with an adverse shock. An increase in the government expenditure indirectly applies a pressure to increase the domestic inflation which appears in the first term in the RHS in Eq.(47), an adverse demand shock applies a pressure to decrease whole of the RHS directly. In addition, as openness increases, the ratio of the domestic inflation to the CPI inflation $1-\nu$ decreases and the effects brought about by the import inflation which has no stickiness increases. Thus, in a small open economy, pressure to increase the real money balance is larger when an adverse demand shock coincides with an increase in the government expenditure.

5.4.1 Sensitivity Analysis

Fig. 5 shows the relationship between the fiscal multipliers and the openness under the *MF* fiscal stimulus to an increase in the government expenditure coinciding with an adverse demand shock, similar to Panel 1, Fig. 2. In Fig. 5, the blue line with diamonds is multipliers to an increase in the government expenditure coinciding with an adverse demand shock and the red line with circles is multipliers to an increase in the government expenditure without an adverse demand shock. The red line with circles is identical with it in Panel 1, Fig. 2 and Fig.3, respectively. As mentioned, as long as there is no adverse demand shock, as the openness increases, the multipliers decrease. The slope is negative (Red line with circles). However, if an adverse demand shock coincides with an increase in the government expenditure, staring from zero in the openness, the multipliers are gradually and slightly increasing. In the benchmark ($\nu = 0.4$), the multiplier is − 0.76 which is higher than it in a closed economy ($\nu = 0$), − 0.81.

An adverse demand shock applies pressure to decrease the CPI inflation. In a small open economy, this decrease applies pressure to appreciate the nominal exchange rate. Thus, as openness increases, the nominal exchange rate appreciates. In other words, with increased openness, CPI inflation decreases because it incorporates import inflation. However, due to the fiscal policy rule under the *MF* fiscal stimulus in the FTPL Eq.(46), an increase in the real money balance is necessary as the CPI inflation decreases. In a small open economy, a decrease in the CPI inflation is more severe so that a greater money injection is necessary. The nominal interest rate then decreases which is larger than that in a small open economy. This larger decrease in the nominal interest rate causes larger decrease in the real consumption interest rate. In addition, as the openness increases, the net export increases. Thus, as the openness increases, the multipliers increase generally, once an adverse demand shock coincides with an increase in the government expenditure.

6 Effects of the Fiscal Stimulus in a Liquidity Trap

This section explores the effectiveness of the *MF* fiscal stimulus in stabilizing the economy in the face of a temporary adverse demand shock. This is by comparing it with the effectiveness of the *DF* fiscal stimulus, similar to Gali[8] and Okano and Eguchi[14]. We assume that the adverse demand shock is sufficiently large to prevent the central bank from fully stabilizing output and inflation, given the ZLB constraint on the nominal interest rate.

Similar to Gali[8] and Eguchi[14], the ZLB constraint takes the form $\hat{i}_t \geq \log\beta$ and the experiment assumes that $\hat{\rho}_t = -\gamma < \log\beta$ for $t = 0, 1, 2, ...T$ and $\hat{\rho}_t = 0$ for $t = T+1, T+2, ...$ This describes a temporary adverse demand shock that takes the natural interest rate to negative territory up to period T. After period T, the shock disappears. We assume $\gamma = -0.01$ and $T = 5$. The shock is assumed to be fully unanticipated; however, once realized, the trajectory of $\{\hat{\rho}_t\}$ and the corresponding policy responses are known with certainty.

The ZLB constraint can be formally incorporated into the set of equilibrium conditions by substituting Eq. (36) under the following complementary slackness conditions:

$$\left(\hat{i}_t - \log\beta\right)\left(\hat{l}_t - \hat{c}_t + \eta\hat{i}_t\right) = 0,$$

for all t, where

$$\hat{l}_t \geq \hat{c}_t - \eta\hat{i}_t, \tag{48}$$

represents the demand for real money balance.

In addition to the previous changes, under the *DF* fiscal stimulus and *no response* benchmark, Eqs. (10) and (11) must be replaced by

$$\left(\hat{i}_t - \log\beta\right)\pi_{H,t} = 0, \tag{49}$$

$$\left(\hat{i}_t - \log\beta\right)\pi_t = 0, \tag{50}$$

for all t, together with Eqs. (10) and (11), which represent the DIT and CIT, respectively. This applies to the period when the ZLB constraint on the nominal interest rate is unavailable. By contrast, in the *MF* fiscal stimulus case, Eq. (9) determines the money supply for all t. If the nominal interest rate is positive, Eq. (48) holds with equality (but with inequality once the nominal interest rate reaches the ZLB and the real money balances overshoot their satiation levels). Therefore, given $\beta = 0.995$, the experiment corresponds to an unanticipated fall in the natural interest rate to –2% (in annual terms) for six quarters and a subsequent revision back to the initial value of 2% (in annual terms).

