EK-140xiv(315)

Tariffs and Monetary Policy in a Low Pass-Through Environment: The Return of the ZLB*

Eiji Okano† Nagoya City University
Yang Zhou Nagoya City University
Masataka Eguchi Nagoya City University

This Draft: Sep., 2026
(First Draft: May, 2026)

Abstract

This paper studies how short-run exchange-rate pass-through changes the welfare-maximizing monetary response to an import-tariff increase. We compare two otherwise comparable environments. In the Perfect Pass-Through Environment, producers use producer-currency pricing (PCP) and there is no independent sticky importer-price block. In the Imperfect Pass-Through Environment, producers continue to use PCP while local importers use local-currency pricing (LCP) and set sticky tariff-exclusive prices in the buyer's currency. This change reverses the optimal interest-rate response from tightening to substantial easing, and accommodation remains optimal when Foreign demand and monetary policy are endogenized in a two-country economy. Under perfect pass-through, the tariff increases demand for Home goods through expenditure switching and raises Home producer real marginal cost, putting upward pressure on Home producer price index (PPI) inflation. Under imperfect pass-through, monetary easing can exploit the open-economy relative-price externality more effectively. In the SOE, depreciation keeps the external relative price higher than under a less accommodative path, supporting Home output and income, while sticky local import prices cause part of the exchange-rate adjustment to appear as a law-of-one-price (LOOP) gap. The associated LOOP-gap adjustment limits the increase in the cum-tariff import price faced by Home households but creates an importer-pricing distortion and additional inflation-stabilization costs. The benchmark imperfect-pass-through economy therefore already calls for substantial accommodation. Changing one economically relevant dimension of the environment at a time—substitutability, preferences, trade-policy interaction, roundabout production, or strategic monetary interaction—can strengthen this already large desired easing enough for the nominal rate to reach the zero lower bound, without an exogenous natural-rate shock.

Keywords: Import tariffs; optimal monetary policy; pricing behavior; local-currency pricing; zero lower bound.

JEL Classification: E31, E52, F13, F41, F42.

*All errors are our own. This research was financially supported by the Japan Society for the Promotion of Science *KAKENHI* (Grant #26K04990).

†Corresponding author. Graduate School of Economics, Nagoya City University, 1, Aza, Yamanobata, Mizuho-cho, Mizuho-ku, Nagoya-shi Aichi, 467-8501, Japan. E-mail: eiji_okano@econ.nagoya-cu.ac.jp

1 Introduction

Should a tariff-imposing country adopt monetary tightening or easing? In an imperfect pass-through environment, a tariff-imposing country should adopt monetary easing. More importantly, the welfare-maximizing easing is already substantial in the benchmark Imperfect Pass-Through Environment. Changing one economically relevant dimension of the environment can strengthen this same accommodative force enough for the nominal interest rate to reach the zero lower bound (ZLB). We obtain this policy implication by analyzing the "constrained-efficient optimal policy" of Monacelli [19], which corresponds to the Ramsey formulation of optimal monetary policy commonly used in the New Keynesian literature.

We introduce local importers that set optimal prices in units of local currency subject to Calvo price rigidity into the small-open-economy (SOE) model developed by Monacelli [19], following Monacelli [21], to generate an imperfect pass-through environment (when we solve the constrained-efficient policy problem, we retain the micro-level production structure and account for price dispersion generated by staggered price setting). We then compare the nominal interest-rate response chosen by the Ramsey social planner to an increase in the import tariff in the Perfect Pass-Through Environment, in which there is no sticky local importer-price block, with that in the Imperfect Pass-Through Environment. Home producers use producer-currency pricing (PCP) in both environments, while the Imperfect Pass-Through Environment additionally contains local importers using local-currency pricing (LCP). Throughout the paper, low pass-through means low short-run exchange-rate pass-through into the tariff-exclusive local-currency import price; it does not refer to statutory tariff incidence. The Ramsey planner chooses monetary tightening under perfect pass-through but pronounced monetary easing under imperfect pass-through. We next extend the model to a two-country economy and let a U.S.-size Home country impose the tariff. The optimal response remains monetary easing in the Imperfect Pass-Through Environment.

We next explain why monetary easing is chosen in the Imperfect Pass-Through Environment. Under Perfect Pass-Through, the law of one price (LOOP) holds, so the open-economy relative-price externality operates through the ex-tariff border relative price of imports alone. Under Imperfect Pass-Through, sticky local import prices allow an exchange-rate movement to affect both that border relative price and the LOOP gap. The same relative-price externality is therefore manifested jointly through the border relative price and the LOOP gap. The LOOP gap is also an importer-pricing distortion because it reflects a discrepancy between the border acquisition cost and the sticky local import price.

Households pay the tariff-inclusive local import price, so a tariff increase raises the price of Foreign goods relative to Home goods. Monetary accommodation and the associated nominal depreciation can support Home demand, output, and income by limiting the tariff-induced decline in the ex-tariff border relative price of imports relative to a less accommodative policy path. Taken by itself, however, that movement raises rather than offsets the household-facing relative import price. Under Perfect Pass-Through, the same exchange-rate movement is transmitted immediately into the domestic import price. Under Imperfect Pass-Through, the sticky local import price breaks this one-for-one link.

A depreciation raises the Home-currency border acquisition price immediately, whereas the local import price adjusts only gradually. Part of the exchange-rate adjustment therefore appears as a larger LOOP gap. The planner can consequently support Home demand, output, and income without generating the full immediate increase in the domestic import price that would occur under Perfect Pass-Through. The LOOP-gap movement also offsets part of the direct tariff-induced increase in the household-facing relative import price. In this sense, imperfect pass-through strengthens the planner's ability to exploit the relative-price externality in response to a tariff increase.

This use of the LOOP gap is costly. It compresses the importer markup and, through the importer Phillips curve, puts upward pressure on ex-tariff import inflation and generates importer-price dispersion. Monetary accommodation also affects Home producer real marginal cost and Home producer price index (PPI) inflation. To make this second-best trade-off transparent, we derive a welfare cost function from a second-order approximation to household utility. The welfare representation shows that the sticky-inflation terms carry large weights, whereas the structural weight on the Welfare Relevant LOOP Gap Deviation is small. The benchmark tariff increase nevertheless creates a sizable Welfare Relevant LOOP Gap Deviation. The planner therefore chooses accommodation that exploits the relative-price externality through the joint adjustment of the border relative price and the LOOP gap while accepting some additional sticky-inflation cost.

We verify this trade-off by comparing welfare costs under the constrained-efficient policy with those under a no-accommodation counterfactual in which the nominal interest rate remains at its steady-state value. The welfare gain from reducing the Welfare Relevant LOOP Gap Deviation exceeds the associated increase in welfare costs from weaker inflation stabilization. The same comparison shows that the consumption of Foreign goods relative to Home goods is higher under the Ramsey response than under the no-accommodation counterfactual. We use this quantity comparison as a direct check on the household allocation implied by the joint relative-price adjustment, rather than as a separate welfare objective.

For completeness, we also note that monetary tightening is chosen in the Perfect Pass-Through Environment. An increase in the import tariff raises the consumer price index (CPI) relative to the Home PPI and reduces the consumption real wage associated with a given nominal wage. To sustain labor supply, the nominal wage must rise, putting upward pressure on Home producer real marginal cost and Home PPI inflation. Because inflation stabilization is an important objective of the constrained-efficient policy and there are no importers, the Ramsey planner raises the nominal interest rate. The direction of the optimal nominal-interest-rate response is therefore opposite to that under imperfect pass-through.

Monetary accommodation is also chosen in the two-country economy, although the Home currency appreciates rather than depreciates. The difference is that the Foreign economy is now endogenous. An increase in the Home import tariff lowers demand for Foreign goods and puts downward pressure on Foreign PPI inflation, so the Foreign monetary authority keeps its nominal interest rate below its steady-state path more persistently than the Home monetary authority. The resulting relative interest-rate path generates a Home-currency appreciation despite the Home rate cut. Home easing works against this appreciation and changes the joint movement of the ex-tariff border relative price of imports and the Home LOOP gap relative to a less accommodative Home policy path. Further Home easing would exploit the national relative-price externality more strongly, but it would worsen other welfare-relevant objectives enough to raise Home welfare cost. Monetary accommodation therefore remains optimal in the two-country Imperfect Pass-Through Environment, even though the equilibrium exchange-rate movement has the opposite sign from the SOE.

Having established monetary accommodation under imperfect pass-through in both the SOE and the two-country economy, we next ask how consequential the size of that accommodation can become. The benchmark Imperfect Pass-Through Environment already requires a large policy-rate reduction. We therefore change one economically relevant feature of the environment at a time and ask whether the additional force is enough to carry the desired easing the rest of the way to the ZLB. No separate recessionary or natural-rate shock is added. Section 5 studies this question through high substitutability, external habit, reciprocal tariffs, roundabout production, and an open-loop Nash policy game.

The mechanisms differ across these cases, but their role in the argument is the same: each strengthens a different part of an accommodation that is already large under imperfect pass-through. Higher substitutability makes a given relative-price movement generate a larger change in the composition of Home consumption and thereby changes the trade-off between exploiting the relative-price externality and stabilizing the other welfare-relevant variables. External habit makes the aggregate-consumption consequences of the accommodation intertemporal and therefore increases the persistence of the policy problem. Reciprocal tariffs add a foreign-demand spillback that places additional downward pressure on Home activity, producer marginal cost, and PPI inflation. Roundabout production adds a material-input spillover–spillback, reduces the effect of Home activity on producer real marginal cost as the direct-labor share falls, and increases the welfare cost of Home producer-price dispersion. Open-loop Nash reorganizes the stabilization problem toward the Home producer-price block. In each case, changing one economically relevant dimension can provide enough additional accommodation for the desired nominal rate to reach the ZLB. The ZLB episodes therefore reveal the quantitative strength of the monetary easing generated by imperfect pass-through rather than a separate source of monetary transmission.

The contribution can therefore be summarized as follows. First, relative to Monacelli [19], we show that changing the short-run pass-through environment can turn the constrained-efficient response to a tariff from monetary tightening into substantial monetary easing. Starting from his Perfect Pass-Through SOE benchmark, we add sticky local importers while retaining the tariff experiment, fiscal closure, PCP producer pricing, and the basic trade structure. This controlled comparison identifies the pass-through environment as the source of the change in the optimal policy response. Second, we identify the relative-price, income-support, welfare, and Phillips-curve mechanisms behind the required accommodation: imperfect pass-through lets the planner use nominal depreciation to support Home output and income without an immediate one-for-one increase in the domestic import price. We also show that monetary easing remains optimal in a two-country national-Ramsey game with endogenous Foreign demand and monetary policy. Third, and more importantly for the quantitative policy implication, the benchmark Imperfect Pass-Through Environment already requires substantial accommodation. Changing one economically relevant dimension of the environment can then add enough further accommodation for the desired rate to reach the ZLB, without adding an exogenous natural-rate shock. Low short-run pass-through can therefore make the optimal easing after a tariff shock large enough that one additional force in the stabilization problem brings the limit of conventional monetary policy into play.

The rest of the paper is organized as follows. Section 2 relates the analysis to the tariff, pass-through, and international policy-game literatures. Section 3 presents the small-open-economy Perfect and Imperfect Pass-Through Environments and their Ramsey problems. Section 4 endogenizes the Foreign economy and studies the two-country national-Ramsey game. Section 5 studies the experiments in which the desired accommodation reaches the ZLB. Section 6 concludes. The Appendix reports the pricing definitions, Ramsey and open-loop first-order conditions, and the Benigno–Woodford welfare objects used in the text.

2 Related Literature

The paper connects the literature on optimal monetary policy after tariff shocks with work on exchange-rate pass-through and pricing currency, and with the literature on strategic interaction between national monetary authorities. The central contribution is a controlled comparison that keeps the tariff process, fiscal closure, producer pricing, and basic trade structure fixed while changing the pass-through environment faced by domestic importers. This comparison shows that the pass-through environment can reverse the sign of the Ramsey interest-rate response to a tariff, from tightening to substantial easing. We then show that this already large accommodation remains optimal when international interaction is endogenous and that changing one economically relevant dimension of the environment can strengthen it enough for the desired nominal rate to reach the ZLB. Finally, the welfare representation identifies why accommodation is desirable: imperfect pass-through strengthens the planner's ability to exploit the open-economy relative-price externality. Nominal depreciation can support Home output and income while the sticky local import price prevents the exchange-rate movement from passing one-for-one into the domestic import price. The resulting LOOP gap is a distortionary consequence of this stronger exploitation of the relative-price externality, which monetary policy trades against output and sticky-inflation costs. These contributions distinguish the paper from work that studies tariff transmission under richer environments without isolating the pass-through environment, and from ZLB analyses in which an additional demand or natural-rate shock creates the liquidity trap.

2.1 Tariffs and optimal monetary policy

Monacelli [19] is the main benchmark for the present paper. He studies import and export tariffs in a small-open-economy New Keynesian model, derives welfare-maximizing monetary policy, and emphasizes that tariff transmission depends crucially on the monetary-policy regime. In particular, canonical CPI-inflation targeting can generate a tightening bias and large welfare costs. We retain his SOE framework and benchmark calibration wherever the model has a direct counterpart, but change the question. Instead of comparing monetary-policy rules, we compare two pass-through environments under Ramsey policy. This isolates the pass-through environment as a source of the sign and magnitude of the optimal interest-rate response itself.

Bergin and Corsetti [12] study cooperative stabilization of tariff shocks in a richer two-country economy with production chains and alternative pricing behavior, including dominant-currency pricing (DCP). Their DCP specification is empirically well motivated and features imperfect short-run exchange-rate pass-through. Their monetary-policy results, however, are obtained in an environment in which pricing behavior and a rich trade and production structure are jointly present. Their exercise therefore does not separately isolate whether the sign of the policy-rate response is due to pricing behavior or to the trade and production structure with which it interacts.

Our approach is designed to isolate that distinction. We begin from the Perfect Pass-Through SOE benchmark corresponding to Monacelli [19] and construct an otherwise comparable Imperfect Pass-Through Environment. The tariff process, fiscal closure, PCP producer pricing, and basic trade structure are unchanged; the Imperfect Pass-Through Environment adds sticky domestic importers using LCP. The resulting Ramsey response changes from tightening to substantial easing. We then endogenize the Foreign economy and examine whether this accommodation survives strategic international interaction and whether changing one economically relevant feature at a time can strengthen it enough to exhaust conventional policy space in the configurations studied in Sec. 5.

Bianchi and Coulibaly [16] focus directly on the tariff-imposing country and obtain an expansionary optimal monetary response over a broad set of tariff experiments. Werning, Lorenzoni, and Guerrieri [22] emphasize the cost-push effect of tariffs on imported intermediate inputs and show that optimal policy should partially accommodate it. Our roundabout-production experiment instead emphasizes the material-input spillback, the weaker effect of Home activity on producer real marginal cost as the direct-labor share falls, and the larger resource cost of producer-price dispersion. Bergin and Corsetti [13] further show that the desired monetary response to a unilateral tariff can vary with the trade elasticity and the price flexibility of tariffed goods. Relative to these contributions, our focus is on how the domestic import-price pass-through environment changes the Ramsey response itself, why the resulting accommodation is already large, how one additional economically relevant force can carry it to the ZLB, and how the welfare representation explains that response.

2.2 Pricing behavior, pass-through, and monetary stabilization

A second strand explains why the importer-pricing block is economically substantive. Betts and Devereux [7] and Devereux and Engel [8] show that local-currency price stickiness weakens the short-run expenditure-switching effect of nominal exchange-rate movements. Devereux and Engel [1] show in a welfare-based analysis that the optimal use of exchange-rate flexibility depends on whether prices are set in producer or local currency. Corsetti, Dedola, and Leduc [6] similarly show that the optimal monetary stance can depend on the degree of exchange-rate pass-through into import prices. These results motivate our distinction between the complete Perfect and Imperfect Pass-Through Environments.

A complementary welfare literature provides the relative-price logic underlying our mechanism. Corsetti and Pesenti [2] show that inward-looking stabilization is not generally optimal when firms' markups are exposed to currency movements: optimal policy trades domestic stabilization against import-price and consumer-purchasing-power considerations, with the trade-off depending

Table 1: Closest literature and the mechanism isolated in this paper

Paper	Monetary-policy question	Pricing/pass-through feature	Difference from the present paper
Monacelli [19]	Optimal monetary policy after tariff shocks in an SOE	Perfect-pass-through SOE benchmark and alternative monetary-policy regimes	Main benchmark for the present paper; we retain the SOE framework and calibration and show that changing the pass-through environment can make the tariff call for substantial Ramsey easing and bring the ZLB into play.
Bergin–Corsetti [12]	Cooperative stabilization of unilateral and reciprocal tariffs	DCP in a richer trade and production structure	Pricing behavior and trade/production structure are jointly present, so their exercise does not isolate which one determines the sign of the policy response; our Monacelli-based controlled comparison changes the pass-through environment while holding the basic trade structure fixed.
Bianchi–Coulibaly [16]	Optimal policy of the tariff-imposing country	Pricing currency is not the central comparison	Finds an expansionary optimal response; we identify how the import-price pass-through environment generates substantial accommodation and how that accommodation can make the ZLB relevant.
Werning–Lorenzoni–Guerrieri [22]	Optimal response to tariffs on imported intermediates	Imported-input cost-push mechanism	Establishes a motive for accommodation; our roundabout experiment adds a material-input spillback, weakens the effect of Home activity on producer real marginal cost, and raises the resource cost of producer-price dispersion.
Bergin–Corsetti [13]	Exchange-rate stabilization after unilateral tariffs	Trade elasticity and price flexibility	Shows that optimal policy is sensitive to transmission; our high-substitution experiment shows that stronger relative-demand transmission can push the desired easing to the ZLB even though the Welfare Relevant LOOP Gap falls.
Auray–Devereux–Eyquem [15]	Macroeconomic transmission of tariff and exchange-rate changes	PCP, LCP, and DCP	Shows that invoicing currency has a first-order effect on tariff transmission; we study the national Ramsey interest-rate response, welfare decomposition, strategic interaction, and the ZLB.

on exchange-rate pass-through. De Paoli [3] emphasizes the terms-of-trade externality in a small open economy: a national planner has an incentive to exploit its ability to influence international relative prices rather than mechanically stabilize domestic inflation. With LCP, Engel [4] shows that currency misalignments generated by deviations from the law of one price are inefficient and enter optimal targeting together with CPI inflation and the output gap. Fujiwara and Wang [5] extend this logic to a noncooperative policy game under LCP, in which deviations from the law of one price create additional stabilization trade-offs through real marginal costs.

Our mechanism combines these insights in the tariff problem. The tariff directly raises the *cum-tariff* relative import price faced by Home households. The Ramsey planner uses nominal depreciation to keep the *ex-tariff* relative price of imports higher than under a less accommodative path, shifting expenditure toward Home goods and supporting Home output and income. Under Perfect Pass-Through, that exchange-rate movement is transmitted one-for-one into the local *ex-tariff* import price. Under Imperfect Pass-Through, sticky local import prices break this link: part of the exchange-rate movement appears instead as a LOOP gap. The planner can therefore exploit the relative-price externality more effectively: keeping the *ex-tariff* relative price of imports higher than under a less accommodative path supports Home income, while the associated LOOP-gap adjustment offsets part of the tariff-induced increase in the household import price. The LOOP deviation remains distortionary and is traded against the sticky-price costs emphasized by Engel [4] and Fujiwara and Wang [5].

The empirical pricing literature gives that distinction a direct interpretation. Gopinath, Itskhoki, and Rigobon [9] show that exchange-rate pass-through into U.S. import prices is closely related to the currency in which those prices are set. Gopinath et al. [10] document the broader role of the U.S. dollar in international invoicing and pass-through. Our use of LCP therefore refers to the behavior of the domestic importer block, not to the entire model. In the Imperfect Pass-Through Environment producers continue to use PCP, while local importers set sticky tariff-exclusive prices in the destination currency. Galí and Monacelli [24] provide the domestic-importer LCP architecture used for this purpose. Okano and Eguchi [20] also introduce LCP as a robustness check in their analysis of money-financed fiscal stimulus in a small open economy.

Tariff-specific work reinforces the point. Hwang and Turnovsky [11] show that tariff transmission differs across PCP and LCP pricing behavior, while Auray, Devereux, and Eyquem [15] show that invoicing currency materially changes the short-run response to tariffs. Our contribution is to connect this pricing literature to the welfare-maximizing interest-rate path after a tariff shock. In the Imperfect Pass-Through Environment the sticky importer block adds import-price inflation, importer-price dispersion, and the Welfare Relevant LOOP Gap Deviation to the stabilization problem. The welfare representation then makes explicit the trade-off that is central to our results. Under Perfect Pass-Through, the relative-price externality operates through the international relative price alone. Under Imperfect Pass-Through, the LOOP gap is an additional importer-pricing distortion that changes how the planner can exploit the same relative-price externality through the joint adjustment of the *ex-tariff* relative price and the LOOP gap. Monetary accommodation can reduce the Welfare Relevant LOOP Gap Deviation as part of this joint adjustment while increasing the Welfare Relevant Output Gap and sticky-inflation costs. This provides a welfare-based explanation for why the pass-through environment can reverse the optimal interest-rate response even when the structural welfare weight on the Welfare Relevant LOOP Gap Deviation is relatively small.

2.3 International policy interaction and the ZLB

The two-country analysis is related to the literature on international monetary-policy games. Faia and Monacelli [23] formulate welfare-maximizing monetary policy in a two-country New Keynesian economy and compare cooperation with Nash competition between national Ramsey planners. We use their national-Ramsey allocation game as the benchmark strategic environment. This extension asks whether the substantial Home accommodation identified in the SOE survives when Foreign demand and Foreign monetary policy are jointly endogenous. It also makes the exchange-rate implication of the relative Home–Foreign interest-rate path part of the equilibrium rather than an exogenous SOE object.

As a strategic-policy experiment, we replace the Faia–Monacelli allocation game with an open-loop Nash policy game in the spirit of Bodenstein, Guerrieri, and LaBriola [29]. Their analysis emphasizes that the equilibrium of an open-loop macroeconomic policy game can depend on the variable used to define the strategy space and that using the nominal interest rate itself can generate non-uniqueness. We therefore use national PPI-inflation paths as the strategic policy paths, leaving nominal interest rates as endogenous equilibrium objects subject to the two ZLBs. This experiment is not a separate source of tariff distortion; it asks whether the strong accommodation survives an alternative strategic-policy concept and how that concept reorganizes the stabilization problem.

Finally, the ZLB experiments are related to work on international monetary transmission when nominal rates are constrained, including Fujiwara et al. [33], Bodenstein, Erceg, and Guerrieri [34], and Billi and Galí [32]. The role of the ZLB here is different. We do not add a demand or natural-rate shock to generate a liquidity trap. The tariff shock already calls for substantial easing in the benchmark Imperfect Pass-Through Environment. The experiments then change one economically relevant feature of the environment at a time and ask whether the resulting additional force is enough to push this already large desired easing to the ZLB. The contribution is therefore not that tariffs mechanically create liquidity traps, but that low short-run pass-through can make the welfare-maximizing easing after a tariff so large that one additional force in the stabilization problem can exhaust conventional policy space.

The ordering of the paper follows this controlled-comparison strategy. We first use the SOE controlled comparison to isolate the role of the pass-through environment, then test the resulting accommodation in a two-country national- Ramsey game, and only then ask whether one additional economically relevant force can carry that accommodation to the ZLB. This separates sticky domestic-importer pricing from other mechanisms that can strengthen monetary accommodation and from other shocks that can independently create a liquidity trap.

3 Small Open Economy

3.1 The Model

The small-open-economy model consists of a representative Home household, monopolistically competitive Home producers, and a monetary authority. Home producers use PCP in both pass-through environments and use Calvo pricing for their producer prices. In the Perfect Pass-Through Environment there is no independent domestic importer-price block and the law of one price holds for imports. In the Imperfect Pass-Through Environment the same PCP producer block is retained and a local importer sector is added; these importers use LCP and use Calvo pricing for tariff-exclusive Home-currency prices. The household consumes a constant-elasticity-of-substitution (CES) composite of Home and Foreign goods and trades state-contingent claims with the rest of the world. The government levies an import tariff and rebates tariff revenue lump sum. No employment subsidy is paid to PCP producers, whereas the importer markup is normalized in steady state as described below. When we solve the constrained-efficient policy problem, we retain the micro-level production structure and account for price dispersion generated by staggered price setting.

3.1.1 Households, demand, and *cum-tariff* import prices

The Home representative household has lifetime utility

$$\mathcal{U} \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right). \tag{1}$$

Here C_t denotes the Home household's composite consumption and $N_t \equiv \int_0^1 N_t(j)dj$ denotes the hours of labor. The parameter $\beta \in (0,1)$ is the discount factor, φ is the inverse Frisch elasticity, and $\upsilon \in (0,1)$ denotes the expenditure weight on Foreign goods, or trade openness.

The consumption aggregator is

$$C_t = \left[(1-\upsilon)^{1/\eta} C_{H,t}^{(\eta-1)/\eta} + \upsilon^{1/\eta} C_{F,t}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}.$$

Here $C_{H,t}$ denotes consumption of Home goods, $C_{F,t}$ denotes consumption of Foreign goods, and η is the elasticity of substitution between Home and Foreign goods.

We use the same international relative-price definition in the SOE and in the two-country model. Specifically, $S_t \equiv \mathcal{E}_t P^*_{F,t}/P_{H,t}$ and $\Psi_t \equiv \mathcal{E}_t P^*_{F,t}/P_{F,t}$. These definitions imply

$$\frac{P_{F,t}}{P_{H,t}} = \frac{S_t}{\Psi_t}. \tag{2}$$

In the Perfect Pass-Through Environment, Eq. (2) reduces to $P_{F,t}/P_{H,t} = S_t$, since there is no LOOP gap, i.e., $\Psi_t = 1$. Here P_t is the Home CPI, $P_{H,t}$ is the Home producer-price index, $P_{F,t}$ is the tariff-exclusive Home-currency import price set by the local importer, $\mathcal{E}_t$ is the price of Foreign currency in units of Home currency, and $P^*_{F,t}$ is the Foreign producer-price index in Foreign currency. Thus S_t is the *ex-tariff* border price of Foreign goods relative to the Home PPI, whereas the left-hand side of Eq. (2) is the *ex-tariff* relative price of imports. Gross Home CPI and PPI inflation are $\Pi_t \equiv P_t/P_{t-1}$ and $\Pi_{H,t} \equiv P_{H,t}/P_{H,t-1}$, respectively. The variable $\tau_{M,t}$ denotes the Home import-tariff rate.

The CPI–PPI ratio is therefore

$$\mathcal{G}\left(\frac{S_t}{\Psi_t}, \tau_{M,t}\right) \equiv \frac{P_t}{P_{H,t}} = \left[(1-\upsilon) + \upsilon\left((1+\tau_{M,t})\frac{S_t}{\Psi_t}\right)^{1-\eta}\right]^{1/(1-\eta)}. \tag{3}$$

Home-good demand is

$$C_{H,t} = (1-\upsilon)\mathcal{G}\left(\frac{S_t}{\Psi_t}, \tau_{M,t}\right)^{\eta} C_t. \tag{4}$$

Import demand is

$$C_{F,t} = \upsilon\left[\frac{(1+\tau_{M,t})S_t}{\Psi_t \mathcal{G}(S_t/\Psi_t, \tau_{M,t})}\right]^{-\eta} C_t. \tag{5}$$

After Eqs. (4) and (5), the household's nominal budget constraint can be written as

$$P_{H,t}C_{H,t} + (1+\tau_{M,t})P_{F,t}C_{F,t} + \sum_{h^{t+1}} \xi_{t,t+1}(h^{t+1}|h^t)B_{t+1}(h^{t+1}) = B_t(h^t) + W_t N_t + \mathcal{D}_t + T_t.$$

Here $B_{t+1}(h^{t+1})$ is a state-contingent nominal claim purchased at date t, $\xi_{t,t+1}(h^{t+1}|h^t)$ is its date-t state price, W_t is the nominal wage, $\mathcal{D}_t$ denotes nominal dividends from firms, and T_t is the lump-sum transfer. Tariff revenue, $\tau_{M,t}P_{F,t}C_{F,t}$, is rebated lump sum.

The household intratemporal optimality condition is

$$\frac{W_t}{P_t} = C_t N_t^{\varphi}. \tag{6}$$

Let i_t denote the Home nominal interest rate. The Euler equation is

$$\frac{1}{1+i_t} = \beta \mathrm{E}_t\left[\frac{C_t \mathcal{G}(S_t/\Psi_t, \tau_{M,t})}{C_{t+1}\mathcal{G}(S_{t+1}/\Psi_{t+1}, \tau_{M,t+1})\,\Pi_{H,t+1}}\right]. \tag{7}$$

The Euler equation follows from the state-contingent condition $\xi_{t,t+1} = \beta(C_t/C_{t+1})(P_t/P_{t+1})$ and $(1+i_t)^{-1} = \mathrm{E}_t\xi_{t,t+1}$. We assume that the Foreign household satisfies the analogous Euler equation. No arbitrage between Home- and Foreign-currency one-period nominal bonds is imposed in levels. With $\xi_{t,t+1}$ denoting the Home-currency nominal state price used above, the Foreign-currency bond satisfies

$$\frac{1}{1+i_t^*} = \mathrm{E}_t\left[\xi_{t,t+1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right],$$

together with $(1+i_t)^{-1} = \mathrm{E}_t\xi_{t,t+1}$. The familiar log-linear uncovered interest parity (UIP) relation is the first-order approximation of these bond-pricing conditions; we do not use that linear approximation separately below.

With the price definitions above, complete international risk sharing implies

$$S_t = C_t\mathcal{G}\left(\frac{S_t}{\Psi_t}, \tau_{M,t}\right). \tag{8}$$

In the Perfect Pass-Through Environment, $\Psi_t = 1$ for all t, so Eq. (8) reduces to $S_t = C_t\mathcal{G}(S_t, \tau_{M,t})$.

Before turning to the pass-through environments, consider the Home production side. Each Home firm $j \in [0,1]$ produces a differentiated good according to

$$Y_t(j) = A_tN_t(j),$$

where A_t denotes labor productivity. Let Y_t denote aggregate Home output and $P_{H,t}(j)$ the price of variety j. Demand for variety j is $Y_t(j) = [P_{H,t}(j)/P_{H,t}]^{-\varepsilon}Y_t$, where $\varepsilon > 1$ is the elasticity of substitution across Home producer varieties. Using this demand function and the definition of aggregate labor gives

$$Y_t = \frac{A_tN_t}{\Delta_{H,t}}. \tag{9}$$

Here $\Delta_{H,t} \equiv \int_0^1[P_{H,t}(j)/P_{H,t}]^{-\varepsilon}dj$ denotes producer-price dispersion. Equation (9) makes explicit the resource cost of relative-price dispersion. If all producers charge the same relative price, $\Delta_{H,t} = 1$ and aggregate production reduces to $Y_t = A_tN_t$. Under Calvo pricing, firms that have reset their prices and firms that have not reset them generally face different relative prices. The CES demand system then allocates different quantities across varieties. Because the differentiated varieties are imperfect substitutes, this uneven allocation of production delivers less aggregate Y_t for a given amount of aggregate labor than the no-dispersion allocation. Equivalently, for a given Y_t, Eq. (9) requires $N_t = Y_t\Delta_{H,t}/A_t$, so $\Delta_{H,t} > 1$ raises the labor required to produce that aggregate output.[1]

3.1.2 Pricing Behavior

Home producers face Calvo price rigidity with non-adjustment probability θ. Following the sequence representation in Monacelli [19], the producer's first-order condition can be written as

$$\widetilde{p}_{H,t} = \mathcal{M}\frac{\mathcal{Z}_{H,t}}{\mathcal{K}_{H,t}}. \tag{10}$$

Here $\widetilde{p}_{H,t} \equiv \overline{P}_{H,t}/P_{H,t}$ is the optimal reset price relative to the current Home PPI and $\mathcal{M} \equiv \varepsilon/(\varepsilon-1)$ is the desired producer markup. No employment subsidy is paid to PCP producers in

[1] Relative to the equal-price allocation, low-price varieties receive disproportionately more demand and therefore require more production and labor input, while high-price varieties receive less. Because demand is convex in the relative price, in aggregate the increase in production and labor input at low-price firms exceeds the reduction at high-price firms. Total labor input therefore rises for a given CES aggregate output Y_t, as summarized by $N_t = Y_t\Delta_{H,t}/A_t$ with $\Delta_{H,t} > 1$.

either pass-through environment; hence this markup is not offset. Let $Q_{t,t+k}$ denote the household's nominal stochastic discount factor from t to $t+k$. The auxiliary objects entering the first-order condition are

$$\begin{aligned}\mathcal{K}_{H,t} &\equiv \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k} \left(\prod_{s=1}^{k} \Pi_{H,t+s} \right)^{\varepsilon}, \\ \mathcal{Z}_{H,t} &\equiv \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k} MC^r_{t+k} \left(\prod_{s=1}^{k} \Pi_{H,t+s} \right)^{1+\varepsilon}.\end{aligned} \tag{11}$$

where $MC^r_t \equiv MC_t/P_{H,t}$ denotes real marginal cost and MC_t denotes its nominal counterpart. Marginal cost is the inverse of the marginal product of labor, so that real marginal cost is given by

$$MC^r_t = \frac{W_t}{A_t P_{H,t}},$$

where we assume $A_t = 1$ for all t throughout the paper. Plugging Eq. (6) into the previous expression yields

$$MC^r_t = C_t N_t^{\varphi} \mathcal{G}\left(\frac{S_t}{\Psi_t}, \tau_{M,t} \right). \tag{12}$$

Using $\Pi_{H,t}$, the producer Calvo price-index and dispersion laws are

$$\begin{aligned} 1 &= \theta \Pi_{H,t}^{\varepsilon-1} + (1-\theta)\widetilde{p}_{H,t}^{\,1-\varepsilon}, \\ \Delta_{H,t} &= \theta \Pi_{H,t}^{\varepsilon} \Delta_{H,t-1} + (1-\theta)\widetilde{p}_{H,t}^{\,-\varepsilon}. \end{aligned} \tag{13}$$

With PCP, Foreign producers set prices in Foreign currency, so the Home-currency border acquisition price is $\mathcal{E}_t P^*_{F,t}$. In the Perfect Pass-Through Environment there is no separate local importer-price block and the Home import price therefore satisfies the law of one price,

$$P_{F,t} = \mathcal{E}_t P^*_{F,t}. \tag{14}$$

The same PCP producer behavior at the border is retained in the Imperfect Pass-Through Environment, but the local importer price $P_{F,t}$ is then set separately and need not equal the border acquisition price in the short run.

With LCP, the additional domestic importer sets the tariff-exclusive Home-currency import price $P_{F,t}(\iota)$ subject to Calvo rigidity with non-adjustment probability θ_M. The import-price index is $P_{F,t}^{1-\varepsilon_M} = \int_0^1 P_{F,t}(\iota)^{1-\varepsilon_M} d\iota$, where ε_M is the elasticity of substitution across import varieties.

If an importer can reset at date t, it chooses a tariff-exclusive price $\overline{P}_{F,t}$. The tariff is not part of the importer's acquisition cost; it is applied to the purchaser price downstream. The parameter τ is a time-invariant importer subsidy rate (a tax if negative). The resetting importer therefore solves

$$\max_{\overline{P}_{F,t}} \mathrm{E}_t \sum_{k=0}^{\infty} \theta_M^k Q_{t,t+k} \left[\overline{P}_{F,t} - (1-\tau)\mathcal{E}_{t+k} P^*_{F,t+k} \right] C_{F,t+k|t}, \tag{15}$$

where $C_{F,t+k|t} = (\overline{P}_{F,t}/P_{F,t+k})^{-\varepsilon_M} C_{F,t+k}$ and $Q_{t,t+k}$ is the household nominal stochastic discount factor defined above.

The local import-price index follows

$$P_{F,t}^{1-\varepsilon_M} = \theta_M P_{F,t-1}^{1-\varepsilon_M} + (1-\theta_M)\overline{P}_{F,t}^{\,1-\varepsilon_M}. \tag{16}$$

The same Calvo transition implies

$$\Delta_{F,t} = \theta_M \Pi_{F,t}^{\varepsilon_M} \Delta_{F,t-1} + (1-\theta_M)\widetilde{p}_{F,t}^{\,-\varepsilon_M}. \tag{17}$$

with $\Pi_{F,t} \equiv P_{F,t}/P_{F,t-1}$, $\widetilde{p}_{F,t} \equiv \overline{P}_{F,t}/P_{F,t}$, and $\Delta_{F,t} \equiv \int_0^1 [P_{F,t}(\iota)/P_{F,t}]^{-\varepsilon_M} d\iota$.

Because $P_{F,t}/P_{H,t} = S_t/\Psi_t$, the *ex-tariff* relative price of imports satisfies

$$\frac{S_t}{\Psi_t} = \frac{S_{t-1}}{\Psi_{t-1}} \frac{\Pi_{F,t}}{\Pi_{H,t}}. \tag{18}$$

In the Perfect Pass-Through Environment, $\Psi_t = 1$ and the *ex-tariff* relative price of imports coincides with the border relative price S_t.

Using $\widetilde{p}_{F,t}$, the first-order condition for the importer reset price can be written as

$$\widetilde{p}_{F,t} = \mathcal{M}_M \frac{\mathcal{Z}_{F,t}}{\mathcal{K}_{F,t}}. \tag{19}$$

where $\mathcal{M}_M \equiv (1-\tau)\varepsilon_M/(\varepsilon_M - 1)$ denotes the constant markup for imports. We set the time-invariant importer subsidy rate to $\tau = 1/\varepsilon_M$, which implies $\mathcal{M}_M = 1$ and hence $\Psi = 1$ in the deterministic zero-inflation steady state. Throughout, a model variable without a time subscript denotes its deterministic steady-state value. This normalization is analogous to that in Okano and Eguchi [20].

The subsidy is paid on the importer's border acquisition cost. Since $\int_0^1 C_{F,t}(\iota)d\iota = C_{F,t}\Delta_{F,t}$, its aggregate fiscal cost is

$$\tau \mathcal{E}_t P^*_{F,t} \int_0^1 C_{F,t}(\iota) d\iota = \tau \mathcal{E}_t P^*_{F,t} C_{F,t} \Delta_{F,t}.$$

Accordingly, the total lump-sum transfer in the household budget constraint is

$$T_t = \tau_{M,t} P_{F,t} C_{F,t} - \tau \mathcal{E}_t P^*_{F,t} C_{F,t} \Delta_{F,t}.$$

Here

$$\begin{aligned}
\mathcal{K}_{F,t} &\equiv \widetilde{p}_{F,t}^{\varepsilon_M} \mathrm{E}_t \sum_{k=0}^{\infty} \theta_M^k Q_{t,t+k} C_{F,t+k|t}, \\
\mathcal{Z}_{F,t} &\equiv \frac{\widetilde{p}_{F,t}^{\varepsilon_M}}{P_{F,t}} \mathrm{E}_t \sum_{k=0}^{\infty} \theta_M^k Q_{t,t+k} C_{F,t+k|t} \mathcal{E}_{t+k} P^*_{F,t+k}.
\end{aligned} \tag{20}$$

The first-order price-setting blocks can be summarized by the familiar New Keynesian Phillips-curve form

$$\pi_{H,t} = \beta \mathrm{E}_t \pi_{H,t+1} + \kappa_H mc_t^r, \tag{21}$$

$$\pi_{F,t} = \beta \mathrm{E}_t \pi_{F,t+1} + \kappa_M \psi_t. \tag{22}$$

Here $\pi_{H,t} \equiv \log \Pi_{H,t}$ and $\pi_{F,t} \equiv \log \Pi_{F,t}$. The Phillips-curve slopes are $\kappa_H \equiv (1-\theta)(1-\beta\theta)/\theta$ and $\kappa_M \equiv (1-\theta_M)(1-\beta\theta_M)/\theta_M$. The log deviation of Home real marginal cost is $mc_t^r \equiv \log(MC_t^r/MC^r)$, where MC^r is its deterministic steady-state value. The logarithmic LOOP gap is $\psi_t \equiv \log \Psi_t$. The second Phillips curve uses the fact that the importer's real acquisition cost relative to its own sticky price is the LOOP gap. Equations (21)–(22) are the key nominal price-setting equations for the interpretation below. The tariff directly raises the purchaser price in Eq. (5), but in the LCP importer block it does not enter the importer's acquisition cost in Eq. (15). This separation is what changes the optimal stabilization problem.

3.1.3 Market Clearing

In the Imperfect Pass-Through Environment, Home goods-market clearing is

$$Y_t = (1-\upsilon)\mathcal{G}\left(\frac{S_t}{\Psi_t}, \tau_{M,t}\right)^{\eta} C_t + \upsilon S_t^{\eta}. \tag{23}$$

In the Perfect Pass-Through Environment, $\Psi_t = 1$ for all t, so Eq. (23) reduces to

$$Y_t = (1-\upsilon)\mathcal{G}(S_t, \tau_{M,t})^{\eta} C_t + \upsilon S_t^{\eta}.$$

3.1.4 Perfect and Imperfect Pass-Through Environments

For clarity, PCP and LCP refer to pricing behavior, whereas the complete model specifications are the Perfect and Imperfect Pass-Through Environments. In the Perfect Pass-Through Environment, the Home household is described by Eqs. (1)–(7), the nominal budget constraint stated after Eqs. (4)–(5), and the Home- and Foreign-currency bond-pricing conditions stated after Eq. (7). Complete risk sharing is given by Eq. (8) with $\Psi_t = 1$. The Home production and PCP producer-pricing blocks are Eqs. (9)–(13). The environment is closed by the law-of-one-price condition in Eq. (14) and goods-market clearing in Eq. (23) with $\Psi_t = 1$. There is no independent Home importer price-setting block, so the only first-order sticky-price equation is the producer Phillips curve in Eq. (21).

The Imperfect Pass-Through Environment keeps all of those household, production, and PCP producer-pricing relations but adds the LCP importer block. The risk-sharing condition in Eq. (8) now contains the endogenous LOOP gap Ψ_t. Domestic importers satisfy the problem in Eq. (15), the Calvo import-price index and dispersion laws in Eqs. (16)–(17), and the reset-price conditions in Eqs. (19)–(20). The local relative-import-price law in Eq. (18) and goods-market clearing in Eq. (23) apply with endogenous Ψ_t. Thus the Imperfect Pass-Through Environment contains both the PCP producer block and the LCP importer block. At first order, Home producer-price inflation continues to satisfy Eq. (21), while the additional importer Phillips curve is Eq. (22).

3.2 Ramsey problem

For each pass-through environment, the monetary authority is a social planner that commits at date zero to a complete contingent plan. In either environment, the planner maximizes Eq. (1).

In the Perfect Pass-Through Environment, the choice variables are C_t, S_t, N_t, Y_t, $\Pi_{H,t}$, $\widetilde{p}_{H,t}$, $\mathcal{K}_{H,t}$, $\mathcal{Z}_{H,t}$, $\Delta_{H,t}$, and i_t for $t \geq 0$. The constraints are Eqs. (3), (6), (7), and (8) with $\Psi_t = 1$, Eqs. (9)–(13), Eq. (14), Eq. (23) with $\Psi_t = 1$, and the ZLB conditions in Eq. (24).

In the Imperfect Pass-Through Environment, the choice variables are C_t, S_t, N_t, Y_t, $\Pi_{H,t}$, $\widetilde{p}_{H,t}$, $\mathcal{K}_{H,t}$, $\mathcal{Z}_{H,t}$, $\Delta_{H,t}$, Ψ_t, $\Pi_{F,t}$, $\widetilde{p}_{F,t}$, $\mathcal{K}_{F,t}$, $\mathcal{Z}_{F,t}$, $\Delta_{F,t}$, and i_t for $t \geq 0$. The constraints are Eqs. (3) and (5), Eqs. (6) and (7), Eq. (8), Eqs. (9)–(13), Eqs. (16), (17), (19), (20), and (18), Eq. (23), and the ZLB condition in Eq. (24).

The feasibility, non-negativity, and complementary-slackness conditions are

$$i_t \geq 0, \qquad \mu_{i,t} \geq 0, \qquad \mu_{i,t} i_t = 0. \tag{24}$$

Here $\mu_{i,t}$ denotes the Kuhn–Tucker multiplier on the ZLB constraint. The tariff process is exogenous and tariff revenue is rebated lump sum. The Lagrangians and first-order conditions for the two pass-through environments are reported in On-line Appendix OA; the first-order conditions are not repeated in the main text. The numerical policy paths are obtained from these nonlinear Ramsey first-order-condition systems. The second-order Welfare Cost Functions below are reporting and interpretation devices rather than policy objectives; when the ZLB binds, OccBin implements the complementary-slackness restriction in the numerical solution.

3.3 Calibration and shocks

The benchmark calibration follows Monacelli [19] wherever the model has a direct counterpart. Table 2 summarizes the benchmark parameterization.

The tariff process is

$$\tau_{M,t} = \rho_\tau \tau_{M,t-1} + \epsilon_{\tau,t}.$$

We set the one-time innovation $\epsilon_{\tau,0} = 0.20$, following Monacelli [19], as our benchmark. Thus the import tariff rises by 20 percentage points on impact. The steady-state tariff is zero. The deterministic quarterly nominal rate is $1/\beta - 1 = 1.0101$ percent, approximately four percent

Table 2: Benchmark calibration

Parameter	Value	Interpretation
β	0.99	Discount factor
υ	0.3	Openness / Home Bias parameter
φ	2.2	Inverse Frisch elasticity
η	1.5	Home–Foreign substitution elasticity
ε	3.8	Elasticity across producer varieties
θ	0.8	Producer Calvo parameter
ε_M	3.8	Elasticity across importer varieties
θ_M	0.8	Importer Calvo parameter
ρ_τ	0.7	Tariff persistence

annualized. Hence a response of -101.01 basis points relative to steady state places the actual nominal rate exactly at the ZLB.

