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Effects of a Money-financed Fiscal Stimulus Without Irredeemability of Money*.

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Abstract

We analyze the effectiveness of a money financed fiscal stimulus (*MF*) without irredeemability of money (IM). Although the effectiveness of the *MF* fiscal stimulus without the IM is weaker than that of the *MF* fiscal stimulus with the IM, that of the *MF* fiscal stimulus without the IM is more substantial than conventional debt financed fiscal (*DF*) fiscal stimulus. This finding is applicable either in normal times or in a liquidity trap. We assume not only a closed economy but also a two-country economy. We find that as the size of home country increases, the effectiveness of the *MF* fiscal stimulus without the IM increases, although that of the *MF* fiscal stimulus with the IM decreases as the size of home country increases. In addition, we find that the effectiveness of global *MF* fiscal stimulus without the IM amid a liquidity trap is more substantial than that of *DF* fiscal stimulus.

Keywords: Fiscal Stimulus, Money Financing, Two-country Economy, Zero Lower Bound
JEL Classification: E31, E32, E52, E62, F41

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1 Introduction

Buiter[7] identifies three conditions that must be satisfied for helicopter money to always boost aggregate demand. One of those is that fiat base money is irredeemable and is viewed as an asset by the holder but not as a liability by the issuer. Fiat money, accompanied by irredeemability, gives net wealth to the private sector. Net wealth is the initial stock of base money plus the present discounted value of all future net base money issuance. Succeeding Buiter[7]'s idea, Gali[15] analyzed the effectiveness of a money-financed (MF) fiscal stimulus in the new Keynesian monetary model, established by Woodford[27] and Gali[13] and [14]. Similar to Buiter[7], Gali[15] assumed a consolidated government issuing money and a class of fiscal feedback rule found by Bohn[4] that suffices a transversality condition (TVC), which implies that fiat base money is an asset (wealth) to the holder, namely, households, but does not constitute in any meaningful sense a liability to the issuer, namely, the central bank which constitutes a part of the consolidated government, following Buiter[7]'s interpretation. Gali[15] was successful in showing how the MF fiscal stimulus is effective, rather than conventional debt-financed (DF) fiscal stimulus. In other words, even Gali[15] admitted that the irredeemability of money (IM) is necessary to make the MF stimulus effective.

What we show in this paper is that the IM is not necessary to make the MF stimulus effective, while Gali[15] admitted that the IM is necessary. To show that, we compare the effectiveness of the MF fiscal stimulus under a fiscal policy rule which suffices the TVC, implying that fiat base money is an asset (wealth) to the holder, namely, households, but does not constitute in any meaningful sense a liability to the issuer, namely, the central bank which constitutes a part of the consolidated government, with that under a fiscal policy rule which suffices a TVC such that fiat base money is an asset to households and dose constitute in any meaningful sense a liability to the central bank. That is, the IM is denied under that TVC.

One of the fiscal policy rules that suffice the TVC for a consolidated government, which denies the IM, is a fiscal policy rule resulting from the fiscal theory of price level (FTPL) with money advocated by Cochrane[8], and we exploit his idea. Our fiscal policy rule resulting from his FTPL suggests that if the redemption of consolidated government's debt (which includes interest payment and the real money balance) and the government expenditure are not covered by lump-sum tax and newly issued debt (including newly issued real money), the government "inflate away" as referred by Cochrane[9]. Although the fiscal policy rule adopted by Gali[15] that can be viewed as a class of Bohn[4] rules intending to redeem just fiscal authority's debt, our fiscal policy rule intends to redeem both fiscal authority's debt and central bank's debt, namely money. Our fiscal policy rule has rejected IM.

As we mentioned, we compare the effectiveness of the MF fiscal stimulus under our fiscal policy rule with that under a class of Bohn[4] rule adopted by Gali[15]. Our fiscal policy rule suggests that an increase in the current real money balance, a renewal of the consolidated government's debt, mitigates the burden of redeeming the consolidated government's debt so that that increase applies pressure to decrease inflation. That increase deprives incentives to "inflate away" the consolidated government's debt. In addition, our fiscal policy rule suggests that fiscal stimulus is financed by taxation (We assume a lump-sum fashion similar to Gali[15]). However, even our fiscal policy rule resulting from Cochrane[8]'s FTPL, the MF fiscal stimulus is more effective than the DF fiscal stimulus. This result is consistent with Gali[15]. It is applicable for normal times when the zero lower bound constraint (ZLB) for the nominal interest rate is not introduced and for a liquidity

trap in which the ZLB prevents pushing away the nominal interest rate negative territory.

Then, concretely, what is the *MF* fiscal stimulus without the IM? According to Buiter[7], the *MF* fiscal stimulus with the IM corresponds to permanent or irreversible quantitative easing (QE), which is viewed as illegal in some countries.[1] Under permanent or irreversible QE, money passed to the private sector is no longer redeemed and has irredeemability. Following this context, it can be said that the *MF* fiscal stimulus without the IM corresponds to temporary or non-permanent QE. Therefore, it can be said that we show the effectiveness of temporary or non-permanent QE while Gali[15], who premises the IM, shows the effectiveness of permanent or irreversible QE. In other words, we show that spending premising the IM, which gives net wealth to the private sector, is unnecessary to boost or bolster the output. Generating or bolstering a decrease in inflation can increase GDP instead of spending.

The global financial crisis (GFC) in 2007 (or 2008) gives incentives to consider the monetary policy as an economic stimulus package, and the *MF* fiscal stimulus has been viewed as one of such packages since the GFC happened (Japanese long-lasting economic slump starting from mid-1990 also gives incentives to consider it). As its name suggests, the GFC was a crisis across borders, so analyzing the effectiveness of the *MF* fiscal stimulus in an open economy is not trivial. So, additionally, we expand a closed economy model in Gali[15] to a two-country economy model following Benigno and Benigno[2] and analyze the effectiveness of the *MF* fiscal stimulus in that two-country economy model. In that two-country economy model, the effectiveness of the *MF* fiscal stimulus is confirmed, even if the IM is not premised. The *MF* fiscal stimulus is more effective than the *DF* fiscal stimulus for normal times and a liquidity trap. Additionally, we find that as the size of the home country increases, the effectiveness of the *MF* fiscal stimulus without the IM also increases, although that of the *MF* fiscal stimulus with the IM decreases as the size of the home country increases. Our results imply that if the effectiveness of large-scale monetary easing measures, such as the QE 3 in the US and the quantitative-qualitative easing (QQE) in Japan, which can be viewed as the *MF* fiscal stimulus are less than that expected, the reason might be not only that those large-scale monetary easing measures are not permanent, but also that the ratio of GDP to the world GDP is small even in the US. If the effectiveness of the QQE is smaller than that of the QEs, it may be due to the ratio of Japan's GDP to the world GDP being lower than that of the US, premising that the QEs and the QQE are not accompanied by IM.

The QE 3 and the QQE were adopted simultaneously from April 2013 for one and a half years. So, we have much curiosity about the effectiveness of the *MF* fiscal stimulus adopted in two countries simultaneously in a liquidity trap, and we analyze that. Although a severe decrease in consumer price index (CPI) inflation is avoided, sufficient money injection does not happen. Thus, the effectiveness to bolster the output is less, irrespective of whether there is the IM, compared with the previous case in which just one country conducts the *MF* fiscal stimulus. However, even the *MF* fiscal stimulus without the IM is more effective than the *DF* fiscal stimulus. Global *MF* fiscal stimulus, which does not depend on the IM, is worth conducting . Based on our analysis, if the QE 3 and the QQE around 2014 seem less effective, the reason might be that both of them were conducted simultaneously in both the US and Japan.

The remainder of the paper proceeds as follows. Section 2 discusses the related literature, while Section 3 shows the fiscal and monetary policy framework we examine. Section 4 shows the

[1] Turner[25] highlights, permanent or irreversible is unlawful (e.g. European Central Bank Article 123.1 and Public Finance Act in Japan Article 5).

effects of a fiscal stimulus in normal times when the ZLB is unavailable by calculating the fiscal multipliers. Section 5 considers the effects of a fiscal stimulus in a liquidity trap where a ZLB is applicable. Section 6 sets forth our conclusions.

2 Related literature

Bernake[3] is a pioneer in discussing the *MF* fiscal stimulus. As a prescription for Japanese economic stagnation, he proposed the *MF* fiscal stimulus. He advocated that the fiscal stimulus, which does not recall an increase in tax in the future, is essential to boost the Japanese economy, and the *MF* fiscal stimulus is such that. Although the *DF* fiscal stimulus recalls an increase in tax in the future, the *MF* fiscal stimulus does not because the fiscal stimulus is financed by the issuance of money, which is irredeemable. Government expenditure certainly increases wealth under the *MF* scheme. Buiter[7] substantiated the *MF* fiscal stimulus as an economic theory precisely. He shows an appropriate TVC that ensures the IM and successfully demonstrates the effectiveness of the *MF* fiscal stimulus. Gali[15] showed the effectiveness of the *MF* in the new Keynesian monetary model in normal times and a liquidity trap. Although he did not highlight it, he assumed the IM (he adopted a fiscal policy rule which sufficed the TVC admitting the IM).

Gali[15] showed not only the effectiveness of an increase in the government expenditure but also that of a reduction in lump-sum tax under the *MF* fiscal stimulus. Generally, changes in lump-sum tax do not affect households' consumption behavior because a decrease in lump-sum tax recalls an increase in taxation in the future and vice versa. Thus, a lump-sum tax reduction seems ineffective (Actually, he showed that a tax cut under the *DF* scheme has no effects). Interestingly, however, his tax cut under the *MF* scheme effectively boosts or bolsters the output. The *MF* fiscal stimulus accompanies money injection, and this money injection depresses the real consumption interest rate through an increase in inflation. Thus, whether it is normal times or a liquidity trap, the *MF* fiscal stimulus is effective. This result implies that the IM is not necessary to make the *MF* fiscal stimulus effective (he inspired us to analyze the *MF* fiscal stimulus without the IM).

Similar to Gali[15], this study relates to the literature on monetary policy in a New Keynesian framework in the presence of a ZLB, in line with Auerbach and Obstfeld[1], Jung et al.[18], Eggertsson and Woodford[11], Werning[26], Buiter[6], Svensson[23], Nakajima[20], and Fujiwara et al.[12]. However, unlike our study, these authors focus only on monetary policy.

We analyze the effectiveness of the *MF* fiscal stimulus in a two-country economy model developed following Benigno and Benigno[2], additionally. Okano and Eguchi[22] and Okano[21] have already analyzed it in a small open economy model based on Gali and Monacelli[16]. Analyzing monetary policy assuming a small open economy is useful and convenient when considering cross-border effects. However, we believe that using a two-country economy model is more suitable for getting policy prescriptions on the GFC in 2007 and the Japanese long-lasting economic slump because the Eurozone, the US, and Japan are not necessarily small open economies. Note that Okano[21] introduced fiscal policy rule resulting from the FTPL with money before us and obtained a result on the effectiveness of the *MF* fiscal stimulus in a small open economy, which is opposite to that obtained by Okano and Eguchi[22].[2]

Several studies focus on the case where the *MF* fiscal stimulus fails to stabilize the economy.

[2]While Okano and Eguchi[22] showed that the effectiveness of the effectiveness of *MF* fiscal stimulus increases as the openness in a small open economy increases, Okano[21] showed that the effectiveness of the effectiveness of *MF* fiscal stimulus decreases as the openness increases.

Drawing on Gali[15], Tsuruga and Wake[24] analyze how an implementation lag modifies the effectiveness of *MF* fiscal stimulus and find that it can result in a recession in normal times. In addition, in a liquidity trap, the recession will deepen. English et al.[10] develop a conventional macroeconomic model and address the central bank's credibility. They show that the *MF* fiscal stimulus is effective if communicated successfully and seen as credible by the public. However, if the public doubts the central bank's commitment to the policy, then the *MF* fiscal stimulus will be ineffective.

Finally, we refer to empirical analysis on the QE and the QQE, which corresponds to the *MF* fiscal stimulus without the IM. Wu and Xia[28] and Kaihatsu et al.[19] forecasted shadow interest rate which is unfastened from the ZLB. Based on their shadow rate, former authors suggested that the unemployment rate in the US was depressed to 1% in 2013, and later authors showed that price level and output in Japan had been boosted since 2013. Regarding the fact that the QE in the US and the QQE in Japan were not necessarily permanent, their finding supports our finding that the IM is not necessary to make the *MF* fiscal stimulus effective.[3]

3 The Fiscal and Monetary Policy Framework

The model consists of policy and non-policy blocks, similar to Gali[15]. Just one thing is different from Gali[15], which appears in the policy blocks. One of the fiscal policy rules is derived following Cochrane[8], which derives iterated government budget constraint with an appropriate TVC, a class of FTPL equation. However, another one is the same as that in Gali[15].

3.1 Government: Budget Constraints and Financing Regimes

We assume that the government (consisting of fiscal and monetary authorities acting coordinated) finances its expenditures through the lump-sum taxes and issuing a riskless nominal one-period bond with a nominal interest rate and (non-interest-bearing) money. Therefore, the consolidated budget constraint is:

$$P_t G_t + B_{t-1}\left(1 + i_{t-1}\right) = P_t TR_t + B_t + \Delta M_t, \tag{1}$$

where P_t denotes the CPI, B_t denotes the nominal riskless one-period domestic government bond, i_t denotes the net nominal interest rate, TR_t denotes the lump-sum tax revenue, M_t denotes the (non-interest bearing) money, Δ is the difference operator, and G_t denotes the (real) government expenditure index.

Dividing both sides of Eq. (1) using the CPI yields:

$$G_t + \mathcal{B}_{t-1}\mathcal{R}_{t-1} = TR_t + \mathcal{B}_t + \frac{\Delta M_t}{P_t}, \tag{2}$$

where $\mathcal{B}_t \equiv \frac{B_t}{P_t}$ denotes real domestic government debt outstanding, $\mathcal{R}_t \equiv (1 + i_t)\,\Pi_{t+1}^{-1}$ denotes the (ex-post) gross real interest rate and $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ denotes the (gross) CPI inflation. The following analysis focuses on the equilibrium near a steady state with zero inflation, no trend growth, and no government expenditure. The constancy of real balances requires $\Delta M = 0$, hence, zero seigniorage in the steady state. Note that the variables without time scripts are the steady-state values of the corresponding variables with a time script.