The scenario for an increase in government expenditure is a 1% increase in the steady state ratio to output in response to the adverse demand shock that lasts for the duration of the adverse shock ($\hat{g}_t = 0.01$, for $t = 0, 1, \ldots, 5$) in the *MF* and *DF* fiscal stimulus cases.

6.1 *No response*

In the case of *no response* to the shock (i.e., $\hat{g}_t = 0$, for $t = 1, 2, 3\ldots$), monetary policy is described by Eqs. (10) and (49) in the DIT and Eqs. (11) and (50) in the CIT. Responses in the FTPL are same as those in the non-FTPL except for responses on fiscal variables. That is, responses are the case of *no response* in Okano and Eguchi[14], even if the FTPL is introduced in the model. As mentioned in Section 5.2, difference on fiscal policy rule between Eqs.(6) and (7) does not change dynamics between the FTPL and the non-FTPL (except for fiscal variables).

First, we refer the response under *no response* with the DIT. In the non-FTPL in a small open economy, an adverse demand shock decreases the domestic inflation. A decrease in the domestic inflation applies a pressure to appreciate the nominal exchange rate so that the import inflation decreases. The import inflation has no stickiness and the CPI inflation decreases remarkably. This remarkable decrease in the CPI inflation causes severe revenue shortfall. This shortfall is financed by issue of bonds. Then, money growth decreases remarkably. Given the ZLB constraint, the nominal interest rate cannot be negative. However, the recovery of CPI inflation is rapid because there is less stickiness in a small open economy, and the real consumption interest rate decreases. The nominal exchange rate appreciates and the TOT improves. Eventually, cumulative output decreases (–12.58). Dynamics in the FTPL is same as that in the non-FTPL, except for fiscal variables. Note that the net export increases after an adverse shock arises and the output is bolstered.

In a closed economy in the FTPL, the CPI inflation is identical with the domestic inflation and a decrease in the CPI inflation is mitigated. However, the CPI inflation is stickier than it in a small open economy and recovery of the CPI inflation is slower. Coupled with the nominal interest rate which sticks to the ZLB, this slower recovery in the CPI inflation makes the real consumption interest rate higher than it in a small open economy. Thus, decrease in cumulative output is more (–17.66).

Next, we refer the responses under *no response* with the CIT. In a small open economy, dynamics is same between the non-FTPL and the FTPL, except for fiscal variables, as mentioned. Under *no response* with the CIT, the domestic inflation is not targeted and it decreases more. Thus, a decrease in the CPI inflation is more severe and the improvement of the TOT is more severe than it under *no response* with the DIT. Thus, cumulative output reaches –31.18, irrespective of the non-FTPL or the FTPL. In the FTPL in a closed economy, the responses are same as those under *no response* with the DIT which have already referred because the CPI inflation is identical with the domestic inflation. The cumulative output is –17.66 as mentioned.

6.2 *MF* Fiscal Stimulus

Fig. 6 shows the dynamic effects of an increase in government expenditure under the *MF* fiscal stimulus in a liquidity trap. To clarify how introducing FTPL changes the result, we refer to the dynamics of non-FTPL in a small open economy, which has already been shown in Okano and Eguchi[14]. An adverse demand shock decreases CPI inflation, causing a revenue shortfall (Panels 3 and 10; Fig. 6). In contrast to the *DF* scheme, this shortfall is financed by money injection, and the real consumption interest rate decreases because of CPI inflation, which has less stickiness, although the nominal interest rate sticks to the ZLB (Panels 2 and 6, Fig. 6). Subsequently, the output recovers (Panel 1, Fig. 6). The cumulative output is –1.78.

Next, we refer to the responses to the *MF* fiscal stimulus in the FTPL. In conclusion, the cumulative output is –6.83 in a small open economy and –7.58 in a closed economy (Panel 1, Fig. 6). In the non-FTPL in a closed economy, the cumulative output is –2.69. Thus, irrespective of openness, the *MF* fiscal stimulus in an FTPL is less effective than that in a non-FTPL in a liquidity trap. The reason for this is discussed in section 5.1. An increase in the current real money balance applies pressure to decrease CPI inflation because it alleviates the burden of redeeming the consolidated government debt, as shown in the fiscal policy rule in the FTPL under the *MF* scheme Eq.(46). In fact, in a small open economy, although an increase in money growth and an increase in the real money balance in the FTPL are higher than those in the non-FTPL, the decrease in CPI inflation is more severe in the FTPL (Panels 3, 6, and 11, Fig. 6). As a result of this more severe decrease in CPI inflation, the decrease in the real consumption interest rate is less and the TOT is more improved in the FTPL (Panels 2 and 7, Fig. 6). Thus, the effectiveness of the *MF* fiscal stimulus in the FTPL is less than that in the non-FTPL.