3.4 Welfare Cost Function

To make the planner's stabilization objectives transparent, we report the second-order Benigno–Woodford welfare-cost representation. In superscripts, PPT and IPT denote the Perfect and Imperfect Pass-Through Environments, respectively. Let $y_t \equiv \log(Y_t/Y)$ and $s_t \equiv \log S_t$. Suppressing terms that are independent of the Home policy allocation, under Perfect Pass-Through,

$$\mathcal{L}_t^{SOE,PPT} \equiv \frac{1}{2}\left[\widehat{\Lambda}_y^{SOE,PPT}(y_t - \widehat{y}_t^{w,SOE,PPT})^2 + \widehat{\Lambda}_s^{SOE,PPT}(s_t - \widehat{s}_t^{w,SOE,PPT})^2 + \Lambda_{\pi,H}^{SOE,PPT}\pi_{H,t}^2\right],$$

whereas under Imperfect Pass-Through it can be written as

$$\begin{aligned}\mathcal{L}_t^{SOE,IPT} \equiv\ & \frac{1}{2}\Big[\widehat{\Lambda}_y^{SOE,IPT}(y_t - \widehat{y}_t^{w,SOE,IPT})^2 + \widehat{\Lambda}_\psi^{SOE,IPT}(\psi_t - \widehat{\psi}_t^{w,SOE,IPT})^2 \\ & + \widehat{\Lambda}_s^{SOE,IPT}(s_t - \widehat{s}_t^{w,SOE,IPT})^2 + \Lambda_{\pi,H}^{SOE,IPT}\pi_{H,t}^2 + \Lambda_{\pi,F}^{SOE,IPT}\pi_{F,t}^2\Big].\end{aligned}$$

Here the hatted Λ coefficients are the completed-square welfare weights associated with y_t, s_t, and, under Imperfect Pass-Through, ψ_t in the expressions above. The corresponding hatted Welfare Relevant reference values are $\widehat{y}_t^{w,SOE,\cdot}$, $\widehat{s}_t^{w,SOE,\cdot}$, and $\widehat{\psi}_t^{w,SOE,IPT}$. Their exact symbolic definitions are given in Appendix A.2.

The contrast between the two expressions shows how the open-economy relative-price externality enters the welfare criterion. Under Perfect Pass-Through, the LOOP holds identically, $\Psi_t = 1$, so the relative-price externality is carried by the Welfare Relevant Ex-tariff Relative Price of Imports Gap $s_t - \widehat{s}_t^{w,SOE,PPT}$. Under Imperfect Pass-Through, $P_{F,t}/P_{H,t} = S_t/\Psi_t$, and the welfare criterion contains both the Welfare Relevant Ex-tariff Relative Price of Imports Gap $s_t - \widehat{s}_t^{w,SOE,IPT}$ and the Welfare Relevant LOOP Gap Deviation $\psi_t - \widehat{\psi}_t^{w,SOE,IPT}$. A nominal exchange-rate movement can therefore change the ex-tariff relative price S_t while, because the local-currency import price $P_{F,t}$ adjusts only gradually, also generating a movement in the LOOP gap Ψ_t. The same relative-price externality is thus manifested jointly through s_t and ψ_t under Imperfect Pass-Through. The LOOP-gap movement additionally creates an importer-pricing distortion and, through the importer Calvo block, import-price dispersion.

Complete international risk sharing together with Home goods-market clearing yield

$$s_t = \frac{1}{\eta + (1-\eta)(1-\upsilon)^2}y_t - \frac{\upsilon(1-\upsilon)(\eta-1)}{\eta + (1-\eta)(1-\upsilon)^2}\tau_{M,t} + \frac{\upsilon(1-\upsilon)(\eta-1)}{\eta + (1-\eta)(1-\upsilon)^2}\psi_t,$$

which follows from log-linearizing Eqs. (8) and (23) and permits s_t to be eliminated from this same SOE welfare criterion. Under Perfect Pass-Through, $\Psi_t = 1$, so the welfare-relevant movement in

s_t is absorbed into the Welfare Relevant Output term. The same period welfare cost can therefore be written as

$$\mathcal{L}_t^{SOE,PPT} \equiv \frac{1}{2}\left[\Lambda_y^{SOE,PPT}(y_t - y_t^{w,SOE,PPT})^2 + \Lambda_{\pi,H}^{SOE,PPT}\pi_{H,t}^2\right], \tag{25}$$

whereas under Imperfect Pass-Through the same elimination of s_t leaves the independently moving LOOP gap generated by sticky local import pricing. The same period welfare cost can be written as

$$\begin{aligned}\mathcal{L}_t^{SOE,IPT} \quad \equiv \quad & \frac{1}{2}\Big[\Lambda_y^{SOE,IPT}(y_t - y_t^{w,SOE,IPT})^2 + \Lambda_\psi^{SOE,IPT}(\psi_t - \psi_t^{w,SOE,IPT})^2 \\ & +\Lambda_{\pi,H}^{SOE,IPT}\pi_{H,t}^2 + \Lambda_{\pi,F}^{SOE,IPT}\pi_{F,t}^2\Big].\end{aligned} \tag{26}$$

The objects $y_t^{w,SOE,PPT}$ and $y_t^{w,SOE,IPT}$ are the (Logarithmic) Welfare Relevant Outputs in the Perfect Pass-Through and Imperfect Pass-Through Environments, respectively, and $y_t - y_t^{w,SOE,PPT}$ and $y_t - y_t^{w,SOE,IPT}$ are the corresponding Welfare Relevant Output Gaps. Under the Imperfect Pass-Through Environment, $\psi_t^{w,SOE,IPT}$ is the (Logarithmic) Welfare Relevant LOOP Gap and $\psi_t - \psi_t^{w,SOE,IPT}$ is the Welfare Relevant LOOP Gap Deviation. The exact definitions of $y_t^{w,SOE,PPT}$, $y_t^{w,SOE,IPT}$, and $\psi_t^{w,SOE,IPT}$ are given in Eqs. (A.6), (A.7), and (A.8), respectively. Thus Eq. (25) contains no separate relative-price term, while Eq. (26) retains the Welfare Relevant LOOP Gap Deviation and the import-inflation term. These are the reduced-form traces of the fact that, before the SOE reduction, the relative-price externality is carried by s_t alone under Perfect Pass-Through but jointly by s_t and ψ_t under Imperfect Pass-Through.

For the benchmark tariff experiment, the exact symbolic definition of the Welfare Relevant LOOP Gap after the SOE reduction is given in Eq. (A.8). Under the benchmark calibration, it reduces to

$$\psi_t^{w,SOE,IPT} = 0.765\,\tau_{M,t}. \tag{27}$$

Thus the tariff moves the welfare-relevant reference value for the LOOP gap; the planner does not attempt to stabilize ψ_t at zero independently of the other welfare-relevant variables. The coefficient in Eq. (27) is below unity because a larger LOOP-gap correction compresses the importer's tariff-exclusive markup and, through Eq. (22), puts upward pressure on ex-tariff import inflation and generates importer-price dispersion. These costs make the Welfare Relevant LOOP Gap rise by less than one-for-one with the tariff.

To interpret why the tariff shifts this welfare-relevant reference value, recall that $S_t \equiv \mathcal{E}_t P_{F,t}^*/P_{H,t}$. The *cum-tariff* price of Foreign goods relative to Home goods is

$$\frac{(1+\tau_{M,t})P_{F,t}}{P_{H,t}} = \frac{1+\tau_{M,t}}{\Psi_t}S_t. \tag{28}$$

Combining Eq. (28) with Eqs. (4)–(5) gives the Home household's relative demand for Foreign and Home goods:

$$\begin{aligned}\frac{C_{F,t}}{C_{H,t}} &= \frac{\upsilon}{1-\upsilon}\left[\frac{(1+\tau_{M,t})P_{F,t}}{P_{H,t}}\right]^{-\eta} \\ &= \frac{\upsilon}{1-\upsilon}\left(\frac{1+\tau_{M,t}}{\Psi_t}S_t\right)^{-\eta}.\end{aligned} \tag{29}$$

Equations (28) and (29) describe how the tariff gives the planner a reason to use the relative-price externality just identified. For given S_t and Ψ_t, an increase in the import tariff raises the *cum-tariff* relative price of Foreign goods and lowers $C_{F,t}/C_{H,t}$. In the Ramsey equilibrium discussed below, the tariff shock lowers S_t. Monetary accommodation induces a depreciation and makes this decline in S_t smaller than under a less accommodative path. Holding Ψ_t fixed, the resulting higher S_t raises the relative price of Foreign goods in Eq. (29), shifts expenditure toward Home goods, and supports Home output and income. Taken by itself, however, the same higher S_t also raises the household-facing *cum-tariff* relative import price in Eq. (28).

Imperfect pass-through allows the planner to separate these two effects more effectively. Because the local-currency import price $P_{F,t}$ adjusts only gradually, the same depreciation raises the border acquisition price $\mathcal{E}_t P^*_{F,t}$ relative to $P_{F,t}$ and therefore raises $\Psi_t = \mathcal{E}_t P^*_{F,t}/P_{F,t}$. Since Ψ_t enters the denominator of $(1+\tau_{M,t})S_t/\Psi_t$, the increase in Ψ_t lowers the household-facing relative import price relative to what the same S_t path would imply and partially offsets the direct tariff-induced purchaser-price increase. Thus the S_t response and the Ψ_t response perform different roles: keeping S_t higher than under a less accommodative path supports Home activity, whereas the rise in Ψ_t limits the purchaser-price increase. At the same time, a nonzero LOOP gap is itself a distortion. An increase in Ψ_t compresses the importer markup below its desired level, and a higher ψ_t puts upward pressure on ex-tariff import inflation through Eq. (22), generating importer-price dispersion. Monetary accommodation also affects Home real marginal cost and Home PPI inflation. The Welfare Cost Function therefore summarizes the second-best trade-off among Home activity, the relative-price externality, the LOOP-gap distortion, and the two sticky inflation rates.

Equation (26) also makes clear that the importance of a welfare-relevant variable cannot be inferred from its weight alone. Each term is the product of a positive weight and the squared deviation of the variable from its Welfare Relevant reference value. A variable with a relatively small weight can therefore remain important when the shock generates a sufficiently large Welfare Relevant gap or deviation. The weight $\Lambda_{\psi}^{SOE,IPT}$ on the Welfare Relevant LOOP Gap Deviation is only 0.49, far below the weights on Home PPI inflation and ex-tariff import inflation. This small weight measures the curvature of the welfare cost around the Welfare Relevant LOOP Gap; it does not measure the size of the tariff-induced shift in that value. As Eq. (A.8), summarized by Eq. (27), shows, the tariff itself shifts $\psi_t^{w,SOE,IPT}$ substantially. A tariff increase can therefore create a large Welfare Relevant LOOP Gap Deviation even when $\Lambda_{\psi}^{SOE,IPT}$ is small. Because monetary accommodation and the associated depreciation raise ψ_t when the local import price is sticky, the planner has an incentive to move the equilibrium LOOP gap toward this tariff-shifted Welfare Relevant LOOP Gap. The planner does not eliminate the deviation because doing so would alter the Welfare Relevant Output Gap and the two sticky-inflation costs. Table 3 reports the benchmark numerical weights entering the reduced SOE Welfare Cost Functions in Eqs. (25) and (26).

Table 3: SOE welfare weights: benchmark values

Environment	Welfare Relevant Variable	Weight	Value
Perfect Pass-Through	$y_t - y_t^{w,SOE,PPT}$	$\Lambda_y^{SOE,PPT}$	1.68
	$\pi_{H,t}$	$\Lambda_{\pi,H}^{SOE,PPT}$	39.87
Imperfect Pass-Through	$y_t - y_t^{w,SOE,IPT}$	$\Lambda_y^{SOE,IPT}$	1.68
	$\psi_t - \psi_t^{w,SOE,IPT}$	$\Lambda_{\psi}^{SOE,IPT}$	0.49
	$\pi_{H,t}$	$\Lambda_{\pi,H}^{SOE,IPT}$	39.87
	$\pi_{F,t}$	$\Lambda_{\pi,F}^{SOE,IPT}$	26.57

3.5 Nominal Interest Rate Responses

Figure 1 compares the benchmark SOE responses. The difference in the policy rate is immediate: the Perfect Pass-Through Environment raises i_t by 20.04 basis points on impact, whereas the Imperfect Pass-Through Environment lowers it by 70.13 basis points. In Figure 1, the blue lines are the impulse response functions (IRFs) under the Imperfect Pass-Through Environment, while the orange lines are those under the Perfect Pass-Through Environment.

In the Perfect Pass-Through Environment, an increase in the import tariff raises the *cum-tariff*

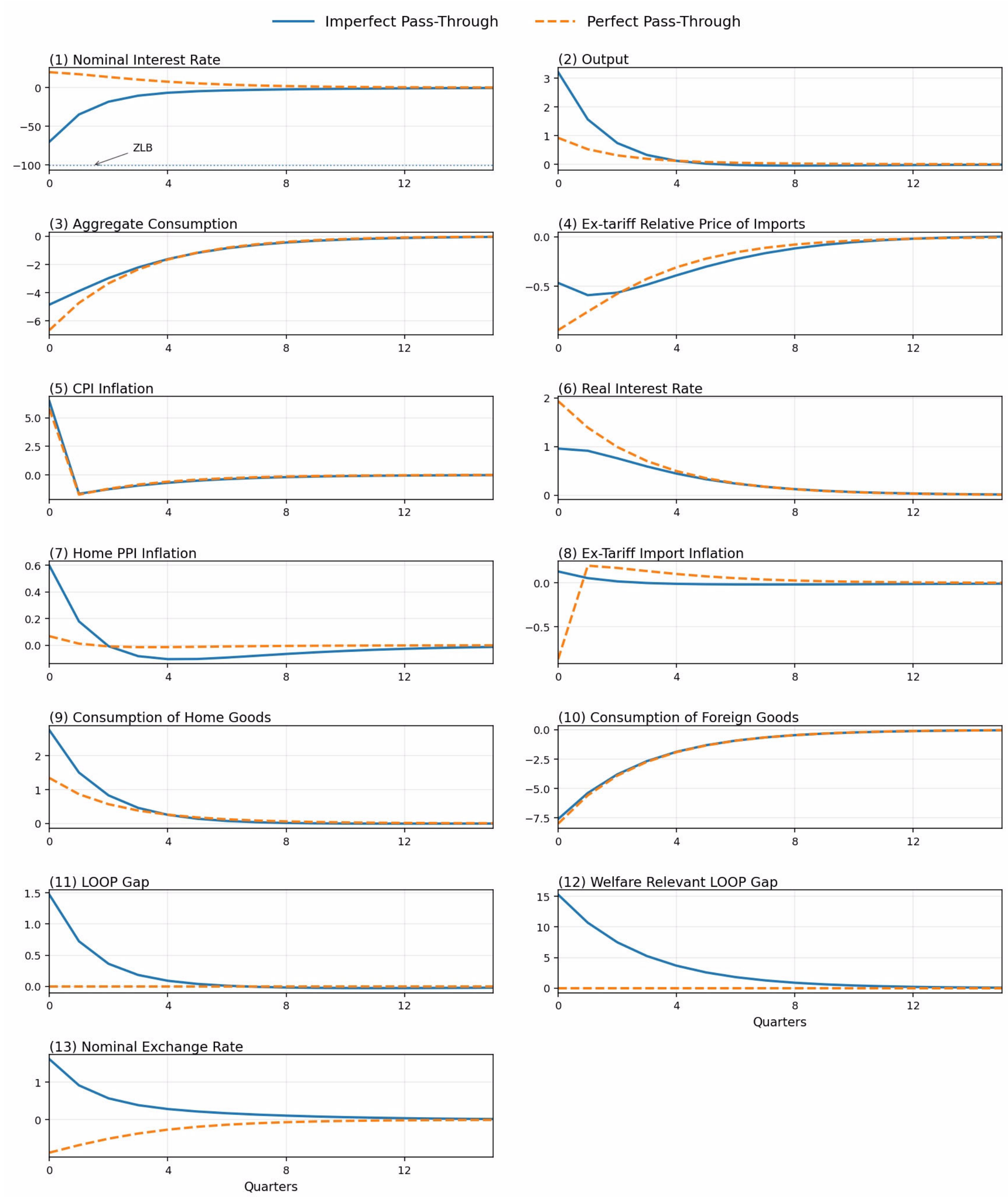

Figure 1: SOE benchmark: (a) Perfect versus Imperfect Pass-Through. The Perfect Pass-Through Environment is shown in orange and the Imperfect Pass-Through Environment in blue. The policy-rate panel reports the deviation of the nominal interest rate from its deterministic steady state in basis points; the horizontal ZLB line is at -101.01 basis points. The other panels report deviations from the deterministic steady state in percentage points. Panel 4 reports the Ex-tariff Relative Price of Imports $P_{F,t}/P_{H,t}$, Panel 8 ex-tariff import inflation $\pi_{F,t}$, Panel 11 the LOOP gap, Panel 12 the Welfare Relevant LOOP Gap $\psi_t^{w,SOE,IPT}$, and Panel 13 the equilibrium nominal exchange rate. The tariff innovation is 20 percentage points with persistence 0.7. Panels 5 and 6 report Home CPI inflation and the Real Consumption Interest Rate (Home), $(1+i_t)/\mathrm{E}_t\Pi_{t+1}$, respectively.

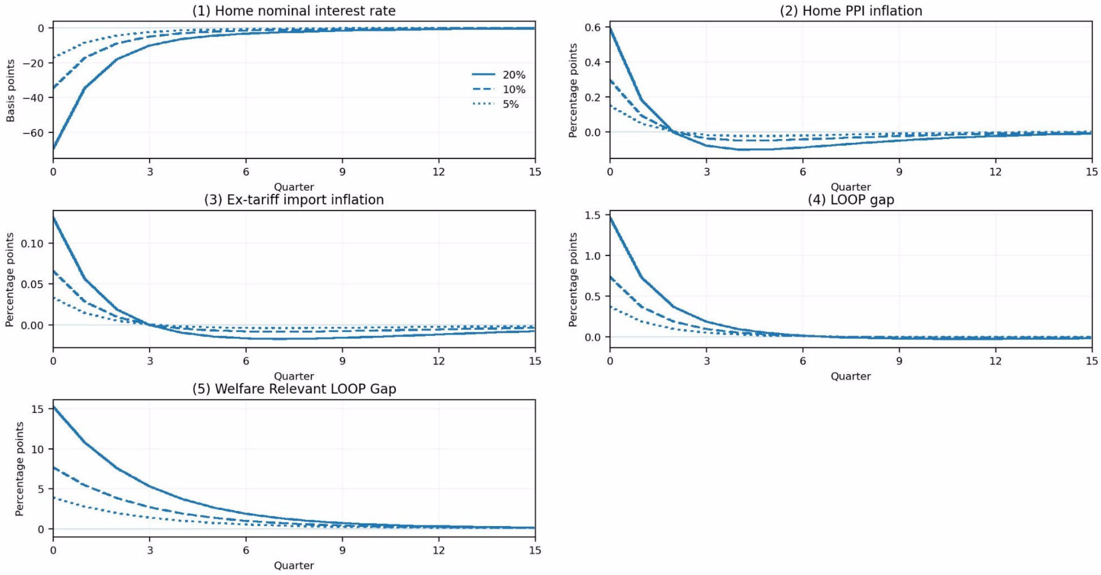

Figure 1: SOE Imperfect Pass-Through Environment: (b) tariff-size comparison. Each panel overlays the IRFs for Home import-tariff innovations of 20, 10, and 5 percentage points. Panels report, respectively, the Home nominal interest rate, Home PPI inflation $\pi_{H,t}$, ex-tariff import inflation $\pi_{F,t}$, the LOOP gap ψ_t, and the Welfare Relevant LOOP Gap $\psi_t^{w,SOE,IPT}$. Each of the three paths is computed separately from the same nonlinear Ramsey first-order-condition system using innovations of 0.20, 0.10, and 0.05, respectively. The ZLB is slack for all three experiments.

price of Foreign goods relative to Home goods and induces expenditure switching toward Home goods. The resulting increase in demand for Home goods raises Home production and labor input. Through Eq. (12), this expansion raises Home producer real marginal cost; through Eq. (21), the higher real marginal cost puts upward pressure on Home PPI inflation. Because stabilizing PPI inflation is the most heavily weighted objective in the Welfare Cost Function, the Ramsey planner raises the nominal interest rate (Panel 1, Fig. 1). Home PPI inflation therefore remains close to zero (Panel 7). Moreover, because $\Psi_t = 1$ under Perfect Pass-Through, relative-price adjustment operates entirely through the external relative price S_t.

The relative-price externality operates differently in the Imperfect Pass-Through Environment. As Eq. (28) shows, for given S_t and Ψ_t a higher import tariff raises the *cum-tariff* price of Foreign goods relative to Home goods. In the equilibrium response, the tariff shock lowers S_t. The Home depreciation induced by the Ramsey rate cut (Panel 13) makes this decline in S_t smaller than under a less accommodative path. Holding Ψ_t fixed, this higher S_t shifts expenditure toward Home goods and supports Home demand, output, and income. It does not offset the tariff-induced purchaser-price increase: taken by itself, the higher S_t raises $(1 + \tau_{M,t})S_t/\Psi_t$ relative to the less accommodative path.

The purchaser-price offset comes from the distinct pass-through channel. Because $P_{F,t}$ is sticky, the border acquisition price $\mathcal{E}_t P^*_{F,t}$ rises relative to $P_{F,t}$ and Ψ_t rises (Panel 11). Since Ψ_t enters the denominator of the household-facing price, the higher Ψ_t lowers S_t/Ψ_t relative to what the same S_t path would imply and partially offsets the direct tariff-induced increase in $(1+\tau_{M,t})S_t/\Psi_t$. The Ramsey easing therefore combines two distinct relative-price effects: the S_t channel supports Home activity by limiting the fall in S_t, while the Ψ_t channel limits the purchaser-price increase. Through this joint adjustment, the planner can exploit the relative-price externality more strongly when pass-through is incomplete.

This exploitation is not costless. A higher ψ_t puts upward pressure on ex-tariff import inflation $\pi_{F,t}$ through Eq. (22) (Panel 8), and resetting importers raise $P_{F,t}$. Monetary accommodation also raises Home real marginal cost and Home PPI inflation (Panel 7). In the benchmark allocation, $P_{H,t}$ rises by more than $P_{F,t}$, so the tariff-exclusive local relative import price $P_{F,t}/P_{H,t} = S_t/\Psi_t$ falls (Panel 4). The Ramsey planner therefore trades the Home-income benefit from keeping S_t higher and the purchaser-price benefit from the associated rise in Ψ_t against the Welfare Relevant Output Gap and the two sticky-inflation costs.

The corresponding household allocation can be checked directly. In the first-order counterfactual allocation, the impact log deviation of $C_{F,t}/C_{H,t}$ is -29.58 percent under the zero policy-rate response and -29.30 percent under the optimal Ramsey response. Monetary accommodation therefore raises the impact relative-consumption ratio by 0.27 percentage points (see Appendix C.1). By Eq. (29), this means that the optimal rate cut leaves the Home household facing a lower *cum-tariff* relative import price than under the zero policy-rate response. Since the easing keeps S_t higher than under that less accommodative counterfactual, the net purchaser-price offset is generated by the accompanying increase in Ψ_t, not by the S_t channel. The quantity comparison is therefore a direct check on the separation of the two relative-price effects.

The structural welfare weights place much greater weight on the sticky-inflation terms than on the Welfare Relevant LOOP Gap Deviation. In the Welfare Cost Function, $\Lambda_{\psi}^{SOE,IPT}$ is smaller than both $\Lambda_{\pi,H}^{SOE,IPT}$ and $\Lambda_{\pi,F}^{SOE,IPT}$. Under the benchmark calibration, the corresponding values are 0.49, 39.87, and 26.57, respectively. Nevertheless, the tariff-induced displacement of the Welfare Relevant LOOP Gap is quantitatively large. On impact, $\psi_0^{w,SOE,IPT} = 15.29$ percent while the equilibrium LOOP gap rises only to 1.47 percent, leaving $\psi_0 - \psi_0^{w,SOE,IPT} = -13.82$ percent (Panels 11–12, Fig. 1). Figure 1 shows that the same pattern holds across tariff sizes: larger tariff shocks generate larger Home rate cuts and larger movements in both the LOOP gap and the Welfare Relevant LOOP Gap. The weight comparison and the IRFs alone do not identify the mechanism. That mechanism is characterized by the relative-price relation in Eq. (28), the relative-demand response, and the two sticky-price Phillips curves. We therefore use the counterfactual decomposition in Appendix C.1 as supporting accounting evidence: it holds the nominal rate at its steady-state path and recomputes the private-equilibrium allocation. We also evaluate the original nonlinear lifetime utility in Eq. (1) directly along the same zero-response, half-optimal, and optimal allocation paths. The resulting total-welfare ranking is the same as the quadratic comparison: lifetime utility is highest under the optimal Ramsey response, next under the half-optimal response, and lowest under the zero-response path; Appendix Table 11 reports the exact utility evaluation. This total-welfare cross-check is kept separate from the completed-square source accounting: exact nonlinear welfare is not assigned source-specific shares.

The accounting diagnostic shows that the optimal Ramsey rate path lowers the discounted Welfare Relevant LOOP Gap Deviation component of the loss from 0.0112 to 0.0096, a reduction of 0.0016. Over the same comparison, the Welfare Relevant Output Gap, Home-PPI-inflation, and import-inflation components rise by 0.0011 in total. The reduction in the Welfare Relevant LOOP Gap Deviation component is therefore about 1.5 times as large as the combined increase in those three components, and aggregate discounted quadratic welfare cost falls by 0.0005, or 4.75 percent. This accounting comparison is consistent with the mechanism identified above: the optimal policy exploits the relative-price externality through the joint adjustment of S_t and the endogenous LOOP gap, using nominal depreciation to keep S_t higher for Home-output and Home-income support while the associated increase in Ψ_t limits the domestic import-price response. It accepts larger realized costs in the output-gap and sticky-inflation terms. The comparison is not a causal decomposition of exact nonlinear welfare, nor a ranking of the structural stabilization weights. The inflation rates remain the higher-weight stabilization terms.

The associated equilibrium adjustment displays the same welfare trade-off. The tariff shock lowers the external relative price S_t, while the nominal depreciation induced by Ramsey easing (Panel 13) makes that decline smaller than under a less accommodative path. The resulting higher S_t supports Home demand, output, and income but, taken alone, raises the household-facing *cum-tariff* relative import price relative to that counterfactual. Because the local import price is sticky, the depreciation also raises the LOOP gap Ψ_t; this second effect works in the opposite direction in

$(1+\tau_{M,t})S_t/\Psi_t$ and partially offsets the tariff-induced purchaser-price increase. In logs, ψ_t moves toward the Welfare Relevant LOOP Gap (Panels 11–12). At the same time, Eq. (22) implies that the higher ψ_t puts upward pressure on ex-tariff import inflation $\pi_{F,t}$ (Panel 8), while monetary accommodation raises Home real marginal cost and Home PPI inflation (Panel 7). The optimal interest-rate cut therefore uses the S_t response for Home-income support and the associated Ψ_t response for the purchaser-price offset, while accepting the resulting sticky-price costs.

4 Two-Country Model

4.1 The Model

The small-open-economy model in Sec. 3 takes the Foreign economy as given. We now endogenize that economy and embed the SOE structure in a unit-mass two-country world. Country size is represented by household mass: households are indexed by $j \in [0,1]$, with $j \in [0,\nu)$ in country H and $j \in [\nu, 1]$ in country F. Because steady-state per-capita output is common across the two countries, ν is also Home's share of steady-state world output and is therefore calibrated to Home's share of world gross domestic product (GDP). Households are identical within each country, so we continue to write the Home allocation in representative-household form. The Home household retains the preferences in Eq. (1), while the Foreign representative household has lifetime utility

$$\mathcal{U}^* \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t^* - \frac{(N_t^*)^{1+\varphi}}{1+\varphi} \right).$$

Here C_t^* and N_t^* denote Foreign composite consumption and labor, respectively. The Foreign household maximizes $\mathcal{U}^*$ subject to the exact starred counterparts of the Home household constraints and optimality conditions.

The two-country extension makes endogenous the Foreign objects that were external to the SOE transmission mechanism. We retain the SOE notation for Home. Country F now has its own consumption, labor, output, producer prices, Calvo price dispersion, and monetary policy. Each country is specialized in a continuum of differentiated goods produced with the same technology and producer Calvo structure as in Sec. 3.1. Home and Foreign have separate monetary authorities with nominal interest rates i_t and i_t^*, respectively, and each government may levy an import tariff, $\tau_{M,t}$ or $\tau_{M,t}^*$. Tariff revenue is rebated lump sum in the country that collects it. Complete international financial markets link the two national household problems and impose international risk sharing. Hence a Home policy or tariff change can alter the Foreign allocation and, through the common world equilibrium, feed back to Home.

Home bias is introduced by scaling each country's expenditure weight on partner-country goods by the common openness parameter υ. Given the country sizes above, the Home and Foreign expenditure weights are $\omega_F = \upsilon(1-\nu)$, $\omega_H = 1-\omega_F$, $\omega_H^* = \upsilon\nu$, and $\omega_F^* = 1-\omega_H^*$. Here ω_F is Home's expenditure weight on Foreign goods and ω_H is Home's expenditure weight on Home goods. Likewise, ω_H^* is Foreign's expenditure weight on Home goods and ω_F^* is Foreign's expenditure weight on Foreign goods. Thus ω_F and ω_H^* are the bilateral import weights, whereas ω_H and ω_F^* are the domestic-good weights. For $\upsilon < 1$, the bilateral import weights are smaller than the corresponding partner-country economic-size shares, generating home bias.

These expenditure weights enter the two national consumption baskets. Home consumption is

$$C_t = \left[\omega_H^{1/\eta} C_{H,t}^{(\eta-1)/\eta} + \omega_F^{1/\eta} C_{F,t}^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}, \tag{30}$$

whereas Foreign consumption is

$$C_t^* = \left[(\omega_F^*)^{1/\eta} (C_{F,t}^*)^{(\eta-1)/\eta} + (\omega_H^*)^{1/\eta} (C_{H,t}^*)^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}.$$

Here $C_{H,t}$ and $C_{F,t}$ denote Home consumption of Home and Foreign goods, respectively, while $C^*_{F,t}$ and $C^*_{H,t}$ denote Foreign consumption of Foreign and Home goods, respectively. Bilateral goods-market clearing therefore replaces the SOE assumption that Foreign demand can be treated as an external object.

The Perfect–Imperfect Pass-Through distinction is extended symmetrically across the two trade directions. We retain exactly the SOE definition $S_t \equiv \mathcal{E}_t P^*_{F,t}/P_{H,t}$ for the Home international relative price. In the Perfect Pass-Through Environment, producer prices obey PCP and the law of one price holds for traded goods in both directions: $P_{F,t} = \mathcal{E}_t P^*_{F,t}$ for Foreign goods sold in Home and $P_{H,t} = \mathcal{E}_t P^*_{H,t}$ for Home goods sold in Foreign. There is no independent local importer-price block. In the Imperfect Pass-Through Environment, the same PCP producer blocks remain in place and local importers additionally use LCP. The Home-currency price of Foreign goods, $P_{F,t}$, and the Foreign-currency price of Home goods, $P^*_{H,t}$, are then subject to Calvo rigidity. Their inflation rates are $\pi_{F,t}$ and $\pi^*_{H,t}$, respectively, and each country has its own LOOP gap and importer-price dispersion. With $\Psi_t \equiv \mathcal{E}_t P^*_{F,t}/P_{F,t}$, the Home local tariff-exclusive relative import price is $P_{F,t}/P_{H,t} = S_t/\Psi_t$. The Foreign LOOP gap is $\Psi^*_t \equiv (P_{H,t}/\mathcal{E}_t)/P^*_{H,t}$, and P^*_t denotes the Foreign CPI. For the welfare representation below, $s_t \equiv \log S_t$, $\psi_t \equiv \log \Psi_t$, and $\psi^*_t \equiv \log \Psi^*_t$, so $\log(P_{F,t}/P_{H,t}) = s_t - \psi_t$. Stars denote Foreign-country or Foreign-currency objects, while the H and F subscripts identify the origin of the good.

This construction makes explicit the step from the SOE to the two-country model. A Home tariff first changes Home absorption, the Home LOOP gap, and Home importer marginal cost. Because Foreign demand and production are now endogenous, the same disturbance also changes Foreign consumption, output, producer and importer prices, and the Foreign policy rate. Those Foreign responses alter Home export demand and international relative prices and therefore feed back into the Home allocation. The resulting spillover and spillback are thus generated inside the common two-country equilibrium rather than appended as reduced-form disturbances.

4.2 National Ramsey policy under commitment

The baseline two-country policy game follows the national Ramsey approach of Faia and Monacelli [23]. Complete international risk sharing and the relevant goods-market relations are imposed before the national first-order conditions are formed. Hence the international relative price is eliminated before national differentiation and is not a choice variable of either national planner.

In the Perfect Pass-Through Environment, the Home planner chooses the paths of C_t, the CPI–PPI ratio $P_t/P_{H,t}$, N_t, Y_t, $\Pi_{H,t}$, $\widetilde{p}_{H,t}$, $\mathcal{K}_{H,t}$, $\mathcal{Z}_{H,t}$, $\Delta_{H,t}$, and i_t. In the Imperfect Pass-Through Environment, the Home importer-price variables $\Pi_{F,t}$, $\widetilde{p}_{F,t}$, $\mathcal{K}_{F,t}$, $\mathcal{Z}_{F,t}$, and $\Delta_{F,t}$ are additionally included in the Home choice set. In particular, S_t is not a Home Ramsey choice variable.

The Foreign planner solves the exact Home–Foreign mirror problem. In the Perfect Pass-Through Environment, it chooses Foreign consumption, the Foreign CPI–PPI ratio, labor, output, the Foreign producer-price block, Foreign producer-price dispersion, and i^*_t. In the Imperfect Pass-Through Environment, its own local importer-price block is additionally included in the Foreign choice set. No additional international-relative-price variable is introduced into the Foreign choice set.

The Home social planner commits to a complete contingent Home plan and maximizes $\mathcal{U}$, taking the complete contingent Foreign plan as given and subject to the common nonlinear two-country private-equilibrium system. The Foreign social planner symmetrically maximizes $\mathcal{U}^*$, taking the Home contingent plan as given and subject to the same common private-equilibrium system. The nominal interest rates i_t and i^*_t are choice variables and are subject to the ZLB in Home and Foreign, respectively.

The two nonlinear national Ramsey problems and the common private-equilibrium conditions are solved jointly at their mutual best response. This Nash–Ramsey fixed point generates all benchmark two-country policy paths reported below. The national Ramsey Lagrangians and first-order conditions, including the ZLB Kuhn–Tucker conditions, are reported in On-line Appendix OA. As in the SOE, the second-order welfare loss functions are not solved to obtain the policy allocation; they are introduced only when interpreting the nominal-interest-rate impulse responses.

4.3 Welfare Cost Function

As in the SOE, the nonlinear national Ramsey problems in Sec. 4.2 are the problems actually solved. The second-order welfare representation is used only to identify the Welfare Relevant variables and to interpret the nominal-interest-rate responses. In the Perfect Pass-Through Environment, the Home period welfare cost is

$$\mathcal{L}_{H,t}^{PPT} \equiv \frac{1}{2}\left[\Lambda_y^{PPT}(y_t - y_t^{w,PPT})^2 + \Lambda_s^{PPT}(s_t - s_t^{w,PPT})^2 + \Lambda_{\pi,H}^{PPT}\pi_{H,t}^2\right], \tag{31}$$

whereas in the Imperfect Pass-Through Environment it is

$$\begin{aligned}\mathcal{L}_{H,t}^{IPT} \quad \equiv \quad & \frac{1}{2}\Big[\Lambda_y^{IPT}(y_t - y_t^{w,IPT})^2 + \Lambda_\psi^{IPT}(\psi_t - \psi_t^{w,IPT})^2 + \Lambda_s^{IPT}(s_t - s_t^{w,IPT})^2 \\ & +\Lambda_{\psi^*}^{IPT}(\psi_t^* - \psi_t^{*,w,IPT})^2 + \Lambda_{\pi,H}^{IPT}\pi_{H,t}^2 + \Lambda_{\pi,F}^{IPT}\pi_{F,t}^2 \\ & +\Lambda_{\pi_H^*}^{IPT}(\pi_{H,t}^*)^2\Big]. \end{aligned} \tag{32}$$

In Eqs. (31)–(32), $y_t^{w,\cdot}$ is the (Logarithmic) Welfare Relevant Output and $y_t - y_t^{w,\cdot}$ is the Welfare Relevant Output Gap. Likewise, $s_t^{w,\cdot}$ is the (Logarithmic) Welfare Relevant Ex-tariff Relative Price of Imports and $s_t - s_t^{w,\cdot}$ is the Welfare Relevant Ex-tariff Relative Price of Imports Gap. In the Imperfect Pass-Through Environment, $\psi_t^{w,IPT}$ is the (Logarithmic) Home Welfare Relevant LOOP Gap and $\psi_t - \psi_t^{w,IPT}$ is the Home Welfare Relevant LOOP Gap Deviation; the starred objects are the corresponding Foreign variables. Under Perfect Pass-Through, $y_t^{w,PPT}$ and $s_t^{w,PPT}$ are defined in Eqs. (A.14) and (A.15), respectively. Under Imperfect Pass-Through, $y_t^{w,IPT}$, $\psi_t^{w,IPT}$, $s_t^{w,IPT}$, and $\psi_t^{*,w,IPT}$ are defined in Eqs. (A.17)– (A.19), respectively.

The relative-price structure is especially transparent in the two-country Welfare Cost Function because s_t remains explicit. Before introducing a tariff shock, the distinction between the two pass-through environments is the same as in the pre-reduction SOE representation. Under Perfect Pass-Through, the LOOP holds identically, so the open-economy relative-price externality is represented directly by the Welfare Relevant Ex-tariff Relative Price of Imports Gap $s_t - s_t^{w,PPT}$. Under Imperfect Pass-Through, the Home household's local relative import price is S_t/Ψ_t, so the same relative-price externality is jointly reflected in s_t and the Home LOOP gap ψ_t. The Foreign LOOP gap enters Home welfare through the endogenous two-country equilibrium. Thus imperfect pass-through adds the importer-pricing distortion associated with the Home LOOP gap without turning that gap into an independent policy objective.

We next introduce the Home import tariff. The Home household's relative-demand condition is

$$\frac{C_{F,t}}{C_{H,t}} = \frac{\omega_F}{\omega_H}\left(\frac{1+\tau_{M,t}}{\Psi_t}S_t\right)^{-\eta}. \tag{33}$$

For given S_t and Ψ_t, a higher Home tariff raises the *cum-tariff* relative price of Foreign goods and lowers $C_{F,t}/C_{H,t}$. For a given Foreign contingent plan, Home monetary easing raises S_t relative to a less accommodative Home policy path, shifting expenditure toward Home goods and supporting Home output and income. Under imperfect pass-through, the same exchange-rate adjustment also raises Ψ_t relative to that less accommodative path because the local-currency import price adjusts only gradually. The increase in Ψ_t limits the increase in the household-facing relative import price that would otherwise be implied by the higher S_t. The Home planner can therefore exploit the national relative-price externality through the joint adjustment of S_t and Ψ_t: the S_t response supports Home activity, while the Ψ_t response limits the purchaser-price effect. A nonzero LOOP gap remains an importer-pricing distortion, and a higher ψ_t puts upward pressure on ex-tariff import inflation through Eq. (22).

The distinction from the SOE is representational as well as economic. In the SOE, complete international risk sharing and Home goods-market clearing permit s_t to be eliminated from the reduced welfare representation. In the two-country economy, s_t remains explicit, and movements in Home relative demand feed back into Foreign demand and the common two-country equilibrium. The national Home planner therefore chooses the equilibrium configuration of s_t and ψ_t jointly rather than seeking to move either variable in isolation.

For the two-country numerical evaluation we retain the benchmark parameterization in Table 2 and set the Home economic-size share to $\nu = 0.27$ to match the U.S. share of world GDP at current prices in U.S. dollars, using the April 2026 International Monetary Fund (IMF) World Economic Outlook Database [17]. Together with $\upsilon = 0.30$, this implies Home's import expenditure weight $\omega_F = \upsilon(1-\nu) = 0.219$. This 21.9 percent share is an implication of country size and openness, not an additional calibration target; it is close to the 22.3 percent average ratio of U.S. imports of goods and services to personal consumption expenditures over 2000–2025 [18]. These values are used in the two-country welfare weights and in the benchmark-parameterization IRFs reported below.

Table 3 provides the corresponding SOE benchmark. Its central implication is that the quantitatively important Welfare Relevant variables are the inflation rates associated with sticky prices. In the Perfect Pass-Through Environment, the weight on Home PPI inflation is 39.87, far above the Welfare Relevant Output Gap weight. In the Imperfect Pass-Through Environment, Home PPI inflation and Home import-price inflation carry weights of 39.87 and 26.57, respectively. Thus the SOE welfare criterion already makes clear that sticky-price inflation, rather than headline CPI inflation, is the main nominal stabilization target.

Table 4 reports the corresponding benchmark weights for Home welfare in the two-country economy.

Table 4: Two-country Home welfare weights: benchmark values

Environment	Welfare Relevant Variable	Weight	Value
Perfect Pass-Through	$y_t - y_t^{w,PPT}$	Λ_y^{PPT}	2.090
	$s_t - s_t^{w,PPT}$	Λ_s^{PPT}	-0.090
	$\pi_{H,t}$	$\Lambda_{\pi,H}^{PPT}$	47.810
Imperfect Pass-Through	$y_t - y_t^{w,IPT}$	Λ_y^{IPT}	2.860
	$\psi_t - \psi_t^{w,IPT}$	Λ_ψ^{IPT}	0.410
	$s_t - s_t^{w,IPT}$	Λ_s^{IPT}	-0.100
	$\psi_t^* - \psi_t^{*,w,IPT}$	$\Lambda_{\psi^*}^{IPT}$	0.350
	$\pi_{H,t}$	$\Lambda_{\pi,H}^{IPT}$	65.310
	$\pi_{F,t}$	$\Lambda_{\pi,F}^{IPT}$	21.280
	$\pi_{H,t}^*$	$\Lambda_{\pi_H^*}^{IPT}$	13.520

The exact symbolic definitions of the Perfect Pass-Through weights and the Imperfect Pass-Through real weights are given in Eqs. (A.16) and (A.20), respectively; the three Imperfect Pass-Through inflation weights are given in Eq. (OB.9).

The exact symbolic definition of the Home Welfare Relevant LOOP Gap in Eq. (A.18) incorporates the tariff together with the strategic relative-price and Foreign-side equilibrium effects. Under the benchmark calibration, it is

$$\psi_t^{w,IPT} = -0.016\, s_t - 0.060\, \psi_t^* + 0.720\, \tau_{M,t} + 0.060\, \tau_{M,t}^*. \tag{34}$$

The negative coefficient on s_t reflects the substitution between movements in the external relative price and the Home LOOP gap when the planner exploits the national relative-price externality. When s_t rises, the strategically favorable expenditure shift toward Home goods reduces the planner's incentive to raise the distortionary Home LOOP gap. Hence the Home Welfare Relevant LOOP Gap falls with s_t under the benchmark calibration. Under a unilateral Home tariff, $\tau_{M,t}^* = 0$.

Table 4 shows that the weight on the Home Welfare Relevant LOOP Gap Deviation is only 0.41, far below the weights on Home PPI inflation and ex-tariff import inflation. As in the SOE, this small weight measures the curvature of the welfare cost around the Welfare Relevant LOOP Gap; it does not measure the size of the movement in the reference value itself. Equation (A.18), summarized by Eq. (34), shows that $\psi_t^{w,IPT}$ depends on the Home tariff and, in the two-country equilibrium, on the endogenous relative price s_t and Foreign LOOP gap ψ_t^*. These forces can therefore move the Home Welfare Relevant LOOP Gap substantially even though Λ_ψ^{IPT} is small. A sufficiently large deviation $\psi_t - \psi_t^{w,IPT}$ can then make the LOOP-gap component quantitatively important. Home monetary easing raises ψ_t relative to a less accommodative Home path and can thereby move the equilibrium LOOP gap toward its shifted Welfare Relevant reference value while simultaneously changing s_t. The planner does not eliminate the deviation because doing so would alter the Welfare Relevant Output Gap, the sticky-price inflation terms, the signed international-relative-price contribution, and the Foreign-side terms in Eq. (32).

Table 4 also shows how the inflation weights change relative to the SOE. In the Imperfect Pass-Through Environment, the weight on Home PPI inflation $\pi_{H,t}$ rises from 39.87 to 65.31 because Foreign demand, Foreign prices, and international risk sharing are endogenous in the two-country model and thereby affect Home production and labor. By contrast, the weight on Home ex-tariff import inflation $\pi_{F,t}$ falls from 26.57 to 21.28 because Home's import expenditure weight falls from 0.30 in the SOE to $\omega_F = 0.219$. Table 4 also shows that Foreign ex-tariff import inflation $\pi_{H,t}^*$ enters the Home Welfare Cost Function with a weight of 13.52. In country F, importers choose prices in Foreign currency when selling Home goods, and their price-setting behavior affects the price of Home goods in Home currency, $P_{H,t}$. This is why $\pi_{H,t}^*$ enters the Home Welfare Cost Function. Note, however, that gross Foreign ex-tariff import inflation $\Pi_{H,t}^*$ is not a choice variable of the Home social planner.