[3]US Federal Reserve assets started to decrease in 2018. Bank of Japan (BOJ)[5] denied that the purchase of government bonds conducted by the BOJ is debt monetization.

By multiplying both sides of Eq.(2) by $1+i_t$, iterating forward k times, plugging Euler equation $U_{c,t}Z_t = \beta\mathcal{R}_t U_{c,t+1}Z_{t+1}$, taking the limit for $k \to \infty$, and imposing an appropriate TVC:

$$\lim_{k\to\infty}\Lambda_{t,t+k}\left(\mathcal{B}_{t+k} + L_{t+k}\right) = 0, \tag{3}$$

One can write:

$$\begin{aligned} U_{c,t}Z_t\mathcal{R}_{t-1}\left(\mathcal{B}_{t-1} + L_{t-1}\right) &= \sum_{j=0}^{\infty}\beta^j U_{c,t+j}Z_{t+j}SP_{t+j} \\ &\quad + \sum_{j=0}^{\infty}\beta^{j-1}U_{c,t+j-1}Z_{t+j-1}\left(\frac{i_{t+j-1}}{1+i_{t+j-1}}\right)L_{t+j-1}, \end{aligned} \tag{4}$$

where $U_{c,t}$ denotes the marginal utility of consumption, Z_t denotes the exogenous preference shifter, $L_t \equiv \frac{M_t}{P_t}$ denotes the real money balance, $\beta \equiv (1+\rho)^{-1}$ denotes the subjective discount factor, ρ denotes the time preference which is identical with steady-state value of the net nominal interest rate, $SP_t \equiv TR_t - G_t$ denotes the (real) fiscal surplus, and $\Lambda_{t,t+k} \equiv \prod_{j=0}^{t+k-1}\mathcal{R}_{t+j}^{-1}$ is the domestic discount factor. $\left(\frac{i_t}{1+i_t}\right)L_t$ is the opportunity cost of holding the real money balance deprived of households so that Eq.(4) shows that the consolidated government liability in terms of the marginal utility of consumption equals the sum of the net present value of the sum of the fiscal surplus in terms of the marginal utility of consumption and the net present value of the sum of the opportunity cost of holding the real money balance.

The TVC Eq.(3) is satisfied for any price level path as long as the discount factor $\Lambda_{t,t+k}$ converges to zero. Buiter[7] imposes a TVC such that $\lim_{k\to\infty}\Lambda_{t,t+k}\mathcal{B}_{t+k} = 0$ for iterated consolidated government's budget constraint although that consolidated government issues money. According to Buiter[7], this TVC suggests that the real money balance L_t is not recognized as consolidated government's debt. However, the consolidated government recognizes the real government bond outstanding $\mathcal{B}_t$ as debt which is redeemed for households or "inflated away" if it cannot be redeemed (even in period $k \to \infty$, the real money balance is neither redeemed nor "inflated away"). Following Buiter[7]'s idea, Eq.(3) can be interpreted as the TVC that is applicable when the consolidated government recognizes both the real government bond outstanding and the real money balance as its debt. Thus, Eq.(3) is the TVC for a consolidated government in an economy without the IM.

Eq.(4) can be rewritten as:

$$\begin{aligned} \frac{U_{c,t}Z_t\left(1+i_{t-1}\right)\left(B_{t-1}+M_{t-1}\right)}{P_t} &= \sum_{j=0}^{\infty}\beta^j U_{c,t+j}Z_{t+j}SP_{t+j} \\ &\quad + \sum_{j=0}^{\infty}\beta^{j-1}U_{c,t+j-1}Z_{t+j-1}\left(\frac{i_{t+j-1}}{1+i_{t+j-1}}\right)L_{t+j-1}. \end{aligned} \tag{5}$$

According to Cochrane[8], FTPL recognizes that nominal debt, including the monetary base, is a residual claim to government primary surpluses. The government must default on or inflate its debt if the surplus is insufficient. Therefore, we can determine the price level using the valuation equation for government debt as follows:

$$\frac{\text{Nominal Government Debt}}{\text{Price Level}} = \text{Expected Present Value of Primary Surpluses.} \tag{6}$$

Eq.(5) is analogous to Eq.(6); therefore, Eq.(4) succeeds the character of Cochrane[8]'s FTPL.

Eq.(4) can be rewritten as:

$$1=\frac{\sum_{j=0}^{\infty}\beta^{j}U_{c,t+j}Z_{t+j}SP_{t+j}+\sum_{j=0}^{\infty}\beta^{j-1}U_{c,t+j-1}Z_{t+j-1}\left(\frac{i_{t+j-1}}{1+i_{t+j-1}}\right)L_{t+j-1}}{U_{c,t}Z_{t}\left(1+i_{t-1}\right)\left(L_{t-1}+\mathcal{B}_{t-1}\right)}\Pi_{t},$$

which implies that fiscal stimulus, such as an increase in government expenditure, which applies pressure to decrease fiscal surplus SP_t can increase the (gross) CPI inflation Π_t premising those changes in the marginal utility of consumption $U_{c,t}$ and exogenous preference shock Z_t are negligible. More importantly, the previous expression implies that the effects of the *MF* scheme on increasing inflation are limited. While an increase in money growth applies pressure to increase inflation, an increase in money growth increases the expected present value of primary surplus through an increase in the opportunity cost for holding real money balance deprived of households $\left(\frac{i_t}{1+i_t}\right)L_t$ and applies pressure to decrease inflation. The pressure to increase inflation is canceled by the pressure to decrease it. Therefore, the *MF* fiscal stimulus's effects weaken in an economy without the IM.

Eq.(4) can be rewritten as the following second-order difference equation:

$$\begin{aligned}U_{c,t}Z_{t}SP_{t}+\beta^{-1}U_{c,t-1}Z_{t-1}\frac{i_{t-1}}{1+i_{t-1}}L_{t-1} &= U_{c,t}Z_{t}\left(1+i_{t-1}\right)\left(\mathcal{B}_{t-1}+L_{t-1}\right)\Pi_{t}^{-1}\\ &\quad-\beta U_{c,t+1}Z_{t+1}\left(1+i_{t}\right)\left(\mathcal{B}_{t}+L_{t}\right)\Pi_{t+1}^{-1}.\end{aligned}$$

In the previous expression, the LHS is consolidated government's revenue in terms of the marginal utility of consumption. The first term in the RHS is the consolidated government's burden to redeem its debt with real interest payments in terms of the marginal utility of consumption. The second term is a renewal of the consolidated government's debt with real interest payment in terms of the marginal utility of consumption (or newly issued consolidated government's debt with real interest payment in terms of the marginal utility of consumption). The previous expression can be log-linearized as:

$$\hat{i}_{t-1}+\frac{b\left(1-\beta\right)}{\chi\beta}\widehat{sp}_{t}=\frac{b+\chi}{\chi}\hat{i}_{t-1}+\frac{1}{\chi}\hat{b}_{t-1}+\frac{b\left(1-\beta\right)^{2}+\chi\beta^{2}}{\chi\beta}\hat{l}_{t-1}-\frac{\beta}{\chi}\hat{b}_{t}-\beta\hat{l}_{t}-\frac{b+\chi\beta}{\chi}\pi_{t},$$

where we use households' intertemporal optimality condition, namely, the Euler equation, to eliminate the marginal utility of consumption before log-linearizing. The definitions of lower-case letters with time subscripts, which are logarithmic variables, are shown in Table 1. In the previous expression, the LHS is (logarithmic) revenue, which consists of interest payment deprived of households and the fiscal surplus (The principal to produce interest payment $\hat{l}_t$ is canceled on both sides). The first to the third term in the RHS is expenditure, which consists of the burden of redeeming government debt with interest payment and the real money balance. The fourth to fifth terms are renewal of government debt (or newly issued government debt) and renewal of real money balance (or newly issued real money balance). The sixth term is the so-called inflation tax, which increases if the consolidated government's revenue or renewal of the consolidated government's debt does not meet the burden to redeem the consolidated government's debt and vice versa.

Plugging the logarithmic definition of the fiscal surplus $\widehat{sp}_t=\widehat{tr}_t-\hat{g}_t$ into the LHS in the previous expression, we have:

$$\widehat{tr}_{t}=b\hat{i}_{t-1}+\hat{b}_{t-1}+\frac{b\left(1-\beta\right)^{2}+\chi\beta^{2}}{\beta}\hat{l}_{t-1}-\beta\hat{b}_{t}-\beta\chi\hat{l}_{t}-\left(b+\chi\beta\right)\pi_{t}+\hat{g}_{t},\tag{7}$$

where $\chi \equiv \frac{L}{Y}$ and $b \equiv \frac{\mathcal{B}}{Y}$ are the steady state inverse velocity and the target debt ratio, respectively. Here, the interest payment deprived of households is canceled on both sides and disappears from the LHS in Eq.(7). That is, a seignorage is less than a burden to pay interest on the government debt. Eq.(7) is the fiscal policy rule that denies the IM and implies that if the burden to redeem consolidated government's debt, including interest payment and the real money balance, and the government expenditure are not covered by the lump-sum tax and newly issued debt, including newly issued real money, the government "inflates away" as referred by Cochrane[9].

As in Gali[15], in an economy with the IM, we assume the following simple tax rule, which is a class of the Bohn rule:

$$\widehat{tr}_t = \psi_b \hat{b}_{t-1}. \tag{8}$$

Eq.(8) shows that tax variations depend on $\psi_b \hat{b}_{t-1}$, which is endogenous and varies in response to deviations in the debt ratio from its long-run target, where ψ_b is a tax adjustment parameter. Note that $\psi_b > \rho$ guarantees that $\lim_{k\to\infty}\left(\hat{b}_{t+k}\right) = 0$; that is, the debt ratio converges to its long-run target. Accordingly, the government's TVC:

$$\lim_{k\to\infty}\Lambda_{t,t+k}\mathcal{B}_{t+k} = 0, \tag{9}$$

is satisfied for any price level path as long as the discount factor $\Lambda_{t,t+k}$ converges to zero as $k \to \infty$, which is the case in all the experiments considered below. Eq.(9) implies that fiat base money is an asset (wealth) to the holder, namely, households, but does not constitute in any meaningful sense a liability to the issuer, namely, the central bank, which constitutes a part of the consolidated government, according to Buiter[7], as mentioned. There is the IM in an economy where Eq.(8) is conducted.

Log-linearizing Eq.(2) gives:

$$\hat{b}_t = \hat{g}_t + (1+\rho)\,\hat{b}_{t-1} + (1+\rho)\,b\hat{i}_{t-1} - (1+\rho)\,b\pi_t - \widehat{tr}_t - \chi\Delta m_t, \tag{10}$$

which describes the evolution of government debt, similar to a first-order approximation of the consolidated budget constraint in Gali [15], where Δm_t denotes money growth.

3.2 Experiments

Below, we analyze two stylized fiscal interventions that take the form of an exogenous increase in government expenditure using the basic New Keynesian model with a small open economy setting as a reference framework. The intervention is announced in period zero and implemented from that period onward, similar to Gali[15]. For concreteness, we assume that:

$$\hat{g}_t = \delta^t > 0,$$

for $t = 0, 1, 2, ...$, where $\delta \in [0, 1)$ measures the persistence of an exogenous fiscal stimulus. We normalize the size of the stimulus to correspond to 1% of the steady-state output in period zero.

Under the *MF* scheme, the real debt $\mathcal{B}_t$ unchanged, similar to Gali[15]. By substituting $\hat{b}_t = 0$ into Eq. (10), we obtain

$$\Delta m_t = \frac{1}{\chi}\left[\hat{g}_t - \widehat{tr}_t + (1+\rho)\,b\left(\hat{i}_{t-1} - \pi_t\right)\right], \tag{11}$$

for $t = 0, 1, 2, ...$ The previous assumptions, combined with Eq. (10) imply that under the *MF* scheme, a monetary policy must give up control of the nominal interest rate and instead adjust the money supply to meet the government's financing needs.

Under the *DF* scheme, the fiscal authority issues debt to finance the fiscal stimulus, eventually adjusting the tax path to attain the long-run debt target $\mathcal{B}$, similar to Gali[15]. We assume that the monetary authority pursues an independent price stability mandate. For concreteness, we assume that, if feasible, it conducts a policy such that

$$\pi_t = 0, \tag{12}$$

for all t. The CPI inflation targeting (CIT) Eq. (12) applies to the *DF* scheme. The money supply adjusts endogenously to bring about the interest rate required to stabilize prices, as well as the regime generally assumed in the New Keynesian literature on the effects of fiscal policy.

3.3 Non-policy Blocks

The non-policy blocks are the same as Gali[15], so we skip to introduce that precisely here. Following Gali[15], we assume a large number of identical infinitely-lived households who line up $[0, 1]$ and maximize their utility, the single final good is produced with a constant returns technology, sticky prices for goods (i.e., Calvo pricing is applied for goods), and flexible wages. See Appendix A for details on non-policy blocks.

3.4 Steady State, Equilibrium Dynamics and Calibration

The following analysis considers the equilibrium in the neighborhood of a steady state with zero inflation and zero government expenditure. Note that the steady state price markups must be at the desired level at zero inflation. This steady state is the same as that shown in Gali[15]. Similarly, equilibrium dynamics is the same as that in Gali[15]. Also, our parameterization is identical with Gali[15] (Tab. 2). See Appendix B for details on steady state and equilibrium dynamics.

4 The Effects of the Fiscal Stimulus in Normal Times

In the present section, we use the basic New Keynesian model, which consists of the policy blocks introduced in Section 3.1 and the non-policy blocks in Gali[15] as a framework for the analysis of the effects of an increase in government purchases under the two financing schemes introduced above, i.e., debt and money financing. We show the responses to the rise in government expenditure during normal times when the ZLB is not applicable in a closed economy without the IM and with the IM.