We now focus on the difference in the effectiveness of the *MF* fiscal stimulus between a closed economy and a small open economy in the FTPL. As previously mentioned, the *MF*fiscal stimulus is more effective in a small open economy than in a closed economy. Causations to make difference are lower real consumption interest rate in a small open economy and an increase in the net export in a small open economy. How these works are referred to in this section. 5.4. where we discuss the effectiveness of the *MF* fiscal stimulus through an increase in government expenditure coinciding with an adverse demand shock in normal times. However, the reason the real consumption interest rate in a small open economy is lower than that in a closed economy is different in a liquidity trap. In a liquidity trap, even if there is a large money injection to comply with the fiscal policy rule under the *MF* fiscal stimulus in the FTPL, Eq.(46), the nominal interest rate cannot fall below zero because of the ZLB constraint (Panels 5 and 6, Fig. 6). Although this huge money growth does not contribute to a further reduction in the nominal interest rate owing to the ZLB constraint, it boosts the CPI (level). Coupled with less stickiness in CPI inflation in a small open economy, recovery from CPI inflation is faster than in a closed economy (Panel 3, Fig. 3). Thus, the decrease in the real consumption interest rate is greater than in a closed economy (Panel 2, Fig. 3). Accompanying an increase in net exports, the cumulative output in a small open economy is larger than that in a closed economy (Panel 1, Fig. 6). Even in the FTPL, the *MF*fiscal stimulus is more effective in a small open economy than in a closed economy. This finding is similar to that reported by Okano and Eguchi[14].

6.3 *DF* Fiscal Stimulus

Except for fiscal variables, the responses under the *DF* fiscal stimulus in the FTPL is same as those in the non-FTPL which has been shown by Okano and Eguchi[14]. In addition, responses under the *DF* fiscal stimulus in the FTPL is not so different from responses under the *no responses* which are described in Section 6.1. However, due to fiscal stimulus, the cumulative output is improved under the *DF* fiscal stimulus. The cumulative output under the *DF* fiscal stimulus with the DIT in a small open economy is –9.80 which is same in both the FTPL and the non-FTPL (Improvement is 2.78). In a closed economy, that is –10.10 (Improvement is 7.56). Under the *DF* fiscal stimulus with the CIT, in a small open economy, the cumulative output is –26.29 which is same in both the FTPL and the non-FTPL (Improvement is 4.89). In a closed economy, that is –10.10, which is same as it under the *DF* fiscal stimulus with the DIT.

6.4 Comparing the Effects of the *MF* Fiscal Stimulus with the *DF* Fiscal Stimulus in a Liquidity Trap in a Small Open Economy

Fig. 7 compares the effectiveness of the *MF* fiscal stimulus with the *DF* fiscal stimulus in a liquidity trap in a small open economy. In Fig. 7, the red line with circles, the blue line with diamonds, and the magenta line with pluses are the responses under the *MF* fiscal stimulus, *DF* fiscal stimulus with DIT, and *DF* fiscal stimulus with CIT, respectively. Similar to Okano and Eguchi[14], even in the FTPL, the *MF* fiscal stimulus is the most effective in terms of recovering output, domestic inflation, and CPI inflation. In normal times, without an adverse demand shock, an increase in government expenditure under an *MF* fiscal stimulus in a small open economy is less effective than in a closed economy. However, it is still stronger in a liquidity trap, as shown in Okano and Eguchi[14]. A huge money injection occurs, which causes faster recovery in CPI inflation and bolsters output in a small open economy.

7 Conclusion

We investigate the effectiveness of the *MF* fiscal stimulus in a small open economy in which the FTPL is premised. This differs from Okano and Eguchi [14]who did not premise FTPL in their analysis. We find that the effectiveness of the *MF* fiscal stimulus depends on the FTPL's fiscal policy rule. In normal times, effectiveness of an *MF* fiscal stimulus decreases as openness increases. This result contradicts that of Okano and Eguchi[14]. The fiscal policy rule in the FTPL prevents a vigorous increase in CPI inflation and depreciation in the nominal exchange rates. Thus, TOT improves in a small open economy. Consequently, the increase in output is smaller in a small open economy than in a closed economy.

In a liquidity trap, however, the result is opposite to that in normal times. An adverse demand shock induces a liquidity trap, along with the ZLB constraint. An adverse demand shock causes a decrease in CPI inflation, and this decrease is greater in a small open economy than in a closed economy. Owing to the FTPL fiscal policy rule, a huge money injection is necessary to a decrease CPI inflation. This injection is more vigorous in a small open economy and causes a larger decrease in the real consumption interest rate. Thus, the effectiveness of the *MF* fiscal stimulus in a small open economy is greater than that in a closed economy. This result is consistent with those of Okano and Eguchi[14].