The negative coefficients on the Welfare Relevant Ex-tariff Relative Price of Imports Gap term, $\Lambda_s^{PPT} = -0.0920$ and $\Lambda_s^{IPT} = -0.0978$, reflect the same strategic relative-price consideration. In the national Ramsey problem, relative-price movements can shift expenditure toward Home goods, support Home output and income, and thereby improve Home welfare. This is the national relative-price externality. The negative sign therefore represents a signed strategic welfare contribution rather than a conventional positive stabilization penalty. Accordingly, we use the term "stabilization objective" for the positive-weight quadratic terms; the negative-weight completed-square term is reported as a signed strategic welfare contribution. This does not make the relative price an independent Ramsey choice variable; it remains an equilibrium outcome that monetary policy can influence strategically.

4.4 Nominal Interest Rate Responses

Figure 2(a) compares the benchmark two-country Home responses. The difference in the Home policy rate is immediate: the Perfect Pass-Through Environment raises i_t by 11.52 basis points on impact, whereas the Imperfect Pass-Through Environment lowers it by 60.64 basis points. In Figure 2(a), the blue lines are the IRFs under the Imperfect Pass-Through Environment, while the orange lines are those under the Perfect Pass-Through Environment.

In the Perfect Pass-Through Environment, as discussed in Sec. 3.5, an increase in the import tariff raises Home real marginal cost through an increase in the CPI–PPI ratio and thereby puts upward pressure on Home PPI inflation. As shown in Table 4, Home PPI inflation carries the largest positive welfare weight. The nominal interest rate is therefore raised (Panel 1, Fig. 2(a)), and Home PPI inflation is well stabilized (Panel 7, Fig. 2(a)).

In the Imperfect Pass-Through Environment, the nominal interest rate instead decreases. Table 4 assigns the Home Welfare Relevant LOOP Gap Deviation a weight of only 0.41, compared with 65.31 on Home PPI inflation and 21.28 on ex-tariff import inflation. As established in Sec. 4.3, however, the small LOOP-gap weight does not imply a small welfare incentive to adjust ψ_t: the tariff and the endogenous movements in s_t and ψ_t^* shift $\psi_t^{w,IPT}$, so the resulting deviation can be

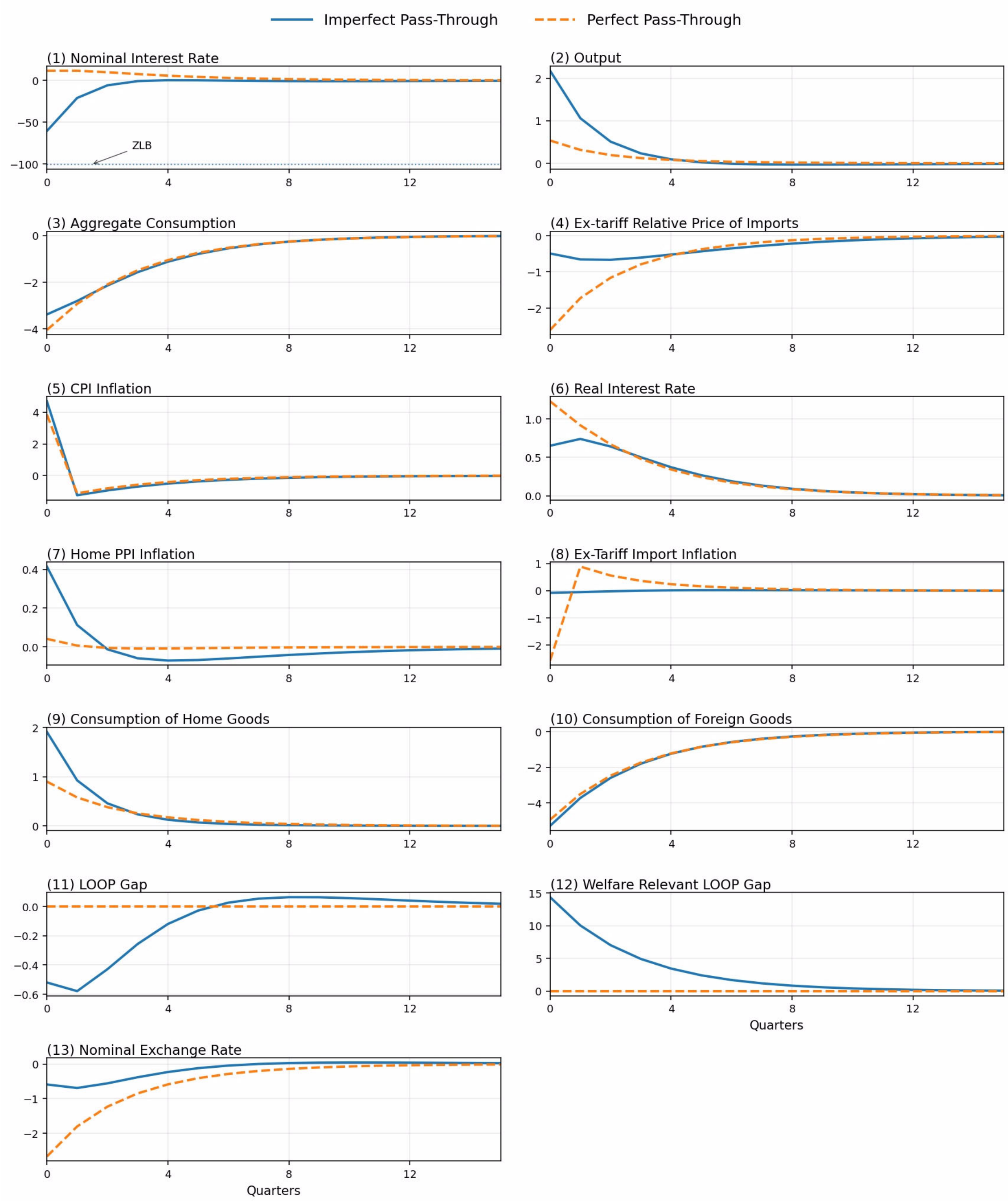

Figure 2: Two-country benchmark parameterization ($\nu = 0.27$): (a) Home responses in the Perfect and Imperfect Pass-Through Environments. The panel composition is the same as in Fig. 1. The Perfect Pass-Through Environment is shown in orange and the Imperfect Pass-Through Environment in blue. The policy-rate panel reports the deviation of the Home nominal interest rate from its deterministic steady state in basis points. The other panels report deviations from deterministic steady state in percentage points. Panel 4 reports the Ex-tariff Relative Price of Imports $P_{F,t}/P_{H,t}$, Panel 8 ex-tariff import inflation $\pi_{F,t}$, Panel 11 the LOOP gap ψ_t, Panel 12 the Welfare Relevant LOOP Gap $\psi_t^{w,IPT}$, and Panel 13 the equilibrium nominal exchange rate. Panels 5 and 6 report Home CPI inflation and the Real Consumption Interest Rate (Home), $(1 + i_t)/\mathrm{E}_t\Pi_{t+1}$, respectively.

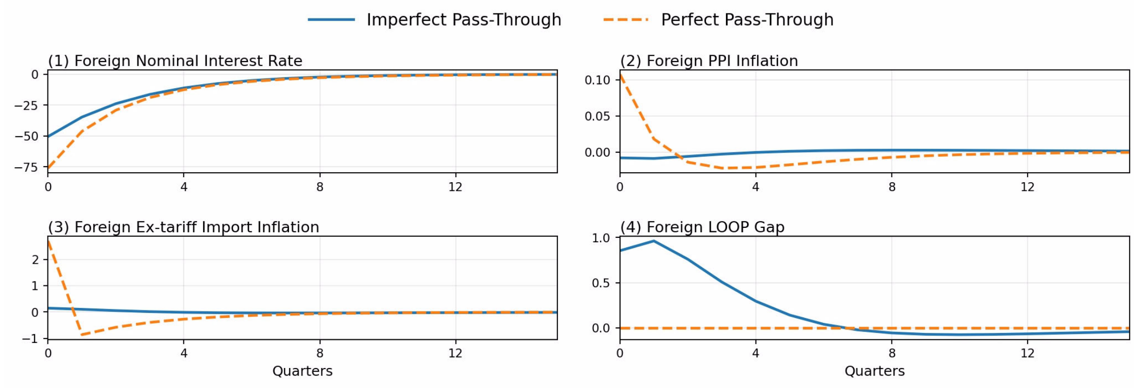

Figure 2: Two-country benchmark parameterization ($\nu = 0.27$): (b) Foreign responses in the Perfect and Imperfect Pass-Through Environments. The Perfect Pass-Through Environment is shown in orange and the Imperfect Pass-Through Environment in blue. Panels 1–4 report the Foreign nominal interest rate, Foreign PPI inflation $\pi^*_{F,t}$, Foreign Ex-tariff Import Inflation, and the Foreign LOOP gap, respectively.

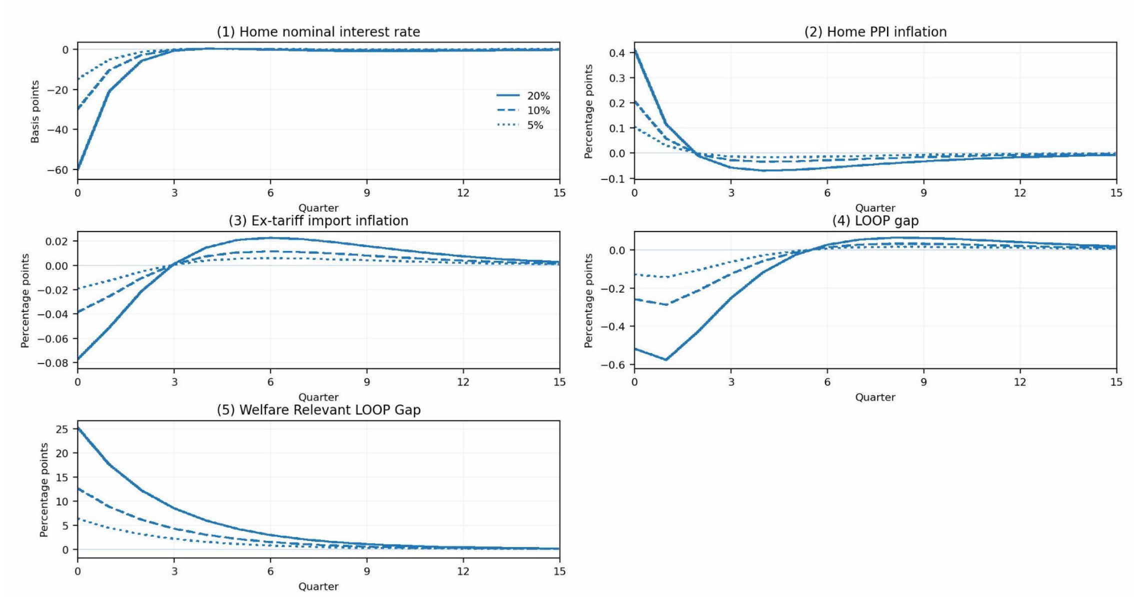

Figure 2: Two-country Imperfect Pass-Through Environment: (c) tariff-size comparison. Each panel overlays the Home IRFs for unilateral Home import-tariff innovations of 20, 10, and 5 percentage points. Panels report, respectively, the Home nominal interest rate, Home PPI inflation $\pi_{H,t}$, ex-tariff import inflation $\pi_{F,t}$, the Home LOOP gap ψ_t, and the Home Welfare Relevant LOOP Gap $\psi_t^{w,IPT}$ in Eq. (34). The three paths correspond to unilateral Home import-tariff innovations of 0.20, 0.10, and 0.05 under the same nonlinear national-Ramsey first-order-condition system. The ZLB is slack for all three experiments.

large. The interest-rate response therefore reflects the full welfare trade-off rather than a ranking of the weights alone.

The behavior of the exchange rate and the LOOP gap is determined jointly with Foreign monetary policy. Following the unilateral Home tariff shock, the Foreign monetary authority lowers its nominal interest rate and keeps it below its steady-state path more persistently than the Home monetary authority (Panel 1, Fig. 2(b)). The resulting relative interest-rate path generates an appreciation of the Home currency despite the Home rate cut. This appreciation contributes to the decline in the Home LOOP gap ψ_t. Home monetary easing works in the opposite direction: it mitigates the appreciation, pushes the Home external relative price upward relative to a less accommodative Home path, and thereby supports demand for Home goods, Home output, and Home income. Because the Home local import price is sticky, the same policy also raises ψ_t relative to what it would be under the less accommodative path, moving the LOOP gap toward its shifted Welfare Relevant reference value and limiting the pass-through of the exchange-rate adjustment into the Home import price.

Consistent with this mechanism, the tariff raises the Home Welfare Relevant LOOP Gap through Eq. (34). As the tariff shock becomes larger, the equilibrium LOOP gap becomes more negative on impact while the Welfare Relevant LOOP Gap becomes larger, as shown in Panels 4 and 5 of Fig. 2. The resulting Welfare Relevant LOOP Gap Deviation $\psi_t - \psi_t^{w,IPT}$ is therefore substantially negative, as implied jointly by Panels 11 and 12 of Fig. 2(a). The stronger Foreign easing generates the Home appreciation and the decline in the actual ψ_t, while Home easing raises ψ_t relative to the less accommodative Home-policy counterfactual.

The corresponding household allocation can be checked directly. Appendix C.2 reports a first-order best-response diagnostic in which the complete Foreign contingent plan is held at its benchmark Nash–Ramsey path and the Home nominal-rate deviation $i_t - i$ is either set to zero or kept at its benchmark Ramsey path. By Eq. (33), the impact log deviation of $C_{F,t}/C_{H,t}$ is -30.05 percent under the zero Home response and -29.26 percent under the optimal Home response. The Home rate cut therefore raises the impact relative-consumption ratio by 0.79 percentage points. The optimal response also leaves the log deviation of $C_{F,t}/C_{H,t}$ less negative than under the zero Home response throughout the 20-quarter diagnostic horizon. This is a direct quantity check on the allocation associated with the joint movement of s_t and ψ_t.

The Home monetary authority does not ease further because doing so worsens other welfare terms. Given the Foreign contingent plan in the Nash–Ramsey equilibrium, additional Home monetary easing would further mitigate the Home currency appreciation and raise ψ_t relative to the benchmark path, thereby exploiting the national relative-price externality through the joint movement of s_t and ψ_t more strongly. The marginal counterfactual reported in Table 16 in Appendix C.2 holds the Foreign contingent plan fixed and makes the benchmark Home rate response 10 percent more accommodative. It further reduces the discounted quadratic contribution associated with the Home Welfare Relevant LOOP Gap Deviation and improves the signed relative-price contribution, but increases the Home PPI-inflation, Welfare Relevant Output Gap, and Foreign Welfare Relevant LOOP Gap Deviation contributions by enough to raise total discounted Home welfare cost. The benchmark Home policy is therefore the optimum of the trade-off between exploiting the national relative-price externality and stabilizing the other welfare-relevant variables; stronger exploitation through the joint movement of s_t and ψ_t has a greater marginal welfare cost elsewhere.

5 ZLB Episodes

The benchmark Imperfect Pass-Through Environment already calls for substantial monetary accommodation in both the SOE and the two-country economy while the ZLB remains slack. The key quantitative question is whether this already large optimal easing needs only one additional economically relevant force to exhaust conventional monetary-policy space. We therefore change one dimension of the environment at a time and ask whether the resulting strengthening of the same tariff-induced stabilization problem is enough to push the desired nominal rate to the ZLB.

The five cases are not five unrelated sources of a liquidity trap. Each adds a different force to an accommodation that is already large under imperfect pass-through. Higher Home–Foreign substitutability magnifies expenditure switching and the subsequent inflation reversal; external habit makes the required stabilization persistent; reciprocal tariffs activate both countries' importer-price systems; roundabout production adds a material-input spillover–spillback, weakens the effect of Home activity on producer real marginal cost, and raises the welfare cost of producer-price dispersion; and open-loop Nash changes the strategic best-response problem while retaining the same physical sticky-price blocks. Their common implication is that the benchmark demand for accommodation is quantitatively strong: changing one economically relevant dimension can be enough to carry the desired easing to the ZLB.

The common element is the Welfare Cost Function. As in Secs. 3.5 and 4.4, the Welfare Cost Function identifies the Welfare Relevant variables, while the producer and importer Phillips curves characterize the corresponding sticky-price inflation paths. Each experiment changes how the tariff shock affects those welfare-relevant real, marginal-cost, and inflation paths. The ZLB becomes relevant when the accommodation required by that stabilization problem pushes the ZLB-relaxed Ramsey rate below zero, so the desired policy can no longer be implemented with a non-negative nominal interest rate.

An important point is that the inflation IRFs shown below are equilibrium outcomes under the reported Ramsey policy. Hence the sign of impact inflation alone does not determine the sign of the policy response. What matters is the discounted path of the welfare-relevant inflation rates and real deviations. Likewise, when the ZLB binds, the actual interest-rate path is not obtained by truncating the ZLB-relaxed path quarter by quarter. The binding constraint changes the continuation Ramsey allocation, so the actual and notional paths can differ after the impact period.

To make the experiments directly comparable, the Home-response figures retain the same core ordering as Figures 1 and 2(a). In all Home-response figures in this section, including Fig. 7(a), Panel 11 reports the Home LOOP gap, Panel 12 the model-specific Welfare Relevant LOOP Gap ψ_t^w, and Panel 13 the nominal exchange rate. Each figure also reproduces the IRF from the economically relevant reference case. Figure 3 uses the SOE benchmark parameterization in Table 2; Figure 4 uses the benchmark SOE Imperfect Pass-Through specification of Sec. 3 with no habit; Figure 5 uses the two-country benchmark parameterization, including $\nu = 0.27$, with the unilateral Home tariff shock; and Figures 6 and 7 use the benchmark two-country Imperfect Pass-Through specification of Sec. 4. Here the benchmark Imperfect Pass-Through specification means the Sec. 3 or Sec. 4 environment before adding the preference, production, shock-vector, or strategic modification studied in the relevant subsection. The focal experiment is shown by a solid line and its reference by a dashed line. Panel 1 reports the corresponding actual nominal-rate paths and the ZLB. The ZLB-relaxed impact rates are already summarized in Table 5 and are therefore not replotted in Figures 3–7.

Table 5: Optimal nominal-rate responses in the ZLB experiments

Case	Home actual (basis points)	Home notional (basis points)	Foreign actual (basis points)	Home ZLB quarters
SOE, High Substitution Case ($\eta = 3$)	-101.010	-119.950	–	1
SOE, External Habit	-101.010	-185.310	–	2
Two-country, Reciprocal Tariffs	-101.010	-210.620	-62.230	3
Two-country, Roundabout Production	-101.010	-106.950	-71.550	1
Two-country, Open-loop Nash	-101.010	-101.310	-83.280	1

All interest-rate entries in Table 5 are impact deviations from the deterministic steady state, measured in basis points. The Home actual response is -101.01 basis points whenever the impact rate is at the ZLB. The Home notional response is the corresponding impact deviation from the ZLB-relaxed Ramsey path obtained from the same OccBin implementation.

5.1 SOE: high elasticity of substitution

5.1.1 Higher substitutability and the relative-price externality

The first experiment changes only the Home–Foreign substitution elasticity: η rises from the benchmark value 1.5 in Table 2 to 3, while the rest of the SOE Imperfect Pass-Through specification is unchanged. The reason this change can generate a ZLB episode is already visible in the relative-demand condition in Eq. (29). A higher η makes a given movement in the household-facing relative price generate a larger change in $C_{F,t}/C_{H,t}$.

5.1.2 Welfare trade-off under high substitution

The Welfare Cost Function retains the SOE Imperfect Pass-Through form in Eq. (26), with its coefficients re-evaluated at the higher substitution elasticity. Differentiating the exact weight definition in Eq. (B.1), while holding the other benchmark parameters fixed, gives

$$\frac{\partial \Lambda_{\psi}^{SOE,IPT}}{\partial \eta} > 0, \qquad 1.5 \leq \eta \leq 3.$$

Table 6 reports the resulting weights.

Table 6: Welfare-cost components: high-substitution SOE

Welfare Relevant Variable	Weight	Value
$y_t - y_t^{w,SOE,IPT}$	$\Lambda_y^{SOE,IPT}$	0.790
$\psi_t - \psi_t^{w,SOE,IPT}$	$\Lambda_\psi^{SOE,IPT}$	1.190
$\pi_{H,t}$	$\Lambda_{\pi,H}^{SOE,IPT}$	19.980
$\pi_{F,t}$	$\Lambda_{\pi,F}^{SOE,IPT}$	34.760

As Table 6 shows, $\Lambda_\psi^{SOE,IPT}$ rises from 0.49 under the benchmark calibration to 1.19 when $\eta = 3$. The economic reason follows directly from the relative-demand mechanism in Sec. 5.1.1. For given S_t, Eq. (28) shows that a higher Ψ_t lowers the household-facing *cum-tariff* Relative Price of Imports. Because a higher η makes $C_{F,t}/C_{H,t}$ more responsive to a given movement in that relative price, increasing Ψ_t to lower the *cum-tariff* Relative Price of Imports has greater welfare value. In the quadratic Welfare Cost Function, this greater value is reflected in the larger weight $\Lambda_\psi^{SOE,IPT}$ on the Welfare Relevant LOOP Gap Deviation.

5.1.3 Nominal interest-rate response and the ZLB

Figure 3 reports the high-substitution allocation together with the SOE benchmark-parameterization allocation.

The IRFs confirm the mechanism. Symbolically, Eq. (29) implies that higher substitutability makes the same household-facing relative-price movement generate a larger change in $C_{F,t}/C_{H,t}$. The Ramsey authority therefore chooses a more aggressive joint adjustment of S_t and Ψ_t, subject to the opposing output and sticky-inflation costs. On impact, the LOOP gap rises by 3.18 percentage points when $\eta = 3$, compared with 1.47 under the benchmark parameterization (Panel 11). In the figure's level-deviation units, $C_{F,t}$ falls by 12.81 rather than 7.61 percentage points (Panel 10), while $C_{H,t}$ rises by 8.94 rather than 2.76 percentage points (Panel 9). Accordingly, the impact log deviation of $C_{F,t}/C_{H,t}$ is -67.74 percent under $\eta = 3$, compared with -33.12 percent under the benchmark parameterization. Output rises by 10.45 rather than 3.21 percentage points (Panel 2), consistent with the stronger relative-demand response.

The resulting desired impact cut rises from 70.13 basis points at $\eta = 1.5$ to 119.95 basis points at $\eta = 3$. Because only 101.01 basis points of conventional policy space are available on impact, the ZLB-relaxed Ramsey rate crosses zero and the actual rate is at the ZLB for one

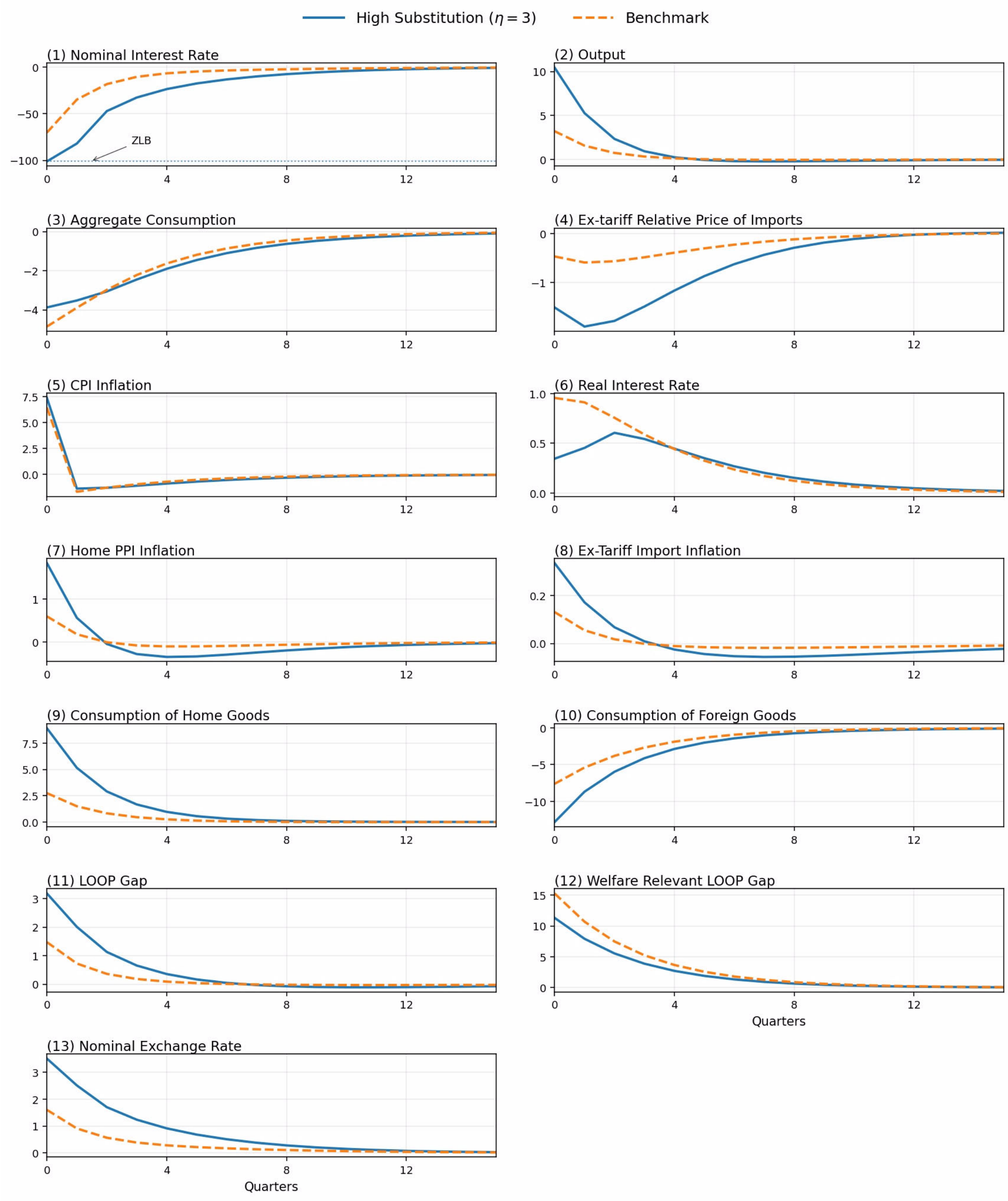

Figure 3: SOE Imperfect Pass-Through Environment with high elasticity of substitution, $\eta = 3$, compared with the SOE benchmark parameterization ($\eta = 1.5$). The solid line reports the high-substitution allocation and the dashed line the benchmark-parameterization IRF. The dotted horizontal line in Panel 1 is the ZLB. The Home ZLB binds for one quarter in the high-substitution case and does not bind under the benchmark parameterization. Panel 4 reports the Ex-tariff Relative Price of Imports $P_{F,t}/P_{H,t}$. Panels 11–13 report the LOOP gap, the Welfare Relevant LOOP Gap $\psi_t^{w,SOE,IPT}$, and the nominal exchange rate, respectively. Panels 5 and 6 report Home CPI inflation and the Real Consumption Interest Rate (Home), $(1 + i_t)/\mathrm{E}_t\Pi_{t+1}$, respectively.

quarter (Panel 1). The ZLB episode is therefore the quantitative consequence of the stronger high-substitution stabilization motive described in the preceding two subsubsections, not a separate source of transmission.

5.2 SOE: external habit

5.2.1 External habit and intertemporal amplification

This experiment changes only household preferences relative to the plain SOE Imperfect Pass-Through specification of Sec. 3. Production, the tariff process, the producer and importer Calvo blocks, goods-market clearing, the lump-sum rebate of tariff revenue, and the ZLB constraint are unchanged. External habit modifies the household's lifetime utility to

$$\mathcal{U} \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log\bigl(C_t - hC_{t-1}^{\mathrm{agg}}\bigr) - \frac{N_t^{1+\varphi}}{1+\varphi} \right]. \tag{35}$$

Here $h \in [0,1)$ denotes the external-habit parameter and C_{t-1}^{agg} denotes lagged aggregate Home consumption. An individual household takes C_{t-1}^{agg} as given when choosing C_t; only after the private first-order conditions are derived is the symmetric-equilibrium condition $C_t^{\mathrm{agg}} = C_t$ imposed. Hence the private marginal utility of consumption is $1/(C_t - hC_{t-1}^{\mathrm{agg}})$, and the household does not internalize the effect of its own C_t on next period's aggregate habit benchmark. The social planner, by contrast, internalizes this aggregate habit externality. We set $h = 0.71$, following Smets and Wouters [25].

5.2.2 Welfare trade-off with external habit

We use $c_t \equiv \log(C_t/C)$ for the log deviation of consumption from its deterministic steady state. The SOE Welfare Cost Function in the Imperfect Pass-Through Environment becomes

$$\begin{aligned}\mathcal{L}_t^{SOE,IPT,h} \equiv\ & \frac{1}{2}\Bigl[\Lambda_y^h (y_t - y_t^{w,h})^2 + \Lambda_\psi^h (\psi_t - \psi_t^{w,h})^2 + \Lambda_{c^-}^h (c_{t-1} - c_{t-1}^{w,h})^2 \\ & + \Lambda_{\pi,H}^h \pi_{H,t}^2 + \Lambda_{\pi,F}^h \pi_{F,t}^2 \Bigr].\end{aligned} \tag{36}$$

The additional lagged-consumption term arises because, after the household optimality conditions are derived and symmetric equilibrium is imposed, aggregate welfare contains $\log(C_t - hC_{t-1}^{\mathrm{agg}})$; under symmetric equilibrium, $C_{t-1}^{\mathrm{agg}} = C_{t-1}$. Its second-order expansion contains policy-dependent terms in c_{t-1}; completing the square therefore yields $(c_{t-1} - c_{t-1}^{w,h})^2$. We refer to $c_{t-1} - c_{t-1}^{w,h}$ as the Welfare Relevant Lagged Consumption Gap. The derivation is given in On-line Appendix OB.2.

At the benchmark calibration the weight ordering is $\Lambda_{c^-}^h < \Lambda_\psi^h < \Lambda_y^h \ll \Lambda_{\pi,H}^h, \Lambda_{\pi,F}^h$. Table 7 reports the corresponding values. The Welfare Relevant Lagged Consumption Gap therefore has the smallest structural stabilization weight; the ZLB episode cannot be attributed to that new term simply because it appears in the Welfare Cost Function.

Table 7: Welfare-cost components: external-habit SOE

Welfare Relevant Variable	Weight	Value
$y_t - y_t^{w,h}$	Λ_y^h	1.820
$\psi_t - \psi_t^{w,h}$	Λ_ψ^h	0.550
$c_{t-1} - c_{t-1}^{w,h}$	$\Lambda_{c^-}^h$	0.160
$\pi_{H,t}$	$\Lambda_{\pi,H}^h$	31.730
$\pi_{F,t}$	$\Lambda_{\pi,F}^h$	30.710

The exact weight definitions and the square-completion construction are reported in Appendix B.2; the two inflation weights are given in Eq. (B.5). More importantly for the mechanism, Eq. (B.7) shows that the inherited consumption state enters not only the Welfare Relevant Lagged

Consumption Gap but also the other Welfare Relevant variables. Under the benchmark calibration with $h = 0.71$, the symbolic definition of Welfare Relevant Output in Eq. (B.7) becomes

$$y_t^{w,h} = -0.073\,\psi_t + 0.287\,c_{t-1} + 0.288\,\tau_{M,t}. \tag{37}$$

The positive coefficient on c_{t-1} in Eq. (37) is the key welfare implication of external habit. Higher current consumption becomes higher inherited consumption in the next period. Because utility depends on $C_t - hC_{t-1}$, higher inherited consumption lowers habit-adjusted consumption for given current consumption. Accordingly, holding y_t, ψ_t, and the tariff fixed, a higher c_{t-1} makes the Welfare Relevant Output Gap $y_t - y_t^{w,h}$ more negative. The planner therefore has an incentive to sustain a higher future output and income path, which requires larger and more persistent monetary accommodation. Since the benchmark Imperfect Pass-Through Environment already calls for substantial easing, this intertemporal amplification can push the desired nominal interest rate to the ZLB.

Notice also that the coefficient on ψ_t is negative. This sign is not the source of the habit amplification; it reflects the unchanged relative-price block. Holding y_t, c_{t-1}, and the tariff fixed, a larger LOOP gap raises the Welfare Relevant Output Gap $y_t - y_t^{w,h}$. This sign is consistent with $P_{F,t}/P_{H,t} = S_t/\Psi_t$: for a given S_t, a larger Ψ_t lowers the relative price of Foreign goods and shifts expenditure toward Foreign goods.

The zero-policy-response accounting diagnostic in Table 13 is consistent with this interpretation. The largest reduction in discounted welfare cost occurs in the Welfare Relevant LOOP Gap Deviation, whereas the contribution of the Welfare Relevant Lagged Consumption Gap itself rises. This does not imply that the intertemporal habit mechanism is quantitatively unimportant. The inherited consumption state affects the welfare criterion not only through its own completed square but also through Welfare Relevant Output in Eq. (37). The stronger Ramsey response therefore sustains the output and income path required by the intertemporal consumption linkage rather than minimizing the lagged-consumption component in isolation.

5.2.3 Nominal interest-rate response and the ZLB

Figure 4 compares the external-habit allocation with the benchmark SOE Imperfect Pass-Through allocation without habit.

As discussed in Sec. 5.2.2, external habit makes the desired monetary accommodation larger and more persistent by linking current consumption support to the future output and income path.

Quantitatively, the ZLB-relaxed impact response is a 185.31-basis-point cut, far beyond the 101.01 basis points available before the nominal rate reaches zero. The actual rate therefore hits the ZLB on impact and remains there for two quarters (Panel 1). The ZLB is thus a constraint on the stronger accommodation implied by the external-habit welfare trade-off; it is not the source of the underlying welfare improvement.

5.3 Two-country model: reciprocal tariffs

5.3.1 Reciprocal tariffs and the foreign-demand spillback

The third experiment changes the shock vector rather than preferences, technology, or price setting. In addition to the 20-percentage-point Home tariff innovation, Foreign receives the same import-tariff innovation. The benchmark comparison remains the two-country Imperfect Pass-Through economy with $\nu = 0.27$ and a unilateral Home tariff shock.

The additional Foreign tariff creates a direct reason for stronger Home accommodation. It reduces Foreign demand for Home goods, weakening Home activity and Home producer real marginal cost. Through the Home producer Phillips curve in Eq. (21), this puts additional downward pressure on Home PPI inflation relative to the unilateral benchmark. The Home Ramsey authority therefore needs stronger monetary easing to offset this additional disinflationary force. If the re-

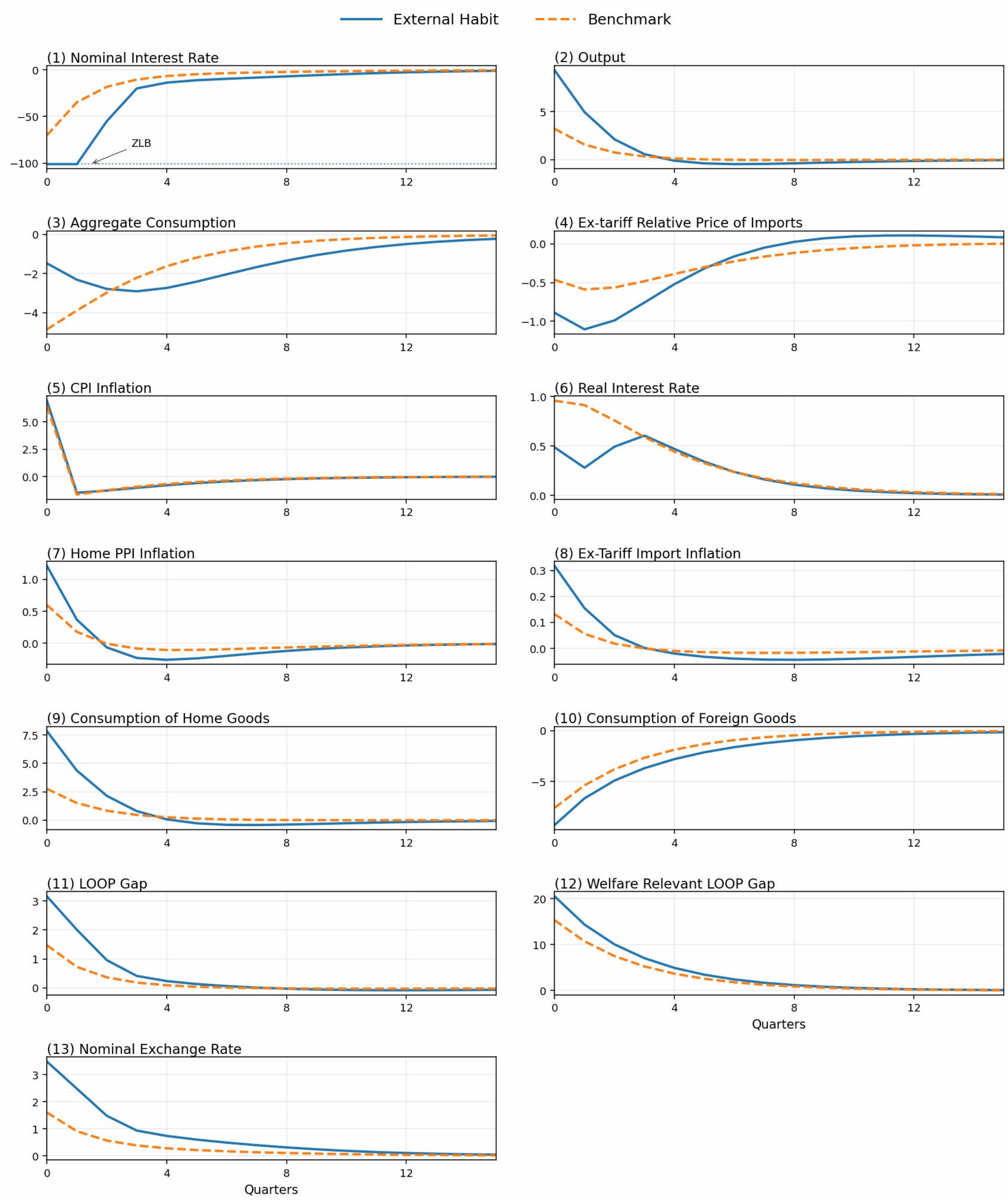

Figure 4: SOE Imperfect Pass-Through Environment with external habit, $h = 0.71$, compared with the benchmark SOE Imperfect Pass-Through specification ($h = 0$). The solid line reports the external-habit allocation and the dashed line the benchmark Imperfect Pass-Through IRF. The dotted horizontal line in Panel 1 is the ZLB. The ZLB binds for two quarters under external habit and does not bind in the benchmark Imperfect Pass-Through specification. Panel 4 reports the Ex-tariff Relative Price of Imports $P_{F,t}/P_{H,t}$. Panels 11–13 report the LOOP gap, the Welfare Relevant LOOP Gap $\psi_t^{w,h}$, and the nominal exchange rate, respectively. Panels 5 and 6 report Home CPI inflation and the Real Consumption Interest Rate (Home), $(1+i_t)/\mathrm{E}_t\Pi_{t+1}$, respectively.

quired accommodation exceeds the available conventional-policy space, the Home nominal rate reaches the ZLB.

5.3.2 Welfare criterion under reciprocal tariffs

The reciprocal experiment leaves the two-country Home Welfare Cost Function in the Imperfect Pass-Through Environment in Eq. (32) unchanged. The undetermined coefficients Θ_n, Θ_H^{AS}, Θ_M^{AS}, and Θ_Y, together with the full two-country Benigno–Woodford welfare derivation, are reported in On-line Appendix OB.3.

This distinction is useful for interpreting the ZLB episode. The stronger Home easing does not come from a new structural welfare weight. It comes from the new shock vector changing the variables entering the unchanged welfare criterion. In particular, the additional Foreign tariff maps into a more contractionary Home activity, marginal-cost, and PPI-inflation path. In addition, Eq. (34) implies $\partial\psi_t^{w,IPT}/\partial\tau_{M,t}^* > 0$ at the benchmark calibration, so the Foreign tariff also raises the Home Welfare Relevant LOOP Gap directly. The foreign-demand spillback nevertheless provides the main additional producer-price stabilization force relative to the unilateral benchmark.

5.3.3 Nominal interest-rate response and the ZLB

Figures 5(a) and 5(b) compare the reciprocal experiment with the unilateral benchmark.

The IRFs confirm the foreign-demand spillback. On impact, Home output changes from +2.18 percentage points under the unilateral tariff to −0.75 percentage points under reciprocal tariffs (Panel 2, Fig. 5(a)). Consistent with the resulting decline in Home activity and mc_t^r, Home PPI inflation falls from 0.41 to 0.27 percentage points on impact and subsequently becomes negative (Panel 7). The ZLB-relaxed Home policy-rate response therefore changes from a 60.64-basis- point cut under the unilateral benchmark to a 210.62-basis-point cut under reciprocal tariffs. The actual Home rate is consequently at the ZLB for three quarters. The Foreign policy rate falls by 62.23 basis points on impact and remains above its own lower bound. Thus the Home ZLB episode is the direct policy consequence of the extra contractionary force generated by Foreign retaliation.

5.4 Two-country model: roundabout production

5.4.1 Roundabout production and the material-input spillback

Following Basu [26], we introduce this roundabout input–output structure.

This experiment extends the benchmark two-country Imperfect Pass-Through specification in Sec. 4 by adding intermediate inputs as an additional factor of production. Preferences, final consumption, importer local-currency pricing, complete international financial markets, the tariff processes, the national Ramsey allocation game, and the two ZLB constraints are unchanged. Home and Foreign producers now use intermediate goods in addition to labor, and those inputs may be sourced domestically or from abroad.

For a Home producer j, the intermediate-input composite is

$$\mathcal{I}_t(j) = \left[\omega_H^{1/\eta}\mathcal{I}_{H,t}(j)^{(\eta-1)/\eta} + \omega_F^{1/\eta}\mathcal{I}_{F,t}(j)^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}, \tag{38}$$

where $\mathcal{I}_{H,t}(j)$ and $\mathcal{I}_{F,t}(j)$ are, respectively, domestically produced and imported intermediate inputs. Because the intermediate-input composite has the same CES structure and Home Bias as final consumption, its unit price is P_t. In the Imperfect Pass-Through Environment the imported component is purchased at the *cum-tariff* local-currency price $(1+\tau_{M,t})P_{F,t}$. Its purchaser price

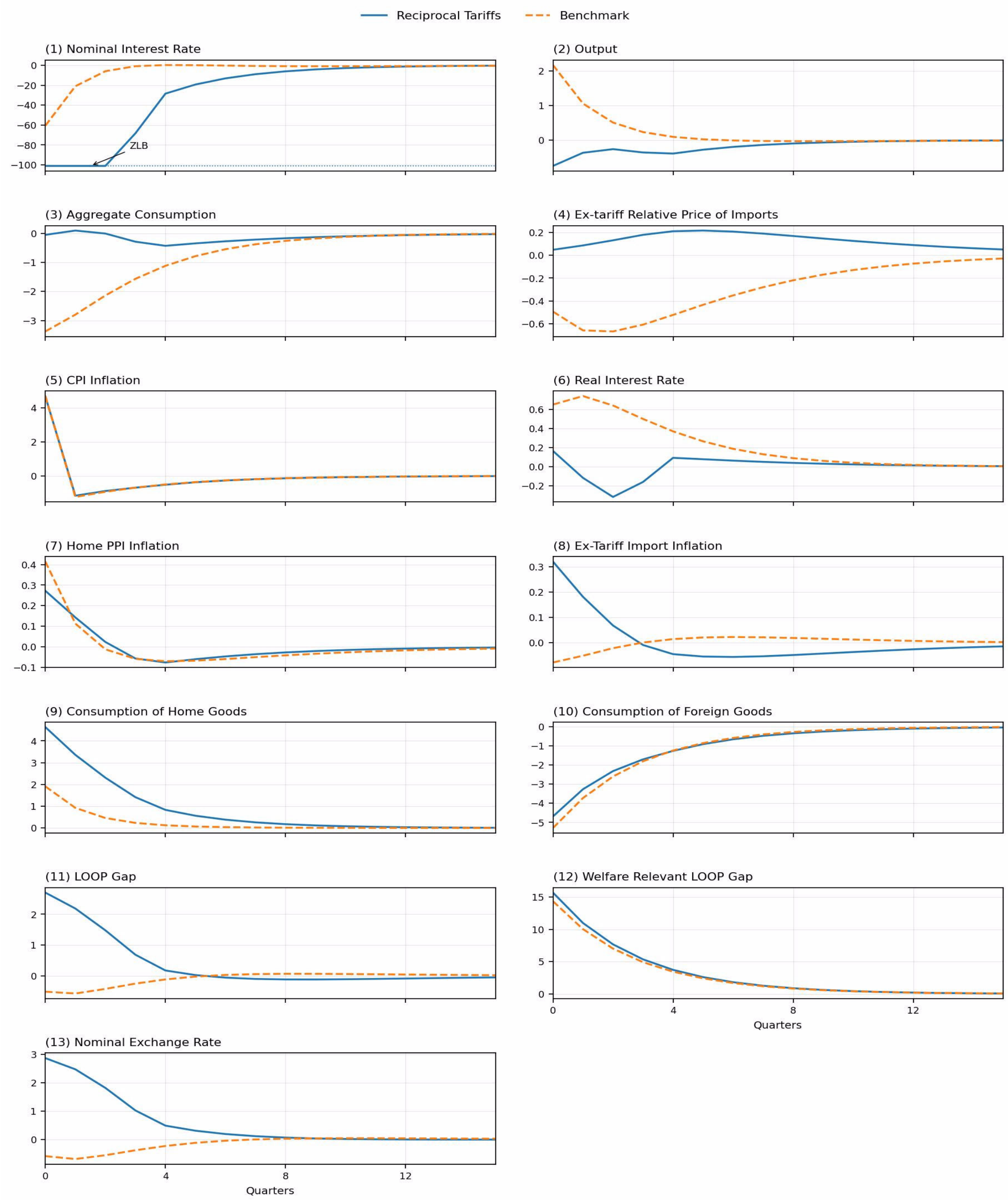

Figure 5: Two-country Imperfect Pass-Through Environment with reciprocal tariff shocks: (a) Home responses, compared with the benchmark parameterization under the unilateral Home tariff shock. Both cases use $\nu = 0.27$. The solid line reports reciprocal tariffs and the dashed line the benchmark-parameterization IRF. The dotted horizontal line in Panel 1 is the ZLB. The Home ZLB binds for three quarters only under reciprocal tariffs. Panel 4 reports the Ex-tariff Relative Price of Imports $P_{F,t}/P_{H,t}$. Panels 11–13 report the LOOP gap, the Welfare Relevant LOOP Gap $\psi_t^{w,IPT}$, and the nominal exchange rate, respectively. Panels 5 and 6 report Home CPI inflation and the Real Consumption Interest Rate (Home), $(1+i_t)/\mathrm{E}_t\Pi_{t+1}$, respectively.

relative to the Home good is therefore given by Eq. (28). Cost minimization gives

$$\begin{aligned}
\mathcal{I}_{H,t}(j) &= \omega_H \left(\frac{P_t}{P_{H,t}}\right)^{\eta} \mathcal{I}_t(j), \\
\mathcal{I}_{F,t}(j) &= \omega_F \left[\frac{(1+\tau_{M,t})S_t}{\Psi_t(P_t/P_{H,t})}\right]^{-\eta} \mathcal{I}_t(j).
\end{aligned} \tag{39}$$

Thus a higher Home tariff raises the purchaser price of Foreign intermediate inputs, other things equal, and reduces Home demand for them.