4.1 *MF* Fiscal Stimulus

Fig. 1 shows the dynamic effects of an increase in the government expenditure under the *MF* fiscal stimulus in normal times. In Fig. 1, the red line with circles and the magenta line with pluses are responses in a closed economy without and with the IM, respectively. Under the *MF* fiscal stimulus, the output and the CPI inflation increase to an increase in the government expenditure, irrespective of whether there is the IM or not (Panels 1 and 3, Fig. 1). An increase in the government expenditure increases the CPI inflation (Panel 3, Fig. 1). This increase in the CPI

inflation decreases the real consumption interest rate (Panel 2, Fig. 1). Then, consumption increases (not shown). Due to an increase in government expenditure, which accompanies an increase in consumption, the output increases in a closed economy with the IM. This is not only an explanation of the responses under the *MF* with the IM but also a review of the *MF* fiscal stimulus in Gali[15].

In a closed economy without the IM, although the output and the CPI inflation increase, an increase in both is smaller (Panels 1 and 3, Fig. 1). There are two reasons. One of them is using lump-sum tax financing to increase government expenditure. Plugging $\hat{b}_t = 0$ into fiscal policy rule which denies the IM Eq.(7), we have:

$$\widehat{tr}_t = b\hat{i}_{t-1} + \frac{b\left(1-\beta\right)^2 + \chi\beta^2}{\beta}\hat{l}_{t-1} - \beta\chi\hat{l}_t - \left(b + \chi\beta\right)\pi_t + \hat{g}_t. \tag{13}$$

This implies that the lump-sum tax varies, and an increase in government expenditure can be financed by an increase in the tax in a closed economy without the IM (See the LHS and the last term in the RHS). Eq.(13) is in contrast to the fiscal policy rule premising the IM under the *MF* fiscal stimulus $\widehat{tr}_t = 0$, which implies that the lump-sum tax is constant and does not finance fiscal stimulus. The tax increases under Eq.(13) (not shown, but its response is 1, 0.479, 0.238, 0.119,..., for $t = 0, 1, 2, 3,...$).

Another one stems from the salient feature of the FTPL. Eq.(13) implies that the CPI inflation negatively relates to the current real money balance in a closed economy without the IM. An increase in government expenditure applies pressure to increase the CPI inflation, which mitigates the burden of redeeming consolidated government's debt. Renewal of its debt is not necessary. Then, the current real money balance corresponding to the renewal of the consolidated government's debt is reduced through a decrease in the money growth (Panels 5 and 6, Fig. 1). This decrease in the money growth applies pressure to suppress the CPI inflation so that an increase in the CPI inflation is less than that in a closed economy with the IM (Panel 3, Fig. 1). Due to a decrease in the real money balance, an increase in the nominal interest rate is higher than that in a closed economy with the IM (Panel 4, Fig. 1).[4] Coupled with lower increase in the CPI inflation, the real consumption interest rate increases in a closed economy without the IM. However, it decreases in a closed economy with the IM (Panel 2, Fig. 1). The consumption decreases in a closed economy without the IM, although it increases in a closed economy with the IM (not shown). Consequently, in a closed economy without the IM, due to stagnated consumption, an output increase is smaller than in a closed economy with the IM (Panel 1, Fig. 1).

Although it is off the subject, we refer to a decrease or inactive increase in the money growth to an increase in the government expenditure under the *MF* fiscal stimulus (Panel 5, Fig. 1). An increase in the CPI inflation works as a so-called inflation tax. Because of the revenue of the inflation tax, which mitigates the burden of redeeming the consolidated government's debt, the fiscal surplus with inflation tax increases (not shown). This increase in the fiscal surplus with inflation tax deprives incentive to increase money growth (Panel 5, Fig. 4). Thus, there is a decrease or inactive increase in the money growth. This result is also found by Gali[15].[5]

[4]Even under fiscal policy rule premising the IM, the nominal interest rate increases because an increase in the CPI inflation decreases the real money balance, although a decrease in the real money balance is smaller than that under the fiscal policy rule that denies the IM (Panel 6, Fig. 1).

[5]Gali[15] showed an inactive increase in the money growth resulting from an increase in the government expenditure in normal times. His analysis is analogous to our analysis of the MF fiscal stimulus premising the IM. In our paper, there is an inactive increase in the money growth to an increase in the government expenditure under the *MF* fiscal stimulus (Panel 5, Fig. 4). Thus, our result is consistent with Gali[15]'s result.

4.2 *DF* Fiscal Stimulus

The *DF* fiscal stimulus is characterized by Eq.(12), namely, the CIT. The new Keynesian Phillips Curve (NKPC) in the non-policy blocks connects the price markup gap, which consists of marginal utility of consumption and output, with the CPI inflation, which is zero under the *DF* fiscal stimulus strongly. Under the *DF* fiscal stimulus, consolidated government's debt is not constrained (although it is constrained by $\hat{b}_t = 0$ under the *MF* fiscal stimulus). Dynamics is featured by the NKPC and the CIT so that differences in fiscal policy rules Eqs.(7) and (8) do not change dynamics between a closed economy with and without the IM (Except for fiscal variables). That is, dynamics in a closed economy without the IM is the same as that in a closed economy with the IM, which is shown in Gali[15] (Responses under the *DF* are not shown in this paper). Irrespective of whether there is an IM, pressure to increase the CPI inflation, resulting from an increase in government expenditure, is absorbed by the rise in the nominal interest rate, and the real consumption interest rate increases. Due to this increase, the consumption decreases, although the output increases (However, an increase in the cumulative output under the *DF* fiscal stimulus is remarkably less than that under the *MF* fiscal stimulus irrespective of whether the IM is premised or not).

As mentioned, responses of fiscal variables differ between those in a closed economy without the IM and those in a closed economy with the IM. When the nominal interest rate is hiked to absorb pressure to increase the CPI inflation resulting from an increase in government expenditure, the real money balance decreases, irrespective of whether there is an IM. In a closed economy without the IM, this decrease, which corresponds to a reduction in the renewal of consolidated government's debt, involves an increase in the lump-sum tax, as shown in the fiscal policy rule, which does not premise the IM Eq.(7). In a closed economy with the IM, that increase in the lump-sum tax is remarkably smaller than in a closed economy without the IM. However, outstanding government debt is higher than that in a closed economy without the IM.

4.3 Sensitivity Analysis

We now discuss the sensitivity of some of these qualitative findings in terms of the effectiveness of fiscal policies. We focus on the parameter measuring the degree of price stickiness θ and the persistence of the shock δ, as Gali[15] does. Following Gali[15], we define the cumulative output multiplier $(1-\delta)\sum_{t=0}^{\infty}\hat{y}_t$.

4.3.1 Fiscal Multipliers: without Irredeemability vs. with Irredeemability of Money

Figs. 2 and 3 depict the cumulative output multipliers for an increase in government expenditure as a function of price stickiness θ and shock persistence δ, respectively. The multipliers are on the vertical axis, and the price stickiness and the shock persistence are on the horizontal axis (The left end is 0.025 in Figs. 2 and 3, the right ends are 0.975 and 1 in Figs. 2 and 3, respectively).[6] The red line with circles depicts the multipliers in a closed economy without the IM, while the magenta line with plusses depicts those in a closed economy with the IM.

On the multipliers for an increase in the government expenditure as a function of the stickiness θ, two observations found by Gali[15] are almost applicable for our results, irrespective of whether there is the IM or not (Fig. 2). First, the multipliers are invariant to the stickiness θ in the case

[6] When the stickiness is 1, the model has no solution and the multipliers under neither *DF* fiscal stimulus nor the *MF* are calculated.

of a *DF* fiscal stimulus but increasing in the case of a *MF* fiscal stimulus. Second, the size of the multiplier for a *MF* fiscal stimulus remains above that for a *DF* fiscal stimulus and converges to it only as prices become fully flexible. However, while the multipliers increase enormously as the stickiness increases in the case of a *MF* fiscal stimulus with the IM, the multipliers do not necessarily increase enormously as the stickiness increases in the case of a *MF* fiscal stimulus without the IM. Under the fiscal policy rule denying the IM Eq.(7), an increase in the money growth accelerates the renewal of consolidated government's debt so that the necessity of "inflate away" declines. Because of this, the real consumption interest rate increases in the case of a *MF* fiscal stimulus without the IM, although it decreases in the case of a *MF* fiscal stimulus with the IM, as mentioned in Section 4.1. Thus, even if the stickiness attains 0.975, the multiplier in the case of a *MF* fiscal stimulus without the IM is 0.997, which is less than that in the case of a *MF* fiscal stimulus with the IM (1.938). Therefore, the multipliers do not necessarily increase enormously as the stickiness increases in the case of a *MF* fiscal stimulus without the IM.

To an increase in the persistency δ, the multiplier attains zero under both the *MF* and the *DF* fiscal stimulus when the persistency attains 1 (the multiplier under the *MF* fiscal stimulus converges to that under the *DF* fiscal stimulus). In the case of a *DF* fiscal stimulus, the multiplier is independent of the persistency. (Fig. 3).[7] Those findings are also reported by Gali[15] and are applicable irrespective of whether there is the IM or not. In a closed economy without the IM, the relationship appears to be monotonic, similar to Gali[15], which premises the IM. As the shock persistence increases, the multipliers decrease. The relationship appears non-monotonic in a closed economy with the IM (Panel 1, Figs. 3). The multiplier increases for values of shock persistence below 0.725 but decreases for larger values of that parameter. Gali[15] reports that the multiplier decreases with the persistence of the shock. Therefore, there is inconsistency between us and Gali[15], although the multipliers range from 1 to 1.4 in both us and Gali[15].

Most importantly, Figs. 2 and 3 confirm the robustness to changes in the degree of shock persistence δ and the stickiness θ of two of the findings above. Although the overall effectiveness of the *MF* fiscal stimulus in a closed economy without the IM is limited, it remains more effective than the *DF* fiscal stimulus, even in the absence of the IM. In other words, the *MF* fiscal stimulus proves effective even without the premise of irredeemability. The IM is not a necessary condition for the *MF* fiscal stimulus to outperform the *DF* fiscal stimulus. Based on this finding, it can be concluded that spending predicated on the IM, which provides net wealth to the private sector, is not essential for boosting output, contrary to the emphasis placed by Bernanke [3] and Buiter [7].

Now, we mention the reason why under the *DF* fiscal stimulus, the multipliers are identical irrespective of whether there is the IM or not, and the multipliers (Panel 2, Figs. 2 and 3). As we refer to in Section 4.2, consolidated government's debt is not constrained under the *DF* fiscal stimulus, and the NKPC and the CIT characterize the dynamics. Thus, the dynamics in a closed economy without the IM is the same as that in a closed economy with the IM, except for fiscal variables. Inevitably, the multipliers under the *DF* with the IM and those without the IM accord.

4.4 An Extension: A Two-country Economy in Normal Times

The GFC has induced vigorous discussions on a *MF* fiscal stimulus. Discussion in Gali[15] is one of them. Although the GFC spread across borders in the world, the effectiveness of the *MF* fiscal stimulus has never been discussed, premising globally spreading adverse demand shock and the

[7] $\delta = 1$ suggests that cumulative increase in the government expenditure is infinite so that the multiplier is zero

effects of fiscal stimulus which go across borders (Okano and Eguchi[22] and Okano[21] have the viewpoint of across borders, but they assume a small open economy where does not affect global economy). Therefore, examining the effectiveness of the *MF* in a two-country economy model is worth . In this subsection, we extend a closed economy model in Gali[15] to a two-country economy model following Benigno and Benigno[2]. Then, we calculate responses in a two-country economy without the IM, compare those with those in a two-country economy with the IM, and show the fiscal multipliers.

Here, we assume that an economy is inhabited by a large number of identical infinitely-lived households, and they line up in $[0, 1]$ similar to them in Section 3; however, $[\nu, 1]$ of households are a foreigner. That is, $[0, \nu)$ of them belong to country H and $[\nu, 1]$ of them belong to country F.

4.4.1 The Fiscal and Monetary Policy Framework

Fiscal policy rules and experiments are the same in Section 3.1.1. Under the *MF* fiscal stimulus, the monetary policy rule in country H is given by Eq. (11). Under the *DF* fiscal stimulus, monetary policy rule in country H follows:

$$\pi_{H,t} = 0, \tag{14}$$

instead of Eq.(12) where $\pi_{H,t}$ denotes the domestic inflation. The definition of the CPI is given by $P_t \equiv P_{H,t}^{\nu} P_{F,t}^{1-\nu}$ where $P_{H,t}$ and $P_{F,t}$ denote the price of goods produced in country H in units of currency in country H (the domestic price in country H) and the price of goods produced in country F in units of currency in country H (the import price in country H), respectively.

Monetary policy in country F is given by:

$$\pi_{F,t}^{*} = 0, \tag{15}$$

where $\pi_{F,t}^{*}$ denotes the domestic inflation in country F. With starred variables, we denote country F's variables. Similar to the definition of the CPI in country H, the definition of the CPI in country F is given by $P_t^{*} \equiv \left(P_{H,t}^{*}\right)^{\nu} \left(P_{H,t}^{*}\right)^{1-\nu}$.

4.4.2 Non-policy Blocks

Like Section 3.3, households maximize their utility; the final good is produced with constant returns technology, sticky prices for goods, and flexible wages. Additionally, we assume that goods produced in each country are perfect substitutes. The marginal utility of consumption in each country is identical. Financial markets are perfect not only at the domestic level but also at the international level, so the uncovered interest rate parity (UIP) is applicable. All the goods produced in each country are tradable. The law of one price is applicable, i.e., $P_{H,t} = \mathcal{E}_t P_{H,t}^{*}$ and $P_{F,t} = \mathcal{E}_t P_{F,t}^{*}$, where $\mathcal{E}_t$ is price of one unit of country F currency in units of country H currency (the nominal exchange rate). Then, the purchasing power parity (PPP) $\mathcal{E}_t = \frac{P_t^{*}}{P_t}$ is applicable. The demands for goods produced in country H are elastic to the terms of trade (TOT) which is the relative price of goods produced in country F in terms of price of goods produced in country H, i.e., $\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}}$ where $\mathcal{S}_t$ is the TOT. Demands for goods produced in each country are elastic to the TOT. For details on non-policy blocks, see Appendix C.