Similar to them, we focus only on openness, although Gali [8] focuses on price stickiness and the persistence of fiscal stimulus. Following Gali[8], we can could extend our research by focusing on both. Another extension is Further studies are warranted in this regard.

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Table 1: Definition of the Logarithmic Variables

Variable	Definition	Variable	Definition
$\hat{y}_t$	$\log\left(\frac{Y_t}{Y}\right)$	$\hat{b}_t$	$\frac{\mathcal{B}_t-\mathcal{B}}{Y}$
$\hat{c}_t$	$\log\left(\frac{C_t}{C}\right)$	$\widehat{tr}_t$	$\frac{TR_t-TR}{Y}$
$\hat{\rho}_t$	$-\log\left(\frac{Z_{t+1}}{Z_t}\right)$	$\widehat{nx}_t$	$\log\left[\left(\frac{NX_t}{P_{H,t}}\right)/Y\right]$
$p_{H,t}$	$\log P_{H,t}$	ζ_t	$-\log\left(\frac{Z_t^*}{Z_t}\right)$
$p_{F,t}$	$\log P_{F,t}$	$\hat{y}_t^*$	$\log\left(\frac{Y_t^*}{Y^*}\right)$
s_t	$\log \mathcal{S}_t$	ξ_t^*	$\log\left(\frac{U_{c,t}^*}{U_c^*}\right)$
$\hat{g}_t$	$\frac{G_t}{Y}$	p_t^*	$\log P_t^*$
ξ_t	$\log\left(\frac{U_{c,t}}{U_c}\right)$	e_t	$\log \mathcal{E}_t$
π_t	$\log \Pi_t$	$\pi_{H,t}$	$\log \Pi_{H,t}$
$\hat{i}_t$	$\log\left(\frac{1+i_t}{1+\rho}\right)$	$\pi_{F,t}$	$\log \Pi_{F,t}$
m_t	$\log M_t$	μ_t	$-\log MC_t$
$\hat{l}_t$	$\log\left(\frac{L_t}{L}\right)$	$\hat{\mu}_t$	$\mu_t - \mu$
$\widehat{sp}_t$	$\log\left(\frac{SP_t}{SP}\right)$		

$MC_t \equiv \frac{MC_t^n}{P_{H,t}}$ denotes the real marginal cost.

$\hat{\mu}_t$ is dubbed the markup gap.

Figure 1: Dynamic Effects of an Increase in the Government Expenditure under the *MF* Fiscal Stimulus in Normal Times