This input-demand response generates the international material-input spillover–spillback that is central to the experiment. The loss of Home demand for Foreign intermediate goods weakens Foreign activity and absorption, which in turn lowers Foreign demand for Home final and intermediate goods. Weaker demand for Home production places downward pressure on Home activity, Home producer real marginal cost, and, through Eq. (21), Home PPI inflation. A larger intermediate-input share ζ increases the importance of this traded-input linkage. The Home Ramsey authority therefore has an additional reason to ease in order to support producer real marginal cost and the Home PPI-inflation path.

The production function is

$$Y_t(j) = \left(\frac{N_t(j)}{1-\zeta}\right)^{1-\zeta} \left(\frac{\mathcal{I}_t(j)}{\zeta}\right)^{\zeta}, \qquad 0 \leq \zeta < 1, \tag{40}$$

where ζ is the intermediate-input share and $1-\zeta$ is the labor share. The term roundabout production reflects the fact that produced goods are used again as inputs into current production rather than production relying only on direct labor input. Aggregating Eq. (40) across Calvo-dispersed Home producers gives

$$Y_t \Delta_{H,t} = \left(\frac{N_t}{1-\zeta}\right)^{1-\zeta} \left(\frac{\mathcal{I}_t}{\zeta}\right)^{\zeta}, \tag{41}$$

where $\Delta_{H,t}$ is the Home producer-price dispersion defined above. The exact Foreign mirror applies. Cost minimization between labor and the intermediate-input composite implies that Home producer real marginal cost can first be written as

$$\begin{aligned}
MC_t^r &= \frac{W_t}{P_{H,t}} \left[\frac{\zeta N_t}{(1-\zeta)\mathcal{I}_t}\right]^{\zeta} \\
&= (C_t N_t^{\varphi})^{1-\zeta} \mathcal{G}(S_t/\Psi_t, \tau_{M,t}).
\end{aligned} \tag{42}$$

The second equality uses the cost-minimizing factor mix, the household intratemporal condition $W_t/P_t = C_t N_t^{\varphi}$, and the CPI–PPI ratio $P_t/P_{H,t} = \mathcal{G}(S_t/\Psi_t, \tau_{M,t})$ defined in Eq. (3). Although $\mathcal{I}_t$ appears in the first expression in Eq. (42), it is not an independent determinant of unit cost after the firm has chosen its cost-minimizing input mix.[2] Since $C_t N_t^{\varphi} = W_t/P_t$ is the real wage and $1-\zeta$ is the labor share, a larger intermediate-input share reduces the effect of a given change in the real wage on producer real marginal cost. Intuitively, produced goods are used again as inputs into current production, so production relies less on direct labor as ζ rises. Consequently, when the material-input spillback weakens Home activity and puts downward pressure on MC_t^r, a larger increase in consumption and labor input is required to restore a given amount of producer-marginal-cost support than in the labor-only economy. This is why stronger monetary easing can be required to support the Home PPI-inflation path.

Since current goods are used both for final consumption and as material inputs, each destination's demand for an origin's goods contains final consumption plus material-input demand. Setting $\zeta = 0$ eliminates intermediate inputs and nests the benchmark two-country Imperfect Pass-Through specification.

[2] The labor–intermediate-input first-order conditions imply $\mathcal{I}_t/N_t = [\zeta/(1-\zeta)](W_t/P_t)$. Substituting this ratio into the first expression in Eq. (42) yields nominal unit cost $W_t^{1-\zeta} P_t^{\zeta}$ and therefore eliminates the intermediate-input quantity from the reduced expression for MC_t^r.

5.4.2 Welfare trade-off and producer-price dispersion

Because the production restrictions change, the Benigno–Woodford coefficients must be recomputed. We use the superscript RA for the roundabout-production welfare objects. The corresponding Home Welfare Cost Function, derived in Appendix B.4, is

$$\begin{aligned}\mathcal{L}^{RA}_{H,t} \equiv \frac{1}{2}\Big[&\Lambda^{RA}_y(y_t - y^{w,RA}_t)^2 + \Lambda^{RA}_\psi(\psi_t - \psi^{w,RA}_t)^2 + \Lambda^{RA}_s(s_t - s^{w,RA}_t)^2 \\ &+\Lambda^{RA}_{\psi^*}(\psi^*_t - \psi^{*,w,RA}_t)^2 + \Lambda^{RA}_{\pi,H}\pi^2_{H,t} + \Lambda^{RA}_{\pi,F}\pi^2_{F,t} + \Lambda^{RA}_{\pi^*_H}(\pi^*_{H,t})^2\Big].\end{aligned}$$

The four real Welfare Relevant reference values $y^{w,RA}_t$, $\psi^{w,RA}_t$, $s^{w,RA}_t$, and $\psi^{*,w,RA}_t$ are defined in Eq. (B.15); their benchmark expressions in the manuscript coordinates are reported in Eq. (B.16). Huang and Liu [27] regard an intermediate-input share between 0.5 and 0.7 as a reasonable range, so we set $\zeta = 0.50$, the lower bound of their range. Table 8 reports the seven Home welfare components.

Table 8: Home welfare-cost components: roundabout production

Welfare Relevant Variable	Weight	Value
$y_t - y^{w,RA}_t$	Λ^{RA}_y	0.910
$\psi_t - \psi^{w,RA}_t$	Λ^{RA}_ψ	0.640
$s_t - s^{w,RA}_t$	Λ^{RA}_s	−0.140
$\psi^*_t - \psi^{*,w,RA}_t$	$\Lambda^{RA}_{\psi^*}$	0.210
$\pi_{H,t}$	$\Lambda^{RA}_{\pi,H}$	109.290
$\pi_{F,t}$	$\Lambda^{RA}_{\pi,F}$	34.080
$\pi^*_{H,t}$	$\Lambda^{RA}_{\pi^*_H}$	15.650

The exact weight definitions are reported in Appendix B.4; the three inflation weights are given in Eq. (B.13). The negative coefficient Λ^{RA}_s has the same strategic interpretation as the negative relative-price coefficient in the benchmark national-Ramsey criterion: it is a signed strategic welfare contribution, not a conventional positive stabilization penalty.

The most important change for the present experiment is the weight on Home PPI inflation in Eq. (B.13). The two Θ objects entering that weight, Θ^{RA}_Y and $\Theta^{AS,RA}_H$, are not primitive structural parameters. They are the Benigno–Woodford undetermined coefficients used to eliminate policy-dependent first-order terms in the second-order welfare expansion. Specifically, Θ^{RA}_Y is attached to the Home gross-output resource restriction, whereas $\Theta^{AS,RA}_H$ is attached to the Home producer aggregate-supply restriction. Equation (B.12) determines them jointly as functions of the primitive structure, including ζ, φ, η, the Home expenditure shares, and the steady-state markup. Their difference is therefore the roundabout-specific coefficient with which the second-order resource cost of Home producer-price dispersion enters augmented welfare before the Calvo scaling factor ε/κ_H is applied.

This interpretation explains why $\Theta^{RA}_Y - \Theta^{AS,RA}_H$ rises with the intermediate-input share. In the labor-only economy, producer-price dispersion requires additional labor to deliver a given aggregate output. In the roundabout economy, Eq. (41) shows that the same dispersion raises the gross-production requirement that must be met with both labor and intermediate inputs. Those intermediate inputs are themselves produced goods, so satisfying the additional input requirement calls for further production and further inputs. Producer-price dispersion therefore absorbs more gross resources as ζ rises, and the coefficient multiplying that resource cost in the Welfare Cost Function increases. Holding the other benchmark structural parameters fixed, the symbolic solution of Eq. (B.12) implies

$$\frac{\partial \Lambda^{RA}_{\pi,H}}{\partial \zeta} = \frac{\varepsilon}{\kappa_H}\left(\frac{\partial \Theta^{RA}_Y}{\partial \zeta} - \frac{\partial \Theta^{AS,RA}_H}{\partial \zeta}\right) > 0, \qquad 0 \le \zeta < 1. \tag{43}$$

Since $\kappa_H = (1-\theta)(1-\beta\theta)/\theta$, the factor ε/κ_H converts Home PPI inflation into the second-order resource cost generated by producer-price dispersion under Calvo pricing. A smaller producer Phillips-curve slope therefore makes a given amount of Home PPI inflation more costly in welfare terms. The producer Calvo block itself is unchanged relative to the benchmark, however, so the increase in the Home PPI-inflation weight comes from the rise in $\Theta_Y^{RA} - \Theta_H^{AS,RA}$, not from a change in κ_H. Table 8 therefore reports $\Lambda_{\pi,H}^{RA} = 109.290$, compared with 65.310 in the benchmark two-country economy.

Equation (42) provides the complementary policy mechanism. Because the real wage enters MC_t^r with exponent $1-\zeta$, the material-input spillback can place downward pressure on Home producer real marginal cost and PPI inflation while a given monetary-policy-induced increase in $C_t N_t^{\varphi}$ provides less marginal-cost support than in the labor-only economy. Supporting the Home PPI-inflation path therefore requires a larger increase in Home activity and stronger monetary easing. This is visible in Panels 2 and 6 of Fig. 6(a), which show a larger Home output response and a lower Real Consumption Interest Rate under roundabout production than in the benchmark economy.

5.4.3 Nominal interest-rate response and the ZLB

Figure 6 compares the roundabout economy with the benchmark two-country Imperfect Pass-Through specification.

The benchmark two-country Imperfect Pass-Through economy does not generate a ZLB episode: in the ZLB-relaxed solution, the Home impact policy rate remains above zero. Roundabout production changes this result through the mechanisms identified in Secs. 5.4.1 and 5.4.2. The material-input spillback weakens Home activity and places additional downward pressure on Home producer real marginal cost and Home PPI inflation. At the same time, Eq. (42) shows that the real wage $C_t N_t^{\varphi}$ enters MC_t^r with exponent $1-\zeta$. Monetary easing must therefore generate a larger increase in Home activity to provide the same support to producer real marginal cost when $\zeta > 0$. The welfare incentive to provide that support is also stronger because the Home PPI-inflation weight $\Lambda_{\pi,H}^{RA}$ is larger under roundabout production, as shown in Sec. 5.4.2.

The stronger accommodation is visible in Fig. 6(a): Home output rises more and the Real Consumption Interest Rate falls more than in the benchmark economy (Panels 2 and 6). Quantitatively, the benchmark Home impact rate cut is 60.64 basis points and remains within the 101.01-basis-point policy space available from the deterministic steady state. Under roundabout production, the ZLB-relaxed impact cut is 106.95 basis points, which exceeds that available policy space. The unconstrained optimum therefore requires a negative Home nominal rate on impact, and the ZLB binds for one quarter.

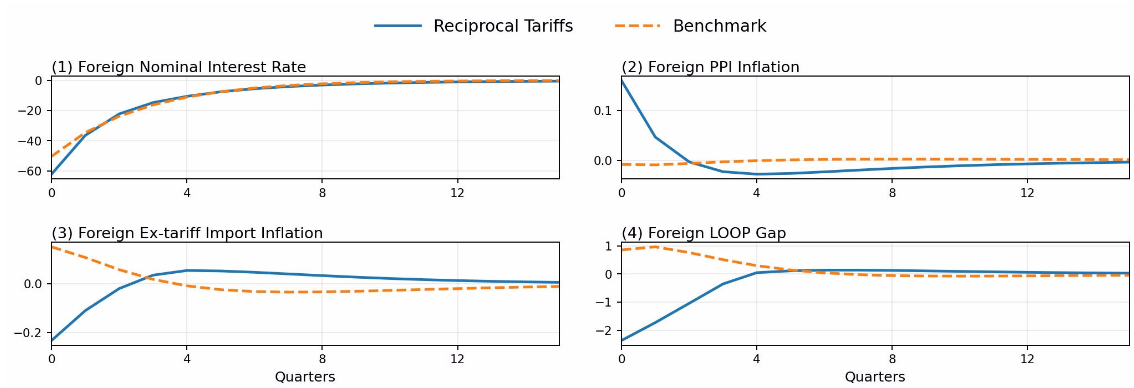

Figure 5: Two-country Imperfect Pass-Through Environment with reciprocal tariff shocks: (b) Foreign responses, compared with the same benchmark-parameterization unilateral-tariff reference. The solid line reports reciprocal tariffs and the dashed line the reference IRF. Panels 1–4 report the Foreign nominal interest rate, Foreign PPI inflation $\pi^*_{F,t}$, Foreign Ex-tariff Import Inflation, and the Foreign LOOP gap, respectively.

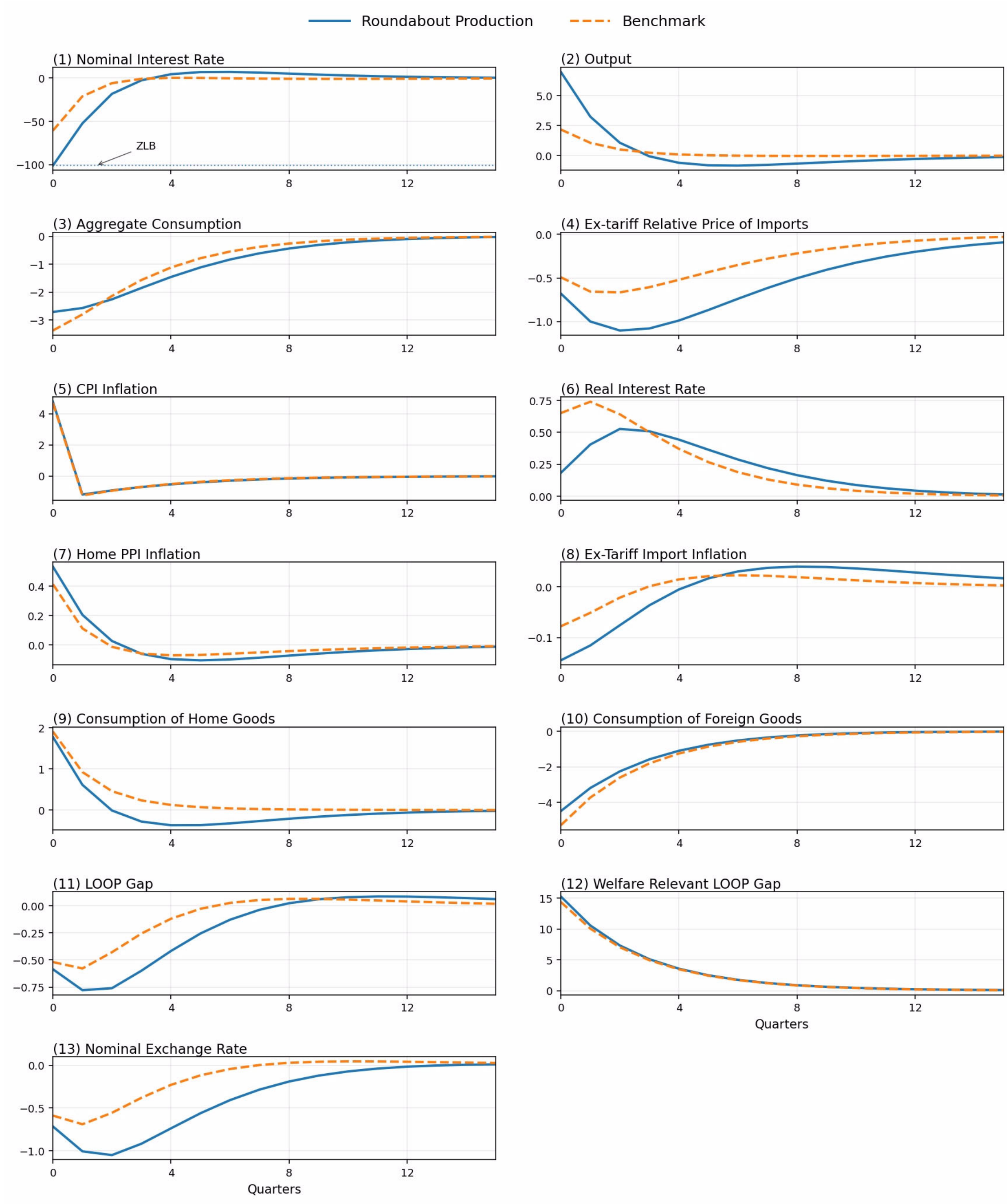

Figure 6: Two-country Imperfect Pass-Through Environment with roundabout production, $\zeta = 0.50$: (a) Home responses, compared with the benchmark two-country Imperfect Pass-Through specification ($\zeta = 0$). The solid line reports roundabout production and the dashed line the benchmark Imperfect Pass-Through IRF. The dotted horizontal line in Panel 1 is the ZLB. The Home ZLB binds for one quarter only under roundabout production. Panels 2, 6, and 7 report Home output, the Real Consumption Interest Rate (Home), $(1 + i_t)/\mathrm{E}_t\Pi_{t+1}$, and Home PPI inflation, respectively.

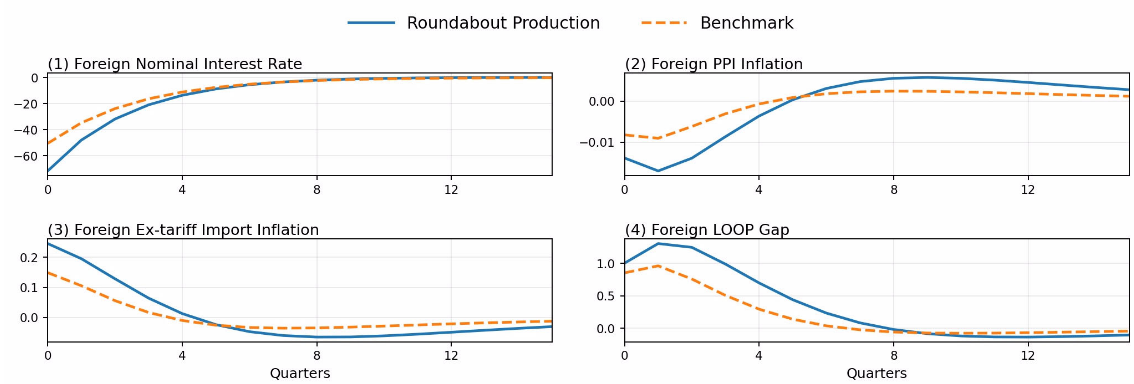

Figure 6: Two-country Imperfect Pass-Through Environment with roundabout production, $\zeta = 0.50$: (b) Foreign responses, compared with the benchmark two-country Imperfect Pass-Through specification. The solid line reports roundabout production and the dashed line the benchmark Imperfect Pass-Through IRF.

5.5 Two-country model: open-loop Nash

5.5.1 Open-loop Nash and the reorganization of the stabilization problem

The final experiment changes the strategic policy concept rather than preferences, technology, tariff processes, or private-equilibrium conditions. The benchmark two-country Imperfect Pass-Through specification in Sec. 4.2 uses the national Ramsey allocation game associated with Faia and Monacelli [23]: Home chooses its own national allocation while taking the complete contingent Foreign plan as given. Under open-loop Nash, following Bodenstein, Guerrieri, and LaBriola [29], the Home strategic policy path is $\{\Pi_{H,t}\}_{t\geq 0}$ and the complete Foreign PPI-inflation path $\{\Pi^*_{F,t}\}_{t\geq 0}$ is taken as given. Conditional on those two strategies, local importer prices, relative prices, private allocations, and the two nominal interest rates adjust endogenously through the common world equilibrium. Following Bodenstein, Guerrieri, and LaBriola, we use producer-price inflation rather than the nominal interest rate to define the strategy space because nominal-rate strategies can be non-unique in this class of open-loop games. The corresponding national Lagrangians and first-order conditions are reported in On-line Appendix OB.5.

This change reorganizes the Home stabilization problem in a way that can push the desired rate to the ZLB. In the benchmark allocation game, the Home importer-price block is part of the Home national plan. Under open-loop Nash, the only Home strategic policy path is $\{\pi_{H,t}\}$, while Home import-price inflation $\pi_{F,t}$ is an endogenous world-equilibrium variable. Through the local relative-import-price identity in Eq. (OA.4) and the Home importer Calvo block, the feasible path of $\pi_{F,t}$ adjusts jointly with the Home PPI-inflation strategy and the endogenous relative-price path. Home import-price inflation remains welfare relevant, but its stabilization is much less independent of the Home PPI strategy. The stabilization problem therefore becomes more concentrated on the Home producer-price block, which can require stronger monetary accommodation.

5.5.2 Welfare criterion under open-loop Nash

For reporting the open-loop Welfare Relevant variables, we retain y_t, ψ_t, s_t, and ψ_t^* as the real coordinates and use the fixed sequential-completion normalization described in On-line Appendix OB.5. We use the superscript OLN for the open-loop Nash welfare objects. The Home Welfare Cost Function is

$$\begin{aligned}\mathcal{L}_{H,t}^{OLN} \quad \equiv \quad & \frac{1}{2}\Big[\Lambda_y^{OLN}(y_t - y_t^{w,OLN})^2 + \Lambda_\psi^{OLN}(\psi_t - \psi_t^{w,OLN})^2 \\ & +\Lambda_s^{OLN}(s_t - s_t^{w,OLN})^2 + \Lambda_{\psi^*}^{OLN}(\psi_t^* - \psi_t^{*,w,OLN})^2 \\ & +\Lambda_{\pi,H}^{OLN}\pi_{H,t}^2 + \Lambda_{\pi,F}^{OLN}\pi_{F,t}^2 + \Lambda_{\pi_H^*}^{OLN}(\pi_{H,t}^*)^2\Big]. \end{aligned} \tag{44}$$

The symbolic square-completion construction is given by Eq. (OB.19) together with Eqs. (OB.2)–(OB.3), and the fixed reporting normalization is reported in Eq. (OB.21). Under the benchmark calibration, the Home Welfare Relevant LOOP Gap is

$$\psi_t^{w,OLN} = -0.645\, s_t - 0.146\, \psi_t^* + 0.138\, \tau_{M,t} + 0.254\, \tau_{M,t}^*. \tag{45}$$

Table 9 reports the corresponding welfare weights.

The four real weights and the three inflation weights are defined in Eqs. (OB.20) and (OB.17), respectively. The real completed-square weights and Welfare Relevant variables are specific to this reporting normalization. In particular, the negative $\Lambda_{\psi^*}^{OLN}$ is a signed strategic welfare contribution rather than a conventional positive stabilization penalty. By contrast, the three inflation weights are pinned down directly by the stationary Home open-loop multipliers on the Calvo price-dispersion equations.

The quantitative change that matters most for the ZLB is the reweighting of the sticky-price blocks. Relative to Table 4, the weight on Home import-price inflation falls from 21.280 to 1.350, whereas the weight on Home PPI inflation falls only from 65.310 to 50.400. The stabilization

Table 9: Home welfare-cost components: open-loop Nash

Welfare Relevant Variable	Weight	Value
$y_t - y_t^{w,OLN}$	Λ_y^{OLN}	3.730
$\psi_t - \psi_t^{w,OLN}$	Λ_ψ^{OLN}	1.530
$s_t - s_t^{w,OLN}$	Λ_s^{OLN}	0.700
$\psi_t^* - \psi_t^{*,w,OLN}$	$\Lambda_{\psi^*}^{OLN}$	-0.470
$\pi_{H,t}$	$\Lambda_{\pi,H}^{OLN}$	50.400
$\pi_{F,t}$	$\Lambda_{\pi,F}^{OLN}$	1.350
$\pi_{H,t}^*$	$\Lambda_{\pi_H^*}^{OLN}$	11.040

problem therefore becomes much more concentrated on the Home producer-price block. Under the fixed reporting normalization, the same reorganization is reflected in the smaller tariff coefficient in $\psi_t^{w,OLN}$, but the real Welfare Relevant variables are normalization-specific. The sharp fall in $\Lambda_{\pi,F}^{OLN}$ is the more direct evidence from the stationary Calvo-dispersion multipliers.

This provides a direct contrast with the benchmark allocation game in Sec. 4.4. There, the marginal counterfactual in Table 16 shows that further easing raises the Home PPI-inflation contribution by more than the additional improvement in the Welfare Relevant LOOP Gap Deviation contribution, so Home producer-price stabilization limits further accommodation. Under open-loop Nash, importer-price and LOOP-gap adjustment are more tightly internalized in the response to the Home PPI strategy, reducing the extent to which the LOOP gap appears as a separate element of the reporting trade-off. Policy is correspondingly governed more strongly by the Home producer-price block.

5.5.3 Nominal interest-rate response and the ZLB

Figures 7(a) and 7(b) compare open-loop Nash with the benchmark two-country national Ramsey allocation game.

The nominal-rate response follows from the reorganization of the stabilization problem. Since the Home producer Phillips curve in Eq. (21) is unchanged, the preferred $\pi_{H,t}$ path requires the corresponding feasible path of Home producer real marginal cost mc_t^r. With importer-price inflation playing a much smaller independent stabilization role, policy is more strongly governed by the Home producer-price block. The monetary policy needed to implement the preferred $\pi_{H,t}$–mc_t^r path is therefore more accommodative in this calibration.

Quantitatively, the benchmark Home rate cut is 60.64 basis points on impact, whereas the ZLB-relaxed open-loop cut is 101.31 basis points, 0.30 basis points beyond the ZLB. The actual Home rate therefore binds at zero for one quarter. Under the reporting normalization, the impact Home Welfare Relevant LOOP Gap is about 3.24 percent under open-loop Nash, compared with about 14.37 percent in the benchmark allocation game. Because these real Welfare Relevant variables are normalization-specific, this comparison is supporting rather than decisive evidence; the reweighting of the sticky-price blocks provides the cleaner explanation for the stronger accommodation. The ZLB episode is a property of the present open-loop equilibrium, not a general result that open-loop Nash must always be more expansionary.

6 Conclusion

This paper shows that low short-run pass-through can make an import tariff call for substantial welfare-maximizing monetary accommodation. Starting from the Perfect Pass-Through SOE benchmark of Monacelli [19], we retain the tariff experiment, fiscal closure, PCP producer pricing, and basic trade structure, and change the pass-through environment by adding sticky domestic importers using LCP. In this controlled comparison the Ramsey response changes from tightening under perfect pass-through to substantial easing under imperfect pass-through, and monetary ac-

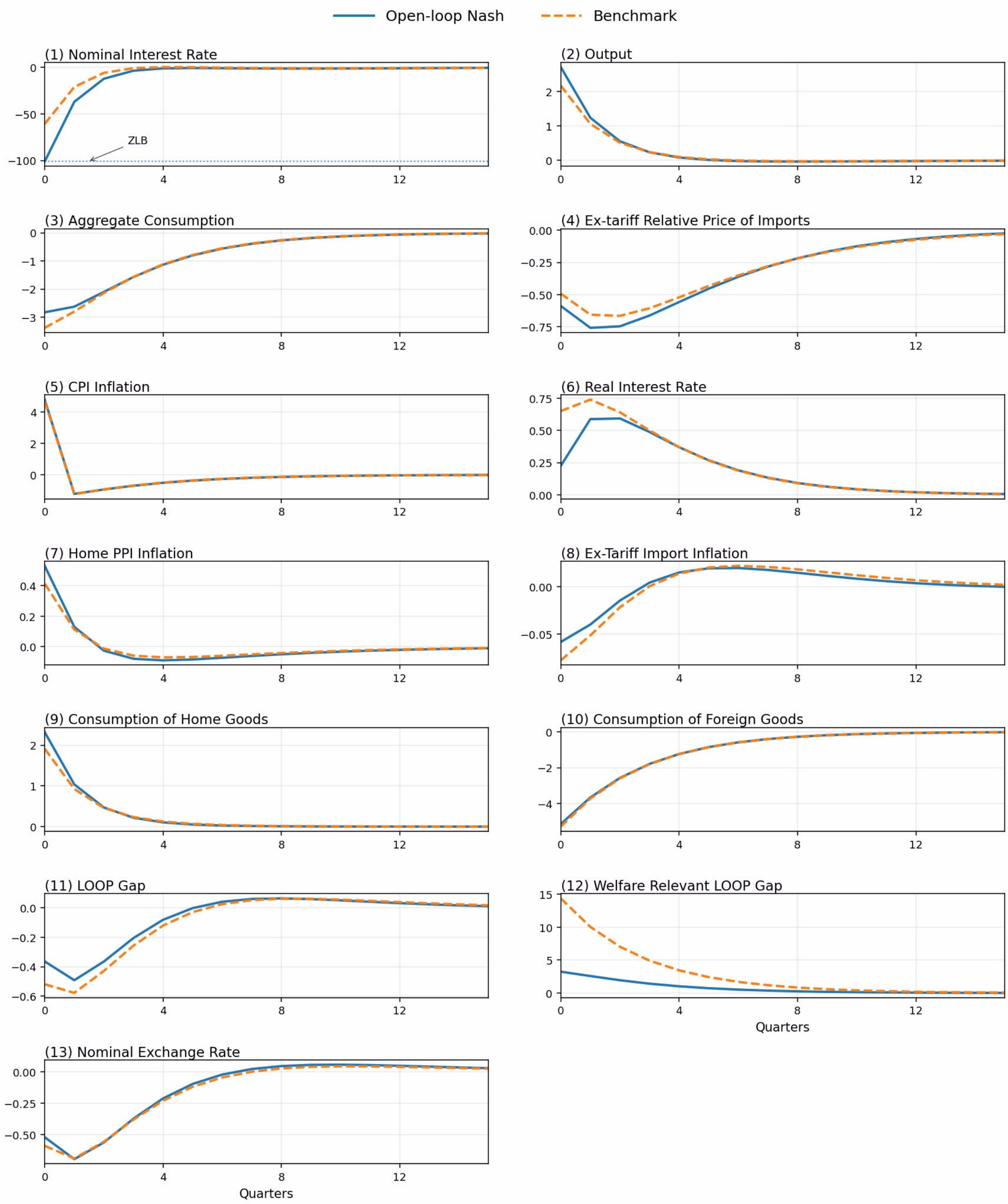

Figure 7: Two-country Imperfect Pass-Through Environment under open-loop Nash, compared with the benchmark two-country national Ramsey allocation game in the Imperfect Pass-Through Environment: (a) Home responses. The solid line reports open-loop Nash and the dashed line the benchmark Imperfect Pass-Through IRF. The dotted horizontal line in Panel 1 is the ZLB. The Home ZLB binds for one quarter only under open-loop Nash. Panels 5 and 6 report Home CPI inflation and the Real Consumption Interest Rate (Home), $(1+i_t)/\mathrm{E}_t\Pi_{t+1}$, respectively. Panel 8 reports ex-tariff Home import inflation $\pi_{F,t}$. Panel 4 reports the Ex-tariff Relative Price of Imports $P_{F,t}/P_{H,t}$. Panels 11–13 report, respectively, the LOOP gap, the Welfare Relevant LOOP Gap, and the nominal exchange rate. In Panel 12, the solid line reports $\psi_t^{w,OLN}$ in Eq. (45) and the dashed line reports $\psi_t^{w,IPT}$ in Eq. (34).

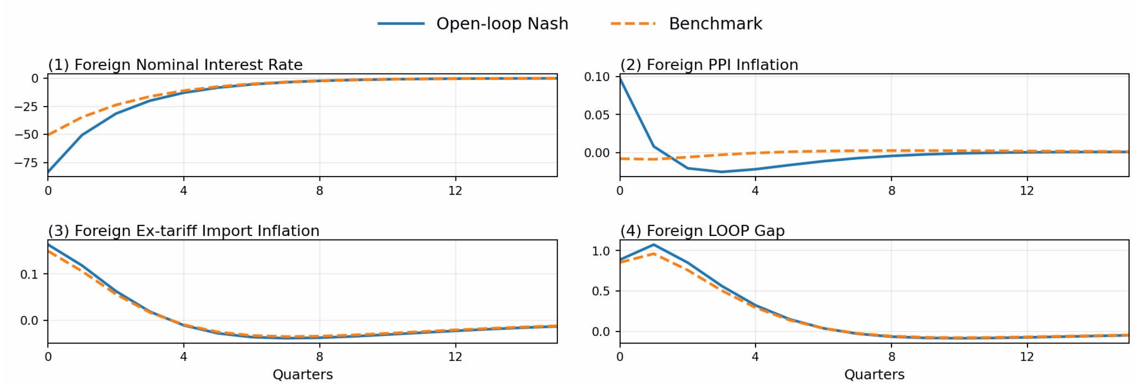

Figure 7: Two-country Imperfect Pass-Through Environment under open-loop Nash, compared with the benchmark two-country national Ramsey allocation game in the Imperfect Pass-Through Environment: (b) Foreign responses. The solid line reports open-loop Nash and the dashed line the benchmark Imperfect Pass-Through IRF. Panels 1–4 report the Foreign nominal interest rate, Foreign PPI inflation $\pi^*_{F,t}$, Foreign Ex-tariff Import Inflation, and the Foreign LOOP gap, respectively.

commodation remains optimal when Foreign demand and monetary policy are endogenized in the two-country economy. Because both sides of the comparison are optimal-policy allocations, this result is distinct from the tightening bias under CPI-inflation targeting.

The mechanism differs across the two pass-through environments. With perfect pass-through, the tariff raises the CPI–PPI ratio and Home producer real marginal cost, putting upward pressure on Home PPI inflation and making monetary tightening optimal. With imperfect pass-through, the planner can exploit the relative-price externality more effectively. In the SOE, monetary easing depreciates the Home currency and pushes the external relative price upward relative to a less accommodative path, shifting expenditure toward Home goods and supporting Home output and income. Because the local import price is sticky, this depreciation does not pass one-for-one into the domestic ex-tariff import price; part of the adjustment appears as a LOOP gap. The planner can therefore support Home income, while the associated rise in the LOOP gap offsets part of the tariff-induced increase in the cum-tariff import price. This gain is traded against importer-price and Home-PPI inflation costs. In the two-country economy, Home easing mitigates the appreciation generated by the relative monetary-policy path and again supports Home output and income relative to a less accommodative Home policy, while Home PPI-inflation stabilization limits further accommodation.

The benchmark imperfect-pass-through economy already requires substantial accommodation while conventional policy remains feasible. The mechanisms that add the final push toward the ZLB differ across the extensions. Greater substitutability makes relative-price movements generate larger changes in the composition of consumption; external habit makes the aggregate-consumption consequences of accommodation intertemporally persistent; reciprocal tariffs add a contractionary foreign-demand spillback; roundabout production adds a material-input spillover–spillback, weakens the effect of Home activity on producer real marginal cost, and raises the welfare cost of Home producer-price dispersion; and open-loop Nash concentrates the stabilization problem more strongly on the Home producer-price block. Each changes one economically relevant dimension of the environment, and each can strengthen the already large desired easing enough for the ZLB to bind, without adding an exogenous natural-rate shock. The central quantitative implication is therefore that low short-run pass-through can make the optimal monetary easing after a tariff shock so substantial that one additional force in the stabilization problem can exhaust the available conventional-policy space.

Appendix

A Pricing and welfare definitions

This Appendix collects the pricing definitions and the symbolic Welfare Relevant variables needed to interpret the results in the main text. The complete SOE and two-country Ramsey Lagrangians and first-order conditions are reported in On-line Appendix OA. The counterfactual welfare accounting used as supporting evidence in Secs. 3.5 and 4.4 is reported in Appendix C. Complete derivations for the extensions used in Sec. 5 are reported in On-line Appendix OB.

A.1 Pricing definitions

Throughout the Appendix, PCP and LCP continue to denote pricing behavior and are retained in established equation superscripts and coefficient labels. The complete model specifications are the Perfect and Imperfect Pass-Through Environments defined in Sec. 3.1.4; the Imperfect Pass-Through Environment retains the PCP producer block and adds the LCP importer block.

Throughout the paper and the Appendix, S_t denotes the same ex-tariff border relative price in the SOE and the two-country model. In particular,

$$S_t = \frac{\mathcal{E}_t P^*_{F,t}}{P_{H,t}}, \qquad \Psi_t = \frac{\mathcal{E}_t P^*_{F,t}}{P_{F,t}}, \qquad \frac{P_{F,t}}{P_{H,t}} = \frac{S_t}{\Psi_t}.$$

In the Perfect Pass-Through Environment, $\Psi_t = 1$. In the Imperfect Pass-Through Environment, the LCP importer price $P_{F,t}$ is the sticky tariff-exclusive Home-currency import price and the tariff is added to the purchaser price through $(1+\tau_{M,t})P_{F,t}$. The Home import-price dispersion is

$$\Delta_{F,t} \equiv \int_0^1 \left(\frac{P_{F,t}(\iota)}{P_{F,t}}\right)^{-\varepsilon_M} d\iota.$$

In the two-country model, the Foreign-country counterpart for Home goods is

$$\Delta^*_{H,t} = \frac{1}{1-\nu}\int_\nu^1 \left(\frac{P^*_{H,t}(\iota)}{P^*_{H,t}}\right)^{-\varepsilon_M} d\iota.$$

where $P^*_{H,t}$ is the Foreign-currency import price of Home goods. Thus stars denote Foreign-country or Foreign-currency objects, while the H and F price subscripts denote the origin of the good.

For completeness, the household nominal stochastic discount factor from t to $t+k$, used in the producer and importer Calvo pricing problems, is

$$Q_{t,t+k} \equiv \beta^k \frac{C_t \mathcal{G}(S_t/\Psi_t, \tau_{M,t})}{C_{t+k}\mathcal{G}(S_{t+k}/\Psi_{t+k}, \tau_{M,t+k})} \left(\prod_{s=1}^{k} \Pi_{H,t+s}\right)^{-1}.$$

A.2 SOE welfare coefficients

Representation with the ex-tariff relative price retained. The two unnumbered welfare representations at the beginning of Sec. 3.4 retain the Welfare Relevant Ex-tariff Relative Price of Imports Gap explicitly. The period welfare cost itself remains $\mathcal{L}_t^{SOE,\cdot}$.

For Perfect Pass-Through, write the real part of the second-order welfare polynomial, up to terms independent of the endogenous allocation, as

$$\begin{aligned}\mathcal{L}^{SOE,PPT}_{t,\mathrm{real}} \quad &= \quad \frac{1}{2}\left[-2\widehat{C}^{SOE,PPT}_{yy} y_t^2 - 2\widehat{C}^{SOE,PPT}_{ss} s_t^2\right] - \widehat{C}^{SOE,PPT}_{ys} y_t s_t \\ &\quad -\widehat{C}^{SOE,PPT}_{ym} y_t \tau_{M,t} - \widehat{C}^{SOE,PPT}_{sm} s_t \tau_{M,t} + \text{s.o.t.i.p.}, \qquad (A.1)\end{aligned}$$

where s.o.t.i.p. denotes terms independent of the endogenous allocation to the order considered. The coefficients $\widehat{C}_{yy}^{SOE,PPT}$, $\widehat{C}_{ss}^{SOE,PPT}$, $\widehat{C}_{ys}^{SOE,PPT}$, $\widehat{C}_{ym}^{SOE,PPT}$, and $\widehat{C}_{sm}^{SOE,PPT}$ are the literal coefficients on the corresponding quadratic or tariff-interaction terms in this welfare polynomial.

The first-order approximations of Eqs. (8) and (23) imply

$$y_t = \left[\eta + (1-\eta)(1-\upsilon)^2\right] s_t + \upsilon(1-\upsilon)(\eta-1)\left(\tau_{M,t} - \psi_t\right). \tag{A.2}$$

Under Perfect Pass-Through, $\psi_t = 0$. Using Eq. (A.2) to remove s_t from the output–relative-price cross term before completing squares gives

$$\begin{aligned}
\widehat{\Lambda}_y^{SOE,PPT} &\equiv -2\widehat{C}_{yy}^{SOE,PPT} - \frac{2\widehat{C}_{ys}^{SOE,PPT}}{\eta + (1-\eta)(1-\upsilon)^2}, \\
\widehat{y}_t^{w,SOE,PPT} &\equiv \frac{1}{\widehat{\Lambda}_y^{SOE,PPT}}\left[\widehat{C}_{ym}^{SOE,PPT} - \frac{\upsilon(1-\upsilon)(\eta-1)}{\eta + (1-\eta)(1-\upsilon)^2}\widehat{C}_{ys}^{SOE,PPT}\right]\tau_{M,t}, \\
\widehat{\Lambda}_s^{SOE,PPT} &\equiv -2\widehat{C}_{ss}^{SOE,PPT}, \\
\widehat{s}_t^{w,SOE,PPT} &\equiv \frac{\widehat{C}_{sm}^{SOE,PPT}}{\widehat{\Lambda}_s^{SOE,PPT}}\tau_{M,t}.
\end{aligned} \tag{A.3}$$

Thus the Welfare Relevant Output appearing in the first unnumbered welfare representation in Sec. 3.4 contains no s_t.

For Imperfect Pass-Through, the corresponding real polynomial is

$$\begin{aligned}
\mathcal{L}_{t,\mathrm{real}}^{SOE,IPT} = \frac{1}{2}&\left[-2\widehat{C}_{yy}^{SOE,IPT} y_t^2 - 2\widehat{C}_{\psi\psi}^{SOE,IPT}\psi_t^2 - 2\widehat{C}_{ss}^{SOE,IPT} s_t^2\right] \\
&-\widehat{C}_{y\psi}^{SOE,IPT} y_t\psi_t - \widehat{C}_{ys}^{SOE,IPT} y_t s_t - \widehat{C}_{\psi s}^{SOE,IPT}\psi_t s_t \\
&-\widehat{C}_{ym}^{SOE,IPT} y_t\tau_{M,t} - \widehat{C}_{\psi m}^{SOE,IPT}\psi_t\tau_{M,t} - \widehat{C}_{sm}^{SOE,IPT} s_t\tau_{M,t} + \text{s.o.t.i.p.}.
\end{aligned} \tag{A.4}$$

The hatted C coefficients again denote the literal coefficients on the corresponding terms in the welfare polynomial. Applying Eq. (A.2) to the $y_t s_t$ term before sequentially completing the y_t, ψ_t, and s_t squares gives

$$\begin{aligned}
\widehat{\Lambda}_y^{SOE,IPT} &\equiv -2\widehat{C}_{yy}^{SOE,IPT} - \frac{2\widehat{C}_{ys}^{SOE,IPT}}{\eta + (1-\eta)(1-\upsilon)^2}, \\
\widehat{y}_t^{w,SOE,IPT} &\equiv \frac{1}{\widehat{\Lambda}_y^{SOE,IPT}}\left\{\left[\widehat{C}_{ym}^{SOE,IPT} - \frac{\upsilon(1-\upsilon)(\eta-1)}{\eta + (1-\eta)(1-\upsilon)^2}\widehat{C}_{ys}^{SOE,IPT}\right]\tau_{M,t}\right. \\
&\qquad\left. + \left[\widehat{C}_{y\psi}^{SOE,IPT} + \frac{\upsilon(1-\upsilon)(\eta-1)}{\eta + (1-\eta)(1-\upsilon)^2}\widehat{C}_{ys}^{SOE,IPT}\right]\psi_t\right\}, \\
\widehat{\Lambda}_\psi^{SOE,IPT} &\equiv -2\widehat{C}_{\psi\psi}^{SOE,IPT} - \frac{1}{\widehat{\Lambda}_y^{SOE,IPT}}\left[\widehat{C}_{y\psi}^{SOE,IPT} + \frac{\upsilon(1-\upsilon)(\eta-1)}{\eta + (1-\eta)(1-\upsilon)^2}\widehat{C}_{ys}^{SOE,IPT}\right]^2, \\
\widehat{\psi}_t^{w,SOE,IPT} &\equiv \frac{1}{\widehat{\Lambda}_\psi^{SOE,IPT}}\left\{\left[\widehat{C}_{\psi m}^{SOE,IPT} + \frac{1}{\widehat{\Lambda}_y^{SOE,IPT}}\left(\widehat{C}_{y\psi}^{SOE,IPT} + \frac{\upsilon(1-\upsilon)(\eta-1)}{\eta + (1-\eta)(1-\upsilon)^2}\widehat{C}_{ys}^{SOE,IPT}\right)\right.\right. \\
&\qquad\left.\left.\times\left(\widehat{C}_{ym}^{SOE,IPT} - \frac{\upsilon(1-\upsilon)(\eta-1)}{\eta + (1-\eta)(1-\upsilon)^2}\widehat{C}_{ys}^{SOE,IPT}\right)\right]\tau_{M,t} + \widehat{C}_{\psi s}^{SOE,IPT} s_t\right\}, \\
\widehat{\Lambda}_s^{SOE,IPT} &\equiv -2\widehat{C}_{ss}^{SOE,IPT} - \frac{(\widehat{C}_{\psi s}^{SOE,IPT})^2}{\widehat{\Lambda}_\psi^{SOE,IPT}}, \\
\widehat{s}_t^{w,SOE,IPT} &\equiv \frac{1}{\widehat{\Lambda}_s^{SOE,IPT}}\left\{\widehat{C}_{sm}^{SOE,IPT} + \frac{\widehat{C}_{\psi s}^{SOE,IPT}}{\widehat{\Lambda}_\psi^{SOE,IPT}}\left[\widehat{C}_{\psi m}^{SOE,IPT} + \frac{1}{\widehat{\Lambda}_y^{SOE,IPT}}\left(\widehat{C}_{y\psi}^{SOE,IPT} + \frac{\upsilon(1-\upsilon)(\eta-1)}{\eta + (1-\eta)(1-\upsilon)^2}\right.\right.\right.
\end{aligned}$$

$$\times \left(\widehat{C}_{ym}^{SOE,IPT} - \frac{\upsilon(1-\upsilon)(\eta-1)}{\eta+(1-\eta)(1-\upsilon)^2} \widehat{C}_{ys}^{SOE,IPT} \right) \Bigg] \Bigg\} \tau_{M,t}.$$

The Welfare Relevant Output in Eq. (A.5) therefore depends on the tariff and the LOOP gap, but not on s_t.