4.4.3 Steady State, Equilibrium Dynamics and Calibration

Steady-state and equilibrium dynamics in the country H is almost the same as in a closed economy. Additionally, parameterization here succeeds in the case of a closed economy. Due to a two-country economy, ν appears in the model, which can be regarded as population size in country H (Therefore, $1-\nu$ is that size in country F). For simplicity, we set ν to 0.5. For details on steady state and equilibrium dynamics in a two-country economy, see Appendix D.

4.4.4 *MF* Fiscal Stimulus

Fig. 4 shows the dynamic effects of an increase in government expenditure under the *MF* fiscal stimulus in normal times. In Fig. 4, similar to Fig. 1, the red line with circles and the magenta line with pluses are responses in the country H in a two-country economy without the IM and with the IM, respectively (unless otherwise stated).

First, we refer to the effectiveness of the *MF* fiscal stimulus in a two-country economy with the IM, in country H. Irrespective of whether a closed economy or a two-country economy, an increase in the government expenditure applies pressure to increase the CPI inflation through an increase in the domestic inflation. In a two-country economy, an increase in domestic inflation applies pressure to depreciate the nominal exchange rate because domestic inflation is part of the CPI inflation, and the PPP is applicable. This depreciation in the nominal exchange rate increases import inflation. The import inflation is identical to that change in the nominal exchange rate because of the DIT in the country F Eq.(15), so the import inflation has no stickiness. Thus, an increase in the CPI inflation is higher than that in a closed economy with the IM (Panel 3, Figs. 1 and 4). Due to this higher CPI inflation, the CPI level increases vigorously. Thus, an increase in the import price $P_{F,t} = \mathcal{E}_t P^*_{F,t}$ is higher than domestic price $P_{H,t}$ in the country H and the TOT deteriorates through depreciation in the nominal exchange rate (The TOT increases). As the TOT deteriorates, the output increases (A deterioration in the TOT corresponds to a relative decrease in the domestic price to the import price, so the domestic output increases through the expenditure switching effect). The output is boosted up, and the cumulative output in a two-country economy with the IM is higher than that in a closed economy with the IM, in country H (Panel 1, Figs. 1 and 4).

Next, we refer to the effectiveness of the *MF* fiscal stimulus in a two-country economy without the IM in country H. Now, we set ν to 0.5, which is less than one. As the size of country H ν is identical with the share of domestic inflation to CPI inflation in country H, pressure to increase the CPI inflation resulting from an increase in the government expenditure is smaller than that in a closed economy without the IM (Note that the definition of CPI inflation is given by $\pi_t = \nu\pi_{H,t} + (1-\nu)\,\pi_{F,t}$ and is not identical with the domestic inflation in a two-country economy, and a pressure to increase in the CPI inflation resulting from an increase in the government expenditure goes through an increase in the domestic inflation). Then, the decrease in the current real money balance is smaller than that in a closed economy (Panel 6, Figs. 1 and 4). This can be understood by paying attention to the definition of $\hat{l}_t$, which appears in the RHS in Eq.(13), and can be rewritten as $m_t - [\nu p_{H,t} + (1-\nu)\,p_{F,t}]$. The current real money balance corresponds to the consolidated government's debt renewal. Thus, a smaller decrease in the current real money balance mitigates the burden of redeeming the consolidated government's debt, and a large "inflate away" is unnecessary. That is, an increase in the CPI inflation in a two-country economy is less than that in a closed economy (Panel 3, Figs. 1 and 4). Because even in a closed economy without

the IM, an increase in the CPI inflation resulting from an increase in the government expenditure is smaller than that in a closed economy with the IM, an increase in the CPI inflation in a two-country economy without the IM is smaller than that in a two-country economy with the IM, understandably (Panel 3, Fig. 4).

Due to a minor increase in the CPI inflation, the real consumption interest rate increases in a two-country economy without the IM, although that in a two-country economy with the IM decreases (Panel 2, Fig. 4). Also, less increase in the CPI inflation to the domestic inflation corresponds to an improvement in the TOT (Due to the PPP, the nominal exchange rate does not sufficiently depreciate so that the domestic price is higher than the import price, and the TOT decreases). As the TOT improves, the output decreases (An improvement in the TOT corresponds to a relative increase in the domestic price to the import price, so the domestic output decreases through the expenditure-switching effect). Consequently, an increase in output is less than that in a two-country economy with the IM in country H. In addition, an increase in the output in country H in a two-country economy is less than that in a closed economy (Panel 1, Figs. 1 and 4).

4.4.5 Sensitivity Analysis

Fig. 5 shows the relationship between the fiscal multipliers and the size of country H ν to increase government expenditure in the country H. In Fig. 5, the red line with circles and the magenta line with pluses are fiscal multipliers in a two-country economy without and with the IM, respectively.

First, we refer to the multipliers under the MF fiscal stimulus. As mentioned, in a two-country economy with the IM, as the size of country H increases, the share of the import inflation, which has less stickiness in the CPI inflation in country H, decreases. Thus, an increase in the CPI inflation is less, and the deterioration in the TOT is less, as the size of country H increases in country H. Consequently, as the size of country H increases, the multiplier decreases in a two-country economy with the IM (Panel 1, Fig. 5). Regarding multipliers as a function of the size of country H ν, its slope is negative.

On the contrary, the slope of a function of the size of country H ν is positive in a two-country economy without the IM (Panel 1, Fig. 5). As the size increases, fiscal multipliers increase. The size of a country H is identical to the share of domestic inflation to the CPI inflation in country H. An increase in government expenditure exerts upward pressure on CPI inflation through an increase in domestic inflation. As the size of the country H grows, the impact of government expenditure on CPI inflation becomes more pronounced. This increase in CPI inflation leads to a significant decline in the real consumption interest rate. Furthermore, rising CPI inflation results in notable depreciation of the nominal exchange rate, as indicated by the PPP. Assuming DIT in country F, this depreciation in the nominal exchange rate translates to higher import inflation in country H. Consequently, as the size of country H increases, the TOT deteriorates. Ultimately, in a two-country economy without the IM, fiscal multipliers in country H increase with the size of the country.

According to International Monetary Fund[17], in 2014, when large-scale monetary easing measures, namely the QE 3 and the QQE, were adopted in both the US and Japan, the ratio of GDP in the US and Japan to the world GDP was 16.32% and 4.52%, respectively. Premising that there is no IM, our two-country model suggests that the multipliers in the US and Japan are 0.58 and 0.56, respectively (The large-scale monetary easing measures in those countries are not permanent,

so the IM is not necessarily premised in those countries, following Buiter[7]). In a closed economy, the multiplier premising the IM and the multiplier that does not premise are 1.397 and 0.739, respectively, under the benchmark parameterization. If the effectiveness of the QE 3 and the QQE is less than expected, the reason might be that those large-scale monetary easing measures are not permanent and that the ratio of GDP to the world GDP is small, even in the US. In addition, if the effectiveness of the QQE is smaller than that of the QQE, the reason might be the ratio of GDP to the world GDP in Japan is less than that in the US.

Even in a two-country economy, although the overall effectiveness of the *MF* fiscal stimulus in a closed economy without the IM is less, the *MF* fiscal stimulus is more effective than the *DF* fiscal stimulus even if the IM is denied, independent from the size of country H ν (Panels 1 and 2, Fig. 5). Even if irredeemability is not premised, the *MF* fiscal stimulus is effective in a two-country economy. Even in a two-country economy, it can be said that spending premising the IM, which gives net wealth to the private sector, is not necessary to boost the output.

Next, we refer to the multipliers under the *DF* fiscal stimulus. Under the *DF* fiscal stimulus, the DIT is conducted in both countries. To an increase in the government expenditure in the country H, the nominal interest rate is hiked in country H. Pressure to increase the output in country H resulting from an increase in the government expenditure is partially canceled by a decrease in the consumption and an improvement in the TOT in country H. Thus, the multipliers are less than those under the *MF* fiscal stimulus irrespective of the size of country H as well as whether there is the IM or not.

In addition, under the *DF* fiscal stimulus, the multipliers are independent of the size of country H. As mentioned, to cope with an increase in the government expenditure, the nominal interest rate is hiked to stabilize domestic inflation by decreasing consumption and improving the TOT. Suppose the size of country H is small. In that case, the contribution to stabilizing the domestic inflation resulting from an improvement in the TOT is more significant than that resulting from a decrease in consumption. Suppose the size of country H is large. In that case, the contribution to stabilizing the domestic inflation resulting from a reduction of consumption is more significant than that resulting from an improvement in the TOT. The sum of contributions resulting from both is unchanged. Eventually, an increase in the output is independent of the size of country H (Panel 2, Fig. 5).

Further, under the *DF* fiscal stimulus, the multipliers as a function of the size of country H ν is the same between those premising the IM and those not premising the IM, similar to the multipliers as a function of the stickiness θ and the persistency δ (Panel 2, Fig. 5). Different from the *MF* fiscal stimulus, consolidated government's debt is not constrained so that dynamics of variables are the same between them, except for fiscal variables, as mentioned. Thus, the multipliers are the same, irrespective of whether an IM exists.

5 The Effects of the Fiscal Stimulus in a Liquidity Trap

This section explores the effectiveness of the *MF* fiscal stimulus in stabilizing a closed economy in the face of a temporary adverse demand shock by comparing it with the effectiveness of the *DF* fiscal stimulus, similar to Gali[15] (Following subsection explores the effectiveness of the *MF* fiscal stimulus in stabilizing a two-country economy). We assume that the adverse demand shock is sufficiently large to prevent the central bank from fully stabilizing the output and the CPI inflation,

given the ZLB constraint on the nominal interest rate.

Similar to Gali[15], the ZLB constraint takes the form $\hat{i}_t \geq \log\beta$ and the experiment assumes that $\hat{\rho}_t = -\gamma < \log\beta$ for $t = 0, 1, 2, ...T$ and $\hat{\rho}_t = 0$ for $t = T+1, T+2, ...$ This describes a temporary adverse demand shock that takes the natural interest rate to negative territory up to period T. After period T, the shock disappears. The shock is assumed to be fully unanticipated; however, once realized, the trajectory of $\{\hat{\rho}_t\}$ and the corresponding policy responses are known with certainty.

The ZLB constraint can be formally incorporated into the set of equilibrium conditions by the real money demand schedule under the following complementary slackness conditions:

$$\left(\hat{i}_t - \log\beta\right)\left(\hat{l}_t - \hat{c}_t + \eta\hat{i}_t\right) = 0, \tag{16}$$

for all t, where

$$\hat{l}_t \geq \hat{c}_t - \eta\hat{i}_t, \tag{17}$$

represents the demand for real money balance.[8]

In addition to the previous changes, under the *DF* fiscal stimulus and *no response* benchmark, Eq. (12) must be replaced by

$$\left(\hat{i}_t - \log\beta\right)\pi_t = 0, \tag{18}$$

for all t, together with Eq. (12) for $t = T+1, T+2, ...$ Thus, the zero CPI inflation is assumed to be met once the shock vanishes; until that happens, the nominal rate is assumed to be kept at the ZLB. By contrast, in the *MF* fiscal stimulus case, Eq. (11) determines the money supply for all t. If the nominal interest rate is positive, Eq. (17) holds with equality (but with inequality once the nominal interest rate reaches the ZLB and the real money balances overshoot their satiation levels). We assume $\gamma = -0.01$ and $T = 5$. Therefore, given $\beta = 0.995$, the experiment corresponds to an unanticipated fall in the natural interest rate to –2% (in annual terms) for six quarters and a subsequent revision back to the initial value of 2% (in annual terms).

The scenario for an increase in government expenditure is a 1% increase in the steady state ratio to output in response to the adverse demand shock that lasts for the duration of the adverse shock ($\hat{g}_t = 0.01$, for $t = 0, 1, \ldots, 5$) in the *MF* and *DF* fiscal stimulus cases.

5.1 *No Response*

We start by considering the case of *no response* in which there is no fiscal stimulus to the adverse demand shock (i.e., $\hat{g}_t = 0$, for $t = 0, 1, 2 \ldots$), monetary policy is described by Eqs. (12) and (18). Responses in a closed economy without the IM are the same as those in a closed economy with the IM, except for responses on fiscal variables. That is, responses are identical with those in the case of *no response* in Gali[15]. Section 4.2 mentions the difference in fiscal policy rules between Eqs. (7) and (8) does not change dynamics between a closed economy with the IM and it without the IM (except for fiscal variables). Arising an adverse demand shock applies pressure to decrease the CPI inflation. This decrease reduces fiscal surplus with inflation tax and revenue shortfall financed by government debt. Coupled with a limited decrease in the nominal interest rate resulting from the ZLB, a decrease in the CPI inflation increases the real consumption interest rate. Then, a decrease in the cumulative output is –17.66, irrespective of whether there is an IM.

[8] The real money demand schedule is given by Eq.(B.8).

5.2 *MF* Fiscal Stimulus

Fig. 6 shows the dynamic effects of an increase in government expenditure under the *MF* fiscal stimulus in a liquidity trap. In Fig. 6, the red line with circles and the magenta line with pluses are responses in a closed economy without and with the IM, respectively.