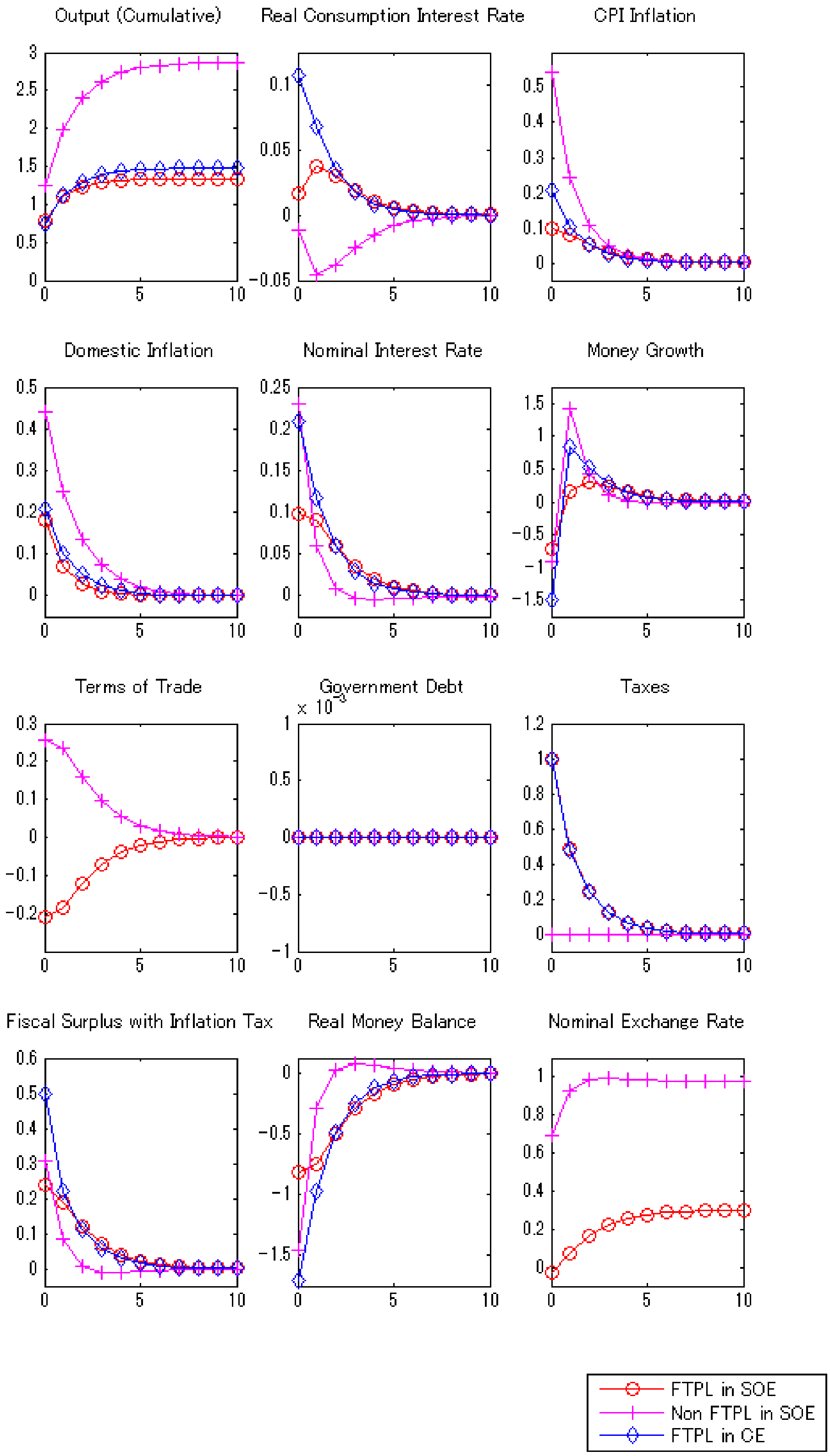

Table 2: Parameterization

Parameter	Description	Value
σ	Relative Risk Aversion	1
ν	Openness	0.4
β	Discount Factor	0.995
φ	Curvature of Labor Disutility	5
α	Index of Decreasing Returns to Labor	0.25
ϵ	Elasticity of Substitution among Goods	9
θ	Calvo Index of Price Rigidities	0.75
χ	Steady state Inverse Velocity	$\frac{1}{3}$
η	Semi-elasticity of Money Demand	7
υ	Separability of Real Balances	0
ψ_b	Tax Adjustment	0.02
b	Target Debt Ratio	2.4
δ	Persistence	0.5