Representation after eliminating the ex-tariff relative price. The Welfare Relevant variables used in the numbered SOE welfare-cost equations in Sec. 3.4 are defined here. Since $\log(1+\tau_{M,t}) = \tau_{M,t}$ to first order, the import-tariff innovation is written directly as $\tau_{M,t}$. In the present SOE experiments there is no export-tariff disturbance, so

$$y_t^{w,SOE,PPT} \equiv \frac{C_{ym}^{SOE,PPT}}{\Lambda_y^{SOE,PPT}} \tau_{M,t}, \tag{A.6}$$

$$y_t^{w,SOE,IPT} \equiv \frac{C_{ym}^{IPT}}{\Lambda_y^{SOE,IPT}} \tau_{M,t} + \frac{C_{y\psi}^{IPT}}{\Lambda_y^{SOE,IPT}} \psi_t. \tag{A.7}$$

After completing the output square, collecting the remaining terms in ψ_t gives the Welfare Relevant LOOP Gap directly as

$$\psi_t^{w,SOE,IPT} \equiv \frac{C_{\psi m}^{IPT} + \dfrac{C_{y\psi}^{IPT} C_{ym}^{IPT}}{\Lambda_y^{SOE,IPT}}}{\Lambda_\psi^{SOE,IPT}} \tau_{M,t}. \tag{A.8}$$

Here $C_{ym}^{SOE,PPT}$, C_{ym}^{IPT}, $C_{\psi m}^{IPT}$, and $C_{y\psi}^{IPT}$ are the corresponding cross coefficients in the second-order welfare expansion; the coefficient notation is retained from the underlying symbolic welfare derivation. Equation (A.8) depends only on the exogenous tariff innovation and therefore is the reference value entering the LOOP-gap square in Eq. (26).

The coefficients Θ_n, Θ_H^{AS}, and Θ_M^{AS} in Table 3 are the undetermined coefficients used to eliminate the first-order terms in the Benigno–Woodford expansion. For the Perfect Pass-Through Environment, temporary coefficients Θ_χ, Θ_r, Θ_Y, Θ_n, $\Theta_{mc,H}$, and Θ_H^{AS} are attached to the corresponding first-order equilibrium restrictions. In the Imperfect Pass-Through Environment, $\Theta_{mc,M}$ and Θ_M^{AS} are additionally attached to the importer marginal-cost identity and the importer aggregate-supply restriction.

For the SOE steady state used in the welfare expansion, $d_H = 1-\upsilon$, $\lambda_M = \upsilon$, $L = \mathcal{M}^{-1}$, where $\mathcal{M} = \varepsilon/(\varepsilon-1)$, and

$$\mathcal{D}_S \equiv 1 + \eta\varphi + d_H\varphi(\eta-1)(\lambda_M - 1).$$

Coefficient matching gives

$$\begin{aligned}
\Theta_n &= \frac{L + \varphi(1-\lambda_M)}{\mathcal{D}_S}, \\
\Theta_\chi &= \frac{L d_H(\eta-1) + \eta\varphi + 1}{\mathcal{D}_S}, \\
\Theta_{mc,H} &= \frac{(1-\lambda_M) - L\eta - L d_H(\eta-1)(\lambda_M-1)}{\mathcal{D}_S}, \\
\Theta_H^{AS} &= -\Theta_{mc,H}.
\end{aligned} \tag{A.9}$$

The producer coefficients are unchanged in the Imperfect Pass-Through Environment. The importer coefficients satisfy

$$\Theta_{mc,M} = \lambda_M \Theta_\chi, \qquad \Theta_M^{AS} = -\lambda_M \Theta_\chi. \tag{A.10}$$

Thus the three coefficients entering the SOE inflation weights are defined by Eqs. (A.9) and (A.10). The exact SOE weight definitions are

$$\begin{aligned}
\Lambda_y^{SOE,PPT} &\equiv -2C_{yy}^{SOE,PPT}, \qquad \Lambda_{\pi,H}^{SOE,PPT} \equiv \frac{\varepsilon}{\kappa_H}(\Theta_n - \Theta_H^{AS}), \\
\Lambda_y^{SOE,IPT} &\equiv -2C_{yy}^{IPT}, \qquad \Lambda_{\psi}^{SOE,IPT} \equiv -2C_{\psi\psi}^{IPT} - \frac{(C_{y\psi}^{IPT})^2}{\Lambda_y^{SOE,IPT}}, \\
\Lambda_{\pi,H}^{SOE,IPT} &\equiv \frac{\varepsilon}{\kappa_H}(\Theta_n - \Theta_H^{AS}), \qquad \Lambda_{\pi,F}^{SOE,IPT} \equiv -\frac{\varepsilon_M}{\kappa_M}\Theta_M^{AS}.
\end{aligned} \tag{A.11}$$

The second-order welfare criteria are expansions around the distorted deterministic steady state. The linear–quadratic construction follows Benigno and Woodford [30, 31]: first-order equilibrium restrictions are used to eliminate terms that would otherwise make a naive second-order utility expansion incorrect around a distorted steady state, and price-dispersion laws generate the inflation-square terms. The Welfare Relevant variables in Eqs. (25), (26), (31), (32), and (44) are obtained by exact sequential completion of squares. They are reference values in the welfare criterion, not new private-equilibrium variables.

The benchmark numerical allocations reported in the main text are not solutions to the quadratic approximations. They are obtained from the nonlinear Ramsey first-order-condition systems; OccBin is used only to implement the occasionally binding ZLB in the local solution. The loss functions are used to interpret those allocations and to identify which nominal and real distortions are welfare-relevant.

A.3 Two-country Welfare Relevant variable definitions

For completeness, this subsection records the two real Welfare Relevant variables appearing in Eq. (31) and the four real Welfare Relevant variables appearing in Eq. (32). To first order,

$$\log(1 + \tau_{M,t}) = \tau_{M,t}, \qquad \log(1 + \tau^*_{M,t}) = \tau^*_{M,t}.$$

Perfect Pass-Through Environment. The PCP Welfare Relevant variables can be obtained directly from the reduced second-order Home-welfare polynomial. Let C_{ym}^{PPT}, $C_{ym^*}^{PPT}$, $C_{y^*m}^{PPT}$, and $C_{y^*m^*}^{PPT}$ denote, consistently with the coefficient notation in Sec. 4.3, the literal coefficients multiplying $y_t\tau_{M,t}$, $y_t\tau^*_{M,t}$, $y^*_t\tau_{M,t}$, and $y^*_t\tau^*_{M,t}$, respectively. Before eliminating Foreign output, the real quadratic terms in the Home period loss are

$$\begin{aligned}
\frac{1}{2}\Big\{ &-2C_{yy}^{PPT}y_t^2 - 2C_{y^*y^*}^{PPT}(y^*_t)^2 - 2C_{yy^*}^{PPT}y_ty^*_t \\
&-2\big(C_{ym}^{PPT}\tau_{M,t} + C_{ym^*}^{PPT}\tau^*_{M,t}\big)y_t \\
&-2\big(C_{y^*m}^{PPT}\tau_{M,t} + C_{y^*m^*}^{PPT}\tau^*_{M,t}\big)y^*_t\Big\} + \text{s.o.t.i.p.}.
\end{aligned} \tag{A.12}$$

where s.o.t.i.p. collects terms independent of the endogenous allocation to the order considered. The first-order world goods-market conditions and complete international risk sharing imply, with $D_S \equiv \eta + (1-\eta)(1-\upsilon)^2$,

$$\begin{aligned}
D_S s_t &= y_t - y^*_t - \upsilon\big[\eta\omega_H - (1-\upsilon)(1-\nu)\big]\tau_{M,t} \\
&\quad + \upsilon\big[\eta\omega^*_F - (1-\upsilon)\nu\big]\tau^*_{M,t},
\end{aligned} \tag{A.13}$$

Substitution of Eq. (A.13) into Eq. (A.12) gives an output–relative-price cross term proportional to $-2D_S[-2C_{y^*y^*}^{PPT} - C_{yy^*}^{PPT}]y_ts_t$. The symbolic Benigno–Woodford coefficients satisfy

$$C_{yy^*}^{PPT} = -2C_{y^*y^*}^{PPT},$$

so this cross term cancels exactly. Completing the remaining y_t and s_t squares yields the final Welfare Relevant variables entering Eq. (31):

$$y_t^{w,PPT} \equiv \frac{C_{ym}^{PPT} + C_{y^*m}^{PPT}}{\Lambda_y^{PPT}} \tau_{M,t} + \frac{C_{ym^*}^{PPT} + C_{y^*m^*}^{PPT}}{\Lambda_y^{PPT}} \tau_{M,t}^*. \tag{A.14}$$

$$\begin{aligned} s_t^{w,PPT} \equiv \frac{1}{D_S} \Bigg\{ & \left[-\upsilon\{\eta\omega_H - (1-\upsilon)(1-\nu)\} + \frac{C_{y^*m}^{PPT}}{2C_{y^*y^*}^{PPT}} \right] \tau_{M,t} \\ & + \left[\upsilon\{\eta\omega_F^* - (1-\upsilon)\nu\} + \frac{C_{y^*m^*}^{PPT}}{2C_{y^*y^*}^{PPT}} \right] \tau_{M,t}^* \Bigg\}. \end{aligned} \tag{A.15}$$

The corresponding Perfect Pass-Through weight definitions are

$$\Lambda_y^{PPT} \equiv -2\left(C_{yy}^{PPT} - C_{y^*y^*}^{PPT}\right), \qquad \Lambda_s^{PPT} \equiv -2D_S^2 C_{y^*y^*}^{PPT}, \qquad \Lambda_{\pi,H}^{PPT} \equiv \frac{\varepsilon}{\kappa_H}(\Theta_n - \Theta_H^{AS}). \tag{A.16}$$

Equation (A.14) defines the (Logarithmic) Welfare Relevant Output, while Eq. (A.15) defines the (Logarithmic) Welfare Relevant Ex-tariff Relative Price of Imports. These are the final, rather than intermediate, PCP Welfare Relevant variables. For the unilateral Home-tariff experiment, $\tau_{M,t}^* = 0$.

Imperfect Pass-Through Environment. Use complete risk sharing and the first-order world equilibrium to eliminate Foreign output. Before collecting the reduced welfare coefficients, express all local-relative-price terms in the manuscript coordinate $s_t = \log(\mathcal{E}_t P_{F,t}^*/P_{H,t})$. In particular, the identity stated in Sec. 4.1, $\log(P_{F,t}/P_{H,t}) = s_t - \psi_t$, is imposed before the real quadratic form is completed. The resulting real Home coordinates are therefore y_t, ψ_t, s_t, and ψ_t^*. The exact sequential completion of this manuscript-coordinate two-country Home-welfare criterion gives

$$\begin{aligned} y_t^{w,IPT} &= b_{ym}^H \tau_{M,t} + b_{ym^*}^H \tau_{M,t}^*, & \text{(A.17)} \\ \psi_t^{w,IPT} &= b_{\psi s}^H s_t + b_{\psi\psi^*}^H \psi_t^* + b_{\psi m}^H \tau_{M,t} + b_{\psi m^*}^H \tau_{M,t}^*, & \text{(A.18)} \\ s_t^{w,IPT} &= b_{s\psi^*}^H \psi_t^* + b_{sm}^H \tau_{M,t} + b_{sm^*}^H \tau_{M,t}^*, & \\ \psi_t^{*,w,IPT} &= b_{\psi^*m}^H \tau_{M,t} + b_{\psi^*m^*}^H \tau_{M,t}^*. & \text{(A.19)} \end{aligned}$$

The corresponding real-weight definitions are

$$\begin{aligned} \Lambda_y^{IPT} &\equiv -2\widehat{C}_{yy}^{IPT}, \qquad \Lambda_\psi^{IPT} \equiv -2\widehat{C}_{\psi\psi}^{IPT}, \\ \Lambda_s^{IPT} &\equiv -2\widehat{C}_{ss}^{IPT} - \frac{(\widehat{C}_{\psi s}^{IPT})^2}{\Lambda_\psi^{IPT}}, \\ \Lambda_{\psi^*}^{IPT} &\equiv -2\widehat{C}_{\psi^*\psi^*}^{IPT} - \frac{(\widehat{C}_{\psi\psi^*}^{IPT})^2}{\Lambda_\psi^{IPT}} \\ &\quad - \frac{1}{\Lambda_s^{IPT}} \left[\widehat{C}_{s\psi^*}^{IPT} + \frac{\widehat{C}_{\psi s}^{IPT} \widehat{C}_{\psi\psi^*}^{IPT}}{\Lambda_\psi^{IPT}} \right]^2. \end{aligned} \tag{A.20}$$

Here the hatted C coefficients are the literal coefficients of the reduced second-order Home-welfare polynomial after the preceding coordinate transformation; the sequential completion is performed in the order y, ψ, s, ψ^*, as detailed in On-line Appendix OB.3. The coefficients $b_\cdot^H$ are the exact completed-square coefficients obtained from that same manuscript-coordinate polynomial and are functions only of the structural parameters. In this representation, $y_t^{w,IPT}$ is the (Logarithmic) Welfare Relevant Output, $\psi_t^{w,IPT}$ is the (Logarithmic) Home Welfare Relevant LOOP Gap, and

$s_t^{w,IPT}$ is the (Logarithmic) Welfare Relevant Ex-tariff Relative Price of Imports; $\psi_t^{*,w,IPT}$ is the corresponding Foreign Welfare Relevant LOOP Gap. Their associated differences in Eq. (32) use the Gap/Deviation terminology defined in Sec. 4.3. For the unilateral Home-tariff experiment, $\tau^*_{M,t} = 0$. In particular, Eq. (A.18) makes explicit that the Home Welfare Relevant LOOP Gap depends not only on the Home tariff but also on the endogenous international relative price s_t and the Foreign LOOP gap ψ_t^*. These Welfare Relevant variables are obtained by completed squares and are not additional private-equilibrium variables.

B Definitions for the ZLB extensions

This Appendix records only the extension-specific definitions needed to read Sec. 5. The complete Ramsey, Benigno–Woodford, and open-loop derivations are collected in On-line Appendix OB. Each subsection below gives the corresponding On-line Appendix location.

B.1 High-substitution SOE

The experiment in Sec. 5.1 changes no private-equilibrium equation. It re-evaluates the benchmark SOE Imperfect Pass-Through Environment at the higher substitution elasticity. The real and inflation weights used in the main text are therefore the benchmark SOE definitions evaluated at that parameter value. Writing the reduced real quadratic coefficients as in Appendix A,

$$\begin{aligned} \Lambda_y^{SOE,IPT} &= -2C_{yy}^{IPT}, \\ \Lambda_\psi^{SOE,IPT} &= -2C_{\psi\psi}^{IPT} - \frac{(C_{y\psi}^{IPT})^2}{\Lambda_y^{SOE,IPT}}. \end{aligned} \tag{B.1}$$

The inflation weights are

$$\begin{aligned} \Lambda_{\pi,H}^{SOE,IPT} &= \frac{\varepsilon}{\kappa_H}(\Theta_n - \Theta_H^{AS}), \\ \Lambda_{\pi,F}^{SOE,IPT} &= -\frac{\varepsilon_M}{\kappa_M}\Theta_M^{AS}. \end{aligned} \tag{B.2}$$

The complete Benigno–Woodford coefficient-matching steps for this experiment are reported in On-line Appendix OB.1.

B.2 External habit

B.2.1 Model and private equilibrium

The external-habit experiment in Sec. 5.2 changes only household preferences and the marginal-utility terms that enter the household, producer, and importer intertemporal conditions. Preferences are those in Eq. (35). In particular, C_{t-1}^{agg} is lagged aggregate consumption and is taken as given by an individual household. Therefore the private marginal utility of consumption is

$$\frac{1}{C_t - hC_{t-1}^{\text{agg}}},$$

with no next-period habit term in the household's private first-order condition. Only after these private conditions have been derived do we impose the symmetric-equilibrium condition $C_t^{\text{agg}} = C_t$.

The intratemporal condition and Home producer marginal cost are

$$\frac{W_t}{P_t} = (C_t - hC_{t-1}^{\text{agg}})N_t^\varphi, \qquad MC_t^r = (C_t - hC_{t-1}^{\text{agg}})N_t^\varphi \mathcal{G}(S_t/\Psi_t, \tau_{M,t}). \tag{B.3}$$

The household nominal stochastic discount factor used by both Calvo blocks is

$$Q^h_{t,t+k} = \beta^k \frac{(C_t - hC^{\text{agg}}_{t-1})\mathcal{G}(S_t/\Psi_t, \tau_{M,t})}{(C_{t+k} - hC^{\text{agg}}_{t+k-1})\mathcal{G}(S_{t+k}/\Psi_{t+k}, \tau_{M,t+k})} \left(\prod_{s=1}^{k} \Pi_{H,t+s}\right)^{-1}.$$

Thus Eqs. (10)–(11) and (15)–(20) are unchanged in form, with $Q_{t,t+k}$ replaced by $Q^h_{t,t+k}$; the MC^r_t entering the producer block is given by Eq. (B.3). The goods-market condition remains Eq. (23). Keeping the price definitions and the complete-markets convention of the main model unchanged, the private risk-sharing and Euler restrictions are

$$\begin{aligned} S_t &= (C_t - hC^{\text{agg}}_{t-1})\mathcal{G}(S_t/\Psi_t, \tau_{M,t}), \\ \frac{1}{1+i_t} &= \beta \mathrm{E}_t \frac{(C_t - hC^{\text{agg}}_{t-1})\mathcal{G}(S_t/\Psi_t, \tau_{M,t})}{(C_{t+1} - hC^{\text{agg}}_{t})\mathcal{G}(S_{t+1}/\Psi_{t+1}, \tau_{M,t+1})\, \Pi_{H,t+1}}. \end{aligned} \tag{B.4}$$

All Calvo price-index laws, price-dispersion laws, and the LCP price laws are those in Sec. 3 and On-line Appendix OA. Equations (B.3)–(B.4) are private-equilibrium conditions: the household treats the aggregate habit benchmark as external.

For the welfare representation, the two sticky-inflation weights used in Sec. 5.2 are

$$\Lambda^h_{\pi,H} = \frac{\varepsilon}{\kappa_H}(\Theta^h_n - \Theta^{AS,h}_H), \qquad \Lambda^h_{\pi,F} = -\frac{\varepsilon_M}{\kappa_M}\Theta^{AS,h}_M. \tag{B.5}$$

Before square completion, the real part of the welfare criterion can be written as

$$\begin{aligned} \mathcal{L}^{h,real}_t &= \frac{1}{2}\left[\Lambda^{(0),h}_y y_t^2 + \Lambda^{(0),h}_\psi \psi_t^2 + \Lambda^{(0),h}_{c^-} c_{t-1}^2\right] \\ &\quad - C^{(0),h}_{y\psi} y_t \psi_t - C^{(0),h}_{yc^-} y_t c_{t-1} - C^{(0),h}_{\psi c^-} \psi_t c_{t-1} \\ &\quad - h^{(0),h}_{y,t} y_t - h^{(0),h}_{\psi,t} \psi_t - h^{(0),h}_{c^-,t} c_{t-1} + \text{s.o.t.i.p.}, \end{aligned} \tag{B.6}$$

Completing the squares in the order y_t, ψ_t, c_{t-1} gives

$$\begin{aligned} y^{w,h}_t &= \frac{h^{(0),h}_{y,t} + C^{(0),h}_{y\psi}\psi_t + C^{(0),h}_{yc^-} c_{t-1}}{\Lambda^{(0),h}_y}, \\ \psi^{w,h}_t &= \frac{h^{(1),h}_{\psi,t} + C^{(1),h}_{\psi c^-} c_{t-1}}{\Lambda^{(1),h}_\psi}, \\ c^{w,h}_{t-1} &= \frac{h^{(2),h}_{c^-,t}}{\Lambda^{(2),h}_{c^-}}. \end{aligned} \tag{B.7}$$

Defining $\Lambda^h_y \equiv \Lambda^{(0),h}_y$, $\Lambda^h_\psi \equiv \Lambda^{(1),h}_\psi$, and $\Lambda^h_{c^-} \equiv \Lambda^{(2),h}_{c^-}$ gives

$$\begin{aligned} \mathcal{L}^{SOE,IPT,h}_t &= \frac{1}{2}\Big[\Lambda^h_y (y_t - y^{w,h}_t)^2 + \Lambda^h_\psi (\psi_t - \psi^{w,h}_t)^2 + \Lambda^h_{c^-}(c_{t-1} - c^{w,h}_{t-1})^2 \\ &\quad + \Lambda^h_{\pi,H}\pi^2_{H,t} + \Lambda^h_{\pi,F}\pi^2_{F,t}\Big], \end{aligned} \tag{B.8}$$

The complete Ramsey Lagrangian and first-order conditions, the Benigno–Woodford coefficient matching, and the comparative-static details are reported in On-line Appendix OB.2.

B.3 Reciprocal tariffs

Reciprocal tariffs change the tariff-shock vector but do not change the underlying two-country private-equilibrium equations or the structural form of the benchmark Home Welfare Cost Function. The complete verification of the unchanged welfare criterion and the reciprocal-tariff Welfare Relevant reference values is reported in On-line Appendix OB.3.

B.4 Roundabout production

B.4.1 Model and national Ramsey problems

The roundabout experiment in Sec. 5.4 uses the intermediate-input composite, imported-input demand, firm technology, and aggregate technology in Eqs. (38)–(41), together with goods-market clearing in which each destination's final demand is replaced by final consumption plus material demand. The remaining unit-cost, marginal-cost, and factor-demand relations are collected here. The exact Foreign mirror applies.

The Home CPI–PPI ratio is not a new state variable. In the Imperfect Pass-Through Environment it is

$$\mathcal{G}\left(\frac{S_t}{\Psi_t}, \tau_{M,t}\right) \equiv \frac{P_t}{P_{H,t}} = \left[\omega_H + \omega_F \left\{\frac{(1+\tau_{M,t})S_t}{\Psi_t}\right\}^{1-\eta}\right]^{1/(1-\eta)}. \tag{B.9}$$

At the zero-tariff deterministic steady state, $S = \Psi = 1$, $\tau_M = 0$, and $P/P_H = 1$. The exact Home producer real marginal cost is given in Eq. (42). Combining that equation with the imported-input demand condition gives the equivalent representation

$$MC_t^r = (C_t N_t^{\varphi})^{1-\zeta} \frac{(1+\tau_{M,t})S_t}{\Psi_t} \left[\frac{\mathcal{I}_{F,t}(j)}{\omega_F \mathcal{I}_t(j)}\right]^{1/\eta}. \tag{B.10}$$

The corresponding factor demands are

$$\begin{aligned} N_t &= (1-\zeta) MC_t^r Y_t \Delta_{H,t} \left[C_t N_t^{\varphi} \mathcal{G}(S_t/\Psi_t, \tau_{M,t})\right]^{-1}, \\ \mathcal{I}_t &= \zeta MC_t^r Y_t \Delta_{H,t} \mathcal{G}(S_t/\Psi_t, \tau_{M,t})^{-1}. \end{aligned} \tag{B.11}$$

The Foreign equations are the exact starred mirrors.

Since final consumption and material purchases use the same CES basket, the plain two-country goods-market equations are modified only by replacing each destination's final demand by final consumption plus material demand; the Foreign equation is the exact mirror. The producer Calvo block uses the roundabout expression for MC_t^r in place of the labor-only expression. The importer reset-price block uses final plus material import demand. The CPI relations, local importer-price laws, acquisition-cost relations, complete markets, household Euler equations, tariff processes, and price-dispersion laws are otherwise exactly those of the benchmark two-country Imperfect Pass-Through specification.

The roundabout-specific undetermined coefficients satisfy

$$\begin{aligned} 0 &= 1 - \omega_H \sigma_C^{\zeta} \Theta_Y^{RA} + \zeta \Theta_N^{RA} - (1-\zeta)\Theta_I^{RA} - (1-\zeta)\Theta_{mc,H}^{RA} - \Theta_{mc,M}^{RA}, \\ 0 &= \Theta_{\mathcal{G}}^{RA} - \eta \omega_H \Theta_Y^{RA} - \Theta_{mc,H}^{RA} - \Theta_{mc,M}^{RA}, \\ 0 &= -L_{\zeta} + (1+\zeta\varphi)\Theta_N^{RA} - (1-\zeta)\varphi\Theta_I^{RA} - (1-\zeta)\varphi\Theta_{mc,H}^{RA}, \\ 0 &= \Theta_Y^{RA} - \Theta_N^{RA} - \Theta_I^{RA}, \\ 0 &= -\omega_H \sigma_I^{\zeta} \Theta_Y^{RA} + \Theta_I^{RA}, \\ 0 &= -\omega_F \Theta_{\mathcal{G}}^{RA} + \Theta_{mc,M}^{RA}, \\ 0 &= \Theta_{mc,H}^{RA} + \Theta_H^{AS,RA}, \\ 0 &= \Theta_{mc,M}^{RA} + \Theta_M^{AS,RA}. \end{aligned} \tag{B.12}$$

and the inflation weights used in Sec. 5.4 are

$$\Lambda_{\pi,H}^{RA} = \frac{\varepsilon}{\kappa_H}(\Theta_Y^{RA} - \Theta_H^{AS,RA}), \qquad \Lambda_{\pi,F}^{RA} = -\frac{\varepsilon_M}{\kappa_M}\Theta_M^{AS,RA}, \qquad \Lambda_{\pi_H^*}^{RA} = \frac{\varepsilon_M}{\kappa_M}\Theta_Y^{RA}\omega_F. \tag{B.13}$$

The comparative static with respect to the intermediate-input share is reported in Eq. (43) in Sec. 5.4.2. After the remaining first-order equilibrium relations are used to eliminate non-reported real variables, the four real coordinates retained for the Home welfare representation are

$$y_t, \qquad \psi_t, \qquad s_t, \qquad \psi_t^*. \tag{B.14}$$

Their sequential-completion reference values are

$$\begin{aligned}
y_t^{w,RA} &= \frac{h_{y,t}^{(0),RA} + C_{y\psi}^{(0),RA}\psi_t + C_{ys}^{(0),RA}s_t + C_{y\psi^*}^{(0),RA}\psi_t^*}{\Lambda_y^{(0),RA}}, \\
\psi_t^{w,RA} &= \frac{h_{\psi,t}^{(1),RA} + C_{\psi s}^{(1),RA}s_t + C_{\psi\psi^*}^{(1),RA}\psi_t^*}{\Lambda_\psi^{(1),RA}}, \\
s_t^{w,RA} &= \frac{h_{s,t}^{(2),RA} + C_{s\psi^*}^{(2),RA}\psi_t^*}{\Lambda_s^{(2),RA}}, \\
\psi_t^{*,w,RA} &= \frac{h_{\psi^*,t}^{(3),RA}}{\Lambda_{\psi^*}^{(3),RA}}.
\end{aligned} \tag{B.15}$$

At the benchmark roundabout calibration, the same reference values in the manuscript coordinates are

$$\begin{aligned}
y_t^{w,RA} &= 0.03614751\,\psi_t - 0.22121298\,s_t - 0.18580954\,\psi_t^* - 0.23703090\,\tau_{M,t} + 0.18580954\,\tau_{M,t}^*, \\
\psi_t^{w,RA} &= 0.050233\,s_t - 0.133375\,\psi_t^* + 0.772507\,\tau_{M,t} + 0.133375\,\tau_{M,t}^*, \\
s_t^{w,RA} &= -0.590373\,\psi_t^* - 1.212884\,\tau_{M,t} + 0.590373\,\tau_{M,t}^*, \\
\psi_t^{*,w,RA} &= -0.45400174\,\tau_{M,t} + \tau_{M,t}^*.
\end{aligned} \tag{B.16}$$

The resulting Home period loss is

$$\begin{aligned}
\mathcal{L}_{H,t}^{RA} = \frac{1}{2}\Big[&\Lambda_y^{RA}(y_t - y_t^{w,RA})^2 + \Lambda_\psi^{RA}(\psi_t - \psi_t^{w,RA})^2 + \Lambda_s^{RA}(s_t - s_t^{w,RA})^2 \\
&+\Lambda_{\psi^*}^{RA}(\psi_t^* - \psi_t^{*,w,RA})^2 + \Lambda_{\pi,H}^{RA}\pi_{H,t}^2 + \Lambda_{\pi,F}^{RA}\pi_{F,t}^2 + \Lambda_{\pi_H^*}^{RA}(\pi_{H,t}^*)^2\Big].
\end{aligned} \tag{B.17}$$

The national Ramsey Lagrangians and first-order conditions and the complete Benigno–Woodford derivation are reported in On-line Appendix OB.4.

B.5 Open-loop Nash

The open-loop experiment changes the strategic policy paths rather than the private-equilibrium equations. Home takes the complete Foreign PPI-inflation path as given and uses the Home PPI-inflation path as its strategic policy path; the two importer-price inflation rates and the two nominal interest rates remain endogenous world-equilibrium variables. The complete choice sets, national Lagrangians, first-order conditions, fixed reporting normalization, and Welfare Cost Function are reported in On-line Appendix OB.5.

C Welfare accounting and counterfactual policy paths

The symbolic Welfare Relevant variables and weights are collected in Paper Appendix A. This section reports the complete counterfactual accounting exercises used only as supporting evidence for the mechanisms discussed in the main text.

C.1 SOE welfare decomposition under counterfactual policy-rate paths

To quantify the policy trade-off discussed in Sec. 3.5, we evaluate the four components of Eq. (26) under three policy-rate paths: a zero policy-rate response, one-half of the optimal rate cut, and the optimal Ramsey response. "Zero policy-rate response" means that the policy rate is held at its deterministic steady-state path, $i_t - i = 0$; it does not mean that the level of the nominal interest rate is set to zero. In the half-optimal case, the deviation of the policy rate from steady state is set period by period to one-half of its optimal deviation. The exercise is a diagnostic decomposition of the welfare trade-off rather than an alternative policy objective. We also evaluate the original nonlinear lifetime utility in Eq. (1) directly along the consumption and labor paths implied by each of these three allocations. This exact total-welfare criterion is separate from the quadratic source accounting below. There is no source-by-source exact decomposition corresponding to the completed-square terms in Eq. (26), so the component decomposition is supporting accounting evidence rather than a decomposition of exact nonlinear welfare.

Table 10: SOE Imperfect Pass-Through Environment: discounted welfare-cost decomposition under counterfactual policy-rate paths.

Welfare component	Zero response	Half-optimal	Optimal	Optimal − zero
Welfare Relevant Output Gap	0.000038	0.000170	0.000407	+0.000369
Welfare Relevant LOOP Gap Deviation	0.011226	0.010401	0.009611	−0.001615
Home PPI inflation	0.000205	0.000482	0.000875	+0.000670
Ex-tariff import inflation	0.000000	0.000008	0.000031	+0.000031
Total	0.011469	0.011060	0.010924	−0.000545

Consistent with Eq. (28) and Eqs. (4)–(5), a lower local *cum-tariff* relative price raises $C_{F,t}/C_{H,t}$. In the first-order counterfactual allocation, the impact log deviation of $C_{F,t}/C_{H,t}$ from its deterministic steady state is -29.58 percent under the zero policy-rate response and -29.30 percent under the optimal Ramsey response. Thus monetary accommodation raises the impact relative-consumption ratio by 0.27 percentage points.

As a total-welfare cross-check, we next evaluate Eq. (1) directly along the same three counterfactual allocations. The first numerical column in Table 11 reports discounted lifetime utility net of the deterministic steady-state path; the second reports the utility gain relative to the zero-response allocation. Because the original utility criterion is used directly, these entries do not allocate welfare to individual completed-square components.

Table 11: SOE Imperfect Pass-Through Environment: original nonlinear lifetime utility under counterfactual policy-rate paths.

Policy-rate path	Lifetime utility relative to steady state	Gain relative to zero response
Zero response	-0.240621	0.000000
Half-optimal	-0.238298	0.002323
Optimal Ramsey response	-0.236134	0.004487

The original nonlinear utility criterion therefore gives the same ordering as the quadratic total-cost comparison: the optimal Ramsey response dominates the half-optimal response, which in turn dominates the zero-response allocation. The total-welfare gain from the optimal response relative to zero response is positive even without assigning that gain to individual welfare sources. The completed-square decomposition in Table 10 is used only to provide supporting accounting evidence about which realized terms move with the policy response.

Each entry in the first three numerical columns is the discounted contribution of the corresponding term in Eq. (26), including the factor 1/2. The final column reports the optimal-response contribution minus the zero-response contribution; hence a negative number denotes a reduction in welfare cost. The Welfare Relevant LOOP Gap Deviation component falls by 0.001615, whereas the Welfare Relevant Output Gap, Home-PPI-inflation, and import-inflation components rise by

0.001070 in total. The former reduction is therefore about 1.5 times as large as the combined increase in the other three components. Consequently, the total quadratic welfare cost is 4.75 percent lower under the optimal response than under a zero policy-rate response. This comparison should not be read as saying that the Welfare Relevant LOOP Gap Deviation has the largest structural stabilization weight: Table 3 shows the opposite. Rather, for this specific counterfactual the accounting representation records a sufficiently large reduction in the realized Welfare Relevant LOOP Gap Deviation component to offset the higher contributions of the other three terms. The economic mechanism is identified in Sec. 3.5 from the purchaser-price relation, relative demand, and the sticky-price Phillips curves; the decomposition here is supporting accounting evidence for that mechanism.

High-substitution ZLB episode ($\eta = 3$)

The same diagnostic can be applied to the high-substitution SOE experiment in Sec. 5.1, where the unconstrained Ramsey rate would fall by 119.95 basis points and the ZLB binds for one quarter. Here the "optimal" column is the actual ZLB-constrained Ramsey allocation. Relative to a zero policy-rate response, the constrained optimum generates the following discounted components of the $\eta = 3$ version of Eq. (26).

Table 12: High-substitution SOE ($\eta = 3$): discounted welfare-cost decomposition under the zero-response and ZLB-constrained optimal policy-rate paths.

Welfare component	Zero response	ZLB optimum	Optimal − zero
Welfare Relevant Output Gap	0.000717	0.000760	+0.000043
Welfare Relevant LOOP Gap Deviation	0.014792	0.008444	−0.006348
Home PPI inflation	0.001172	0.004223	+0.003050
Ex-tariff import inflation	0.000000	0.000290	+0.000290
Total	0.016681	0.013716	−0.002965

The ZLB-constrained optimum lowers total discounted welfare cost by 0.002965, or 17.8 percent. The Welfare Relevant LOOP Gap Deviation component falls by 0.006348, while the Welfare Relevant Output Gap, Home-PPI-inflation, and import-inflation components rise by 0.003383 in total. Thus the central bank does not attempt to set the Welfare Relevant LOOP Gap Deviation to zero. It accepts larger losses in the sticky-inflation and Welfare Relevant Output Gap because eliminating the Welfare Relevant LOOP Gap Deviation further would require a different interest-rate path and would raise total welfare cost. As in the benchmark decomposition, this is a realized-loss comparison, not a ranking of structural weights; indeed Table 6 assigns much larger weights to the two sticky-inflation terms than to the LOOP gap. Moreover, the unconstrained $\eta = 3$ Ramsey allocation itself has $\psi_0 = 3.27$ percent and $\psi_0^w = 11.32$ percent, leaving an impact Welfare Relevant LOOP Gap Deviation of about -8.06 percent. Hence the planner would not eliminate the Welfare Relevant LOOP Gap Deviation even in the absence of the ZLB.

External-habit ZLB episode

For the external-habit experiment, we use the same zero-policy-response diagnostic as above. The zero-response allocation holds the nominal policy rate at its deterministic steady-state path and recomputes the private equilibrium using the external-habit optimality conditions. The second column reports the ZLB-constrained Ramsey allocation. The zero-response welfare contributions are evaluated with the numerical weights reported in Table 7; all entries are rounded to six decimals.

Relative to the zero policy-rate response, the ZLB-constrained Ramsey allocation lowers total discounted welfare cost by about 0.00221. The largest reduction is the 0.00548 decline in the Welfare Relevant LOOP Gap Deviation component. The Welfare Relevant Output Gap component also falls, while the Welfare Relevant Lagged Consumption Gap, Home-PPI-inflation, and import-inflation components rise. In particular, the Welfare Relevant Lagged Consumption Gap does not

Table 13: External-habit SOE: discounted welfare-cost decomposition under the zero-response and ZLB-constrained optimal policy-rate paths.

Welfare component	Zero response	ZLB optimum	Optimal − zero
Welfare Relevant Output Gap	0.000733	0.000358	−0.000375
Welfare Relevant LOOP Gap Deviation	0.022451	0.016974	−0.005477
Welfare Relevant Lagged Consumption Gap	0.033769	0.034968	+0.001199
Home PPI inflation	0.000682	0.002911	+0.002229
Ex-tariff import inflation	0.000000	0.000217	+0.000217
Total	0.057635	0.055429	−0.002206

account for the large monetary easing. For comparison, the plain SOE Imperfect Pass-Through diagnostic in Table 10 lowers the Welfare Relevant LOOP Gap Deviation component by only 0.001615. The external-habit experiment therefore makes the welfare consequences of the policy-induced LOOP-gap adjustment substantially more persistent, which is the mechanism emphasized in Sec. 5.2.

C.2 Two-country welfare decomposition under a counterfactual Home policy-rate path

To quantify the Home policy trade-off discussed in Sec. 4.4, we use the first-order private-equilibrium system associated with the benchmark two-country Imperfect Pass-Through Environment. The complete Foreign contingent plan is held at its benchmark Nash–Ramsey first-order path. In the zero-Home-response counterfactual, the Home nominal interest rate is held at its deterministic steady-state path, $i_t - i = 0$, and the remaining Home private-equilibrium variables are recomputed conditional on that Foreign plan. The optimal-response column uses the benchmark Home Ramsey path in the same first-order system. This is the natural best-response diagnostic for the Home national Ramsey problem, which takes the Foreign contingent plan as given. The second-order Welfare Cost Function is then evaluated from these first-order allocations over the same 20-quarter horizon.

The relative-demand implication can be checked without using the Welfare Relevant LOOP Gap as a proxy. Log-linearizing Eq. (33) around the zero-tariff deterministic steady state gives

$$\log\left(\frac{C_{F,t}/C_{H,t}}{C_F/C_H}\right) = -\eta\left(\tau_{M,t} + s_t - \psi_t\right).$$

For the benchmark tariff innovation, the impact local relative import-price term $s_0 - \psi_0$ is $+0.0341$ percent under the zero Home response and -0.4925 percent under the optimal Home response. The corresponding impact log deviations of $C_{H,t}$ and $C_{F,t}$ are reported in Table 14. Hence the impact log deviation of $C_{F,t}/C_{H,t}$ is -30.0511 percent under the zero Home response and -29.2613 percent under the optimal response. The optimal Home rate cut therefore raises the impact value of the relative-demand response by 0.7898 percentage points relative to the zero-response counterfactual. The same ordering holds in every quarter of the 20-quarter diagnostic horizon, so the result is not an impact-only sign reversal.

Table 14: Two-country Imperfect Pass-Through Environment: impact first-order relative-demand diagnostic.

Impact log deviation (percent)	Zero Home response	Optimal response	Optimal − zero
$\log(C_{H,0}/C_H)$	−1.0735	+2.6915	+3.7651
$\log(C_{F,0}/C_F)$	−31.1246	−26.5697	+4.5549
$\log[(C_{F,0}/C_{H,0})/(C_F/C_H)]$	−30.0511	−29.2613	+0.7898
$s_0 - \psi_0$	+0.0341	−0.4925	−0.5265

All entries are first-order log deviations from the deterministic steady state, reported in percent; the final column is the optimal response minus the zero Home response. The positive difference in the relative-consumption row is the direct quantity measure of the policy-induced change in relative consumption. The separate $C_{H,0}$ and $C_{F,0}$ rows also show that the result is not an artifact of movements in only one component of Home consumption.

Table 15 reports the corresponding completed-square accounting using the current Welfare Relevant variables and weights in Eq. (32). The original Home lifetime utility $\mathcal{U}$ remains the model's total-welfare criterion; the completed-square terms are used here only to interpret which realized welfare components move with the policy path. We do not assign source-specific shares to exact nonlinear Home utility.

Table 15: Two-country Imperfect Pass-Through Environment: discounted Home welfare-cost decomposition under a counterfactual Home policy-rate path.

Welfare Relevant Variable	Zero Home response	Optimal response	Optimal − zero
$y_t - y_t^{w,IPT}$	0.000204	0.001068	+0.000864
$\psi_t - \psi_t^{w,IPT}$	0.014880	0.008871	−0.006010
$s_t - s_t^{w,IPT}$	−0.002222	−0.004311	−0.002089
$\psi_t^* - \psi_t^{*,w,IPT}$	0.000012	0.000556	+0.000544
$\pi_{H,t}$	0.008542	0.000677	−0.007865
$\pi_{F,t}$	0.002511	0.000012	−0.002498
$\pi_{H,t}^*$	0.000030	0.000030	0.000000
Total	0.023957	0.006903	−0.017054

Each entry in the first two numerical columns is the discounted contribution of the corresponding term in Eq. (32), including the factor 1/2. The final column reports the optimal-response contribution minus the zero-response contribution, so a negative number denotes an improvement in Home welfare. Because $\Lambda_s^{IPT} < 0$, the relative-price row is a signed strategic welfare contribution rather than a conventional positive loss. The optimal response lowers total discounted quadratic welfare cost by 0.017054, or 71.2 percent. The Home PPI-inflation contribution falls by 0.007865, the Home Welfare Relevant LOOP Gap Deviation contribution by 0.006010, the ex-tariff import-inflation contribution by 0.002498, and the signed strategic relative-price contribution by 0.002089. These improvements are partly offset by increases of 0.000864 in the Welfare Relevant Output Gap component and 0.000544 in the Foreign Welfare Relevant LOOP Gap Deviation component. Thus the zero-versus-optimal diagnostic confirms that the benchmark Home easing raises the relative-consumption ratio documented above and improves the realized Home welfare criterion; it is not used as a causal decomposition of exact nonlinear Home welfare.

To identify the marginal welfare consideration that prevents still greater Home monetary easing, we conduct an additional counterfactual around the benchmark optimum. The Foreign contingent plan is again held fixed at its benchmark Nash–Ramsey path. The counterfactual Home policy-rate path is defined by increasing the deviation of the optimal Home nominal interest rate from its deterministic steady state by 10 percent at every horizon,

$$i_t^{CF} - i = 1.1\left(i_t^{OPT} - i\right).$$

Here i_t^{CF} denotes the Home nominal interest rate under the additional-easing counterfactual, i_t^{OPT} denotes the benchmark optimal Home nominal interest rate, and i denotes its deterministic steady-state value. Thus the experiment makes the benchmark Home policy response uniformly more accommodative without allowing the Home policymaker to internalize an additional Foreign policy response. At impact, the log deviation of $C_{F,t}/C_{H,t}$ moves from −29.2613 percent under the benchmark optimum to −29.1823 percent under this additional-easing path, an additional increase of 0.0790 percentage points in the relative-consumption ratio.