An adverse demand shock decreases the CPI inflation, which causes revenue shortfall through a decrease in the inflation tax, irrespective of whether there is an IM. Different from the case of *no response*, this shortfall is financed by money injection, and the money growth increases, irrespective of whether there is the IM or not (Panel 5, Fig. 6). As shown in Eq.(13), a decrease in the CPI inflation, which increases the burden of redeeming consolidated government's debt, and an increase in the current real money balance, which is a renewal of the debt, is necessary. Then, the current real money balance increases through an increase in the money growth (Panel 5 and 6, Fig. 6). However, this real money balance increase deprives incentive to "inflate away" its debt in a closed economy without the IM. For this reason, a decrease in the CPI inflation is more severe in a closed economy without the IM than a decrease in a closed economy with the IM (Panel 3, Fig. 6).

Although the nominal interest rate decreases irrespective of whether there is an IM or not, the nominal interest rate does not decrease beyond zero due to the ZLB constraint, and that sticks to zero (Panel 4, Fig. 6). Combined with a more significant decrease in the CPI inflation, this nominal interest rate, which sticks to zero, makes the real consumption interest rate in a closed economy without the IM higher than in a closed economy with the IM. In addition, which is positive when an adverse shock arises (Panel 2, Fig. 6). Consequently, in a closed economy without the IM, the cumulative output is –7.58, less than that in a closed economy with the IM, which is –2.69 (Panel 1, Fig. 6).

5.3 *DF* Fiscal Stimulus

Except for fiscal variables, the responses under the *DF* fiscal stimulus in a closed economy without the IM are the same as those in a closed economy with the IM because outstanding government debt is not constrained under the *DF* fiscal stimulus. In addition, responses under the *DF* fiscal stimulus in a closed economy without the IM are not so different from responses in the case of *no responses*. However, the cumulative output is improved under the *DF* fiscal stimulus due to an increase in the government expenditure. The cumulative output under the *DF* fiscal stimulus is –10.10, which is the same irrespective of whether there is an IM or not.

5.4 Comparing the Effects of the *MF* Fiscal Stimulus with the *DF* Fiscal Stimulus in a Liquidity Trap

Fig. 7 compares the effectiveness of the *MF* fiscal stimulus with the *DF* fiscal stimulus in a liquidity trap. In Fig. 7, the red line with circles, the magenta line with plusses and the blue line with diamonds are the responses under the *MF* fiscal stimulus without the IM, the *MF* fiscal stimulus with the IM and the *DF* fiscal stimulus, respectively (Irrespective of whether there is the IM or not, responses under the *DF* fiscal stimulus is the same except for those of fiscal variables so that responses under the *DF* fiscal stimulus with and without the IM are not distinguished in Fig. 7). Irrespective of whether there is the IM, the *MF* fiscal stimulus is more effective from the viewpoint of recovering the output and the CPI inflation. Although *MF* fiscal stimulus in a closed economy without the IM is less effective than that in a closed economy with the IM, the *MF* fiscal stimulus

in a closed economy without the IM is still more effective than the *DF* fiscal stimulus in a closed economy without the IM.

5.5 An Extension: A Two-country Economy in a Liquidity Trap

Similar to Section 4.4, we show the effectiveness of the *MF* fiscal stimulus in a liquidity trap in a two-country economy. The scenario of a liquidity trap is the same as above. Slackness condition in country H is given by Eq.(16), while that in country F is given by:

$$\left(\hat{i}_t^* - \log\beta\right)\left(\hat{l}_t^* - \hat{c}_t^* + \eta\hat{i}_t^*\right) = 0.$$

The demand for real money balance in country H is given by Eq.(17) while that in country F is given by:

$$\hat{l}_t^* \geq \hat{c}_t^* - \eta\hat{i}_t^*. \tag{19}$$

Under the *MF* fiscal stimulus, Eq.(11) determines the money supply in country H. Under the *DF* fiscal stimulus, Eq.(14) is replaced by:

$$\left(\hat{i}_t - \log\beta\right)\pi_{H,t} = 0.$$

In the country, F, the ZLB constraint is introduced, and the monetary policy is given by:

$$\left(\hat{i}_t - \log\beta\right)\pi_{F,t}^* = 0,$$

which replaces Eq.(15). The scenario for an increase in government expenditure is the same as above.

5.5.1 *MF* Fiscal Stimulus

Fig. 8 shows the dynamic effects of an increase in government expenditure under the *MF* fiscal stimulus in a liquidity trap in a two-country economy. In Fig. 8, like Fig. 1, the red line with circles and the magenta line with pluses are responses in a two-country economy without the IM and in a two-country economy with the IM, respectively. In response to an adverse demand shock, the CPI inflation in the country H decreases whether there is an IM or not. However, that decrease in a two-country economy without the IM is more significant (Panel 3, Fig. 8). Reducing CPI inflation increases the current real money balance, renewing the consolidated government's debt (Panel 6, Fig. 8). This increase in the current real money balance mitigates the burden of redeeming the government debt and reduces the necessity of "inflating away." Rather than positive CPI inflation, negative CPI inflation occurs, as the fiscal policy rule Eq.(13) implies. Thus, a decrease in the CPI inflation in a two-country economy without the IM is more significant than in a two-country economy with the IM. Due to a severe reduction in the CPI inflation, the real consumption interest rate in a two-country economy without the IM is higher than in a two-country economy with the IM (Panel 2, Fig. 8).

Pressure to decrease the CPI inflation resulting from an adverse demand shock appreciates the nominal exchange rate, and this appreciation decreases the CPI inflation more through a decrease in import inflation. Therefore, the TOT is improved more in a two-country economy without the IM (Panel 8, Fig. 8). Thus, cumulative output in a two-country economy without the IM is –6.76, much smaller than in a two-country economy with the IM –1.55 (Panel 1, Fig. 8).

5.5.2 *MF* Fiscal Stimulus in both Countries

The QE 3 was adopted from September 2012 to October 2014 in the US, while the QQE started in April 2013 in Japan. For one and a half years, large-scale monetary easing measures, which can be regarded as the *MF* fiscal stimulus, were adopted by two economic powers. So, we are curious about the effectiveness of the *MF* fiscal stimulus adopted in two countries simultaneously. Then, we assume that country H and country F increase the government expenditure under the *MF* fiscal stimulus to an adverse demand shock in a liquidity trap. The scenario of a liquidity trap is the same as above, while an adverse demand shock arises even in country F ($\hat{\rho}_t^* = -\gamma$ for $t = 0, 1, 2, ...T$ and $\hat{\rho}_t^* = 0$ for $t = T+1, T+2, ...$). In country F, the monetary policy rule is given by an equation similar to Eq.(11). Under this monetary policy, the government debt ratio to the output is constant, like country H ($\hat{b}_t^* = 0$). Fiscal policy rule is given by an equality analogous to Eq.(7), which does not premise the IM. Comparing how the IM affects the result, we consider another fiscal policy rule analogous to Eq.(8), which premises the IM. The scenario of the *MF* fiscal stimulus is the same as above, which applies even in country F ($\hat{g}_t^* = 0.01$, for $t = 0$, 1,..., 5).

Fig. 9 shows the dynamic effects of an increase in the government expenditure under the *MF* fiscal stimulus in a liquidity trap in a two-country economy (both countries suffer a liquidity trap and conduct the *MF* fiscal stimulus simultaneously). In Fig. 9, like Fig. 1, the red line with circles and the magenta line with pluses are responses in a two-country economy without the IM and in a two-country economy with the IM, respectively. Country F suffers adverse demand shock and adopts the *MF* fiscal stimulus similar to country H so that the response of cumulative output in country F is identical to that in-country H (Panel 9, Fig. 9). Now, the government expenditure increases under *MF* fiscal stimulus, and the extent of monetary easing in both countries is the same. The responses of CPI inflation are the same between both countries. Therefore, the nominal exchange rate is unchanged (not shown) due to the PPP. The TOT is also constant, so the output in both countries is not affected by the TOT. The *MF* fiscal stimulus is conducted in both countries so that a decrease in the CPI inflation is mitigated in comparison with the case that just country H suffers a liquidity trap and increases the government expenditure under the *MF* fiscal stimulus (which is analyzed in Section 5.5.1 and called the case 1, hereafter), irrespective of whether there is the IM or not. (Panel 3, Figs. 8 and 9).

However, due to a smaller decrease in the CPI inflation, the consolidated government's revenue shortfall is less than that in case 1. Hence, the money growth is less than that in case 1, irrespective of whether there is an IM or not (Panel 6, Figs. 8 and 9). Less money growth delays recovering the CPI inflation, so the real consumption interest rate does not decrease sufficiently in the country H with the IM. The real consumption interest rate in country H without the IM increases although that in country H with the IM decreases (Panel 2, Figs. 8 and 9). Thus, the effectiveness of bolstering the output is less, irrespective of whether there is an IM or not (Panel 1, Figs. 8 and 9). As mentioned, cumulative output in a two-country economy without the IM is –6.76, and that in a two-country economy with the IM is –1.55, in case 1. Now, those are –7.58 and –2.69, respectively. Although the global *MF* fiscal stimulus is effective in preventing a severe decrease in CPI inflation, that fiscal stimulus is less effective in bolstering the output (in addition, recovery of the CPI inflation delays under the *MF* fiscal stimulus conducted in both countries). Based on this result, it can be said that if the QE 3 and the QQE around 2014 seem less effective than we expected, the reason might stem from the fact that both were conducted simultaneously in both the US and Japan.

5.5.3 Comparing the Effects of the *MF* Fiscal Stimulus with the *DF* Fiscal Stimulus in a Liquidity Trap in a Two-country Economy

Figs. 10 and 11 compare the effectiveness of the *MF* fiscal stimulus with the *DF* fiscal stimulus in a liquidity trap. In Figs. 10 and 11, the red line with circles, the magenta line with plusses, and the blue line with diamonds are the responses under the *MF* fiscal without the IM, the *MF* fiscal stimulus with the IM and *DF* fiscal stimulus, respectively (Irrespective of whether there is the IM or not, responses under the *DF* fiscal stimulus is the same except for those of fiscal variables so that responses under the *DF* fiscal stimulus with and without the IM are not distinguished in Figs. 10 and 11). Fig. 10 shows responses in case 1, and 11 shows those in the case of both countries suffering a liquidity trap and increasing the government expenditure under the *MF* fiscal stimulus simultaneously (which is analyzed in Section 5.5.2 and called the case 2, hereafter).

In case 1, *MF* fiscal stimulus is more effective than that *DF* fiscal stimulus, irrespective of whether there is the IM or not (Fig. 10). As mentioned, a decrease in the CPI inflation is more severe in a two-country economy without the IM in country H. Then, the money is injected vigorously in country H to increase the real money balance (Panel 5, Fig. 8). This money injection makes a recovery in the CPI inflation faster. Consequently, cumulative output in a two-country economy without the IM under the *MF* fiscal stimulus is higher than that under the *DF* fiscal stimulus. In a two-country economy, although deflationary pressure is more severe when the IM is denied, the effectiveness of *MF* fiscal stimulus admits of no doubt.

Even in case 2, still the *MF* fiscal stimulus is more effective than the *DF* fiscal stimulus, irrespective of whether there is an IM or not (Fig. 11). As we show, the *MF* fiscal stimulus conducted in both countries is less effective in bolstering the output. In case 1, the gap between the cumulative output under the *MF* fiscal stimulus without the IM and that under the *DF* fiscal stimulus without the IM is 2.97. In contrast, in the case 2, that gap is 2.52. Although the superiority of the *MF* fiscal stimulus is weaker in case 2, the effectiveness of the *MF* fiscal stimulus to bolster the output is still more substantial than that of the *DF* fiscal stimulus, even if there is not the IM. Global *MF* fiscal stimulus is worth conducting amid a liquidity trap.

6 Conclusion

While Gali[15] implicitly admitted that the IM is necessary, we show that the IM is not required to make the *MF* fiscal stimulus effective. Although the effectiveness of the *MF* fiscal stimulus without the IM is weaker than that of the *MF* fiscal stimulus with the IM, that of the *MF* fiscal stimulus without the IM is stronger than the *DF* fiscal stimulus. This finding is applicable either in normal times or in a liquidity trap.

We assume not only a closed economy but also a two-country economy. By assuming a two-country economy, we find that as the home country's size increases, the effectiveness of the *MF* fiscal stimulus without the IM increases, although that of the *MF* fiscal stimulus with the IM decreases as the size of the home country increases. In addition, we find that the effectiveness of global *MF* fiscal stimulus without the IM amid a liquidity trap is still more substantial than that of *DF* fiscal stimulus.

Based on our result that the IM is not essential to make the *MF* fiscal stimulus more effective than the *DF* fiscal stimulus, it can be said that spending premising the IM, which gives net wealth to the private sector is not necessary to boost up or bolster the output. Generating the CPI

inflation or recovering the CPI inflation can play a role in boosting or strengthening the GDP instead of spending, which premises the IM. We will be unleashed from a dispute on the illegality of *MF* fiscal stimulus that depends on the IM to escape from a liquidity trap.

If recent large-scale monetary easing measures such as the QEs in the US and the QQE in Japan are less effective than we expected, we could explain why. If the effectiveness of the QEs and the QQE is less than expected, the reason might be not only that those large-scale monetary easing measures are not permanent but also that the ratio of GDP to the world GDP is small even in the US. If the effectiveness of the QQE is smaller than that of the QEs, the reason might be the ratio of GDP to the world GDP in Japan is less than that in the US. If the QE 3 and the QQE around 2014 seem less effective, they might be that both were conducted simultaneously in both the US and Japan. Of course, those should be verified empirically.

Appendices

A Non-policy Blocks

Households maximize its utility given by:

$$\sum_{t=0}^{\infty} \beta^t \mathcal{U}\left(C_t, L_t, N_t; Z_t\right), \tag{A.1}$$

with $U\left(C_t, L_t, N_t; Z_t\right) \equiv \left(U\left(C_t, L_t\right) - V\left(N_t\right)\right) Z_t$ subject to a sequence of budget constraints:

$$P_t C_t + B_t + M_t = B_{t-1}\left(1 + i_{t-1}\right) + M_{t-1} + W_t N_t + D_t - P_t T R_t, \tag{A.2}$$

where C_t denotes consumption and N_t is employment.