Figure 2: Fiscal Multipliers: FTPL vs. Non-FTPL

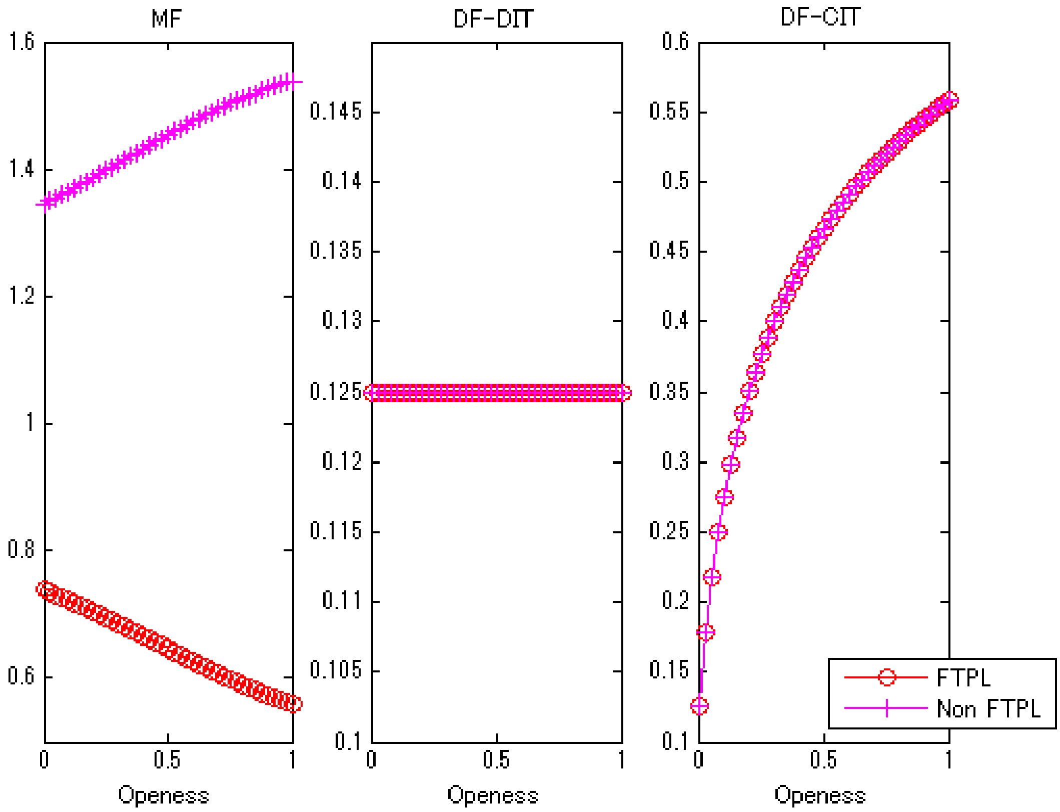

Figure 3: Fiscal Multipliers: *MF* vs. *DF*

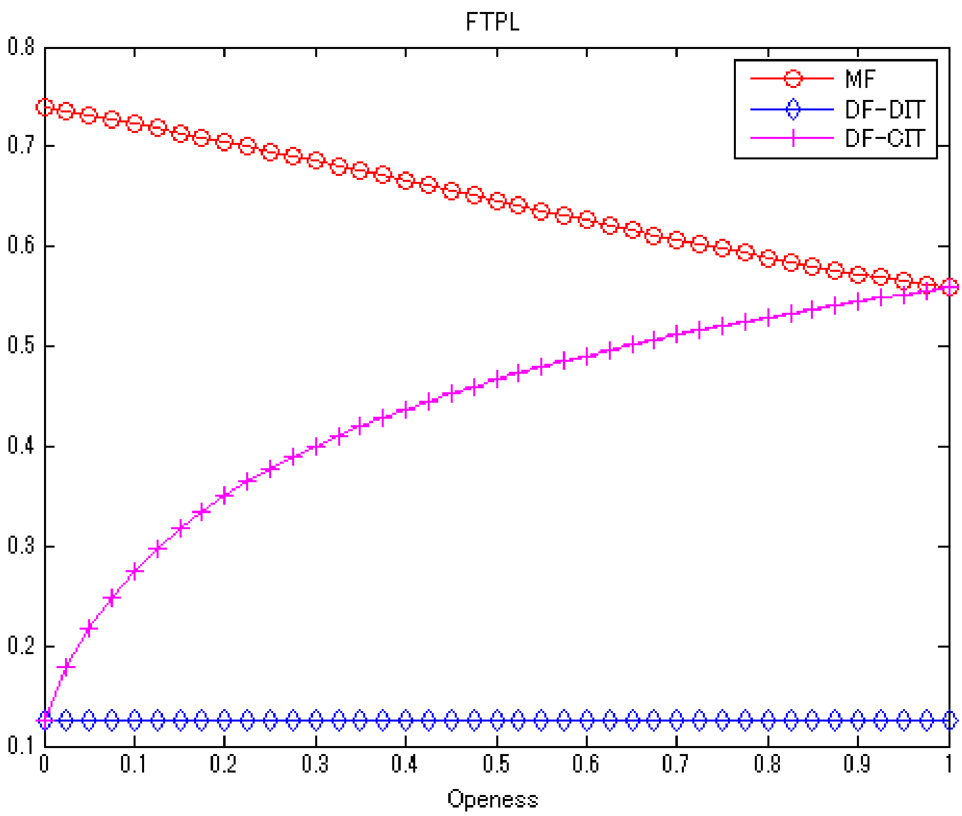

Figure 4: Dynamic Effects of an Increase in the Government Expenditure under the *MF* Fiscal Stimulus in Normal Times with an Adverse Demand Shock