The marginal counterfactual provides the complementary reason why the Home policymaker does not ease further. Additional easing lowers the discounted Home Welfare Relevant LOOP Gap Deviation contribution by 0.000514 and improves the signed Welfare Relevant Ex-tariff Relative Price of Imports Gap contribution by 0.000247. The largest single offsetting deterioration is the 0.000527 increase in the Home PPI-inflation contribution. The Welfare Relevant Output Gap and

Table 16: Two-country Imperfect Pass-Through Environment: marginal welfare effects of additional Home monetary easing.

Welfare component	Optimal response	10% additional easing	Additional easing − optimal
$y_t - y_t^{w,IPT}$	0.001068	0.001370	+0.000301
$\psi_t - \psi_t^{w,IPT}$	0.008871	0.008356	−0.000514
$s_t - s_t^{w,IPT}$	−0.004311	−0.004558	−0.000247
$\psi_t^* - \psi_t^{*,w,IPT}$	0.000556	0.000656	+0.000101
$\pi_{H,t}$	0.000677	0.001204	+0.000527
$\pi_{F,t}$	0.000012	0.000015	+0.000003
$\pi_{H,t}^*$	0.000030	0.000030	0.000000
Total	0.006903	0.007073	+0.000171

Foreign Welfare Relevant LOOP Gap Deviation contributions also rise by 0.000301 and 0.000101, respectively, and the ex-tariff import-inflation contribution rises slightly. These costs more than offset the two improvements, so total discounted Home welfare cost increases by 0.000171. Thus a further rate cut would continue to exploit the national relative-price externality through the joint movement of s_t and ψ_t and improve the Home LOOP-gap and signed relative-price contributions, but the marginal welfare cost elsewhere exceeds those additional gains.

Reciprocal-tariff ZLB episode

For reciprocal tariffs, comparing the unconstrained Ramsey allocation with the ZLB-constrained allocation gives The decomposition in Table 17 is evaluated in the manuscript coordinates, with

Table 17: Reciprocal tariffs: discounted Home welfare-cost decomposition, unconstrained versus ZLB-constrained Ramsey allocations.

Variable	Notional optimum	ZLB optimum	ZLB − notional
$y_t - y_t^{w,IPT}$	0.000168	0.000203	+0.000035
$\psi_t - \psi_t^{w,IPT}$	0.006476	0.006714	+0.000238
$s_t - s_t^{w,IPT}$	−0.017468	−0.017088	+0.000380
$\psi_t^* - \psi_t^{*,w,IPT}$	0.025893	0.025373	−0.000519
$\pi_{H,t}$	0.000378	0.000365	−0.000013
$\pi_{F,t}$	0.000159	0.000167	+0.000009
$\pi_{H,t}^*$	0.000054	0.000053	−0.000001
Total	0.015660	0.015788	+0.000128

$s_t = \log(\mathcal{E}_t P_{F,t}^* / P_{H,t})$ throughout. The corresponding real weights are $\Lambda_\psi^{IPT} = 0.40634284$ and $\Lambda_s^{IPT} = -0.09782778$. The coordinate change only redistributes the Home Welfare Relevant LOOP Gap Deviation and signed Welfare Relevant Ex-tariff Relative Price of Imports Gap contributions: their sum, and hence total Home welfare cost, is invariant.

On impact the unconstrained optimum has $\psi_0 = 3.58$ percent and, using Eq. (34) in the manuscript coordinates, $\psi_0^{w,IPT} = 15.74$ percent, leaving a Home Welfare Relevant LOOP Gap Deviation of −12.16 percent. Under the ZLB-constrained allocation the corresponding Home Welfare Relevant LOOP Gap is 15.70 percent and the displayed-value deviation is −12.99 percent. Thus even the unconstrained optimum deliberately leaves a sizable Welfare Relevant LOOP Gap Deviation. Closing it further would alter the signed Welfare Relevant Ex-tariff Relative Price of Imports Gap contribution, the Foreign Welfare Relevant LOOP Gap Deviation, and the sticky-inflation paths; the planner chooses the combination that minimizes the full Home criterion.

Roundabout-production ZLB episode

For $\zeta = 0.50$, the same notional-versus-constrained comparison, expressed in the current manuscript relative-price coordinate, is The change from the previously reported LOOP and

Table 18: Roundabout production: discounted Home welfare-cost decomposition, unconstrained versus ZLB-constrained Ramsey allocations.

Variable	Notional optimum	ZLB optimum	ZLB − notional
$y_t - y_t^{w,RA}$	0.006694	0.006682	−0.000012
$\psi_t - \psi_t^{w,RA}$	0.015923	0.015931	+0.000009
$s_t - s_t^{w,RA}$	−0.007296	−0.007294	+0.000001
$\psi_t^* - \psi_t^{*,w,RA}$	0.002288	0.002290	+0.000002
$\pi_{H,t}$	0.002062	0.002063	+0.000001
$\pi_{F,t}$	0.000086	0.000086	0.000000
$\pi_{H,t}^*$	0.000113	0.000113	0.000000
Total	0.019871	0.019872	+0.000001

signed-relative-price entries is a coordinate decomposition: their sum, and therefore total welfare cost, is unchanged. On impact the unconstrained optimum has $\psi_0 = -0.56$ percent and $\psi_0^{w,RA} = 15.26$ percent, a deviation of -15.81 percent. Under the one-quarter ZLB episode the corresponding Welfare Relevant LOOP Gap is 15.25 percent and the deviation is -15.84 percent. The ZLB is only marginally binding in this experiment, so the welfare-component differences are correspondingly tiny. The unconstrained planner nevertheless accepts a sizable Welfare Relevant LOOP Gap Deviation because reducing it further would alter the signed Welfare Relevant Ex-tariff Relative Price of Imports Gap component, the Foreign Welfare Relevant LOOP Gap Deviation, Welfare Relevant Output Gap, and the three sticky-inflation paths.

Open-loop Nash ZLB episode

Using the fixed OLN reporting normalization in On-line Appendix OB.5, the stored notional and constrained solutions give The notional rate is only 0.30 basis points beyond the ZLB, so the two

Table 19: Open-loop Nash: discounted Home welfare-cost decomposition, unconstrained versus ZLB-constrained allocations.

Variable	Notional optimum	ZLB optimum	ZLB − notional
$y_t - y_t^{w,OLN}$	0.000653407	0.000653105	−0.000000302
$\psi_t - \psi_t^{w,OLN}$	0.002092445	0.002092656	+0.000000211
$s_t - s_t^{w,OLN}$	0.002320517	0.002320571	+0.000000054
$\psi_t^* - \psi_t^{*,w,OLN}$	−0.000011304	−0.000011277	+0.000000027
$\pi_{H,t}$	0.000838515	0.000838501	−0.000000013
$\pi_{F,t}$	0.000000470	0.000000469	−0.000000000
$\pi_{H,t}^*$	0.000029361	0.000029359	−0.000000002
Total	0.005923410	0.005923384	−0.000000026

allocations are numerically indistinguishable at the scale of the welfare criterion. Nevertheless, both are informative about the welfare trade-off: the unconstrained impact LOOP gap is -0.36 percent while $\psi_0^{w,OLN} = 3.01$ percent, leaving a deviation of -3.37 percent. The constrained deviation is also -3.37 percent. Thus the fact that the Welfare Relevant LOOP Gap Deviation is not driven to zero is not caused by the ZLB; it is already optimal in the unconstrained open-loop policy game.

References

[1] Devereux, Michael B., and Charles Engel (2003), "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange-Rate Flexibility," *Review of Economic Studies*, 70(4), 765–783.

[2] Corsetti, Giancarlo, and Paolo Pesenti (2005), "International Dimensions of Optimal Monetary Policy," *Journal of Monetary Economics*, 52(2), 281–305.

[3] De Paoli, Bianca (2009), "Monetary Policy and Welfare in a Small Open Economy," *Journal of International Economics*, 77(1), 11–22.

[4] Engel, Charles (2011), "Currency Misalignments and Optimal Monetary Policy: A Reexamination," *American Economic Review*, 101(6), 2796–2822.

[5] Fujiwara, Ippei, and Jiao Wang (2017), "Optimal Monetary Policy in Open Economies Revisited," *Journal of International Economics*, 108, 300–314.

[6] Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc (2023), "Exchange Rate Misalignment and External Imbalances: What Is the Optimal Monetary Policy Response?," *Journal of International Economics*, 144, 103771.

[7] Betts, Caroline, and Michael B. Devereux (2000), "Exchange Rate Dynamics in a Model of Pricing-to-Market," *Journal of International Economics*, 50(1), 215–244.

[8] Devereux, Michael B., and Charles Engel (2002), "Exchange Rate Pass-Through, Exchange Rate Volatility, and Exchange Rate Disconnect," *Journal of Monetary Economics*, 49(5), 913–940.

[9] Gopinath, Gita, Oleg Itskhoki, and Roberto Rigobon (2010), "Currency Choice and Exchange Rate Pass-Through," *American Economic Review*, 100(1), 304–336.

[10] Gopinath, Gita, Emine Boz, Camila Casas, Federico J. Díez, Pierre-Olivier Gourinchas, and Mikkel Plagborg-Møller (2020), "Dominant Currency Paradigm," *American Economic Review*, 110(3), 677–719.

[11] Hwang, Yu-Ning, and Stephen J. Turnovsky (2013), "Exchange Rate Pass-through and the Effects of Tariffs on Economic Performance and Welfare," *Journal of International Money and Finance*, 33, 81–102.

[12] Bergin, Paul R., and Giancarlo Corsetti (2023), "The Macroeconomic Stabilization of Tariff Shocks: What Is the Optimal Monetary Response?," *Journal of International Economics*, 143, 103758.

[13] Bergin, Paul R., and Giancarlo Corsetti (2025, revised 2026), "Exchange Rates and Monetary Stabilization of Tariff Shocks," *NBER Working Paper* No. 33845.

[14] Auray, Stéphane, Michael B. Devereux, and Aurélien Eyquem (2025), "Trade Wars, Nominal Rigidities, and Monetary Policy," *Review of Economic Studies*, 92, 2228–2270.

[15] Auray, Stéphane, Michael B. Devereux, and Aurélien Eyquem (2026), "The Pass-through of Tariffs and Exchange Rates," *NBER Working Paper* No. 35398.

[16] Bianchi, Javier, and Louphou Coulibaly (2025), "The Optimal Monetary Policy Response to Tariffs," *NBER Working Paper* No. 33560.

[17] International Monetary Fund (2026), "World Economic Outlook Database," April 2026, GDP at current prices in U.S. dollars, World and United States.

[18] U.S. Bureau of Economic Analysis (2026), "National Income and Product Accounts: Imports of Goods and Services and Personal Consumption Expenditures," annual series, retrieved through FRED, Federal Reserve Bank of St. Louis, July 28.

[19] Monacelli, Tommaso (2025), "Tariffs and Monetary Policy," *CEPR Discussion Paper* No. 20142, thoroughly revised November 2025.

[20] Okano, Eiji, and Masataka Eguchi (2024), "The Effects of a Money-Financed Fiscal Stimulus in a Small Open Economy," *IMF Economic Review*, 72(3), 1212–1237.

[21] Monacelli, Tommaso (2004), "Into the Mussa Puzzle: Monetary Policy Regimes and the Real Exchange Rate in a Small Open Economy," *Journal of International Economics*, 62, 191–217.

[22] Werning, Iván, Guido Lorenzoni, and Veronica Guerrieri (2025), "Tariffs as Cost-Push Shocks: Implications for Optimal Monetary Policy," *NBER Working Paper* No. 33772.

[23] Faia, Ester, and Tommaso Monacelli (2004), "Ramsey Monetary Policy and International Relative Prices," *International Finance Discussion Papers* No. 798, Board of Governors of the Federal Reserve System.

[24] Galí, Jordi, and Tommaso Monacelli (2016), "Understanding the Gains from Wage Flexibility: The Exchange Rate Connection," *American Economic Review*, 106, 3829–3868.

[25] Smets, Frank, and Rafael Wouters (2007), "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

[26] Basu, Susanto (1995), "Intermediate Goods and Business Cycles: Implications for Productivity and Welfare," *American Economic Review*, 85(3), 512–531.

[27] Huang, Kevin X.D., and Zheng Liu (2004), "Input–Output Structure and Nominal Rigidity: The Persistence Problem Revisited," *Macroeconomic Dynamics*, 8(2), 188–206.

[28] Jones, Charles I. (2011), "Intermediate Goods and Weak Links in the Theory of Economic Development," *American Economic Journal: Macroeconomics*, 3(2), 1–28.

[29] Bodenstein, Martin, Luca Guerrieri, and Joe LaBriola (2019), "Macroeconomic Policy Games," *Journal of Monetary Economics*, 101, 64–81.

[30] Benigno, Pierpaolo, and Michael Woodford (2003), "Optimal Monetary and Fiscal Policy: A Linear-Quadratic Approach," *NBER Working Paper* No. 9905.

[31] Benigno, Pierpaolo, and Michael Woodford (2005), "Inflation Stabilization and Welfare: The Case of a Distorted Steady State," *Journal of the European Economic Association*, 3(6), 1185–1236.

[32] Billi, Roberto M., and Jordi Galí (2020), "Gains from Wage Flexibility and the Zero Lower Bound," *Oxford Bulletin of Economics and Statistics*, 82, 1239–1261.

[33] Fujiwara, Ippei, Tomoyuki Nakajima, Nao Sudo, and Yuki Teranishi (2013), "Global Liquidity Trap," *Journal of Monetary Economics*, 60, 936–949.

[34] Bodenstein, Martin, Christopher J. Erceg, and Luca Guerrieri (2017), "The Effects of Foreign Shocks When Interest Rates Are at Zero," *Canadian Journal of Economics*, 50, 660–684.

On-line Appendix for

Tariffs and Monetary Policy in a Low Pass-Through Environment: The Return of the ZLB

Eiji Okano[1] Yang Zhou Masataka Eguchi

OA Complete Ramsey systems

Pricing definitions used by these systems are collected in Paper Appendix A. This section reports the complete nonlinear Ramsey Lagrangians and first-order conditions used for the SOE and two-country allocations.

OA.1 SOE Ramsey Lagrangian and first-order conditions

Following Monacelli [19], we write the Ramsey problem in terms of the equilibrium relations stated in the text, substitute the reset-price relations before differentiation, and then display the first-order conditions explicitly. In particular, no residual vector or generic derivative operator is used below.

OA.1.1 Perfect Pass-Through Environment

In the Perfect Pass-Through Environment, the social planner maximizes $\mathcal{U}$ subject to Eqs. (6), (7), (8), (9), (10)–(13), (14), (23) with $\Psi_t = 1$, and (24). After Eqs. (10)–(11) have been substituted, the Lagrangian is

$$
\begin{aligned}
\mathcal{L}^{SOE,PPT} \quad = \quad & \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \\
& + \lambda_{p,t} \left[1 - \theta \Pi_{H,t}^{\varepsilon-1} - (1-\theta) \widetilde{p}_{H,t}^{1-\varepsilon} \right] \\
& + \lambda_{\Delta,t} \left[\Delta_{H,t} - \theta \Pi_{H,t}^{\varepsilon} \Delta_{H,t-1} - (1-\theta) \widetilde{p}_{H,t}^{-\varepsilon} \right] \\
& + \lambda_{n,t} \left[N_t - Y_t \Delta_{H,t} \right] \\
& + \lambda_{y,t} \left[Y_t - (1-\upsilon) \mathcal{G}(S_t, \tau_{M,t})^{\eta} C_t - \upsilon S_t^{\eta} \right] \\
& + \lambda_{r,t} \left[S_t - C_t \mathcal{G}(S_t, \tau_{M,t}) \right] \\
& + \lambda_{e,t} \left[\frac{1}{1+i_t} - \beta \mathrm{E}_t \frac{C_t \mathcal{G}(S_t, \tau_{M,t})}{C_{t+1} \mathcal{G}(S_{t+1}, \tau_{M,t+1}) \Pi_{H,t+1}} \right] + \mu_{i,t} i_t \Bigg\}. \qquad \text{(OA.1)}
\end{aligned}
$$

Here $\lambda_{p,t}$, $\lambda_{\Delta,t}$, $\lambda_{n,t}$, $\lambda_{y,t}$, $\lambda_{r,t}$, and $\lambda_{e,t}$ are the multipliers on the price-index, price-dispersion, production, goods-market, risk-sharing, and Euler conditions shown in the same order in Eq. (OA.1). The multiplier $\mu_{i,t}$ is the ZLB multiplier defined after Eq. (24).

[1]Corresponding author. Graduate School of Economics, Nagoya City University, 1, Aza, Yamanobata, Mizuho-cho, Mizuho-ku, Nagoya-shi Aichi, 467-8501, Japan. E-mail: eiji_okano@econ.nagoya-cu.ac.jp

As on pp. 74–75 of Monacelli [19], it is useful only for taking derivatives to write

$$\begin{aligned}
\mathcal{K}'_{H,t-j}(x_t) &\equiv \frac{\partial \mathcal{K}_{H,t-j}}{\partial x_t}, \qquad \mathcal{Z}'_{H,t-j}(x_t) \equiv \frac{\partial \mathcal{Z}_{H,t-j}}{\partial x_t}, \\
\widetilde{p}'_{H,t-j}(x_t) &\equiv \frac{\partial \widetilde{p}_{H,t-j}}{\partial x_t} = \mathcal{M}\frac{\mathcal{Z}'_{H,t-j}(x_t)\mathcal{K}_{H,t-j} - \mathcal{K}'_{H,t-j}(x_t)\mathcal{Z}_{H,t-j}}{(\mathcal{K}_{H,t-j})^2}.
\end{aligned} \tag{OA.2}$$

This is derivative notation for the reset price already defined in Eqs. (10)–(11); it introduces no new economic variable.

The first-order condition for C_t is

$$\begin{aligned}
0 &= \frac{1}{C_t} - (1-\theta)(1-\varepsilon)\left[\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(C_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(C_t)\right] \\
&\quad +(1-\theta)\varepsilon\left[\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(C_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(C_t)\right] \\
&\quad -(1-\upsilon)\lambda_{y,t}\mathcal{G}(S_t,\tau_{M,t})^{\eta} - \lambda_{r,t}\mathcal{G}(S_t,\tau_{M,t}) \\
&\quad -\beta\lambda_{e,t}\mathrm{E}_t\left[\frac{\mathcal{G}(S_t,\tau_{M,t})}{C_{t+1}\mathcal{G}(S_{t+1},\tau_{M,t+1})\Pi_{H,t+1}}\right] \\
&\quad +\lambda_{e,t-1}\frac{C_{t-1}\mathcal{G}(S_{t-1},\tau_{M,t-1})}{C_t^2\mathcal{G}(S_t,\tau_{M,t})\Pi_{H,t}}.
\end{aligned}$$

The first-order condition for N_t is

$$\begin{aligned}
0 &= -N_t^{\varphi} + \lambda_{n,t} \\
&\quad -(1-\theta)(1-\varepsilon)\left[\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(N_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(N_t)\right] \\
&\quad +(1-\theta)\varepsilon\left[\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(N_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(N_t)\right].
\end{aligned}$$

The first-order condition for Y_t is

$$\begin{aligned}
0 &= -\lambda_{n,t}\Delta_{H,t} + \lambda_{y,t} \\
&\quad -(1-\theta)(1-\varepsilon)\left[\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(Y_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(Y_t)\right] \\
&\quad +(1-\theta)\varepsilon\left[\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(Y_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(Y_t)\right].
\end{aligned}$$

The first-order condition for S_t is

$$\begin{aligned}
0 &= -(1-\theta)(1-\varepsilon)\left[\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(S_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(S_t)\right] \\
&\quad +(1-\theta)\varepsilon\left[\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(S_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(S_t)\right] \\
&\quad -\lambda_{y,t}\left[(1-\upsilon)\eta\mathcal{G}(S_t,\tau_{M,t})^{\eta-1}\frac{\partial\mathcal{G}(S_t,\tau_{M,t})}{\partial S_t}C_t + \upsilon\eta S_t^{\eta-1}\right]
\end{aligned}$$

$$
\begin{aligned}
&+\lambda_{r,t}\left[1-C_t\frac{\partial\mathcal{G}(S_t,\tau_{M,t})}{\partial S_t}\right]\\
&-\beta\lambda_{e,t}\mathrm{E}_t\left[\frac{C_t\,\partial\mathcal{G}(S_t,\tau_{M,t})/\partial S_t}{C_{t+1}\mathcal{G}(S_{t+1},\tau_{M,t+1})\Pi_{H,t+1}}\right]\\
&+\lambda_{e,t-1}\frac{C_{t-1}\mathcal{G}(S_{t-1},\tau_{M,t-1})}{C_t\mathcal{G}(S_t,\tau_{M,t})^2\Pi_{H,t}}\frac{\partial\mathcal{G}(S_t,\tau_{M,t})}{\partial S_t}.
\end{aligned}
$$

The first-order condition for $\Pi_{H,t}$ is

$$
\begin{aligned}
0 \;=\;& -\theta(\varepsilon-1)\lambda_{p,t}\Pi_{H,t}^{\varepsilon-2}-\theta\varepsilon\lambda_{\Delta,t}\Pi_{H,t}^{\varepsilon-1}\Delta_{H,t-1}\\
&-(1-\theta)(1-\varepsilon)\left[\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(\Pi_{H,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(\Pi_{H,t})\right]\\
&+(1-\theta)\varepsilon\left[\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(\Pi_{H,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(\Pi_{H,t})\right]\\
&+\lambda_{e,t-1}\frac{C_{t-1}\mathcal{G}(S_{t-1},\tau_{M,t-1})}{C_t\mathcal{G}(S_t,\tau_{M,t})\Pi_{H,t}^2}.
\end{aligned}
$$

The first-order condition for $\Delta_{H,t}$ is

$$
0=\lambda_{\Delta,t}-\lambda_{n,t}Y_t-\beta\theta\lambda_{\Delta,t+1}\Pi_{H,t+1}^{\varepsilon}.
$$

Finally, the interest-rate condition and the ZLB conditions are

$$
-\frac{\lambda_{e,t}}{(1+i_t)^2}+\mu_{i,t}=0,\qquad i_t\geq 0,\qquad \mu_{i,t}\geq 0,\qquad \mu_{i,t}i_t=0.
$$

OA.1.2 Imperfect Pass-Through Environment

In the Imperfect Pass-Through Environment, the producer block remains unchanged and the local importer block in Eqs. (16)–(20) is added. Dividing Eq. (16) by $P_{F,t}^{1-\varepsilon_M}$ gives

$$
1=\theta_M\Pi_{F,t}^{\varepsilon_M-1}+(1-\theta_M)\widetilde{p}_{F,t}^{1-\varepsilon_M}.
$$

To keep the manuscript definition $S_t=\mathcal{E}_tP_{F,t}^{*}/P_{H,t}$ fixed, this subsection writes the tariff-exclusive local relative import price directly as $P_{F,t}/P_{H,t}=S_t/\Psi_t$ when taking derivatives; no additional relative-price symbol is introduced. Equivalently, the displayed conditions use $P_{F,t}/P_{H,t}$ and Ψ_t as differentiation coordinates, with $S_t=\Psi_t(P_{F,t}/P_{H,t})$. After the producer and importer reset-price relations have been substituted, the Lagrangian is

$$
\begin{aligned}
\mathcal{L}^{SOE,IPT} \;=\;& \mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\Bigg\{\log C_t-\frac{N_t^{1+\varphi}}{1+\varphi}\\
&+\lambda_{p,t}\left[1-\theta\Pi_{H,t}^{\varepsilon-1}-(1-\theta)\widetilde{p}_{H,t}^{1-\varepsilon}\right]\\
&+\lambda_{\Delta,t}\left[\Delta_{H,t}-\theta\Pi_{H,t}^{\varepsilon}\Delta_{H,t-1}-(1-\theta)\widetilde{p}_{H,t}^{-\varepsilon}\right]\\
&+\lambda_{p,t}^{F}\left[1-\theta_M\Pi_{F,t}^{\varepsilon_M-1}-(1-\theta_M)\widetilde{p}_{F,t}^{1-\varepsilon_M}\right]\\
&+\lambda_{\Delta,t}^{F}\left[\Delta_{F,t}-\theta_M\Pi_{F,t}^{\varepsilon_M}\Delta_{F,t-1}-(1-\theta_M)\widetilde{p}_{F,t}^{-\varepsilon_M}\right]\\
&+\lambda_{n,t}\left[N_t-Y_t\Delta_{H,t}\right]\\
&+\lambda_{y,t}\left[Y_t-(1-\upsilon)\mathcal{G}((P_{F,t}/P_{H,t}),\tau_{M,t})^{\eta}C_t-\upsilon(\Psi_t\,(P_{F,t}/P_{H,t}))^{\eta}\right]
\end{aligned}
$$

$$
\begin{aligned}
&+\lambda_{r,t}\left[\Psi_t\left(P_{F,t}/P_{H,t}\right) - C_t\mathcal{G}(\left(P_{F,t}/P_{H,t}\right), \tau_{M,t})\right] \\
&+\lambda_{s,t}\left[\left(P_{F,t}/P_{H,t}\right) - \left(P_{F,t-1}/P_{H,t-1}\right)\frac{\Pi_{F,t}}{\Pi_{H,t}}\right] \\
&+\lambda_{e,t}\left[\frac{1}{1+i_t} - \beta\mathrm{E}_t\frac{C_t\mathcal{G}(\left(P_{F,t}/P_{H,t}\right), \tau_{M,t})}{C_{t+1}\mathcal{G}(\left(P_{F,t+1}/P_{H,t+1}\right), \tau_{M,t+1})\Pi_{H,t+1}}\right] + \mu_{i,t}i_t\Bigg\}.
\end{aligned}
$$

For the importer reset price, define the derivative notation

$$
\begin{aligned}
\mathcal{K}'_{F,t-j}(x_t) &\equiv \frac{\partial \mathcal{K}_{F,t-j}}{\partial x_t}, \qquad \mathcal{Z}'_{F,t-j}(x_t) \equiv \frac{\partial \mathcal{Z}_{F,t-j}}{\partial x_t}, \\
\widetilde{p}'_{F,t-j}(x_t) &\equiv \frac{\partial \widetilde{p}_{F,t-j}}{\partial x_t} = \mathcal{M}_M\frac{\mathcal{Z}'_{F,t-j}(x_t)\mathcal{K}_{F,t-j} - \mathcal{K}'_{F,t-j}(x_t)\mathcal{Z}_{F,t-j}}{(\mathcal{K}_{F,t-j})^2}.
\end{aligned}
$$

The first-order condition for C_t is

$$
\begin{aligned}
0 \;=\;& \frac{1}{C_t} - (1-\theta)(1-\varepsilon)\left[\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(C_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(C_t)\right] \\
&+(1-\theta)\varepsilon\left[\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(C_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(C_t)\right] \\
&-(1-\theta_M)(1-\varepsilon_M)\left[\lambda_{p,t}^F\widetilde{p}_{F,t}^{-\varepsilon_M}\widetilde{p}'_{F,t}(C_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}^F\widetilde{p}_{F,t-j}^{-\varepsilon_M}\widetilde{p}'_{F,t-j}(C_t)\right] \\
&+(1-\theta_M)\varepsilon_M\left[\lambda_{\Delta,t}^F\widetilde{p}_{F,t}^{-\varepsilon_M-1}\widetilde{p}'_{F,t}(C_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}^F\widetilde{p}_{F,t-j}^{-\varepsilon_M-1}\widetilde{p}'_{F,t-j}(C_t)\right] \\
&-(1-\upsilon)\lambda_{y,t}\mathcal{G}(\left(P_{F,t}/P_{H,t}\right), \tau_{M,t})^{\eta} - \lambda_{r,t}\mathcal{G}(\left(P_{F,t}/P_{H,t}\right), \tau_{M,t}) \\
&-\beta\lambda_{e,t}\mathrm{E}_t\left[\frac{\mathcal{G}(\left(P_{F,t}/P_{H,t}\right), \tau_{M,t})}{C_{t+1}\mathcal{G}(\left(P_{F,t+1}/P_{H,t+1}\right), \tau_{M,t+1})\Pi_{H,t+1}}\right] \\
&+\lambda_{e,t-1}\frac{C_{t-1}\mathcal{G}(\left(P_{F,t-1}/P_{H,t-1}\right), \tau_{M,t-1})}{C_t^2\mathcal{G}(\left(P_{F,t}/P_{H,t}\right), \tau_{M,t})\Pi_{H,t}}.
\end{aligned}
$$

The first-order condition for N_t is

$$
\begin{aligned}
0 \;=\;& -N_t^{\varphi} + \lambda_{n,t} \\
&-(1-\theta)(1-\varepsilon)\left[\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(N_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(N_t)\right] \\
&+(1-\theta)\varepsilon\left[\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(N_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(N_t)\right].
\end{aligned}
$$

The first-order condition for Y_t is

$$
\begin{aligned}
0 \;=\;& -\lambda_{n,t}\Delta_{H,t} + \lambda_{y,t} \\
&-(1-\theta)(1-\varepsilon)\left[\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(Y_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(Y_t)\right] \\
&+(1-\theta)\varepsilon\left[\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(Y_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(Y_t)\right].
\end{aligned}
$$

The first-order condition for the local relative import price $P_{F,t}/P_{H,t}$ is

$$\begin{aligned}
0 &= -(1-\theta)(1-\varepsilon)\left[\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}((P_{F,t}/P_{H,t})) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}((P_{F,t}/P_{H,t}))\right] \\
&+(1-\theta)\varepsilon\left[\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}((P_{F,t}/P_{H,t})) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}((P_{F,t}/P_{H,t}))\right] \\
&-(1-\theta_M)(1-\varepsilon_M)\left[\lambda_{p,t}^{F}\widetilde{p}_{F,t}^{-\varepsilon_M}\widetilde{p}'_{F,t}((P_{F,t}/P_{H,t})) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}^{F}\widetilde{p}_{F,t-j}^{-\varepsilon_M}\widetilde{p}'_{F,t-j}((P_{F,t}/P_{H,t}))\right] \\
&+(1-\theta_M)\varepsilon_M\left[\lambda_{\Delta,t}^{F}\widetilde{p}_{F,t}^{-\varepsilon_M-1}\widetilde{p}'_{F,t}((P_{F,t}/P_{H,t})) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}^{F}\widetilde{p}_{F,t-j}^{-\varepsilon_M-1}\widetilde{p}'_{F,t-j}((P_{F,t}/P_{H,t}))\right] \\
&-\lambda_{y,t}\left[(1-\upsilon)\eta\mathcal{G}((P_{F,t}/P_{H,t}),\tau_{M,t})^{\eta-1}\frac{\partial\mathcal{G}((P_{F,t}/P_{H,t}),\tau_{M,t})}{\partial(P_{F,t}/P_{H,t})}C_t + \upsilon\eta\Psi_t(\Psi_t(P_{F,t}/P_{H,t}))^{\eta-1}\right] \\
&+\lambda_{r,t}\left[\Psi_t - C_t\frac{\partial\mathcal{G}((P_{F,t}/P_{H,t}),\tau_{M,t})}{\partial(P_{F,t}/P_{H,t})}\right] + \lambda_{s,t} \\
&-\beta\lambda_{s,t+1}\frac{\Pi_{F,t+1}}{\Pi_{H,t+1}} \\
&-\beta\lambda_{e,t}\mathrm{E}_t\left[\frac{C_t\,\partial\mathcal{G}((P_{F,t}/P_{H,t}),\tau_{M,t})/\partial(P_{F,t}/P_{H,t})}{C_{t+1}\mathcal{G}((P_{F,t+1}/P_{H,t+1}),\tau_{M,t+1})\Pi_{H,t+1}}\right] \\
&+\lambda_{e,t-1}\frac{C_{t-1}\mathcal{G}((P_{F,t-1}/P_{H,t-1}),\tau_{M,t-1})}{C_t\mathcal{G}((P_{F,t}/P_{H,t}),\tau_{M,t})^2\Pi_{H,t}}\frac{\partial\mathcal{G}((P_{F,t}/P_{H,t}),\tau_{M,t})}{\partial(P_{F,t}/P_{H,t})}.
\end{aligned}$$

The first-order condition for Ψ_t is

$$\begin{aligned}
0 &= -(1-\theta_M)(1-\varepsilon_M)\left[\lambda_{p,t}^{F}\widetilde{p}_{F,t}^{-\varepsilon_M}\widetilde{p}'_{F,t}(\Psi_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}^{F}\widetilde{p}_{F,t-j}^{-\varepsilon_M}\widetilde{p}'_{F,t-j}(\Psi_t)\right] \\
&+(1-\theta_M)\varepsilon_M\left[\lambda_{\Delta,t}^{F}\widetilde{p}_{F,t}^{-\varepsilon_M-1}\widetilde{p}'_{F,t}(\Psi_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}^{F}\widetilde{p}_{F,t-j}^{-\varepsilon_M-1}\widetilde{p}'_{F,t-j}(\Psi_t)\right] \\
&-\upsilon\eta\lambda_{y,t}(P_{F,t}/P_{H,t})(\Psi_t(P_{F,t}/P_{H,t}))^{\eta-1} \\
&+\lambda_{r,t}(P_{F,t}/P_{H,t}).
\end{aligned}$$

The first-order condition for $\Pi_{H,t}$ is

$$\begin{aligned}
0 &= -\theta(\varepsilon-1)\lambda_{p,t}\Pi_{H,t}^{\varepsilon-2} - \theta\varepsilon\lambda_{\Delta,t}\Pi_{H,t}^{\varepsilon-1}\Delta_{H,t-1} \\
&-(1-\theta)(1-\varepsilon)\left[\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(\Pi_{H,t}) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(\Pi_{H,t})\right] \\
&+(1-\theta)\varepsilon\left[\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(\Pi_{H,t}) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(\Pi_{H,t})\right] \\
&-(1-\theta_M)(1-\varepsilon_M)\left[\lambda_{p,t}^{F}\widetilde{p}_{F,t}^{-\varepsilon_M}\widetilde{p}'_{F,t}(\Pi_{H,t}) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}^{F}\widetilde{p}_{F,t-j}^{-\varepsilon_M}\widetilde{p}'_{F,t-j}(\Pi_{H,t})\right] \\
&+(1-\theta_M)\varepsilon_M\left[\lambda_{\Delta,t}^{F}\widetilde{p}_{F,t}^{-\varepsilon_M-1}\widetilde{p}'_{F,t}(\Pi_{H,t}) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}^{F}\widetilde{p}_{F,t-j}^{-\varepsilon_M-1}\widetilde{p}'_{F,t-j}(\Pi_{H,t})\right]
\end{aligned}$$

$$+\lambda_{s,t}\left(P_{F,t-1}/P_{H,t-1}\right)\frac{\Pi_{F,t}}{\Pi_{H,t}^2}+\lambda_{e,t-1}\frac{C_{t-1}\mathcal{G}(\left(P_{F,t-1}/P_{H,t-1}\right),\tau_{M,t-1})}{C_t\mathcal{G}(\left(P_{F,t}/P_{H,t}\right),\tau_{M,t})\Pi_{H,t}^2}.$$

The first-order condition for $\Pi_{F,t}$ is

$$\begin{aligned}
0 \;=\; & -\theta_M(\varepsilon_M-1)\lambda_{p,t}^F\Pi_{F,t}^{\varepsilon_M-2}-\theta_M\varepsilon_M\lambda_{\Delta,t}^F\Pi_{F,t}^{\varepsilon_M-1}\Delta_{F,t-1} \\
& -(1-\theta_M)(1-\varepsilon_M)\left[\lambda_{p,t}^F\widetilde{p}_{F,t}^{-\varepsilon_M}\widetilde{p}'_{F,t}(\Pi_{F,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}^F\widetilde{p}_{F,t-j}^{-\varepsilon_M}\widetilde{p}'_{F,t-j}(\Pi_{F,t})\right] \\
& +(1-\theta_M)\varepsilon_M\left[\lambda_{\Delta,t}^F\widetilde{p}_{F,t}^{-\varepsilon_M-1}\widetilde{p}'_{F,t}(\Pi_{F,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}^F\widetilde{p}_{F,t-j}^{-\varepsilon_M-1}\widetilde{p}'_{F,t-j}(\Pi_{F,t})\right] \\
& -\lambda_{s,t}\left(P_{F,t-1}/P_{H,t-1}\right)\frac{1}{\Pi_{H,t}}.
\end{aligned}$$

The two dispersion conditions are

$$\begin{aligned}
0 &= \lambda_{\Delta,t}-\lambda_{n,t}Y_t-\beta\theta\lambda_{\Delta,t+1}\Pi_{H,t+1}^{\varepsilon}, \\
0 &= \lambda_{\Delta,t}^F-\beta\theta_M\lambda_{\Delta,t+1}^F\Pi_{F,t+1}^{\varepsilon_M}.
\end{aligned}$$

Finally,

$$-\frac{\lambda_{e,t}}{(1+i_t)^2}+\mu_{i,t}=0,\qquad i_t\geq 0,\qquad \mu_{i,t}\geq 0,\qquad \mu_{i,t}i_t=0.$$

OA.2 Two-country national Ramsey Lagrangians and first-order conditions

We now write the two national problems explicitly. Complete international risk sharing is imposed before either national planner differentiates. With the manuscript definition of S_t, complete risk sharing implies

$$S_t=\frac{C_t}{C_t^*}\frac{P_t/P_{H,t}}{P_t^*/P_{F,t}^*}. \tag{OA.3}$$

Equation (OA.3) is a substitution implied by complete markets, not a new definition. It is used to eliminate S_t from the national differentiation problems. No separate starred international-relative-price variable is introduced.

OA.2.1 Perfect Pass-Through Environment

In the Perfect Pass-Through Environment, after Eq. (OA.3) has been substituted, the Home CPI–PPI relation is

$$\left(\frac{P_t}{P_{H,t}}\right)^{1-\eta} \;=\; \omega_H+\omega_F(1+\tau_{M,t})^{1-\eta}\left[\frac{C_t}{C_t^*}\frac{P_t/P_{H,t}}{P_t^*/P_{F,t}^*}\right]^{1-\eta},$$

and Home goods-market clearing is

$$\begin{aligned}
Y_t \;=\; & \omega_H\left(\frac{P_t}{P_{H,t}}\right)^{\eta}C_t \\
& +\frac{1-\nu}{\nu}\omega_H^*\left[\frac{C_t(P_t/P_{H,t})}{C_t^*(1+\tau_{M,t}^*)}\right]^{\eta}C_t^*.
\end{aligned}$$

The Foreign counterparts, used in the Foreign national problem, are

$$
\begin{aligned}
\left(\frac{P_t^*}{P_{F,t}^*}\right)^{1-\eta} &= \omega_F^* + \omega_H^*(1+\tau_{M,t}^*)^{1-\eta}\left[\frac{C_t^*}{C_t}\frac{P_t^*/P_{F,t}^*}{P_t/P_{H,t}}\right]^{1-\eta}, \\
Y_t^* &= \omega_F^*\left(\frac{P_t^*}{P_{F,t}^*}\right)^{\eta} C_t^* \\
&\quad + \frac{\nu}{1-\nu}\omega_F\left[\frac{C_t^*(P_t^*/P_{F,t}^*)}{C_t(1+\tau_{M,t})}\right]^{\eta} C_t.
\end{aligned}
$$

The Home planner takes the complete Foreign contingent plan as given. After the Home reset-price relation has been substituted, its national Lagrangian is

$$
\begin{aligned}
\mathcal{L}_H^{PPT} &= \mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\Bigg\{\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \\
&\quad + \lambda_{g,t}^H\left[\left(\frac{P_t}{P_{H,t}}\right)^{1-\eta} - \omega_H - \omega_F(1+\tau_{M,t})^{1-\eta}\left[\frac{C_t}{C_t^*}\frac{P_t/P_{H,t}}{P_t^*/P_{F,t}^*}\right]^{1-\eta}\right] \\
&\quad + \lambda_{y,t}^H\left[Y_t - \omega_H\left(\frac{P_t}{P_{H,t}}\right)^{\eta} C_t - \frac{1-\nu}{\nu}\omega_H^*\left[\frac{C_t(P_t/P_{H,t})}{C_t^*(1+\tau_{M,t}^*)}\right]^{\eta} C_t^*\right] \\
&\quad + \lambda_{n,t}^H\left[N_t - Y_t\Delta_{H,t}\right] \\
&\quad + \lambda_{p,t}^H\left[1 - \theta\Pi_{H,t}^{\varepsilon-1} - (1-\theta)\widetilde{p}_{H,t}^{1-\varepsilon}\right] \\
&\quad + \lambda_{\Delta,t}^H\left[\Delta_{H,t} - \theta\Pi_{H,t}^{\varepsilon}\Delta_{H,t-1} - (1-\theta)\widetilde{p}_{H,t}^{-\varepsilon}\right] \\
&\quad + \lambda_{e,t}^H\left[\frac{1}{1+i_t} - \beta\mathrm{E}_t\frac{C_t(P_t/P_{H,t})}{C_{t+1}(P_{t+1}/P_{H,t+1})\Pi_{H,t+1}}\right] + \mu_{i,t}^H i_t\Bigg\}.
\end{aligned}
$$

In A.3, a superscript H or F on a multiplier identifies the national planner, while the subscript M identifies that planner's local importer LCP local importer-price block.