The optimality conditions are given by:

$$U_{c,t} = \beta\left(1 + i_t\right)\Pi_{t+1} U_{c,t+1}, \tag{A.3}$$

$$\frac{W_t}{P_t} = \frac{V_{n,t}}{U_{c,t}}, \tag{A.4}$$

$$\frac{U_{l,t}}{U_{c,t}} = \frac{i_t}{1 + i_t}. \tag{A.5}$$

Profit maximization under perfect competition leads to a demand schedule as follows:

$$Y_t\left(j\right) = \left(\frac{P_t\left(j\right)}{P_t}\right)^{-\epsilon}, \tag{A.6}$$

where $Y_t\left(j\right)$ denotes the quantity of good $j \in [0, 1]$. Note that the aggregator is given by:

$$Y_t \equiv \left(\int_0^1 Y_t\left(j\right)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}, \tag{A.7}$$

Each firm produces a differentiated good with a technology as follows:

$$Y_t\left(j\right) = N_t\left(j\right)^{1-\alpha}. \tag{A.8}$$

Each firm can reset the price of its good with probability $1 - \theta$ in any given period, subject to the isoelastic demand schedule Eq.(A.6). The FONC for firms is given by:

$$\sum_{k=0}^{\infty} \theta^k \left[\Lambda_{t,t+k}\left(\frac{1}{P_{t+k}}\right) Y_{t+k|t}\left(\tilde{P}_t - \mathcal{M}MC^n_{t+k|t}\right)\right] = 0, \tag{A.9}$$

with $\mathcal{M} \equiv \frac{\varepsilon}{\varepsilon-1}$ and:

$$Y_{t+k|t} \equiv \left(\frac{\tilde{P}_t}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}, \tag{A.10}$$

where $\tilde{P}_t$ denotes the price chosen by firms when they have a chance to change their prices, $MC^n_{t+k|t}$ denote the nominal marginal cost under nominal rigidities. Market clearing condition is given by:

$$Y_t(j) = C_t(j) + G_t(j), \tag{A.11}$$

Plugging Eq.(7) into Eq.(A.11) yields:

$$Y_t = C_t + G_t, \tag{A.12}$$

where we assume that aggregator of consumption and government expenditure is analogous to aggregator of output.

B Steady State and Equilibrium Dynamics

Steady state output and real balances, which are given by the system:

$$(1-\alpha) U_c = \mathcal{M} V_n N^{\alpha}, \tag{B.1}$$

$$\frac{U_l}{U_c} = \frac{\rho}{1+\rho}. \tag{B.2}$$

The equilibrium around the steady state can be approximated by the following system:

$$\hat{y}_t = \hat{c}_t + \hat{g}_t, \tag{B.3}$$

$$\hat{\xi}_t = \hat{\xi}_{t+1} + \hat{i}_t - \pi_{t+1} - \hat{\rho}_t, \tag{B.4}$$

$$\hat{\xi}_t = -\sigma \hat{c}_t + \upsilon \hat{l}_t, \tag{B.5}$$

$$\pi_t = \beta \pi_{t+1} - \kappa_t \hat{\mu}_t, \tag{B.6}$$

$$\hat{\mu}_t = \hat{\xi}_t - \frac{\alpha+\varphi}{1-\alpha}\hat{y}_t, \tag{B.7}$$

$$\hat{l}_t = \hat{c}_t - \eta \hat{i}_t, \tag{B.8}$$

$$\hat{l}_{t-1} = \hat{l}_t + \pi_t \Delta m_t, \tag{B.9}$$

as well as Eq.(10) with $\sigma \equiv -\frac{U_{cc}C}{U_c}$, $\varphi \equiv \frac{V_{nn}N}{V_n}$, $\upsilon \equiv \frac{U_{cl}L}{U_c}$, $\eta \equiv \frac{\epsilon_{lc}}{\rho}$, $\epsilon_{lc} \equiv -\frac{1}{h'}\frac{\rho}{1+\rho}V$ and $h\left(\frac{C}{L}\right) \equiv \frac{U_l}{U_c}$.

Eq.(B.3) results from log-linearizing Eq.(A.12). Eqs.(B.4) and (B.8) result from Eqs.(A.3) and (A.5), respectively. Eq.(B.5) is derived by log-linearizing $U_{c,t}$. Eq.(B.6) results from log-linearizing Eq.(A.9). Eq.(B.9) results from log-linearizing the marginal cost $MC_t = \frac{V_{n,t}}{U_{c,t}}\frac{N_t^{\alpha}}{1-\alpha}$. Eq.(B.9) results from log-linearizing the definition of the real money balance.

C Non-policy Blocks in a Two-country Economy

Infinitely lived households in country H maximize Eq.(A.1) subject to:

$$P_t C_t + B_{H,t} + \mathcal{E}_t B^*_{H,t} + M_t = B_{H,t-1}(1+i_{t-1}) + \mathcal{E}_t B^*_{H,t-1}\left(1+i^*_{t-1}\right) + M_{t-1} + W_t N_t - P_t TR_t,$$

where $B_{H,t}$ denotes the nominal country H's government debt held by households in country H and $B^*_{H,t}$ denotes the nominal country F's government debt held by households in country H. Note that $B_{H,t} + B_{F,t} = B_t$ where $B_{F,t}$ is the nominal country H's government debt held by households in country H.

Infinitely lived households in country F maximize the counterpart of Eq.(A.1) subject to the previous expression. Optimality conditions are given by Eqs.(A.3) to (A.5) and counterparts of them.

Financial markets are complete internationally so that $\mathcal{U}_{c,t} = \mathcal{U}^*_{c,t}$.

We assume the law of one price such that $P_t(h) = \mathcal{E}_t P^*_t(h)$ with $h \in [0, \nu)$ and its counterpart. Plugging those expressions into $P_{H,t} \equiv \left[\frac{1}{\nu}\int_0^\nu P_t(h)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$ and its counterpart in country F, we have $P_{H,t} = \mathcal{E}_t P^*_{H,t}$ and $P_{F,t} = \mathcal{E}_t P^*_{F,t}$. Plugging previous expressions into the definition of the CPI, we have purchasing power parity condition (PPP) $P_t = \mathcal{E}_t P^*_t$.

Plugging the PPP into the definition of the real exchange rate $\mathcal{Q}_t \equiv \frac{\mathcal{E}_t P^*_t}{P_t}$, we have:

$$\mathcal{Q}_t = 1, \tag{C.1}$$

Combining Eq.(A.3), its counterpart in country F and Eq.(C.1), we have:

$$1 + i_t = (1 + i^*_t)\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t},$$

which is the UIP.

Analogous to Eq.(A.6), demand schedule for generic good h is given by:

$$Y_t(h) = \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\epsilon}. \tag{C.2}$$

Demand schedule for generic good f is given similarly. The aggregator is now given by:

$$Y_t \equiv \left[\left(\frac{1}{\nu}\right)^{\frac{1}{\epsilon}}\int_0^\nu Y_t(h)^{\frac{\epsilon-1}{\epsilon}}dh\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{C.3}$$

instead of Eq.(A.7). The aggregator of Y^*_t is given similarly.

Domestic producers in country H has technology $Y_t(h) = N_t(h)^{1-\alpha}$ similar to Eq.(A.8). Its counterpart in country F is similarly.

Similar to Appendix A, each firm can reset the price of its good with probability $1-\theta$ in any given period. However, not $\tilde{P}_t$ but $\tilde{P}_{H,t}$ is chosen to reset so that the FONC for firms is given by:

$$\sum_{k=0}^{\infty}\theta^k\left[\Lambda_{t,t+k}\left(\frac{1}{P_{t+k}}\right)Y_{t+k|t}\left(\tilde{P}_{H,t} - \mathcal{M}MC^n_{t+k|t}\right)\right] = 0, \tag{C.4}$$

instead of Eq.(A.9). Note that Eq.(A.10) is replaced by $Y_{t+k|t} \equiv \frac{\tilde{P}_{H,t}}{P_{H,t+k}}Y_{t+k}$. There is a counterpart of Eq.(C.4) in country F.

Market clearing condition is given by Eq.(A.11). Plugging Eq.(C.2) into Eq.(A.11) yields:

$$Y_t = \mathcal{S}_t^{1-\nu}C^W_t + G_t, \tag{C.5}$$

where C^W_t is aggregate consumption in the whole economy. Counterpart of Eq.(C.5) in country F is analogous to Eq.(C.5).

D Steady State and Equilibrium Dynamics in a Two-country Economy

Steady state in a two-country economy is described by Eqs.(B.1) and (B.2), counterparts of Eqs.(B.1) and (B.2) and $\mathcal{Q} = 1$. Note that even in country F, steady state government expenditure is zero.

The equilibrium around the steady state can be approximated by Eq.(10), Eqs.(B.4) and (B.5), Eqs.(B.8) to (B.9) and counterparts of them. Instead of Eqs.(B.3), (B.6) and (B.7), following log-linearized equalities describe equilibrium dynamics:

$$\nu\hat{y}_t + (1-\nu)\,\hat{y}_t^* = \nu\hat{c}_t + (1-\nu)\,\hat{c}_t^* + \nu\hat{g}_t + (1-\nu)\,\hat{g}_t^*, \tag{D.1}$$

$$\pi_{H,t} = \beta\pi_{H,t+1} - \kappa\hat{\mu}_t, \tag{D.2}$$

$$\hat{\mu}_t = \hat{\xi}_t - \frac{\varphi+\alpha}{1-\alpha}\hat{y}_t - (1-\nu)\,s_t, \tag{D.3}$$

and counterparts of Eqs.(D.2) and (D.3). Eq.(D.1) results from log-linearizing Eq.(C.5) and its counterpart in country F. Eq.(C.4) results from log-linearizing Eq.(C.4). Eq.(D.3) results from log-linearizing the marginal cost in country H $MC_t = \frac{V_{n,t}}{U_{c,t}}\frac{P_t}{P_{H,t}}\frac{N_t^\alpha}{1-\alpha}$. In addition:

$$s_t = \hat{y}_t - \hat{y}_t^* - \hat{g}_t + \hat{g}_t^*, \tag{D.4}$$

$$\hat{\xi}_t = \hat{\xi}_t^* - \hat{\rho}_t + \hat{\rho}_t^*, \tag{D.5}$$

are essential to describe equilibrium dynamics. Eq.(D.4) results from log-linearizing Eq.(C.5) and its counterpart in country F. Eq.(D.5) results from log-linearizing $\mathcal{U}_{c,t} = \mathcal{U}_{c,t}^*$.

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Table 1: Definition of the Logarithmic Variables

Variable	Definition	Variable	Definition
$\hat{y}_t$	$\log\left(\frac{Y_t}{Y}\right)$	$\hat{b}_t$	$\frac{\mathcal{B}_t-\mathcal{B}}{Y}$
$\hat{c}_t$	$\log\left(\frac{C_t}{C}\right)$	$\widehat{tr}_t$	$\frac{TR_t-TR}{Y}$
$\hat{\rho}_t$	$-\log\left(\frac{Z_{t+1}}{Z_t}\right)$	$\widehat{nx}_t$	$\log\left[\left(\frac{NX_t}{P_{H,t}}\right)/Y\right]$
$p_{H,t}$	$\log P_{H,t}$	ζ_t	$-\log\left(\frac{Z_t^*}{Z_t}\right)$
$p_{F,t}$	$\log P_{F,t}$	$\hat{y}_t^*$	$\log\left(\frac{Y_t^*}{Y^*}\right)$
s_t	$\log\mathcal{S}_t$	ξ_t^*	$\log\left(\frac{U_{c,t}^*}{U_c^*}\right)$
$\hat{g}_t$	$\frac{G_t}{Y}$	$\pi_{F,t}^*$	$\log\Pi_{F,t}^*$
ξ_t	$\log\left(\frac{U_{c,t}}{U_c}\right)$	e_t	$\log\mathcal{E}_t$
π_t	$\log\Pi_t$	$\pi_{H,t}$	$\log\Pi_{H,t}$
$\hat{i}_t$	$\log\left(\frac{1+i_t}{1+\rho}\right)$	$\pi_{F,t}$	$\log\Pi_{F,t}$
m_t	$\log M_t$	μ_t	$-\log MC_t$
$\hat{l}_t$	$\log\left(\frac{L_t}{L}\right)$	$\hat{\mu}_t$	$\mu_t-\mu$
$\widehat{sp}_t$	$\log\left(\frac{SP_t}{SP}\right)$		

$MC_t \equiv \frac{MC_t^n}{P_{H,t}}$ denotes the real marginal cost.

$\hat{\mu}_t$ is dubbed the markup gap.