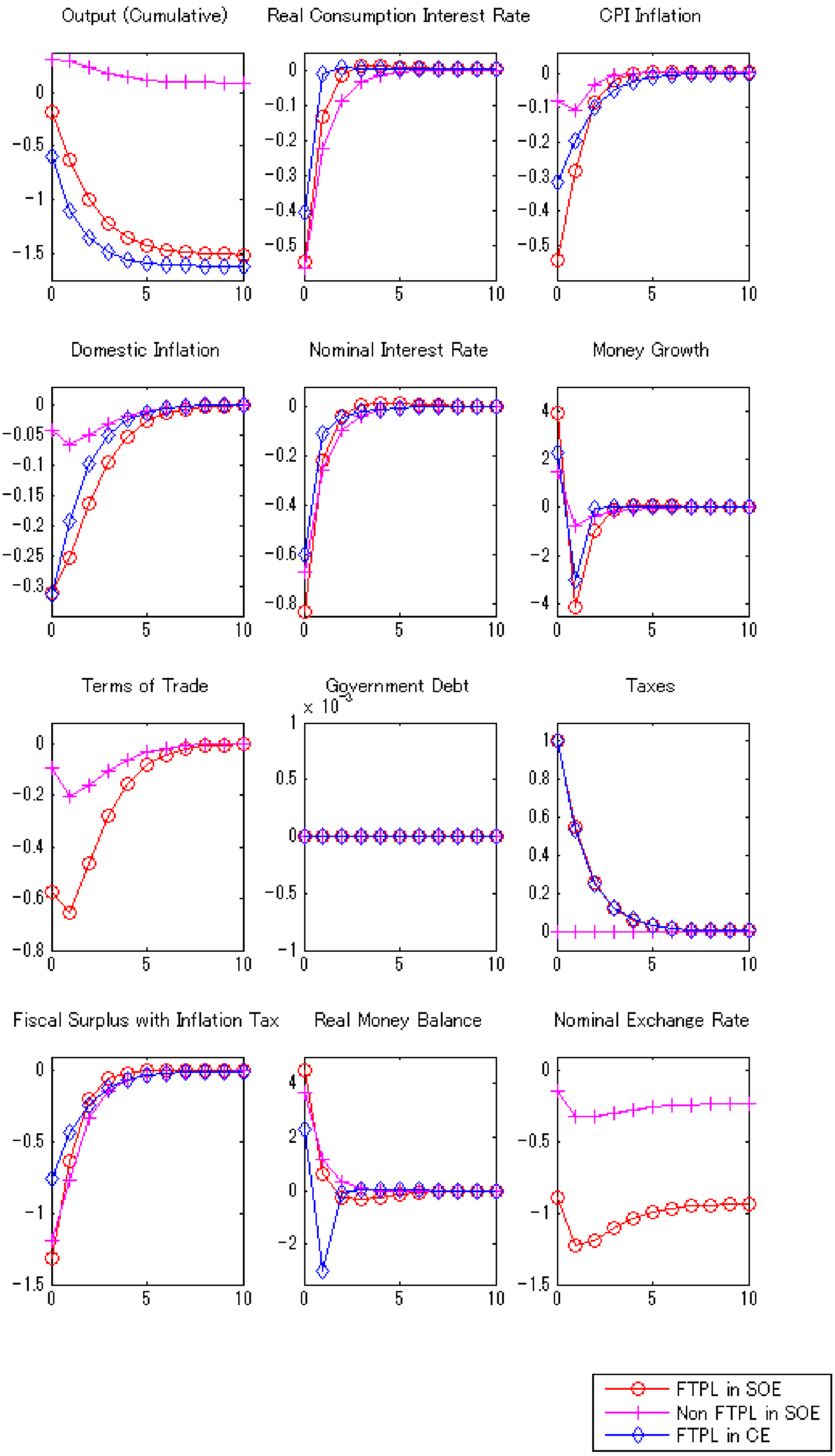

Figure 5: Fiscal Multipliers: with Adverse Demand Shock vs. without Adverse Demand Shock

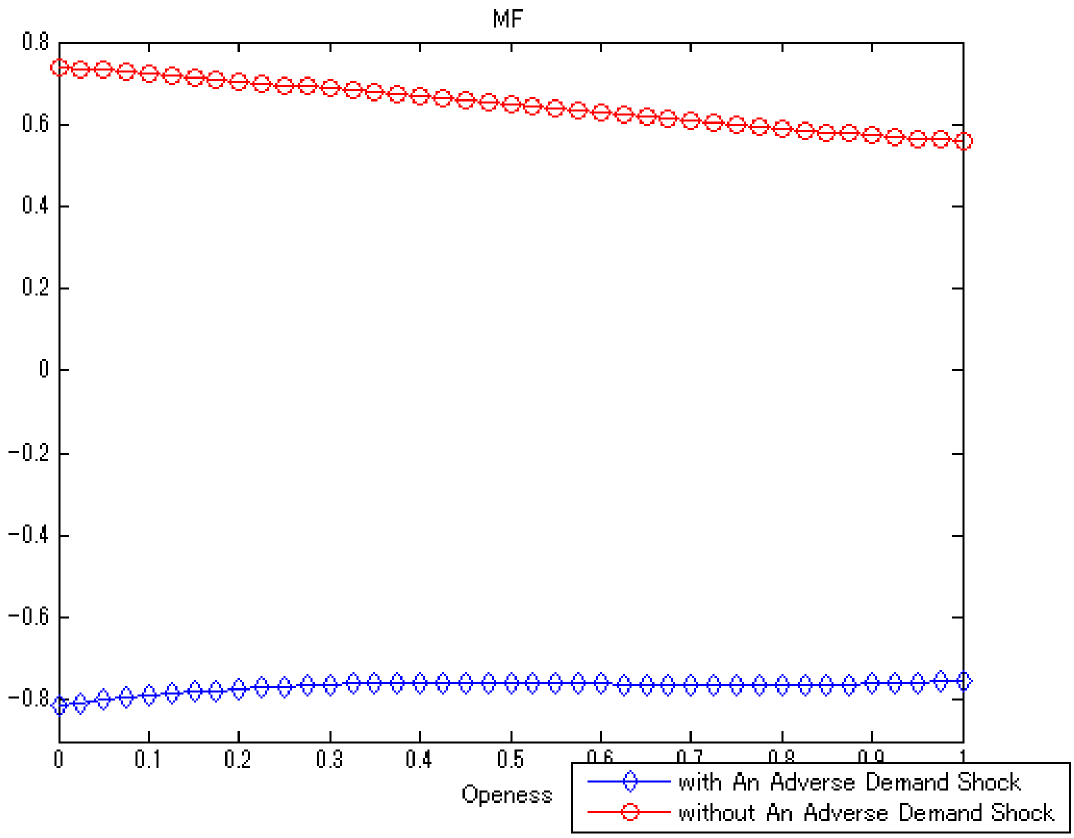

Figure 6: Dynamic Effects of an Increase in the Government Expenditure under the *MF* Fiscal Stimulus in a Liquidity Trap