The Foreign planner takes the complete Home contingent plan as given. Its national Lagrangian is

$$
\begin{aligned}
\mathcal{L}_F^{PPT} &= \mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\Bigg\{\log C_t^* - \frac{(N_t^*)^{1+\varphi}}{1+\varphi} \\
&\quad + \lambda_{g,t}^F\left[\left(\frac{P_t^*}{P_{F,t}^*}\right)^{1-\eta} - \omega_F^* - \omega_H^*(1+\tau_{M,t}^*)^{1-\eta}\left[\frac{C_t^*}{C_t}\frac{P_t^*/P_{F,t}^*}{P_t/P_{H,t}}\right]^{1-\eta}\right] \\
&\quad + \lambda_{y,t}^F\left[Y_t^* - \omega_F^*\left(\frac{P_t^*}{P_{F,t}^*}\right)^{\eta} C_t^* - \frac{\nu}{1-\nu}\omega_F\left[\frac{C_t^*(P_t^*/P_{F,t}^*)}{C_t(1+\tau_{M,t})}\right]^{\eta} C_t\right] \\
&\quad + \lambda_{n,t}^F\left[N_t^* - Y_t^*\Delta_{F,t}^*\right] \\
&\quad + \lambda_{p,t}^F\left[1 - \theta(\Pi_{F,t}^*)^{\varepsilon-1} - (1-\theta)(\widetilde{p}_{F,t}^*)^{1-\varepsilon}\right] \\
&\quad + \lambda_{\Delta,t}^F\left[\Delta_{F,t}^* - \theta(\Pi_{F,t}^*)^{\varepsilon}\Delta_{F,t-1}^* - (1-\theta)(\widetilde{p}_{F,t}^*)^{-\varepsilon}\right] \\
&\quad + \lambda_{e,t}^F\left[\frac{1}{1+i_t^*} - \beta\mathrm{E}_t\frac{C_t^*(P_t^*/P_{F,t}^*)}{C_{t+1}^*(P_{t+1}^*/P_{F,t+1}^*)\Pi_{F,t+1}^*}\right] + \mu_{i,t}^F i_t^*\Bigg\}.
\end{aligned}
$$

For the Home reset price the derivative notation is that in Eq. (OA.2). The Foreign reset price uses the exact starred counterpart,

$$(\widetilde{p}^*_{F,t-j})'(x_t) = \mathcal{M}\frac{(\mathcal{Z}^*_{F,t-j})'(x_t)\mathcal{K}^*_{F,t-j} - (\mathcal{K}^*_{F,t-j})'(x_t)\mathcal{Z}^*_{F,t-j}}{(\mathcal{K}^*_{F,t-j})^2}.$$

The Home first-order condition for C_t is

$$\begin{aligned}
0 \;=\; & \frac{1}{C_t} - (1-\theta)(1-\varepsilon)\left[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(C_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(C_t)\right] \\
& +(1-\theta)\varepsilon\left[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(C_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(C_t)\right] \\
& -\lambda^H_{g,t}\omega_F(1+\tau_{M,t})^{1-\eta}(1-\eta)\frac{1}{C_t}\left[\frac{C_t}{C^*_t}\frac{P_t/P_{H,t}}{P^*_t/P^*_{F,t}}\right]^{1-\eta} \\
& -\lambda^H_{y,t}\omega_H\left(\frac{P_t}{P_{H,t}}\right)^{\eta} \\
& -\lambda^H_{y,t}\frac{1-\nu}{\nu}\omega^*_H\eta\frac{C^*_t}{C_t}\left[\frac{C_t(P_t/P_{H,t})}{C^*_t(1+\tau^*_{M,t})}\right]^{\eta} \\
& -\beta\lambda^H_{e,t}\mathrm{E}_t\left[\frac{P_t/P_{H,t}}{C_{t+1}(P_{t+1}/P_{H,t+1})\Pi_{H,t+1}}\right] \\
& +\lambda^H_{e,t-1}\frac{C_{t-1}(P_{t-1}/P_{H,t-1})}{C^2_t(P_t/P_{H,t})\Pi_{H,t}}.
\end{aligned}$$

The Home first-order condition for $P_t/P_{H,t}$ is

$$\begin{aligned}
0 \;=\; & -(1-\theta)(1-\varepsilon)\left[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(P_t/P_{H,t}) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(P_t/P_{H,t})\right] \\
& +(1-\theta)\varepsilon\left[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(P_t/P_{H,t}) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(P_t/P_{H,t})\right] \\
& +(1-\eta)\lambda^H_{g,t}\left(\frac{P_t}{P_{H,t}}\right)^{-\eta} \\
& -\lambda^H_{g,t}\omega_F(1+\tau_{M,t})^{1-\eta}(1-\eta)\left(\frac{P_t}{P_{H,t}}\right)^{-1}\left[\frac{C_t}{C^*_t}\frac{P_t/P_{H,t}}{P^*_t/P^*_{F,t}}\right]^{1-\eta} \\
& -\eta\lambda^H_{y,t}\omega_H\left(\frac{P_t}{P_{H,t}}\right)^{\eta-1}C_t \\
& -\eta\lambda^H_{y,t}\frac{1-\nu}{\nu}\omega^*_H\left(\frac{P_t}{P_{H,t}}\right)^{-1}C^*_t\left[\frac{C_t(P_t/P_{H,t})}{C^*_t(1+\tau^*_{M,t})}\right]^{\eta} \\
& -\beta\lambda^H_{e,t}\mathrm{E}_t\left[\frac{C_t}{C_{t+1}(P_{t+1}/P_{H,t+1})\Pi_{H,t+1}}\right] \\
& +\lambda^H_{e,t-1}\frac{C_{t-1}(P_{t-1}/P_{H,t-1})}{C_t(P_t/P_{H,t})^2\Pi_{H,t}}.
\end{aligned}$$

The remaining Home conditions in the Perfect Pass-Through Environment are

$$0 \;=\; -N^{\varphi}_t + \lambda^H_{n,t} - (1-\theta)(1-\varepsilon)\left[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(N_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(N_t)\right]$$

$$
\begin{aligned}
&\quad +(1-\theta)\varepsilon\left[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(N_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(N_t)\right],\\
0 &= -\lambda^H_{n,t}\Delta_{H,t}+\lambda^H_{y,t}-(1-\theta)(1-\varepsilon)\left[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(Y_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(Y_t)\right]\\
&\quad +(1-\theta)\varepsilon\left[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(Y_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(Y_t)\right],\\
0 &= -\theta(\varepsilon-1)\lambda^H_{p,t}\Pi^{\varepsilon-2}_{H,t}-\theta\varepsilon\lambda^H_{\Delta,t}\Pi^{\varepsilon-1}_{H,t}\Delta_{H,t-1}\\
&\quad -(1-\theta)(1-\varepsilon)\left[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(\Pi_{H,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(\Pi_{H,t})\right]\\
&\quad +(1-\theta)\varepsilon\left[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(\Pi_{H,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(\Pi_{H,t})\right]\\
&\quad +\lambda^H_{e,t-1}\frac{C_{t-1}(P_{t-1}/P_{H,t-1})}{C_t(P_t/P_{H,t})\Pi^2_{H,t}},\\
0 &= \lambda^H_{\Delta,t}-\lambda^H_{n,t}Y_t-\beta\theta\lambda^H_{\Delta,t+1}\Pi^{\varepsilon}_{H,t+1},\\
0 &= -\frac{\lambda^H_{e,t}}{(1+i_t)^2}+\mu^H_{i,t}.
\end{aligned}
$$

The Home ZLB conditions are

$$
i_t\geq 0,\qquad \mu^H_{i,t}\geq 0,\qquad \mu^H_{i,t}i_t=0.
$$

The Foreign first-order conditions in the Perfect Pass-Through Environment are written explicitly as

$$
\begin{aligned}
0 &= \frac{1}{C^*_t}-(1-\theta)(1-\varepsilon)\left[\lambda^F_{p,t}(\widetilde{p}^*_{F,t})^{-\varepsilon}(\widetilde{p}^*_{F,t})'(C^*_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{p,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon}(\widetilde{p}^*_{F,t-j})'(C^*_t)\right]\\
&\quad +(1-\theta)\varepsilon\left[\lambda^F_{\Delta,t}(\widetilde{p}^*_{F,t})^{-\varepsilon-1}(\widetilde{p}^*_{F,t})'(C^*_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{\Delta,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon-1}(\widetilde{p}^*_{F,t-j})'(C^*_t)\right]\\
&\quad -\lambda^F_{g,t}\omega^*_H(1+\tau^*_{M,t})^{1-\eta}(1-\eta)\frac{1}{C^*_t}\left[\frac{C^*_t}{C_t}\frac{P^*_t/P^*_{F,t}}{P_t/P_{H,t}}\right]^{1-\eta}\\
&\quad -\lambda^F_{y,t}\omega^*_F\left(\frac{P^*_t}{P^*_{F,t}}\right)^{\eta}\\
&\quad -\lambda^F_{y,t}\frac{\nu}{1-\nu}\omega_F\eta\frac{C_t}{C^*_t}\left[\frac{C^*_t(P^*_t/P^*_{F,t})}{C_t(1+\tau_{M,t})}\right]^{\eta}\\
&\quad -\beta\lambda^F_{e,t}\mathrm{E}_t\left[\frac{P^*_t/P^*_{F,t}}{C^*_{t+1}(P^*_{t+1}/P^*_{F,t+1})\Pi^*_{F,t+1}}\right]\\
&\quad +\lambda^F_{e,t-1}\frac{C^*_{t-1}(P^*_{t-1}/P^*_{F,t-1})}{(C^*_t)^2(P^*_t/P^*_{F,t})\Pi^*_{F,t}},\\
0 &= -(1-\theta)(1-\varepsilon)\left[\lambda^F_{p,t}(\widetilde{p}^*_{F,t})^{-\varepsilon}(\widetilde{p}^*_{F,t})'(P^*_t/P^*_{F,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{p,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon}(\widetilde{p}^*_{F,t-j})'(P^*_t/P^*_{F,t})\right]
\end{aligned}
$$

$$
\begin{aligned}
&+(1-\theta)\varepsilon\left[\lambda^F_{\Delta,t}(\widetilde{p}^*_{F,t})^{-\varepsilon-1}(\widetilde{p}^*_{F,t})'(P^*_t/P^*_{F,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{\Delta,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon-1}(\widetilde{p}^*_{F,t-j})'(P^*_t/P^*_{F,t})\right]\\
&+(1-\eta)\lambda^F_{g,t}\left(\frac{P^*_t}{P^*_{F,t}}\right)^{-\eta}\\
&-\lambda^F_{g,t}\omega^*_H(1+\tau^*_{M,t})^{1-\eta}(1-\eta)\left(\frac{P^*_t}{P^*_{F,t}}\right)^{-1}\left[\frac{C^*_t}{C_t}\frac{P^*_t/P^*_{F,t}}{P_t/P_{H,t}}\right]^{1-\eta}\\
&-\eta\lambda^F_{y,t}\omega^*_F\left(\frac{P^*_t}{P^*_{F,t}}\right)^{\eta-1}C^*_t\\
&-\eta\lambda^F_{y,t}\frac{\nu}{1-\nu}\omega_F\left(\frac{P^*_t}{P^*_{F,t}}\right)^{-1}C_t\left[\frac{C^*_t(P^*_t/P^*_{F,t})}{C_t(1+\tau_{M,t})}\right]^{\eta}\\
&-\beta\lambda^F_{e,t}\mathrm{E}_t\left[\frac{C^*_t}{C^*_{t+1}(P^*_{t+1}/P^*_{F,t+1})\Pi^*_{F,t+1}}\right]\\
&+\lambda^F_{e,t-1}\frac{C^*_{t-1}(P^*_{t-1}/P^*_{F,t-1})}{C^*_t(P^*_t/P^*_{F,t})^2\Pi^*_{F,t}},
\end{aligned}
$$

together with

$$
\begin{aligned}
0 &= -(N^*_t)^{\varphi}+\lambda^F_{n,t}-(1-\theta)(1-\varepsilon)\left[\lambda^F_{p,t}(\widetilde{p}^*_{F,t})^{-\varepsilon}(\widetilde{p}^*_{F,t})'(N^*_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{p,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon}(\widetilde{p}^*_{F,t-j})'(N^*_t)\right]\\
&\quad+(1-\theta)\varepsilon\left[\lambda^F_{\Delta,t}(\widetilde{p}^*_{F,t})^{-\varepsilon-1}(\widetilde{p}^*_{F,t})'(N^*_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{\Delta,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon-1}(\widetilde{p}^*_{F,t-j})'(N^*_t)\right],\\
0 &= -\lambda^F_{n,t}\Delta^*_{F,t}+\lambda^F_{y,t}-(1-\theta)(1-\varepsilon)\left[\lambda^F_{p,t}(\widetilde{p}^*_{F,t})^{-\varepsilon}(\widetilde{p}^*_{F,t})'(Y^*_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{p,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon}(\widetilde{p}^*_{F,t-j})'(Y^*_t)\right]\\
&\quad+(1-\theta)\varepsilon\left[\lambda^F_{\Delta,t}(\widetilde{p}^*_{F,t})^{-\varepsilon-1}(\widetilde{p}^*_{F,t})'(Y^*_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{\Delta,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon-1}(\widetilde{p}^*_{F,t-j})'(Y^*_t)\right],\\
0 &= -\theta(\varepsilon-1)\lambda^F_{p,t}(\Pi^*_{F,t})^{\varepsilon-2}-\theta\varepsilon\lambda^F_{\Delta,t}(\Pi^*_{F,t})^{\varepsilon-1}\Delta^*_{F,t-1}\\
&\quad-(1-\theta)(1-\varepsilon)\left[\lambda^F_{p,t}(\widetilde{p}^*_{F,t})^{-\varepsilon}(\widetilde{p}^*_{F,t})'(\Pi^*_{F,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{p,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon}(\widetilde{p}^*_{F,t-j})'(\Pi^*_{F,t})\right]\\
&\quad+(1-\theta)\varepsilon\left[\lambda^F_{\Delta,t}(\widetilde{p}^*_{F,t})^{-\varepsilon-1}(\widetilde{p}^*_{F,t})'(\Pi^*_{F,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{\Delta,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon-1}(\widetilde{p}^*_{F,t-j})'(\Pi^*_{F,t})\right]\\
&\quad+\lambda^F_{e,t-1}\frac{C^*_{t-1}(P^*_{t-1}/P^*_{F,t-1})}{C^*_t(P^*_t/P^*_{F,t})(\Pi^*_{F,t})^2},\\
0 &= \lambda^F_{\Delta,t}-\lambda^F_{n,t}Y^*_t-\beta\theta\lambda^F_{\Delta,t+1}(\Pi^*_{F,t+1})^{\varepsilon},\\
0 &= -\frac{\lambda^F_{e,t}}{(1+i^*_t)^2}+\mu^F_{i,t}.
\end{aligned}
$$

The Foreign ZLB conditions are

$$
i^*_t\geq 0,\qquad \mu^F_{i,t}\geq 0,\qquad \mu^F_{i,t}i^*_t=0.
$$

OA.2.2 Imperfect Pass-Through Environment

In the Imperfect Pass-Through Environment, Eq. (OA.3) continues to eliminate the international relative price before national differentiation. The remaining local import-price ratios are written directly as $P_{F,t}/P_{H,t}$ in Home and $P^*_{H,t}/P^*_{F,t}$ in Foreign; no new relative-price symbol is introduced. The two CPI–PPI relations are

$$\begin{aligned}
\left(\frac{P_t}{P_{H,t}}\right)^{1-\eta} &= \omega_H + \omega_F\left[(1+\tau_{M,t})\frac{P_{F,t}}{P_{H,t}}\right]^{1-\eta}, \\
\left(\frac{P^*_t}{P^*_{F,t}}\right)^{1-\eta} &= \omega^*_F + \omega^*_H\left[(1+\tau^*_{M,t})\frac{P^*_{H,t}}{P^*_{F,t}}\right]^{1-\eta}.
\end{aligned}$$

The goods-market conditions relevant for the two national problems are

$$\begin{aligned}
Y_t &= \omega_H\left(\frac{P_t}{P_{H,t}}\right)^{\eta} C_t \\
&\quad + \frac{1-\nu}{\nu}\omega^*_H\left[\frac{(1+\tau^*_{M,t})(P^*_{H,t}/P^*_{F,t})}{P^*_t/P^*_{F,t}}\right]^{-\eta} C^*_t\Delta^*_{H,t}, \\
Y^*_t &= \omega^*_F\left(\frac{P^*_t}{P^*_{F,t}}\right)^{\eta} C^*_t \\
&\quad + \frac{\nu}{1-\nu}\omega_F\left[\frac{(1+\tau_{M,t})(P_{F,t}/P_{H,t})}{P_t/P_{H,t}}\right]^{-\eta} C_t\Delta_{F,t}.
\end{aligned}$$

Here $\Delta_{F,t}$ is the Home importer-price dispersion already denoted by $\Delta_{F,t}$ in the SOE importer block, and $\Delta^*_{H,t}$ is its exact Foreign counterpart. These superscripts only distinguish importer-price dispersion from producer-price dispersion inside this appendix.

The local relative import prices obey the identities

$$\begin{aligned}
\frac{P_{F,t}}{P_{H,t}} &= \frac{P_{F,t-1}}{P_{H,t-1}}\frac{\Pi_{F,t}}{\Pi_{H,t}}, \\
\frac{P^*_{H,t}}{P^*_{F,t}} &= \frac{P^*_{H,t-1}}{P^*_{F,t-1}}\frac{\Pi^*_{H,t}}{\Pi^*_{F,t}}.
\end{aligned} \tag{OA.4}$$

After complete risk sharing has been imposed, the acquisition-cost ratios entering the two importer reset-price problems are

$$\begin{aligned}
\frac{\mathcal{E}_t P^*_{F,t}}{P_{F,t}} &= \frac{C_t}{C^*_t}\frac{P_t/P_{H,t}}{P^*_t/P^*_{F,t}}\left(\frac{P_{F,t}}{P_{H,t}}\right)^{-1}, \\
\frac{P_{H,t}/\mathcal{E}_t}{P^*_{H,t}} &= \frac{C^*_t}{C_t}\frac{P^*_t/P^*_{F,t}}{P_t/P_{H,t}}\left(\frac{P^*_{H,t}}{P^*_{F,t}}\right)^{-1}.
\end{aligned}$$

Thus no separate international relative-price choice is required in either importer block.

With the Home and Foreign reset-price relations substituted as in A.2, the Home national Lagrangian in the Imperfect Pass-Through Environment is

$$\begin{aligned}
\mathcal{L}^{IPT}_H &= \mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\Bigg\{\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \\
&\quad + \lambda^H_{g,t}\left[\left(\frac{P_t}{P_{H,t}}\right)^{1-\eta} - \omega_H - \omega_F\left[(1+\tau_{M,t})\frac{P_{F,t}}{P_{H,t}}\right]^{1-\eta}\right]
\end{aligned}$$

$$
\begin{aligned}
&+\lambda^H_{y,t}\left[Y_t - \omega_H \left(\frac{P_t}{P_{H,t}}\right)^{\eta} C_t - \frac{1-\nu}{\nu}\omega^*_H \left[\frac{(1+\tau^*_{M,t})(P^*_{H,t}/P^*_{F,t})}{P^*_t/P^*_{F,t}}\right]^{-\eta} C^*_t \Delta^*_{H,t}\right] \\
&+\lambda^H_{n,t}[N_t - Y_t\Delta_{H,t}] \\
&+\lambda^H_{p,t}\left[1-\theta\Pi^{\varepsilon-1}_{H,t} - (1-\theta)\widetilde{p}^{1-\varepsilon}_{H,t}\right] \\
&+\lambda^H_{\Delta,t}\left[\Delta_{H,t} - \theta\Pi^{\varepsilon}_{H,t}\Delta_{H,t-1} - (1-\theta)\widetilde{p}^{-\varepsilon}_{H,t}\right] \\
&+\lambda^H_{q,t}\left[\frac{P_{F,t}}{P_{H,t}} - \frac{P_{F,t-1}}{P_{H,t-1}}\frac{\Pi_{F,t}}{\Pi_{H,t}}\right] \\
&+\lambda^H_{Mp,t}\left[1-\theta_M\Pi^{\varepsilon_M-1}_{F,t} - (1-\theta_M)\widetilde{p}^{1-\varepsilon_M}_{F,t}\right] \\
&+\lambda^H_{M\Delta,t}\left[\Delta_{F,t} - \theta_M\Pi^{\varepsilon_M}_{F,t}\Delta_{F,t-1} - (1-\theta_M)\widetilde{p}^{-\varepsilon_M}_{F,t}\right] \\
&+\lambda^H_{e,t}\left[\frac{1}{1+i_t} - \beta \mathrm{E}_t \frac{C_t(P_t/P_{H,t})}{C_{t+1}(P_{t+1}/P_{H,t+1})\Pi_{H,t+1}}\right] + \mu^H_{i,t} i_t \Bigg\}.
\end{aligned}
$$

The Foreign national Lagrangian in the Imperfect Pass-Through Environment is

$$
\begin{aligned}
\mathcal{L}^{IPT}_F \;=\; & \mathrm{E}_0 \sum_{t=0}^{\infty}\beta^t \Bigg\{ \log C^*_t - \frac{(N^*_t)^{1+\varphi}}{1+\varphi} \\
&+\lambda^F_{g,t}\left[\left(\frac{P^*_t}{P^*_{F,t}}\right)^{1-\eta} - \omega^*_F - \omega^*_H \left[(1+\tau^*_{M,t})\frac{P^*_{H,t}}{P^*_{F,t}}\right]^{1-\eta}\right] \\
&+\lambda^F_{y,t}\left[Y^*_t - \omega^*_F \left(\frac{P^*_t}{P^*_{F,t}}\right)^{\eta} C^*_t - \frac{\nu}{1-\nu}\omega_F \left[\frac{(1+\tau_{M,t})(P_{F,t}/P_{H,t})}{P_t/P_{H,t}}\right]^{-\eta} C_t\Delta_{F,t}\right] \\
&+\lambda^F_{n,t}[N^*_t - Y^*_t\Delta^*_{F,t}] \\
&+\lambda^F_{p,t}\left[1-\theta(\Pi^*_{F,t})^{\varepsilon-1} - (1-\theta)(\widetilde{p}^*_{F,t})^{1-\varepsilon}\right] \\
&+\lambda^F_{\Delta,t}\left[\Delta^*_{F,t} - \theta(\Pi^*_{F,t})^{\varepsilon}\Delta^*_{F,t-1} - (1-\theta)(\widetilde{p}^*_{F,t})^{-\varepsilon}\right] \\
&+\lambda^F_{q,t}\left[\frac{P^*_{H,t}}{P^*_{F,t}} - \frac{P^*_{H,t-1}}{P^*_{F,t-1}}\frac{\Pi^*_{H,t}}{\Pi^*_{F,t}}\right] \\
&+\lambda^F_{Mp,t}\left[1-\theta_M(\Pi^*_{H,t})^{\varepsilon_M-1} - (1-\theta_M)(\widetilde{p}^*_{H,t})^{1-\varepsilon_M}\right] \\
&+\lambda^F_{M\Delta,t}\left[\Delta^*_{H,t} - \theta_M(\Pi^*_{H,t})^{\varepsilon_M}\Delta^*_{H,t-1} - (1-\theta_M)(\widetilde{p}^*_{H,t})^{-\varepsilon_M}\right] \\
&+\lambda^F_{e,t}\left[\frac{1}{1+i^*_t} - \beta \mathrm{E}_t \frac{C^*_t(P^*_t/P^*_{F,t})}{C^*_{t+1}(P^*_{t+1}/P^*_{F,t+1})\Pi^*_{F,t+1}}\right] + \mu^F_{i,t} i^*_t \Bigg\}.
\end{aligned}
$$

The Home first-order condition in the Imperfect Pass-Through Environment for C_t is

$$
\begin{aligned}
0 \;=\; & \frac{1}{C_t} - (1-\theta)(1-\varepsilon)\left[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(C_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(C_t)\right] \\
&+(1-\theta)\varepsilon\left[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(C_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(C_t)\right] \\
&-(1-\theta_M)(1-\varepsilon_M)\left[\lambda^H_{Mp,t}\widetilde{p}^{-\varepsilon_M}_{F,t}\widetilde{p}'_{F,t}(C_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{Mp,t-j}\widetilde{p}^{-\varepsilon_M}_{F,t-j}\widetilde{p}'_{F,t-j}(C_t)\right]
\end{aligned}
$$

$$
\begin{aligned}
&+(1-\theta_M)\varepsilon_M\left[\lambda^H_{M\Delta,t}\widetilde{p}_{F,t}^{-\varepsilon_M-1}\widetilde{p}'_{F,t}(C_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{M\Delta,t-j}\widetilde{p}_{F,t-j}^{-\varepsilon_M-1}\widetilde{p}'_{F,t-j}(C_t)\right]\\
&-\lambda^H_{y,t}\omega_H\left(\frac{P_t}{P_{H,t}}\right)^{\eta}\\
&-\beta\lambda^H_{e,t}\mathrm{E}_t\left[\frac{P_t/P_{H,t}}{C_{t+1}(P_{t+1}/P_{H,t+1})\Pi_{H,t+1}}\right]+\lambda^H_{e,t-1}\frac{C_{t-1}(P_{t-1}/P_{H,t-1})}{C_t^2(P_t/P_{H,t})\Pi_{H,t}}.
\end{aligned}
$$

The Home first-order condition in the Imperfect Pass-Through Environment for $P_t/P_{H,t}$ is

$$
\begin{aligned}
0 \quad = \quad & -(1-\theta)(1-\varepsilon)\left[\lambda^H_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(P_t/P_{H,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(P_t/P_{H,t})\right]\\
&+(1-\theta)\varepsilon\left[\lambda^H_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(P_t/P_{H,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(P_t/P_{H,t})\right]\\
&-(1-\theta_M)(1-\varepsilon_M)\left[\lambda^H_{Mp,t}\widetilde{p}_{F,t}^{-\varepsilon_M}\widetilde{p}'_{F,t}(P_t/P_{H,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{Mp,t-j}\widetilde{p}_{F,t-j}^{-\varepsilon_M}\widetilde{p}'_{F,t-j}(P_t/P_{H,t})\right]\\
&+(1-\theta_M)\varepsilon_M\left[\lambda^H_{M\Delta,t}\widetilde{p}_{F,t}^{-\varepsilon_M-1}\widetilde{p}'_{F,t}(P_t/P_{H,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{M\Delta,t-j}\widetilde{p}_{F,t-j}^{-\varepsilon_M-1}\widetilde{p}'_{F,t-j}(P_t/P_{H,t})\right]\\
&+(1-\eta)\lambda^H_{g,t}\left(\frac{P_t}{P_{H,t}}\right)^{-\eta}-\eta\lambda^H_{y,t}\omega_H\left(\frac{P_t}{P_{H,t}}\right)^{\eta-1}C_t\\
&-\beta\lambda^H_{e,t}\mathrm{E}_t\left[\frac{C_t}{C_{t+1}(P_{t+1}/P_{H,t+1})\Pi_{H,t+1}}\right]+\lambda^H_{e,t-1}\frac{C_{t-1}(P_{t-1}/P_{H,t-1})}{C_t(P_t/P_{H,t})^2\Pi_{H,t}}.
\end{aligned}
$$

The Home first-order condition in the Imperfect Pass-Through Environment for the local import-price ratio is

$$
\begin{aligned}
0 \quad = \quad & -(1-\eta)\lambda^H_{g,t}\omega_F(1+\tau_{M,t})^{1-\eta}\left(\frac{P_{F,t}}{P_{H,t}}\right)^{-\eta}\\
&-(1-\theta_M)(1-\varepsilon_M)\left[\lambda^H_{Mp,t}\widetilde{p}_{F,t}^{-\varepsilon_M}\widetilde{p}'_{F,t}(P_{F,t}/P_{H,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{Mp,t-j}\widetilde{p}_{F,t-j}^{-\varepsilon_M}\widetilde{p}'_{F,t-j}(P_{F,t}/P_{H,t})\right]\\
&+(1-\theta_M)\varepsilon_M\left[\lambda^H_{M\Delta,t}\widetilde{p}_{F,t}^{-\varepsilon_M-1}\widetilde{p}'_{F,t}(P_{F,t}/P_{H,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{M\Delta,t-j}\widetilde{p}_{F,t-j}^{-\varepsilon_M-1}\widetilde{p}'_{F,t-j}(P_{F,t}/P_{H,t})\right]\\
&+\lambda^H_{q,t}-\beta\lambda^H_{q,t+1}\frac{\Pi_{F,t+1}}{\Pi_{H,t+1}}.
\end{aligned}
$$

The remaining Home conditions in the Imperfect Pass-Through Environment are

$$
\begin{aligned}
0 \quad = \quad & -N_t^{\varphi}+\lambda^H_{n,t}-(1-\theta)(1-\varepsilon)\left[\lambda^H_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(N_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(N_t)\right]\\
&+(1-\theta)\varepsilon\left[\lambda^H_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(N_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(N_t)\right],\\
0 \quad = \quad & -\lambda^H_{n,t}\Delta_{H,t}+\lambda^H_{y,t}-(1-\theta)(1-\varepsilon)\left[\lambda^H_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(Y_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(Y_t)\right]
\end{aligned}
$$

$$
\begin{aligned}
& +(1-\theta)\varepsilon\left[\lambda_{\Delta,t}^{H}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}_{H,t}'(Y_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}^{H}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}_{H,t-j}'(Y_t)\right], \\
0 \quad = \quad & -\theta(\varepsilon-1)\lambda_{p,t}^{H}\Pi_{H,t}^{\varepsilon-2}-\theta\varepsilon\lambda_{\Delta,t}^{H}\Pi_{H,t}^{\varepsilon-1}\Delta_{H,t-1} \\
& -(1-\theta)(1-\varepsilon)\left[\lambda_{p,t}^{H}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}_{H,t}'(\Pi_{H,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}^{H}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}_{H,t-j}'(\Pi_{H,t})\right] \\
& +(1-\theta)\varepsilon\left[\lambda_{\Delta,t}^{H}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}_{H,t}'(\Pi_{H,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}^{H}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}_{H,t-j}'(\Pi_{H,t})\right] \\
& -(1-\theta_M)(1-\varepsilon_M)\left[\lambda_{Mp,t}^{H}\widetilde{p}_{F,t}^{-\varepsilon_M}\widetilde{p}_{F,t}'(\Pi_{H,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{Mp,t-j}^{H}\widetilde{p}_{F,t-j}^{-\varepsilon_M}\widetilde{p}_{F,t-j}'(\Pi_{H,t})\right] \\
& +(1-\theta_M)\varepsilon_M\left[\lambda_{M\Delta,t}^{H}\widetilde{p}_{F,t}^{-\varepsilon_M-1}\widetilde{p}_{F,t}'(\Pi_{H,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{M\Delta,t-j}^{H}\widetilde{p}_{F,t-j}^{-\varepsilon_M-1}\widetilde{p}_{F,t-j}'(\Pi_{H,t})\right] \\
& +\lambda_{q,t}^{H}\frac{P_{F,t-1}}{P_{H,t-1}}\frac{\Pi_{F,t}}{\Pi_{H,t}^{2}}+\lambda_{e,t-1}^{H}\frac{C_{t-1}(P_{t-1}/P_{H,t-1})}{C_t(P_t/P_{H,t})\Pi_{H,t}^{2}}, \\
0 \quad = \quad & -\theta_M(\varepsilon_M-1)\lambda_{Mp,t}^{H}\Pi_{F,t}^{\varepsilon_M-2}-\theta_M\varepsilon_M\lambda_{M\Delta,t}^{H}\Pi_{F,t}^{\varepsilon_M-1}\Delta_{F,t-1} \\
& -(1-\theta_M)(1-\varepsilon_M)\left[\lambda_{Mp,t}^{H}\widetilde{p}_{F,t}^{-\varepsilon_M}\widetilde{p}_{F,t}'(\Pi_{F,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{Mp,t-j}^{H}\widetilde{p}_{F,t-j}^{-\varepsilon_M}\widetilde{p}_{F,t-j}'(\Pi_{F,t})\right] \\
& +(1-\theta_M)\varepsilon_M\left[\lambda_{M\Delta,t}^{H}\widetilde{p}_{F,t}^{-\varepsilon_M-1}\widetilde{p}_{F,t}'(\Pi_{F,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{M\Delta,t-j}^{H}\widetilde{p}_{F,t-j}^{-\varepsilon_M-1}\widetilde{p}_{F,t-j}'(\Pi_{F,t})\right] \\
& -\lambda_{q,t}^{H}\frac{P_{F,t-1}}{P_{H,t-1}}\frac{1}{\Pi_{H,t}}, \\
0 \quad = \quad & \lambda_{\Delta,t}^{H}-\lambda_{n,t}^{H}Y_t-\beta\theta\lambda_{\Delta,t+1}^{H}\Pi_{H,t+1}^{\varepsilon}, \\
0 \quad = \quad & \lambda_{M\Delta,t}^{H}-\beta\theta_M\lambda_{M\Delta,t+1}^{H}\Pi_{F,t+1}^{\varepsilon_M}, \\
0 \quad = \quad & -\frac{\lambda_{e,t}^{H}}{(1+i_t)^2}+\mu_{i,t}^{H}.
\end{aligned}
$$

The Home ZLB conditions are

$$
i_t \geq 0, \qquad \mu_{i,t}^{H} \geq 0, \qquad \mu_{i,t}^{H} i_t = 0.
$$

The Foreign first-order conditions in the Imperfect Pass-Through Environment are the fully written mirror system:

$$
\begin{aligned}
0 \quad = \quad & \frac{1}{C_t^*}-(1-\theta)(1-\varepsilon)\left[\lambda_{p,t}^{F}(\widetilde{p}_{F,t}^{*})^{-\varepsilon}(\widetilde{p}_{F,t}^{*})'(C_t^*)+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}^{F}(\widetilde{p}_{F,t-j}^{*})^{-\varepsilon}(\widetilde{p}_{F,t-j}^{*})'(C_t^*)\right] \\
& +(1-\theta)\varepsilon\left[\lambda_{\Delta,t}^{F}(\widetilde{p}_{F,t}^{*})^{-\varepsilon-1}(\widetilde{p}_{F,t}^{*})'(C_t^*)+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}^{F}(\widetilde{p}_{F,t-j}^{*})^{-\varepsilon-1}(\widetilde{p}_{F,t-j}^{*})'(C_t^*)\right] \\
& -(1-\theta_M)(1-\varepsilon_M)\left[\lambda_{Mp,t}^{F}(\widetilde{p}_{H,t}^{*})^{-\varepsilon_M}(\widetilde{p}_{H,t}^{*})'(C_t^*)+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{Mp,t-j}^{F}(\widetilde{p}_{H,t-j}^{*})^{-\varepsilon_M}(\widetilde{p}_{H,t-j}^{*})'(C_t^*)\right]
\end{aligned}
$$

$$
\begin{aligned}
& +(1-\theta_M)\varepsilon_M \left[\lambda^F_{M\Delta,t}(\widetilde{p}^*_{H,t})^{-\varepsilon_M-1}(\widetilde{p}^*_{H,t})'(C^*_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{M\Delta,t-j}(\widetilde{p}^*_{H,t-j})^{-\varepsilon_M-1}(\widetilde{p}^*_{H,t-j})'(C^*_t)\right] \\
& -\lambda^F_{y,t}\omega^*_F\left(\frac{P^*_t}{P^*_{F,t}}\right)^{\eta} \\
& -\beta\lambda^F_{e,t}\mathrm{E}_t\left[\frac{P^*_t/P^*_{F,t}}{C^*_{t+1}(P^*_{t+1}/P^*_{F,t+1})\Pi^*_{F,t+1}}\right] + \lambda^F_{e,t-1}\frac{C^*_{t-1}(P^*_{t-1}/P^*_{F,t-1})}{(C^*_t)^2(P^*_t/P^*_{F,t})\Pi^*_{F,t}}, \\
0 \quad = \quad & -(1-\theta)(1-\varepsilon)\left[\lambda^F_{p,t}(\widetilde{p}^*_{F,t})^{-\varepsilon}(\widetilde{p}^*_{F,t})'(P^*_t/P^*_{F,t}) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{p,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon}(\widetilde{p}^*_{F,t-j})'(P^*_t/P^*_{F,t})\right] \\
& +(1-\theta)\varepsilon\left[\lambda^F_{\Delta,t}(\widetilde{p}^*_{F,t})^{-\varepsilon-1}(\widetilde{p}^*_{F,t})'(P^*_t/P^*_{F,t}) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{\Delta,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon-1}(\widetilde{p}^*_{F,t-j})'(P^*_t/P^*_{F,t})\right] \\
& -(1-\theta_M)(1-\varepsilon_M)\left[\lambda^F_{Mp,t}(\widetilde{p}^*_{H,t})^{-\varepsilon_M}(\widetilde{p}^*_{H,t})'(P^*_t/P^*_{F,t}) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{Mp,t-j}(\widetilde{p}^*_{H,t-j})^{-\varepsilon_M}(\widetilde{p}^*_{H,t-j})'(P^*_t/P^*_{F,t})\right] \\
& +(1-\theta_M)\varepsilon_M\left[\lambda^F_{M\Delta,t}(\widetilde{p}^*_{H,t})^{-\varepsilon_M-1}(\widetilde{p}^*_{H,t})'(P^*_t/P^*_{F,t}) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{M\Delta,t-j}(\widetilde{p}^*_{H,t-j})^{-\varepsilon_M-1}(\widetilde{p}^*_{H,t-j})'(P^*_t/P^*_{F,t})\right] \\
& +(1-\eta)\lambda^F_{g,t}\left(\frac{P^*_t}{P^*_{F,t}}\right)^{-\eta} - \eta\lambda^F_{y,t}\omega^*_F\left(\frac{P^*_t}{P^*_{F,t}}\right)^{\eta-1}C^*_t \\
& -\beta\lambda^F_{e,t}\mathrm{E}_t\left[\frac{C^*_t}{C^*_{t+1}(P^*_{t+1}/P^*_{F,t+1})\Pi^*_{F,t+1}}\right] + \lambda^F_{e,t-1}\frac{C^*_{t-1}(P^*_{t-1}/P^*_{F,t-1})}{C^*_t(P^*_t/P^*_{F,t})^2\Pi^*_{F,t}}, \\
0 \quad = \quad & -(1-\eta)\lambda^F_{g,t}\omega^*_H(1+\tau^*_{M,t})^{1-\eta}\left(\frac{P^*_{H,t}}{P^*_{F,t}}\right)^{-\eta} \\
& -(1-\theta_M)(1-\varepsilon_M)\left[\lambda^F_{Mp,t}(\widetilde{p}^*_{H,t})^{-\varepsilon_M}(\widetilde{p}^*_{H,t})'(P^*_{H,t}/P^*_{F,t}) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{Mp,t-j}(\widetilde{p}^*_{H,t-j})^{-\varepsilon_M}(\widetilde{p}^*_{H,t-j})'(P^*_{H,t}/P^*_{F,t})\right] \\
& +(1-\theta_M)\varepsilon_M\left[\lambda^F_{M\Delta,t}(\widetilde{p}^*_{H,t})^{-\varepsilon_M-1}(\widetilde{p}^*_{H,t})'(P^*_{H,t}/P^*_{F,t}) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{M\Delta,t-j}(\widetilde{p}^*_{H,t-j})^{-\varepsilon_M-1}(\widetilde{p}^*_{H,t-j})'(P^*_{H,t}/P^*_{F,t})\right] \\
& +\lambda^F_{q,t} - \beta\lambda^F_{q,t+1}\frac{\Pi^*_{H,t+1}}{\Pi^*_{F,t+1}},
\end{aligned}
$$

and

$$
\begin{aligned}
0 \quad = \quad & -(N^*_t)^{\varphi} + \lambda^F_{n,t} - (1-\theta)(1-\varepsilon)\left[\lambda^F_{p,t}(\widetilde{p}^*_{F,t})^{-\varepsilon}(\widetilde{p}^*_{F,t})'(N^*_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{p,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon}(\widetilde{p}^*_{F,t-j})'(N^*_t)\right] \\
& +(1-\theta)\varepsilon\left[\lambda^F_{\Delta,t}(\widetilde{p}^*_{F,t})^{-\varepsilon-1}(\widetilde{p}^*_{F,t})'(N^*_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{\Delta,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon-1}(\widetilde{p}^*_{F,t-j})'(N^*_t)\right], \\
0 \quad = \quad & -\lambda^F_{n,t}\Delta^*_{F,t} + \lambda^F_{y,t} - (1-\theta)(1-\varepsilon)\left[\lambda^F_{p,t}(\widetilde{p}^*_{F,t})^{-\varepsilon}(\widetilde{p}^*_{F,t})'(Y^*_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{p,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon}(\widetilde{p}^*_{F,t-j})'(Y^*_t)\right]
\end{aligned}
$$

$$
\begin{aligned}
& +(1-\theta)\varepsilon\left[\lambda^F_{\Delta,t}(\widetilde{p}^*_{F,t})^{-\varepsilon-1}(\widetilde{p}^*_{F,t})'(Y^*_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{\Delta,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon-1}(\widetilde{p}^*_{F,t-j})'(Y^*_t)\right],\\
0 \quad = \quad & -\theta(\varepsilon-1)\lambda^F_{p,t}(\Pi^*_{F,t})^{\varepsilon-2}-\theta\varepsilon\lambda^F_{\Delta,t}(\Pi^*_{F,t})^{\varepsilon-1}\Delta^*_{F,t-1}\\
& -(1-\theta)(1-\varepsilon)\left[\lambda^F_{p,t}(\widetilde{p}^*_{F,t})^{-\varepsilon}(\widetilde{p}^*_{F,t})'(\Pi^*_{F,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{p,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon}(\widetilde{p}^*_{F,t-j})'(\Pi^*_{F,t})\right]\\
& +(1-\theta)\varepsilon\left[\lambda^F_{\Delta,t}(\widetilde{p}^*_{F,t})^{-\varepsilon-1}(\widetilde{p}^*_{F,t})'(\Pi^*_{F,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{\Delta,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon-1}(\widetilde{p}^*_{F,t-j})'(\Pi^*_{F,t})\right]\\
& -(1-\theta_M)(1-\varepsilon_M)\left[\lambda^F_{Mp,t}(\widetilde{p}^*_{H,t})^{-\varepsilon_M}(\widetilde{p}^*_{H,t})'(\Pi^*_{F,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{Mp,t-j}(\widetilde{p}^*_{H,t-j})^{-\varepsilon_M}(\widetilde{p}^*_{H,t-j})'(\Pi^*_{F,t})\right]\\
& +(1-\theta_M)\varepsilon_M\left[\lambda^F_{M\Delta,t}(\widetilde{p}^*_{H,t})^{-\varepsilon_M-1}(\widetilde{p}^*_{H,t})'(\Pi^*_{F,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{M\Delta,t-j}(\widetilde{p}^*_{H,t-j})^{-\varepsilon_M-1}(\widetilde{p}^*_{H,t-j})'(\Pi^*_{F,t})\right]\\
& +\lambda^F_{q,t}\frac{P^*_{H,t-1}}{P^*_{F,t-1}}\frac{\Pi^*_{H,t}}{(\Pi^*_{F,t})^2}+\lambda^F_{e,t-1}\frac{C^*_{t-1}(P^*_{t-1}/P^*_{F,t-1})}{C^*_t(P^*_t/P^*_{F,t})(\Pi^*_{F,t})^2},\\
0 \quad = \quad & -\theta_M(\varepsilon_M-1)\lambda^F_{Mp,t}(\Pi^*_{H,t})^{\varepsilon_M-2}-\theta_M\varepsilon_M\lambda^F_{M\Delta,t}(\Pi^*_{H,t})^{\varepsilon_M-1}\Delta^*_{H,t-1}\\
& -(1-\theta_M)(1-\varepsilon_M)\left[\lambda^F_{Mp,t}(\widetilde{p}^*_{H,t})^{-\varepsilon_M}(\widetilde{p}^*_{H,t})'(\Pi^*_{H,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{Mp,t-j}(\widetilde{p}^*_{H,t-j})^{-\varepsilon_M}(\widetilde{p}^*_{H,t-j})'(\Pi^*_{H,t})\right]\\
& +(1-\theta_M)\varepsilon_M\left[\lambda^F_{M\Delta,t}(\widetilde{p}^*_{H,t})^{-\varepsilon_M-1}(\widetilde{p}^*_{H,t})'(\Pi^*_{H,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{M\Delta,t-j}(\widetilde{p}^*_{H,t-j})^{-\varepsilon_M-1}(\widetilde{p}^*_{H,t-j})'(\Pi^*_{H,t})\right]\\
& -\lambda^F_{q,t}\frac{P^*_{H,t-1}}{P^*_{F,t-1}}\frac{1}{\Pi^*_{F,t}},\\
0 \quad = \quad & \lambda^F_{\Delta,t}-\lambda^F_{n,t}Y^*_t-\beta\theta\lambda^F_{\Delta,t+1}(\Pi^*_{F,t+1})^{\varepsilon},\\
0 \quad = \quad & \lambda^F_{M\Delta,t}-\beta\theta_M\lambda^F_{M\Delta,t+1}(\Pi^*_{H,t+1})^{\varepsilon_M},\\
0 \quad = \quad & -\frac{\lambda^F_{e,t}}{(1+i^*_t)^2}+\mu^F_{i,t}.
\end{aligned}
$$

The Foreign ZLB conditions are

$$
i^*_t \geq 0, \qquad \mu^F_{i,t}\geq 0, \qquad \mu^F_{i,t}i^*_t=0.
$$

The Home and Foreign first-order conditions, together with the common private-equilibrium relations, are solved jointly. Their mutual best response is the nonlinear national Ramsey equilibrium used in the two-country simulations.

OB Complete derivations for the ZLB extensions

Paper Appendix B contains only the definitions needed to read Sec. 5. This section provides the complete extension-specific derivations and implementation details.

This section gives the model modifications where needed and the detailed coefficient definitions and second-order welfare derivations behind Tables 6–9. Throughout, the Welfare Cost Functions are derived by the linear–quadratic method of Benigno and Woodford [30, 31]: policy-dependent first-order terms are eliminated with the first-order equilibrium restrictions before the second-order criterion is reduced and completed into squares. For the external-habit, roundabout-production, and open-loop Nash experiments it also reports the corresponding Ramsey or open-loop Lagrangians and first-order conditions. The high-substitution and reciprocal-tariff experiments change parameters or shock paths but not the underlying private-equilibrium equations. The pricing convention, tariff timing, fiscal closure, Home-Bias expenditure weights, and complete- markets structure are exactly those used in the main text and in On-line Appendix OA. In particular, tariffs are rebated lump sum, importer prices are tariff-exclusive, and the tariff is applied downstream to the purchaser price. The derivations below do not alter any equilibrium equation used in the numerical models.

For later use, it is convenient to record the scalar square-completion step. Suppose a reduced policy-dependent quadratic loss has the form

$$\mathcal{L}_t \quad = \quad \frac{1}{2}\sum_{k=1}^{n}\Lambda_k^{(0)}z_{k,t}^2 - \sum_{k<j}C_{kj}^{(0)}z_{k,t}z_{j,t} - \sum_{k=1}^{n}h_{k,t}^{(0)}z_{k,t} + \text{s.o.t.i.p.}. \tag{OB.1}$$

At step k, after the preceding $k-1$ squares have been completed, define

$$z_{k,t}^{w} = \frac{h_{k,t}^{(k-1)} + \sum_{j=k+1}^{n} C_{kj}^{(k-1)}z_{j,t}}{\Lambda_k^{(k-1)}}. \tag{OB.2}$$

Completing the $z_{k,t}$ square updates, for $j, l > k$,

$$\begin{aligned}
\Lambda_j^{(k)} &= \Lambda_j^{(k-1)} - \frac{[C_{kj}^{(k-1)}]^2}{\Lambda_k^{(k-1)}}, \\
C_{jl}^{(k)} &= C_{jl}^{(k-1)} + \frac{C_{kj}^{(k-1)}C_{kl}^{(k-1)}}{\Lambda_k^{(k-1)}}, \\
h_{j,t}^{(k)} &= h_{j,t}^{(k-1)} + \frac{h_{k,t}^{(k-1)}C_{kj}^{(k-1)}}{\Lambda_k^{(k-1)}}.
\end{aligned} \tag{OB.3}$$

These operations are algebraic only. The Welfare Relevant variables are not additional private-equilibrium variables.

OB.1 High-substitution SOE: complete welfare construction

The weight definitions needed to read Sec. 5.1 are reported in Paper Appendix B.1. For completeness, let $F_{j,t}^{IPT} = 0$ denote the nonlinear SOE private-equilibrium restrictions in the Imperfect Pass-Through Environment and let λ_j^{IPT} solve the stationary undetermined-coefficient system that eliminates all policy-dependent linear terms,

$$0 = U_v^{(1)} + \sum_j \lambda_j^{IPT}\mathcal{E}_v F_j^{IPT} \qquad \text{for every endogenous scalar } v.$$

The policy-dependent second-order contribution is

$$\mathcal{W}_t^{SOE,IPT} = U_t^{(2)} + \sum_j \lambda_j^{IPT}\mathcal{Q}_t(F_j^{IPT}) + \text{s.o.t.i.p.},$$

where $\mathcal{Q}_t(F_j^{IPT})$ is the date-t quadratic contribution of restriction j after discounted lead and lag terms have been shifted to date t. Completing the output–LOOP square gives the same SOE

Imperfect Pass-Through period loss as in the benchmark, with every coefficient evaluated at the high-substitution calibration:

$$\begin{aligned}\mathcal{L}_t^{SOE,IPT}\Big|_{\eta=3} &= \frac{1}{2}\Big[\Lambda_y^{SOE,IPT}(y_t - y_t^{w,SOE,IPT})^2 + \Lambda_\psi^{SOE,IPT}(\psi_t - \psi_t^{w,SOE,IPT})^2 \\ &\quad + \Lambda_{\pi,H}^{SOE,IPT}\pi_{H,t}^2 + \Lambda_{\pi,F}^{SOE,IPT}\pi_{F,t}^2\Big]_{\eta=3}.\end{aligned}$$

Thus the high-substitution experiment introduces no new welfare object; it re-evaluates the benchmark SOE criterion at the altered substitution elasticity.