Table 2: Parameterization

Parameter	Description	Value
σ	Relative Risk Aversion	1
β	Discount Factor	0.995
φ	Curvature of Labor Disutility	5
α	Index of Decreasing Returns to Labor	0.25
ϵ	Elasticity of Substitution among Goods	9
θ	Calvo Index of Price Rigidities	0.75
χ	Steady state Inverse Velocity	$\frac{1}{3}$
η	Semi-elasticity of Money Demand	7
υ	Separability of Real Balances	0
ψ_b	Tax Adjustment	0.02
b	Target Debt Ratio	2.4
δ	Persistence	0.5

Figure 1: Dynamic Effects of an Increase in the Government Expenditure under the *MF* Fiscal Stimulus in Normal Times

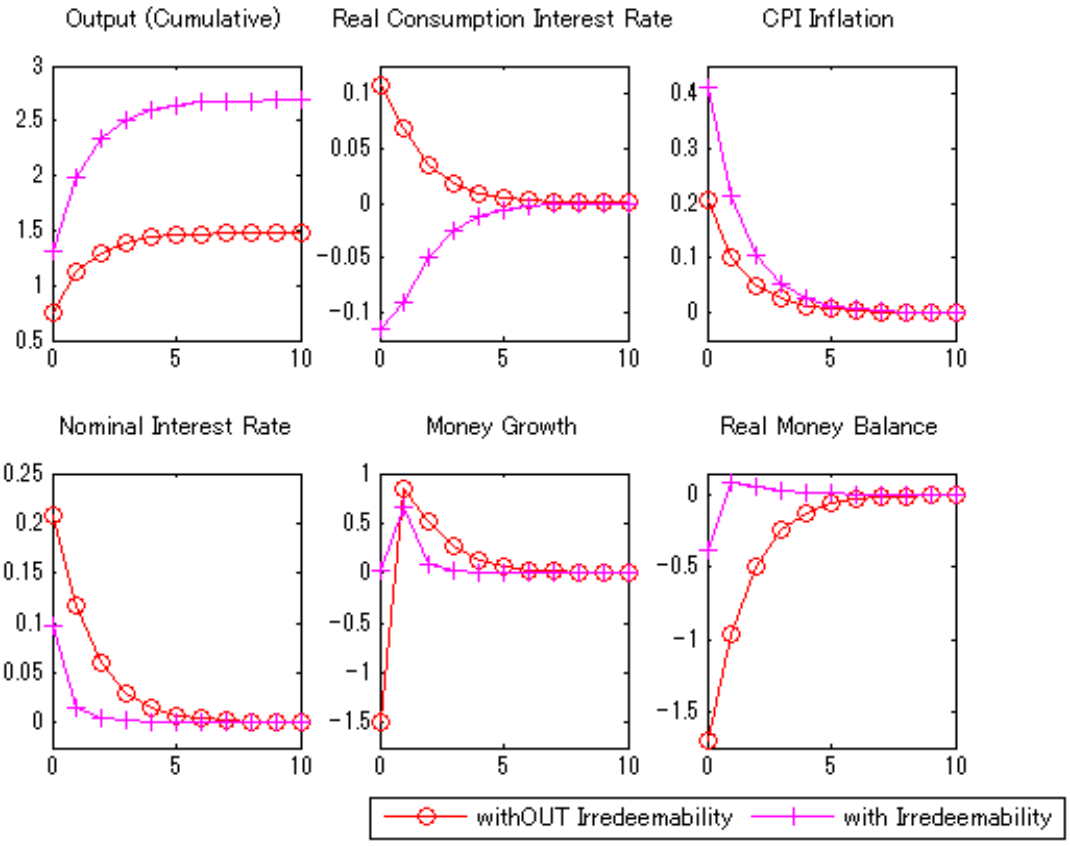

Figure 2: Fiscal Multipliers: The Role of Price Stickiness

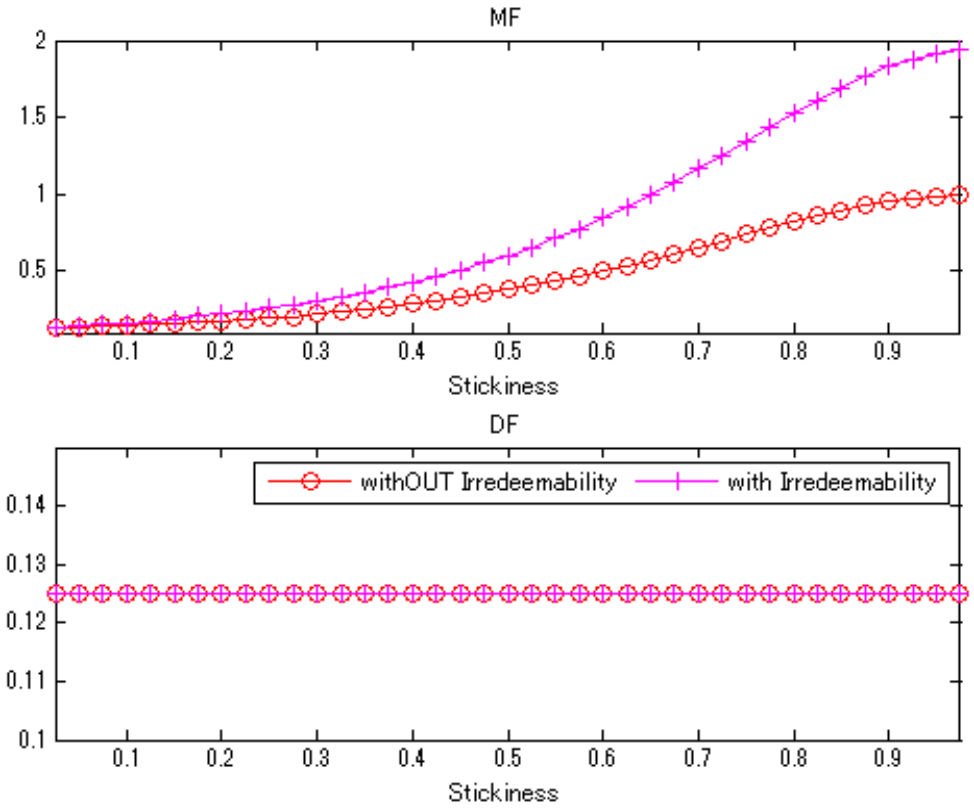

Figure 3: Fiscal Multipliers: The Role of Shock Persistence

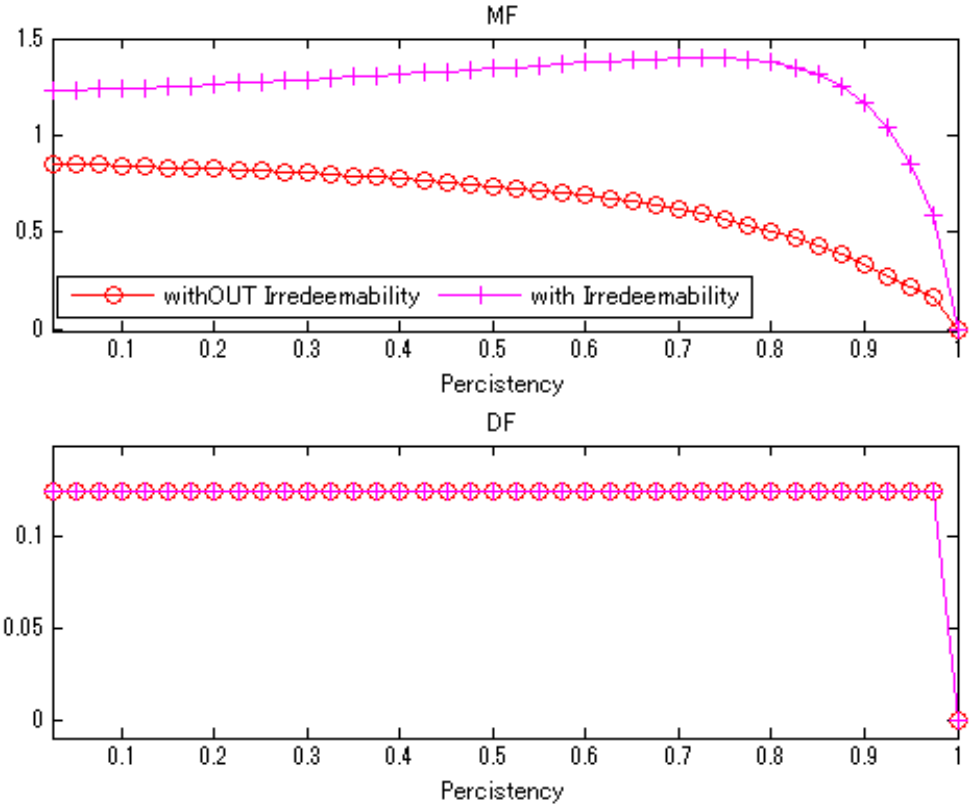

Figure 4: Dynamic Effects of an Increase in the Government Expenditure under the *MF* Fiscal Stimulus in Normal Times in a Two-country Economy

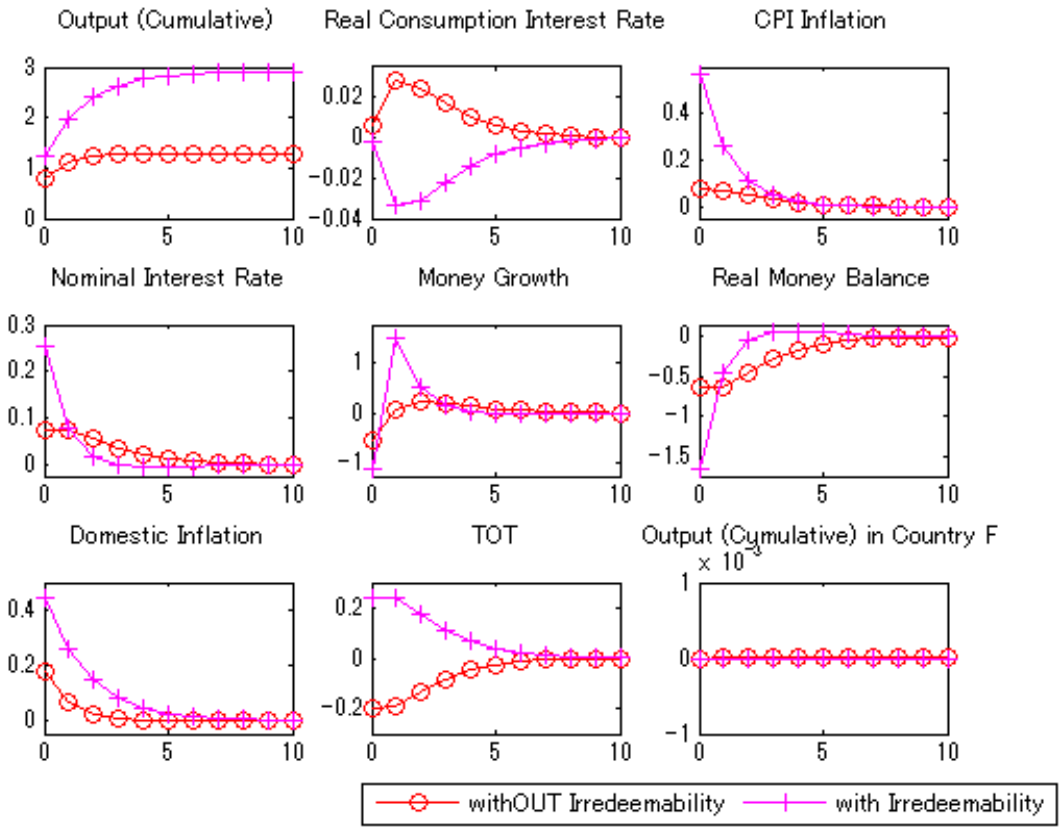

Figure 5: Fiscal Multipliers: The Role of Size of Country H

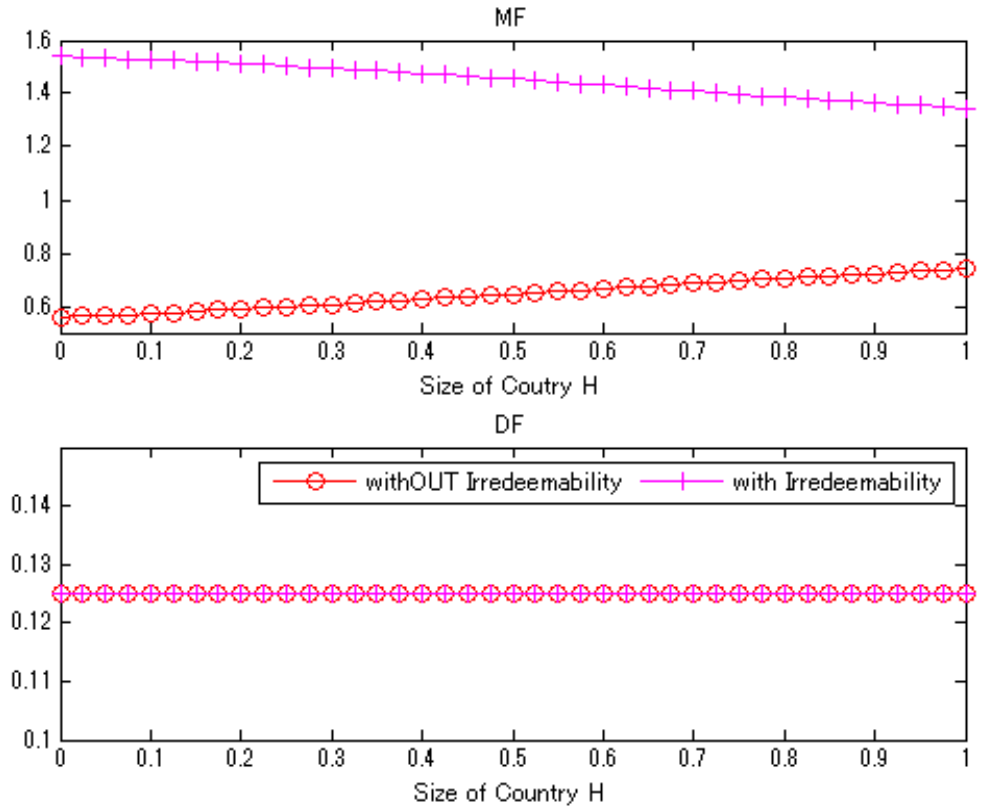

Figure 6: Dynamic Effects of an Increase in the Government Expenditure under the *MF* Fiscal Stimulus in a Liquidity Trap