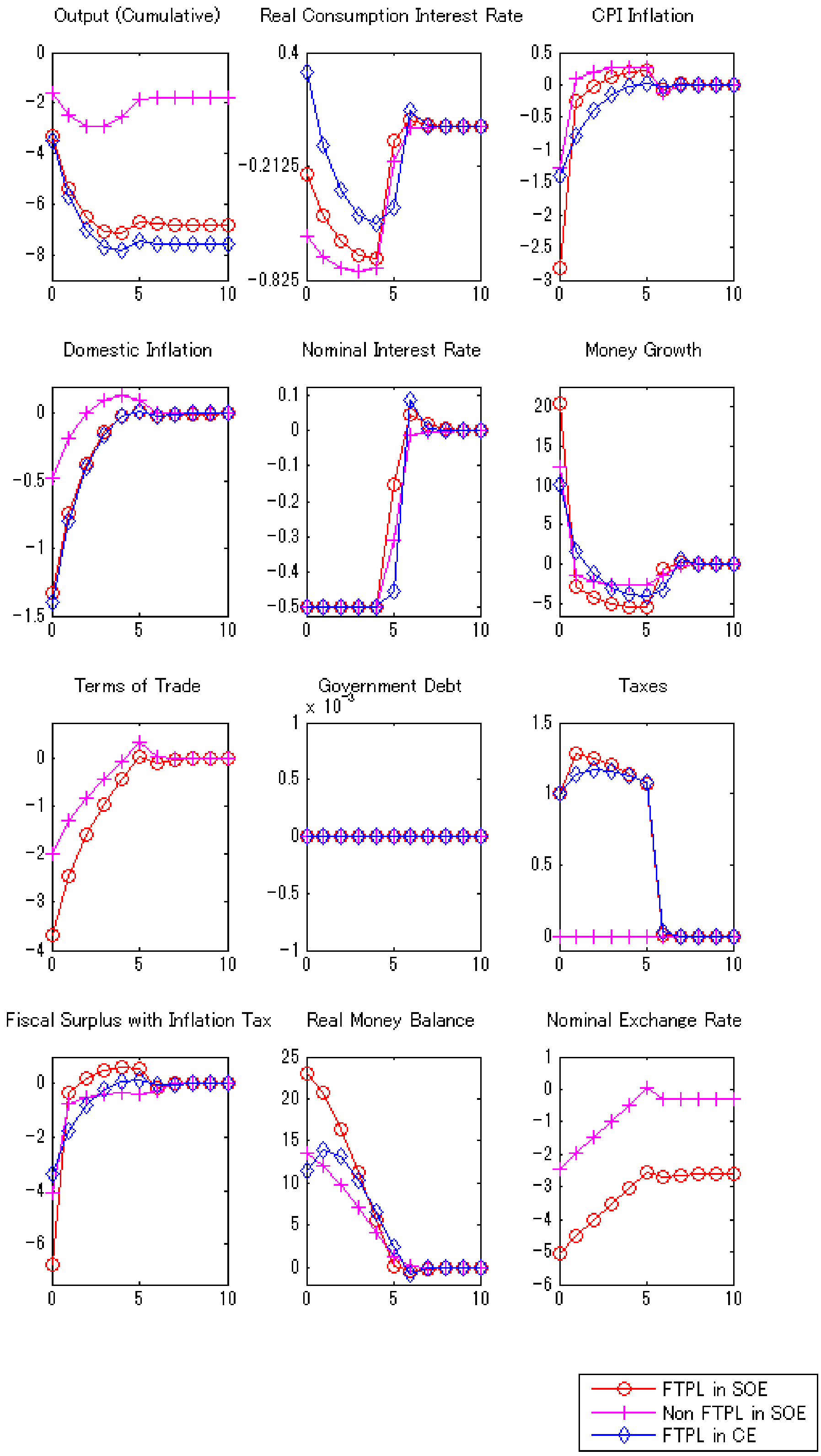

Figure 7: Dynamic Effects of an Increase in the Government Expenditure in a Liquidity Trap – Comparison of the *MF*, the *DF* (DIT), and the *DF* (CIT) Fiscal Stimulus