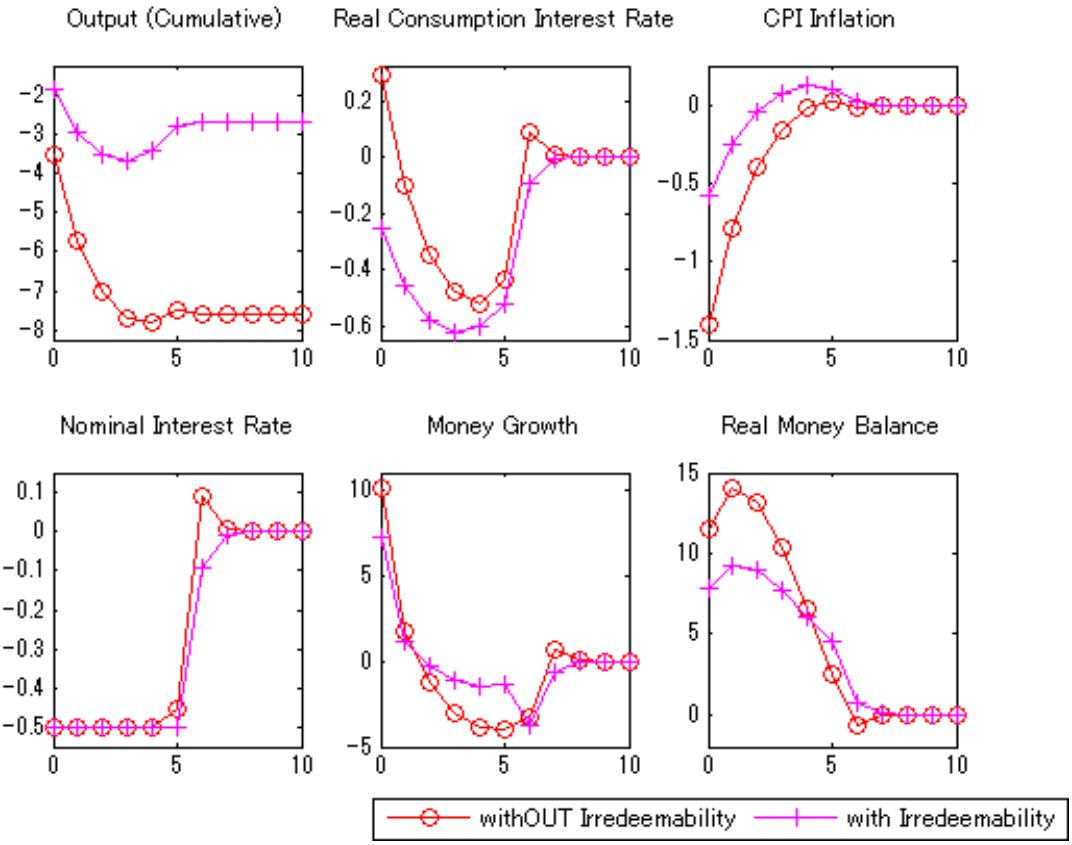

Figure 7: Dynamic Effects of an Increase in Government Expenditure in a Liquidity trap: Comparison of the *MF* scheme and *DF* scheme

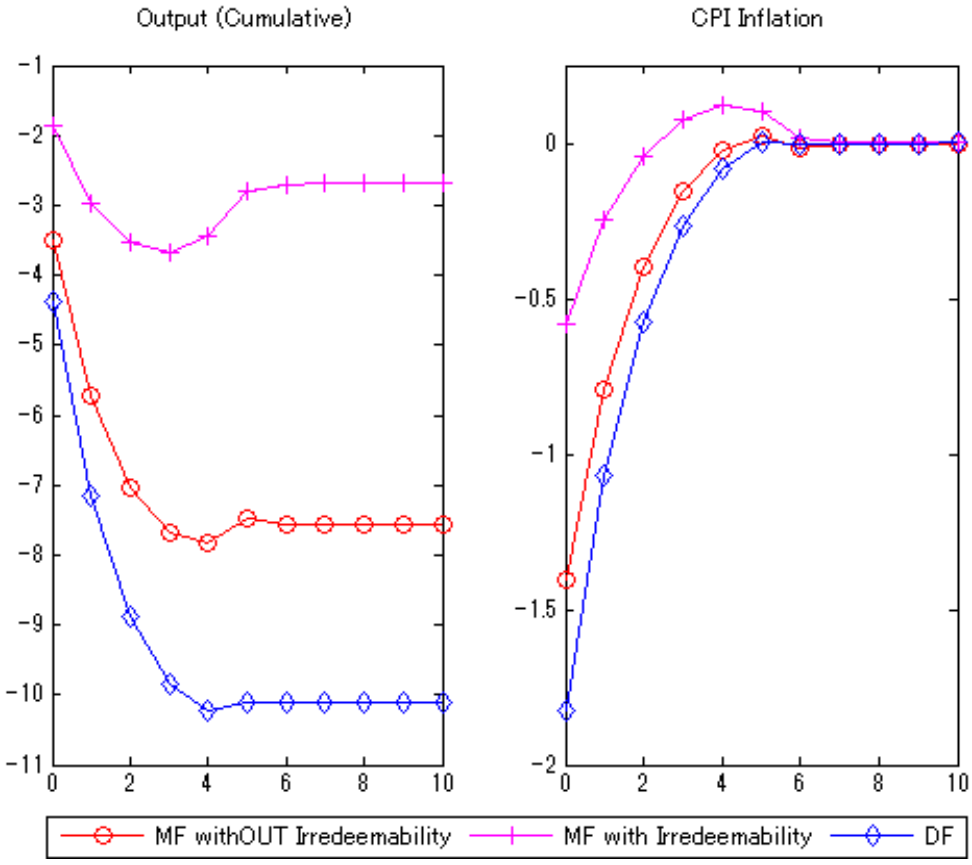

Figure 8: Dynamic Effects of an Increase in the Government Expenditure under the *MF* Fiscal Stimulus in a Liquidity Trap in a Two-country Economy

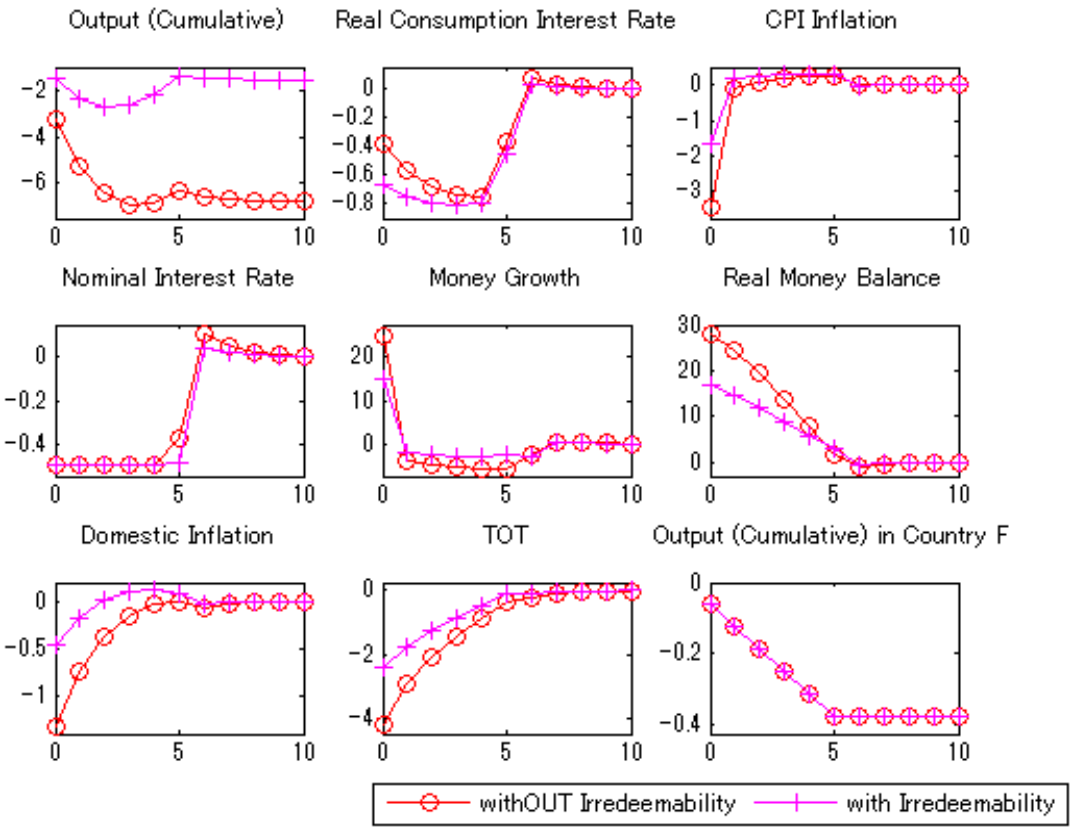

Figure 9: Dynamic Effects of Increases in the Government Expenditure under the *MF* Fiscal Stimulus in a Liquidity Trap in Both Countries

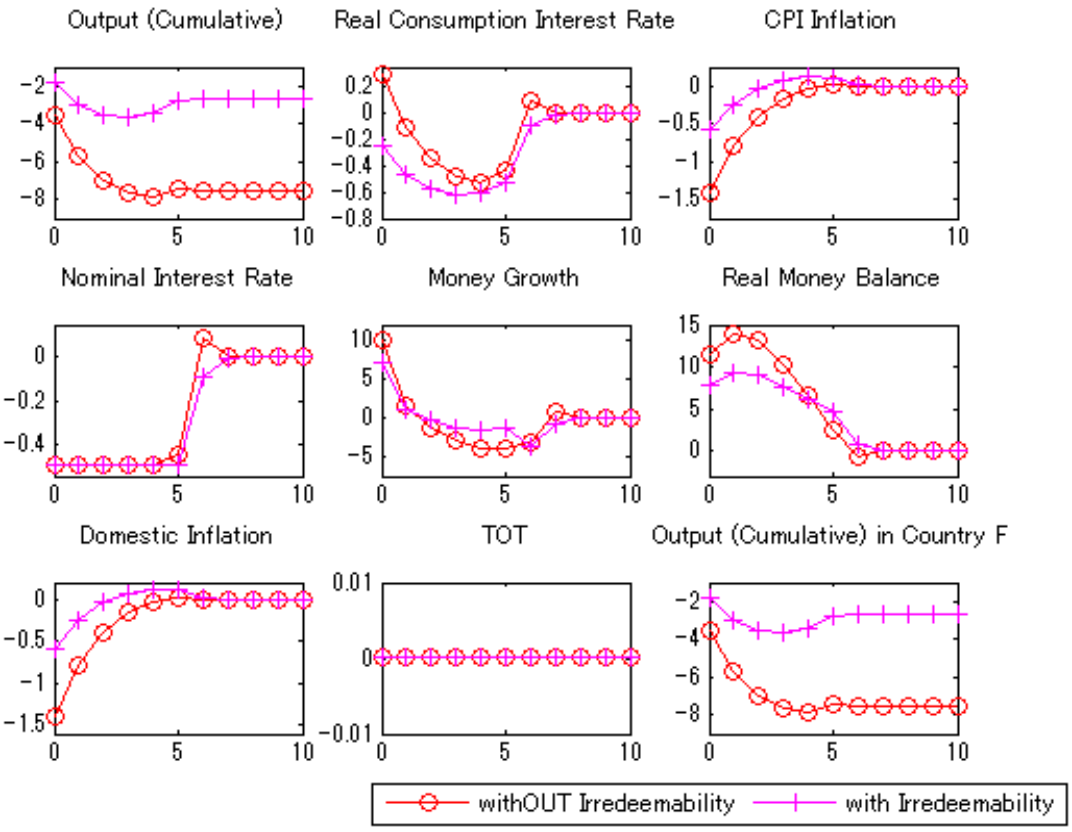

Figure 10: Dynamic Effects of an Increase in Government Expenditure in a Liquidity trap in a Two-country Economy: Comparison of the *MF* scheme and *DF* scheme

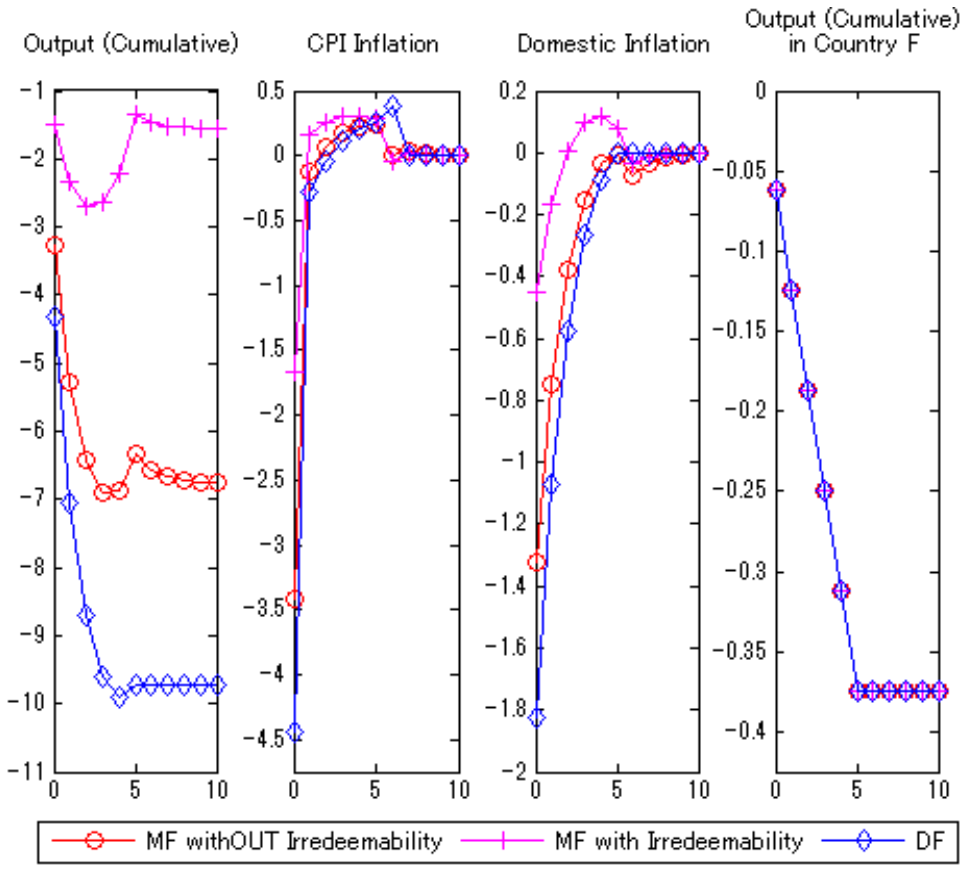

Figure 11: Dynamic Effects of Increases in Government Expenditure in a Liquidity trap in Both Countries: Comparison of the *MF* scheme and *DF* scheme