OB.2 External habit: complete Ramsey and welfare derivation

The modified private-equilibrium conditions and the compact welfare representation are collected in Paper Appendix B.2. The complete Ramsey and Benigno–Woodford derivations are as follows.

OB.2.1 Ramsey Lagrangian and first-order conditions

The social planner evaluates aggregate welfare. After the private first-order conditions have been derived and symmetry is imposed, $C_t^{\mathrm{agg}} = C_t$, so aggregate welfare can be written as $\log(C_t - hC_{t-1}^{\mathrm{agg}})$, with $C_{t-1}^{\mathrm{agg}} = C_{t-1}$ under symmetry. The planner therefore internalizes the effect of aggregate current consumption on the next period's aggregate habit benchmark. This is the external-habit externality; it should not be confused with an internal-habit term in the private Euler equation.

Move every nonlinear SOE equilibrium condition in the Imperfect Pass-Through Environment just described to the left-hand side and denote its residual by $F_{j,t}^h$. The index set $\mathcal{J}_h$ contains the Home producer price-index, reset-price, auxiliary, and dispersion conditions, the Home importer price-index, reset-price, auxiliary, and dispersion conditions, aggregate production, goods-market clearing, risk sharing, the LCP relative-price law, and the Euler equation. No additional habit state or choice variable is introduced.

After the reset-price conditions are imposed, the Ramsey Lagrangian can be written as

$$\mathcal{L}^{SOE,IPT,h} = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t - hC_{t-1}^{\mathrm{agg}}) - \frac{N_t^{1+\varphi}}{1+\varphi} + \sum_{j\in\mathcal{J}_h} \lambda_{j,t}^h F_{j,t}^h + \mu_{i,t}^h i_t\right].$$

For any Ramsey choice x_t, define the exact discounted constraint derivative

$$\mathcal{D}_{x_t}^h \equiv \beta^{-t}\frac{\partial}{\partial x_t}\mathrm{E}_0 \sum_{s=0}^{\infty} \beta^s \sum_{j\in\mathcal{J}_h} \lambda_{j,s}^h F_{j,s}^h.$$

Because the definition differentiates the full discounted constraint sum, it includes every current, lagged, and lead occurrence of x_t, including those generated by the Calvo auxiliary recursions.

The consumption and labor first-order conditions are

$$\begin{aligned} 0 &= \frac{1}{C_t - hC_{t-1}} - \beta\frac{h}{C_{t+1} - hC_t} + \mathcal{D}_{C_t}^h, \qquad \text{(OB.4)} \\ 0 &= -N_t^{\varphi} + \mathcal{D}_{N_t}^h. \end{aligned}$$

The future-habit term in Eq. (OB.4) is a *planner* term arising from the aggregate externality. It is absent from the household Euler equation (B.4). For every other non-interest-rate Ramsey choice in the SOE Imperfect Pass-Through Environment in the relative-price, output, producer-price, and importer-price blocks,

$$0 = \mathcal{D}_{x_t}^h.$$

With the Euler residual signed as in Eq. (B.4), the interest-rate condition and the ZLB Kuhn–Tucker conditions are

$$-\frac{\lambda_{e,t}^{h}}{(1+i_t)^2}+\mu_{i,t}^{h}=0, \qquad i_t\geq 0, \qquad \mu_{i,t}^{h}\geq 0, \qquad \mu_{i,t}^{h}i_t=0. \tag{OB.5}$$

Equations (OB.4)–(OB.5), together with the nonlinear private equilibrium, are the Ramsey system underlying the external-habit IRFs. The structural parameter h is left symbolic here; the experiment in Sec. 5.2 evaluates the system at $h=0.71$.

OB.2.2 Second-order Welfare Cost Function

The deterministic zero-tariff, zero-inflation steady state satisfies

$$C=Y=N=\left[\frac{1}{\mathcal{M}(1-h)}\right]^{1/(1+\varphi)}, \qquad L_h\equiv N^{1+\varphi}=\frac{1}{\mathcal{M}(1-h)}.$$

No auxiliary "habit-surplus" variable is introduced. Let $c_t\equiv\log(C_t/C)$. After imposing $C_{t-1}^{\text{agg}}=C_{t-1}$, expanding the habit term directly in the original consumption variables gives

$$\begin{aligned}\log(C_t-hC_{t-1}) \quad &= \quad \log[(1-h)C]+\frac{c_t-hc_{t-1}}{1-h}\\ &\quad +\frac{c_t^2-hc_{t-1}^2}{2(1-h)}-\frac{(c_t-hc_{t-1})^2}{2(1-h)^2}+O(3).\end{aligned} \tag{OB.6}$$

Hence the lagged-consumption terms enter the second-order welfare expansion directly. In particular, $\sum_t\beta^t hc_{t-1}^2=\beta h\sum_t\beta^t c_t^2+\text{t.i.p.}$, so the endogenous lagged-consumption contribution cannot be dropped as a policy-independent term.

Following Benigno and Woodford [30, 31], the linear terms are eliminated before the quadratic loss is interpreted. We apply the stationary scalar coefficient-matching system directly to the original equilibrium restrictions in $C_t, C_{t-1}, Y_t, N_t, \chi_t$, producer marginal cost, importer marginal cost, and the two Calvo price-setting blocks. Thus no additional habit state or auxiliary choice variable is created for the welfare derivation. Denote by Θ_n^h, $\Theta_H^{AS,h}$, and $\Theta_M^{AS,h}$ the coefficients on, respectively, the labor term and the producer- and importer-price aggregate-supply restrictions obtained from that direct scalar matching. The remaining coefficients are determined simultaneously from the same original SOE restrictions in the Imperfect Pass-Through Environment and the direct expansion in Eq. (OB.6).

Accordingly the two sticky-inflation weights reported in Table 7 are Equation (B.5) is reported in Paper Appendix B.2.

Let λ_j^h denote the stationary multipliers that eliminate the policy-dependent first-order terms in the distorted-steady-state expansion. For every scalar endogenous variable v, they solve

$$0=U_v^{(1),h}+\sum_{j\in\mathcal{J}_h}\lambda_j^h\mathcal{E}_vF_j^h, \tag{OB.7}$$

where $\mathcal{E}_vF_j^h$ denotes the stationary discounted Euler–Lagrange derivative of residual j with respect to v; explicitly it is the sum of the current derivative plus the appropriately discounted derivatives of all lead and lag occurrences. Equation (OB.7) is the scalar undetermined-coefficient system. No numerical value of h is used in solving it.

Let $\mathcal{Q}_t(F_j^h)$ denote the date-t quadratic contribution obtained by expanding the discounted sum of residual j to second order and moving its lead and lag terms to date t. The Benigno–Woodford contribution is then

$$\mathcal{W}_t^{SOE,IPT,h}=U_t^{(2),h}+\sum_{j\in\mathcal{J}_h}\lambda_j^h\mathcal{Q}_t(F_j^h)+\text{s.o.t.i.p.}. \tag{OB.8}$$

The producer and importer dispersion equations in this sum generate the $\pi_{H,t}^2$ and $\pi_{F,t}^2$ terms. Solving the first-order restrictions and substituting them into Eq. (OB.8) leaves the three real policy coordinates y_t, ψ_t, and c_{t-1}, plus the two sticky-price inflation rates. Write the real part before square completion as Equation (B.6) is reported in Paper Appendix B.2. where the $h_{u,t}^{(0),h}$ terms are the symbolic linear combinations of the exogenous tariff process generated by the expansion. Complete the squares in the order y_t, ψ_t, c_{t-1} using Eqs. (OB.2)–(OB.3). Thus Equation (B.7) is reported in Paper Appendix B.2. Here $y_t^{w,h}$ is the (Logarithmic) Welfare Relevant Output and $\psi_t^{w,h}$ is the (Logarithmic) Welfare Relevant LOOP Gap; their corresponding differences in Eq. (36) are the Welfare Relevant Output Gap and Welfare Relevant LOOP Gap Deviation, respectively. For the unilateral benchmark experiment with $h = 0.71$ and no Foreign tariff, the evaluated Welfare Relevant variables are

$$
\begin{aligned}
y_t^{w,h} &= -0.07291610\,\psi_t + 0.28677487\,c_{t-1} + 0.28803833\,\tau_{M,t},\\
\psi_t^{w,h} &= -0.00513035\,c_{t-1} + 1.02448559\,\tau_{M,t},\\
c_{t-1}^{w,h} &= -2.45211505\,\tau_{M,t}.
\end{aligned}
$$

These numerical coefficients are used to construct the Welfare Relevant LOOP Gap IRF in Panel 12 of Fig. 4; they are evaluations of the symbolic Welfare Relevant variables above, not additional assumptions. For completeness, hold all other benchmark parameters fixed and re-evaluate the symbolic Welfare Relevant variables in Eq. (B.7) as h varies. Numerical differentiation of the resulting coefficient function confirms $\partial\psi_0^{w,h}/\partial h > 0$ throughout $0 \le h \le 0.735$ for a positive impact tariff. The corresponding tariff loadings $\partial\psi_0^{w,h}/\partial\tau_{M,0}$ are 0.76454951, 0.79228376, 0.86181720, 0.94951798, 1.02448559, 1.02545414, 1.02596798, and 1.02604120 for $h = 0$, 0.10, 0.30, 0.50, 0.71, 0.72, 0.73, and 0.735, respectively. The derivative reaches zero numerically at approximately $h = 0.735354$, where the tariff loading is 1.02604153, and becomes negative immediately above that point. Thus the positive-impact comparative static holds over the conservative closed range $0 \le h \le 0.735$ rather than globally. The impact statement is kept separate from later periods because c_{t-1} in Eq. (B.7) is endogenous once $t > 0$. Defining $\Lambda_y^h \equiv \Lambda_y^{(0),h}$, $\Lambda_\psi^h \equiv \Lambda_\psi^{(1),h}$, and $\Lambda_{c^-}^h \equiv \Lambda_{c^-}^{(2),h}$, and denoting by $\Lambda_{\pi,H}^h$ and $\Lambda_{\pi,F}^h$ the literal inflation-square coefficients generated by the two Calvo dispersion laws, gives Equation (B.8) is reported in Paper Appendix B.2. which is Eq. (36) in the text. The additional c_{t-1} square is therefore a genuine welfare term generated by the habit state; apart from the inherited initial condition at $t = 0$, it is not a policy-independent term.

OB.3 Reciprocal tariffs: complete welfare definitions

The reciprocal-tariff experiment changes the exogenous shock vector, not the private economy or the Home national-Ramsey objective. The Home Welfare Cost Function is therefore the benchmark two-country criterion in the Imperfect Pass-Through Environment with the first-order tariff terms $\log(1+\tau_{M,t}) = \tau_{M,t}$ and $\log(1+\tau_{M,t}^*) = \tau_{M,t}^*$ entering the Welfare Relevant variables. Its derivation follows Benigno and Woodford [30, 31]: the stationary multipliers on the first-order equilibrium restrictions are first chosen to eliminate all policy-dependent linear terms, after which the second-order augmented welfare is reduced and completed into squares.

For the coefficient matching, attach $\Theta_\chi, \Theta_Y, \Theta_n, \Theta_{mc,H}, \Theta_{mc,M}, \Theta_H^{AS}, \Theta_M^{AS}$ to the Home CPI equation, Home goods-market condition, labor identity, the two marginal-cost identities, and the two aggregate-supply restrictions. With the Home expenditure shares ω_H and ω_F, coefficient matching gives

$$
\begin{aligned}
0 &= 1 - \omega_H\Theta_Y - \Theta_{mc,H} - \Theta_{mc,M},\\
0 &= \Theta_\chi - \eta\omega_H\Theta_Y - \Theta_{mc,H} - \Theta_{mc,M},\\
0 &= -L + \Theta_n - \varphi\Theta_{mc,H},\\
0 &= \Theta_Y - \Theta_n,
\end{aligned}
$$

$$
\begin{aligned}
0 &= -\omega_F \Theta_\chi + \Theta_{mc,M}, \\
0 &= \Theta_{mc,H} + \Theta_H^{AS}, \\
0 &= \Theta_{mc,M} + \Theta_M^{AS}.
\end{aligned}
$$

Hence, defining

$$\mathcal{D}_I \equiv 1 + \eta\omega_H\varphi + (1-\eta)\omega_H^2\varphi,$$

we obtain

$$
\begin{aligned}
\Theta_Y = \Theta_n &= \frac{L + \omega_H\varphi}{\mathcal{D}_I}, \\
\Theta_\chi &= \frac{1 + \omega_H\eta\varphi + \omega_H L(\eta-1)}{\mathcal{D}_I}, \\
\Theta_{mc,M} &= \omega_F\Theta_\chi, \qquad \Theta_M^{AS} = -\omega_F\Theta_\chi, \\
\Theta_{mc,H} &= 1 - \omega_H\Theta_Y - \omega_F\Theta_\chi, \qquad \Theta_H^{AS} = -\Theta_{mc,H}.
\end{aligned}
$$

The three sticky-inflation weights in Table 4 therefore satisfy

$$\Lambda_{\pi,H}^{IPT} = \frac{\varepsilon}{\kappa_H}(\Theta_n - \Theta_H^{AS}), \qquad \Lambda_{\pi,F}^{IPT} = -\frac{\varepsilon_M}{\kappa_M}\Theta_M^{AS}, \qquad \Lambda_{\pi_H^*}^{IPT} = \frac{\varepsilon_M}{\kappa_M}\Theta_Y\omega_F. \tag{OB.9}$$

To make the Welfare Cost Function itself explicit, use complete risk sharing and the first-order world equilibrium to eliminate Foreign output. Before collecting the reduced coefficients, use the identity stated in Sec. 4.1, $\log(P_{F,t}/P_{H,t}) = s_t - \psi_t$, to express every local-relative-price term in the manuscript coordinate $s_t = \log(\mathcal{E}_t P_{F,t}^*/P_{H,t})$. Write the remaining real Home coordinates as $z_t = (y_t, \psi_t, s_t, \psi_t^*)'$. If $\widehat{C}_{uv}^{IPT}$ denotes the literal coefficient on $u_t v_t$ in the resulting second-order Home-welfare polynomial, define the initial loss coefficients by

$$\Lambda_u^{(0),IPT} \equiv -2\widehat{C}_{uu}^{IPT}, \qquad C_{uv}^{(0),IPT} \equiv \widehat{C}_{uv}^{IPT}, \qquad h_{u,t}^{(0),IPT} \equiv \widehat{C}_{um}^{IPT}\tau_{M,t} + \widehat{C}_{um^*}^{IPT}\tau_{M,t}^*.$$

Applying Eqs. (OB.2)–(OB.3) in the order y, ψ, s, ψ^* defines the four Welfare Relevant variables and the final real weights

$$\Lambda_y^{IPT} \equiv \Lambda_1^{(0),IPT}, \quad \Lambda_\psi^{IPT} \equiv \Lambda_2^{(1),IPT}, \quad \Lambda_s^{IPT} \equiv \Lambda_3^{(2),IPT}, \quad \Lambda_{\psi^*}^{IPT} \equiv \Lambda_4^{(3),IPT}.$$

The resulting Home period loss is

$$
\begin{aligned}
\mathcal{L}_{H,t}^{IPT} &= \frac{1}{2}\Big[\Lambda_y^{IPT}(y_t - y_t^{w,IPT})^2 + \Lambda_\psi^{IPT}(\psi_t - \psi_t^{w,IPT})^2 + \Lambda_s^{IPT}(s_t - s_t^{w,IPT})^2 \\
&\qquad + \Lambda_{\psi^*}^{IPT}(\psi_t^* - \psi_t^{*,w,IPT})^2 + \Lambda_{\pi,H}^{IPT}\pi_{H,t}^2 + \Lambda_{\pi,F}^{IPT}\pi_{F,t}^2 + \Lambda_{\pi_H^*}^{IPT}(\pi_{H,t}^*)^2\Big].
\end{aligned}
$$

Thus reciprocity changes the conditioning tariff paths and hence the Welfare Relevant variables, but it does not create a new set of welfare weights. The numerical inflation weights are therefore the benchmark two-country Imperfect Pass-Through weights reported in Table 4.

OB.4 Roundabout production: complete Ramsey and welfare derivation

The model equations and welfare definitions needed to read Sec. 5.4 are collected in Paper Appendix B.4. The remaining national Ramsey and Benigno–Woodford derivations are reported here.

As in the plain national Ramsey allocation game, complete international risk sharing is imposed before national differentiation. Let $F_{H,j,t}^{RA} = 0$ denote the resulting Home equilibrium residuals after the roundabout-production modifications in Sec. 5.4 and in Eqs. (B.9)– (B.11), and let $\mathcal{J}_H^{RA}$ be their scalar index set. The Home planner maximizes Home welfare taking the complete Foreign contingent allocation as given. Its Lagrangian is

$$\mathcal{L}_H^{RA} = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} + \sum_{j \in \mathcal{J}_H^{RA}} \lambda_{H,j,t}^{RA} F_{H,j,t}^{RA} + \mu_{H,t}^{RA} i_t \right].$$

The Foreign planner takes the complete Home contingent allocation as given and solves the exact mirror problem,

$$\mathcal{L}_F^{RA} = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t^* - \frac{(N_t^*)^{1+\varphi}}{1+\varphi} + \sum_{j \in \mathcal{J}_F^{RA}} \lambda_{F,j,t}^{RA} F_{F,j,t}^{RA} + \mu_{F,t}^{RA} i_t^* \right].$$

The roundabout allocation is the mutual best-response fixed point of these two national commitment problems and the common private equilibrium.

OB.4.1 First-order conditions

For a Home own-plan variable x_t, define

$$\mathcal{D}_{H,x_t}^{RA} \equiv \beta^{-t} \frac{\partial}{\partial x_t} \mathrm{E}_0 \sum_{s=0}^{\infty} \beta^s \sum_{j \in \mathcal{J}_H^{RA}} \lambda_{H,j,s}^{RA} F_{H,j,s}^{RA}. \qquad \text{(OB.10)}$$

The Home consumption, labor, and intermediate-input conditions are

$$\begin{aligned} 0 &= \frac{1}{C_t} + \mathcal{D}_{H,C_t}^{RA}, \\ 0 &= -N_t^{\varphi} + \mathcal{D}_{H,N_t}^{RA}, \\ 0 &= \mathcal{D}_{H,\mathcal{I}_t}^{RA}. \end{aligned}$$

For every other non-interest-rate Home own-plan variable appearing in the benchmark Imperfect Pass-Through choice set, including output, the Home CPI–PPI ratio, the Home producer-price block and dispersion, and the Home importer-price block and dispersion,

$$0 = \mathcal{D}_{H,x_t}^{RA}.$$

No Home first-order condition is taken with respect to a Foreign own-plan variable. With the same Euler-residual sign convention as On-line Appendix OA,

$$-\frac{\lambda_{H,e,t}^{RA}}{(1+i_t)^2} + \mu_{H,t}^{RA} = 0, \qquad i_t \geq 0, \qquad \mu_{H,t}^{RA} \geq 0, \qquad \mu_{H,t}^{RA} i_t = 0.$$

The Foreign FOCs and KKT conditions are the exact starred mirror. Because Eq. (OB.10) differentiates the full discounted residual sum, these conditions contain all lagged and lead multipliers implied by commitment. The structural share ζ is kept symbolic; Sec. 5.4 evaluates the system at $\zeta = 0.50$.

OB.4.2 Second-order Welfare Cost Function

At the zero-tariff, zero-inflation deterministic steady state, steady-state values are written without time subscripts. The implemented roundabout system implies

$$\sigma_C^{\zeta} \equiv \frac{C}{Y} = 1 - \frac{\zeta}{\mathcal{M}}, \qquad \sigma_I^{\zeta} \equiv \frac{\mathcal{I}}{Y} = \frac{\zeta}{\mathcal{M}}, \qquad L_{\zeta} \equiv N^{1+\varphi} = \frac{1-\zeta}{\mathcal{M}-\zeta}.$$

Let $\iota_t^I \equiv \log(\mathcal{I}_t/\mathcal{I})$ denote the log deviation of the material-input quantity. The log-linearized factor-demand conditions are

$$n_t = \frac{y_t - \zeta c_t}{1+\zeta\varphi}, \qquad \iota_t^I = \frac{(1-\zeta)c_t + (1+\varphi)y_t}{1+\zeta\varphi}. \tag{OB.11}$$

From Eq. (42) and $P/P_H = 1$ at the steady state, the log deviation of producer real marginal cost satisfies

$$mc_t^r = (1-\zeta)c_t + (1-\zeta)\varphi n_t + \omega_F\big(s_t + \tau_{M,t} - \psi_t\big),$$

where $s_t = \log S_t$ uses the manuscript definition $S_t = \mathcal{E}_t P_{F,t}^*/P_{H,t}$. The tariff term is an exogenous conditioning path; the $-\omega_F\psi_t$ term is required because the sticky local import price differs from the border acquisition price in the LCP importer block. Using the labor-demand relation in Eq. (OB.11) gives the equivalent output form

$$mc_t^r = \frac{1-\zeta}{1+\zeta\varphi}c_t + \frac{(1-\zeta)\varphi}{1+\zeta\varphi}y_t + \omega_F\big(s_t + \tau_{M,t} - \psi_t\big).$$

For gross absorption $C_t + \mathcal{I}_t$, the first-order log component is

$$d_t^{RA} = \sigma_C^\zeta c_t + \sigma_I^\zeta \iota_t^I,$$

and its second-order expansion contains the additional term $\frac{1}{2}\sigma_C^\zeta\sigma_I^\zeta(c_t - \iota_t^I)^2$. This is the channel through which roundabout production changes the importer-demand expansion as well as producer marginal cost.

The detailed definitions behind Table 8 can be written with the Benigno–Woodford undetermined coefficients. Attach $\Theta_{\mathcal{G}}^{RA}$, Θ_Y^{RA}, Θ_N^{RA}, Θ_I^{RA}, $\Theta_{mc,H}^{RA}$, $\Theta_{mc,M}^{RA}$, $\Theta_H^{AS,RA}$, and $\Theta_M^{AS,RA}$ to the CPI–PPI relation, gross-output, factor-demand, marginal-cost, and aggregate-supply restrictions. Coefficient matching gives Equation (B.12) is reported in Paper Appendix B.4. These coefficients are solved symbolically for general ζ before the numerical value $\zeta = 0.50$ is imposed. The inflation weights are then Equation (B.13) is reported in Paper Appendix B.4. Differentiating the Home PPI-inflation weight with respect to the intermediate-input share gives Eq. (43) in Sec. 5.4.2 when all remaining structural parameters are held at their benchmark values in Table 2. The inequality is obtained from the symbolic solution of Eq. (B.12) before imposing the roundabout evaluation $\zeta = 0.50$; it is therefore a calibration-conditional comparative static, not a parameter-free general result. The second-order construction below follows Benigno and Woodford [30, 31].

Let $\lambda_{H,j}^{RA}$ denote the stationary values of $\lambda_{H,j,t}^{RA}$ that eliminate all policy-dependent first-order terms. They solve, scalar by scalar,

$$0 = U_{H,v}^{(1)} + \sum_{j\in\mathcal{J}_H^{RA}} \lambda_{H,j}^{RA}\mathcal{E}_v F_{H,j}^{RA}$$

for every Home best-response variable v. The second-order augmented Home welfare contribution is

$$\mathcal{W}_{H,t}^{RA} = U_{H,t}^{(2)} + \sum_{j\in\mathcal{J}_H^{RA}} \lambda_{H,j}^{RA}\mathcal{Q}_t(F_{H,j}^{RA}) + \text{s.o.t.i.p.}. \tag{OB.12}$$

The producer and two importer-dispersion terms in the common equilibrium produce the three inflation squares $\pi_{H,t}^2$, $\pi_{F,t}^2$, and $(\pi_{H,t}^*)^2$ in Home welfare.

Using Eq. (OB.11), the Home and Foreign goods-market equations, the two CPI equations, complete risk sharing, and the existing price identities to eliminate consumption, labor, intermediate inputs, and Foreign output leaves the four real Home policy coordinates used in the main text: Equation (B.14) is reported in Paper Appendix B.4. Here s_t and the LOOP gaps are exactly the variables already used in Eq. (32); no alternative relative-price definition is introduced. The real part of the reduced loss can therefore be written as Eq. (OB.1) with $(z_1, z_2, z_3, z_4) = (y, \psi, s, \psi^*)$. Its scalar coefficients are the literal symbolic coefficients generated by Eq. (OB.12); the linear terms depend on the exogenous Home and Foreign tariff processes.

Complete the four real squares in the order y, ψ, s, ψ^*. Equation (OB.2) gives Equation (B.15) is reported in Paper Appendix B.4. These are, respectively, the (Logarithmic) Welfare Relevant Output, Home Welfare Relevant LOOP Gap, Welfare Relevant Ex-tariff Relative Price of Imports, and Foreign Welfare Relevant LOOP Gap for the roundabout economy. At the benchmark $\zeta = 0.50$, expressing the same reduced quadratic form in the manuscript coordinate $s_t = \log(\mathcal{E}_t P^*_{F,t}/P_{H,t})$ gives Equation (B.16) is reported in Paper Appendix B.4. The corresponding real weights are approximately $\Lambda_y^{RA} = 0.9060$, $\Lambda_\psi^{RA} = 0.6357$, $\Lambda_s^{RA} = -0.1441$, and $\Lambda_{\psi^*}^{RA} = 0.2061$. For the unilateral Home-tariff experiment $\tau^*_{M,t} = 0$, the constrained impact roundabout Welfare Relevant LOOP Gap is $\psi_0^{w,RA} = 15.25$ percent and the Welfare Relevant LOOP Gap Deviation is $\psi_0 - \psi_0^{w,RA} = -15.84$ percent. Panel 12 of Fig. 6 reports this current-coordinate Welfare Relevant LOOP Gap. Define the final real weights by $\Lambda_y^{RA} = \Lambda_y^{(0),RA}$, $\Lambda_\psi^{RA} = \Lambda_\psi^{(1),RA}$, $\Lambda_s^{RA} = \Lambda_s^{(2),RA}$, and $\Lambda_{\psi^*}^{RA} = \Lambda_{\psi^*}^{(3),RA}$. If $\Lambda_{\pi,H}^{RA}$, $\Lambda_{\pi,F}^{RA}$, and $\Lambda_{\pi_H^*}^{RA}$ denote the three inflation-square coefficients obtained from the corresponding Calvo dispersion laws, the Home period loss is Equation (B.17) is reported in Paper Appendix B.4. All weights and Welfare Relevant variables in Eq. (B.17) are symbolic functions of structural parameters and the conditioning tariff paths. The value $\zeta = 0.50$ used in Sec. 5.4 is an evaluation of this criterion, not an assumption used in its derivation.

OB.5 Open-loop Nash: complete strategy-space, first-order, and welfare derivation

OB.5.1 Choice variables, Lagrangian, and first-order conditions

The open-loop experiment changes the strategic policy path and what is taken as given in a national best response. Home takes the complete path $\{\Pi^*_{F,t}\}_{t\geq 0}$ as given and does not vary it in its best-response problem. The Home strategic policy path is $\{\Pi_{H,t}\}_{t\geq 0}$. The endogenous variables with respect to which the Home open-loop Lagrangian is differentiated are, for every $t \geq 0$,

$$
\begin{aligned}
&C_t,\ C_t^*,\quad \frac{P_t}{P_{H,t}},\ \frac{P_t^*}{P_{F,t}^*},\quad N_t,\ N_t^*,\quad Y_t,\ Y_t^*,\\
&\frac{\mathcal{E}_t P_{F,t}^*}{P_{H,t}},\quad \frac{P_{F,t}}{P_{H,t}},\quad \frac{P_{H,t}^*}{P_{F,t}^*},\\
&\Pi_{H,t},\ \widetilde{p}_{H,t},\ \mathcal{K}_{H,t},\ \mathcal{Z}_{H,t},\ \Delta_{H,t},\quad \widetilde{p}_{F,t}^*,\ \mathcal{K}_{F,t}^*,\ \mathcal{Z}_{F,t}^*,\ \Delta_{F,t}^*,\\
&\Pi_{F,t},\ \widetilde{p}_{F,t},\ \mathcal{K}_{F,t},\ \mathcal{Z}_{F,t},\ \Delta_{F,t},\quad \Pi_{H,t}^*,\ \widetilde{p}_{H,t}^*,\ \mathcal{K}_{H,t}^*,\ \mathcal{Z}_{H,t}^*,\ \Delta_{H,t}^*,\\
&i_t,\quad i_t^*.
\end{aligned}
$$

Foreign takes $\{\Pi_{H,t}\}_{t\geq 0}$ as given and does not vary it in its best-response problem. Its strategic policy path is $\{\Pi^*_{F,t}\}_{t\geq 0}$, and its endogenous differentiated variables are the exact Home–Foreign mirror of those listed above. The tariff paths are exogenous. Initial price-dispersion states are predetermined.

To keep the derivation in the notation of the model, we write the national Lagrangian directly rather than introducing a vector of equilibrium residuals. As in the numerical implementation, complete risk sharing is substituted into the two importer acquisition-cost equations before the national derivatives are taken; the border relative price therefore appears only in the complete risk-sharing condition. For Home,

$$
\begin{aligned}
\mathcal{L}_H^{OLN} = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg[&\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \\
&+ \lambda_{gH,t}^H \left[\left(\frac{P_t}{P_{H,t}} \right)^{1-\eta} - \omega_H - \omega_F \left((1+\tau_{M,t}) \frac{P_{F,t}}{P_{H,t}} \right)^{1-\eta} \right]
\end{aligned}
$$

$$+ \lambda^H_{yH,t}\left[Y_t - \omega_H \left(\frac{P_t}{P_{H,t}}\right)^{\eta} C_t - \frac{1-\nu}{\nu}\omega^*_H \left(\frac{P^*_t}{P^*_{F,t}}\right)^{\eta} C^*_t \Delta^*_{H,t} \left((1+\tau^*_{M,t})\frac{P^*_{H,t}}{P^*_{F,t}}\right)^{-\eta}\right]$$

$$+ \lambda^H_{nH,t}[N_t - Y_t \Delta_{H,t}]$$

$$+ \lambda^H_{pH,t}\left[1 - \theta \Pi^{\varepsilon-1}_{H,t} - (1-\theta)\widetilde{p}^{1-\varepsilon}_{H,t}\right]$$

$$+ \lambda^H_{kH,t}\left[\mathcal{K}_{H,t} - Y_t - \beta\theta \mathrm{E}_t \frac{C_t(P_t/P_{H,t})}{C_{t+1}(P_{t+1}/P_{H,t+1})} \Pi^{\varepsilon-1}_{H,t+1} \mathcal{K}_{H,t+1}\right]$$

$$+ \lambda^H_{zH,t}\left[\mathcal{Z}_{H,t} - C_t \left(\frac{P_t}{P_{H,t}}\right) N^{\varphi}_t Y_t - \beta\theta \mathrm{E}_t \frac{C_t(P_t/P_{H,t})}{C_{t+1}(P_{t+1}/P_{H,t+1})} \Pi^{\varepsilon}_{H,t+1} \mathcal{Z}_{H,t+1}\right]$$

$$+ \lambda^H_{rH,t}\left[\widetilde{p}_{H,t} - \mathcal{M}\frac{\mathcal{Z}_{H,t}}{\mathcal{K}_{H,t}}\right]$$

$$+ \lambda^H_{dH,t}\left[\Delta_{H,t} - \theta \Pi^{\varepsilon}_{H,t} \Delta_{H,t-1} - (1-\theta)\widetilde{p}^{-\varepsilon}_{H,t}\right]$$

$$+ \lambda^H_{qH,t}\left[\frac{P_{F,t}}{P_{H,t}} - \frac{P_{F,t-1}}{P_{H,t-1}}\frac{\Pi_{F,t}}{\Pi_{H,t}}\right]$$

$$+ \lambda^H_{pMH,t}\left[1 - \theta_M \Pi^{\varepsilon_M-1}_{F,t} - (1-\theta_M)\widetilde{p}^{1-\varepsilon_M}_{F,t}\right]$$

$$+ \lambda^H_{kMH,t}\Bigg[\mathcal{K}_{F,t} - \omega_F C_t \left(\frac{P_t}{P_{H,t}}\right)^{\eta}\left((1+\tau_{M,t})\frac{P_{F,t}}{P_{H,t}}\right)^{-\eta}$$

$$- \beta\theta_M \mathrm{E}_t \frac{C_t(P_t/P_{H,t})}{C_{t+1}(P_{t+1}/P_{H,t+1})}\frac{(P_{F,t+1}/P_{H,t+1})}{(P_{F,t}/P_{H,t})} \Pi^{\varepsilon_M-1}_{F,t+1}\mathcal{K}_{F,t+1}\Bigg]$$

$$+ \lambda^H_{zMH,t}\Bigg[\mathcal{Z}_{F,t} - \frac{C_t(P_t/P_{H,t})}{C^*_t(P^*_t/P^*_{F,t})}\frac{1}{P_{F,t}/P_{H,t}}\omega_F C_t \left(\frac{P_t}{P_{H,t}}\right)^{\eta}\left((1+\tau_{M,t})\frac{P_{F,t}}{P_{H,t}}\right)^{-\eta}$$

$$- \beta\theta_M \mathrm{E}_t \frac{C_t(P_t/P_{H,t})}{C_{t+1}(P_{t+1}/P_{H,t+1})}\frac{(P_{F,t+1}/P_{H,t+1})}{(P_{F,t}/P_{H,t})} \Pi^{\varepsilon_M}_{F,t+1}\mathcal{Z}_{F,t+1}\Bigg]$$

$$+ \lambda^H_{rMH,t}\left[\widetilde{p}_{F,t} - \mathcal{M}_M\frac{\mathcal{Z}_{F,t}}{\mathcal{K}_{F,t}}\right]$$

$$+ \lambda^H_{dMH,t}\left[\Delta_{F,t} - \theta_M \Pi^{\varepsilon_M}_{F,t}\Delta_{F,t-1} - (1-\theta_M)\widetilde{p}^{-\varepsilon_M}_{F,t}\right]$$

$$+ \lambda^H_{eH,t}\left[\frac{1}{1+i_t} - \beta \mathrm{E}_t \frac{C_t(P_t/P_{H,t})}{C_{t+1}(P_{t+1}/P_{H,t+1})\Pi_{H,t+1}}\right]$$

$$+ \lambda^H_{gF,t}\left[\left(\frac{P^*_t}{P^*_{F,t}}\right)^{1-\eta} - \omega^*_F - \omega^*_H\left((1+\tau^*_{M,t})\frac{P^*_{H,t}}{P^*_{F,t}}\right)^{1-\eta}\right]$$

$$+ \lambda^H_{yF,t}\left[Y^*_t - \omega^*_F\left(\frac{P^*_t}{P^*_{F,t}}\right)^{\eta} C^*_t - \frac{\nu}{1-\nu}\omega_F\left(\frac{P_t}{P_{H,t}}\right)^{\eta} C_t \Delta_{F,t}\left((1+\tau_{M,t})\frac{P_{F,t}}{P_{H,t}}\right)^{-\eta}\right]$$

$$+ \lambda^H_{nF,t}[N^*_t - Y^*_t \Delta^*_{F,t}]$$

$$+ \lambda^H_{pF,t}\left[1 - \theta(\Pi^*_{F,t})^{\varepsilon-1} - (1-\theta)(\widetilde{p}^*_{F,t})^{1-\varepsilon}\right]$$

$$+ \lambda^H_{kF,t}\left[\mathcal{K}^*_{F,t} - Y^*_t - \beta\theta \mathrm{E}_t \frac{C^*_t(P^*_t/P^*_{F,t})}{C^*_{t+1}(P^*_{t+1}/P^*_{F,t+1})}(\Pi^*_{F,t+1})^{\varepsilon-1}\mathcal{K}^*_{F,t+1}\right]$$

$$+ \lambda^H_{zF,t}\left[\mathcal{Z}^*_{F,t} - C^*_t\left(\frac{P^*_t}{P^*_{F,t}}\right)(N^*_t)^{\varphi} Y^*_t - \beta\theta \mathrm{E}_t \frac{C^*_t(P^*_t/P^*_{F,t})}{C^*_{t+1}(P^*_{t+1}/P^*_{F,t+1})}(\Pi^*_{F,t+1})^{\varepsilon}\mathcal{Z}^*_{F,t+1}\right]$$

$$
\begin{aligned}
&+ \lambda^H_{rF,t}\left[\widetilde{p}^*_{F,t} - \mathcal{M}\frac{\mathcal{Z}^*_{F,t}}{\mathcal{K}^*_{F,t}}\right] \\
&\quad + \lambda^H_{dF,t}\left[\Delta^*_{F,t} - \theta(\Pi^*_{F,t})^{\varepsilon}\Delta^*_{F,t-1} - (1-\theta)(\widetilde{p}^*_{F,t})^{-\varepsilon}\right] \\
&+ \lambda^H_{qF,t}\left[\frac{P^*_{H,t}}{P^*_{F,t}} - \frac{P^*_{H,t-1}}{P^*_{F,t-1}}\frac{\Pi^*_{H,t}}{\Pi^*_{F,t}}\right] \\
&+ \lambda^H_{pMF,t}\left[1 - \theta_M(\Pi^*_{H,t})^{\varepsilon_M-1} - (1-\theta_M)(\widetilde{p}^*_{H,t})^{1-\varepsilon_M}\right] \\
&+ \lambda^H_{kMF,t}\Bigg[\mathcal{K}^*_{H,t} - \omega^*_H C^*_t\left(\frac{P^*_t}{P^*_{F,t}}\right)^{\eta}\left((1+\tau^*_{M,t})\frac{P^*_{H,t}}{P^*_{F,t}}\right)^{-\eta} \\
&\qquad - \beta\theta_M \mathrm{E}_t\frac{C^*_t(P^*_t/P^*_{F,t})}{C^*_{t+1}(P^*_{t+1}/P^*_{F,t+1})}\frac{(P^*_{H,t+1}/P^*_{F,t+1})}{(P^*_{H,t}/P^*_{F,t})}(\Pi^*_{H,t+1})^{\varepsilon_M-1}\mathcal{K}^*_{H,t+1}\Bigg] \\
&+ \lambda^H_{zMF,t}\Bigg[\mathcal{Z}^*_{H,t} - \frac{C^*_t(P^*_t/P^*_{F,t})}{C_t(P_t/P_{H,t})}\frac{1}{P^*_{H,t}/P^*_{F,t}}\omega^*_H C^*_t\left(\frac{P^*_t}{P^*_{F,t}}\right)^{\eta}\left((1+\tau^*_{M,t})\frac{P^*_{H,t}}{P^*_{F,t}}\right)^{-\eta} \\
&\qquad - \beta\theta_M \mathrm{E}_t\frac{C^*_t(P^*_t/P^*_{F,t})}{C^*_{t+1}(P^*_{t+1}/P^*_{F,t+1})}\frac{(P^*_{H,t+1}/P^*_{F,t+1})}{(P^*_{H,t}/P^*_{F,t})}(\Pi^*_{H,t+1})^{\varepsilon_M}\mathcal{Z}^*_{H,t+1}\Bigg] \\
&+ \lambda^H_{rMF,t}\left[\widetilde{p}^*_{H,t} - \mathcal{M}_M\frac{\mathcal{Z}^*_{H,t}}{\mathcal{K}^*_{H,t}}\right] \\
&+ \lambda^H_{dMF,t}\left[\Delta^*_{H,t} - \theta_M(\Pi^*_{H,t})^{\varepsilon_M}\Delta^*_{H,t-1} - (1-\theta_M)(\widetilde{p}^*_{H,t})^{-\varepsilon_M}\right] \\
&+ \lambda^H_{eF,t}\left[\frac{1}{1+i^*_t} - \beta\mathrm{E}_t\frac{C^*_t(P^*_t/P^*_{F,t})}{C^*_{t+1}(P^*_{t+1}/P^*_{F,t+1})\Pi^*_{F,t+1}}\right] \\
&\quad + \lambda^H_{rs,t}\left[\frac{\mathcal{E}_t P^*_{F,t}}{P_{H,t}} - \frac{C_t(P_t/P_{H,t})}{C^*_t(P^*_t/P^*_{F,t})}\right] + \mu^H_{i,t}i_t\Bigg].
\end{aligned} \tag{OB.13}
$$

The Foreign Lagrangian is the exact Home–Foreign mirror of Eq. (OB.13): its objective is $\log C^*_t - (N^*_t)^{1+\varphi}/(1+\varphi)$, every multiplier carries the superscript F, the Home and Foreign blocks are interchanged, and the ZLB term is $\mu^F_{i,t}i^*_t$. Foreign treats $\Pi_{H,t}$ as given, whereas $\Pi^*_{F,t}$ is one of its choice variables.

Because Eq. (OB.13) is the discounted Lagrangian over the entire path, its derivatives automatically include the lead and lag terms from the Calvo recursions and Euler equations. The Home first-order conditions are therefore the following conditions, one for each Home choice variable:

$$
\begin{gathered}
\frac{\partial\mathcal{L}^{OLN}_H}{\partial C_t} = 0, \quad \frac{\partial\mathcal{L}^{OLN}_H}{\partial C^*_t} = 0, \quad \frac{\partial\mathcal{L}^{OLN}_H}{\partial(P_t/P_{H,t})} = 0, \quad \frac{\partial\mathcal{L}^{OLN}_H}{\partial(P^*_t/P^*_{F,t})} = 0, \\
\frac{\partial\mathcal{L}^{OLN}_H}{\partial N_t} = 0, \quad \frac{\partial\mathcal{L}^{OLN}_H}{\partial N^*_t} = 0, \quad \frac{\partial\mathcal{L}^{OLN}_H}{\partial Y_t} = 0, \quad \frac{\partial\mathcal{L}^{OLN}_H}{\partial Y^*_t} = 0, \\
\frac{\partial\mathcal{L}^{OLN}_H}{\partial(\mathcal{E}_t P^*_{F,t}/P_{H,t})} = 0, \quad \frac{\partial\mathcal{L}^{OLN}_H}{\partial(P_{F,t}/P_{H,t})} = 0, \quad \frac{\partial\mathcal{L}^{OLN}_H}{\partial(P^*_{H,t}/P^*_{F,t})} = 0,
\end{gathered} \tag{OB.14}
$$

$$
\begin{gathered}
\frac{\partial\mathcal{L}^{OLN}_H}{\partial \Pi_{H,t}} = 0, \quad \frac{\partial\mathcal{L}^{OLN}_H}{\partial \widetilde{p}_{H,t}} = 0, \quad \frac{\partial\mathcal{L}^{OLN}_H}{\partial \mathcal{K}_{H,t}} = 0, \quad \frac{\partial\mathcal{L}^{OLN}_H}{\partial \mathcal{Z}_{H,t}} = 0, \quad \frac{\partial\mathcal{L}^{OLN}_H}{\partial \Delta_{H,t}} = 0, \\
\frac{\partial\mathcal{L}^{OLN}_H}{\partial \widetilde{p}^*_{F,t}} = 0, \quad \frac{\partial\mathcal{L}^{OLN}_H}{\partial \mathcal{K}^*_{F,t}} = 0, \quad \frac{\partial\mathcal{L}^{OLN}_H}{\partial \mathcal{Z}^*_{F,t}} = 0, \quad \frac{\partial\mathcal{L}^{OLN}_H}{\partial \Delta^*_{F,t}} = 0,
\end{gathered}
$$

$$
\frac{\partial\mathcal{L}^{OLN}_H}{\partial \Pi_{F,t}} = 0, \quad \frac{\partial\mathcal{L}^{OLN}_H}{\partial \widetilde{p}_{F,t}} = 0, \quad \frac{\partial\mathcal{L}^{OLN}_H}{\partial \mathcal{K}_{F,t}} = 0, \quad \frac{\partial\mathcal{L}^{OLN}_H}{\partial \mathcal{Z}_{F,t}} = 0, \quad \frac{\partial\mathcal{L}^{OLN}_H}{\partial \Delta_{F,t}} = 0,
$$

$$\frac{\partial \mathcal{L}_H^{OLN}}{\partial \Pi_{H,t}^*} = 0, \quad \frac{\partial \mathcal{L}_H^{OLN}}{\partial \widetilde{p}_{H,t}^*} = 0, \quad \frac{\partial \mathcal{L}_H^{OLN}}{\partial \mathcal{K}_{H,t}^*} = 0, \quad \frac{\partial \mathcal{L}_H^{OLN}}{\partial \mathcal{Z}_{H,t}^*} = 0, \quad \frac{\partial \mathcal{L}_H^{OLN}}{\partial \Delta_{H,t}^*} = 0. \quad \text{(OB.15)}$$

For transparency, the economically important stationarity conditions can also be written out directly. The conditions for labor and output are

$$\begin{aligned}
0 &= -N_t^{\varphi} + \lambda_{nH,t}^H - \varphi C_t \left(\frac{P_t}{P_{H,t}}\right) N_t^{\varphi-1} Y_t \lambda_{zH,t}^H, \\
0 &= \lambda_{nF,t}^H - \varphi C_t^* \left(\frac{P_t^*}{P_{F,t}^*}\right) (N_t^*)^{\varphi-1} Y_t^* \lambda_{zF,t}^H, \\
0 &= \lambda_{yH,t}^H - \lambda_{kH,t}^H - \Delta_{H,t}\lambda_{nH,t}^H - C_t \left(\frac{P_t}{P_{H,t}}\right) N_t^{\varphi} \lambda_{zH,t}^H, \\
0 &= \lambda_{yF,t}^H - \lambda_{kF,t}^H - \Delta_{F,t}^*\lambda_{nF,t}^H - C_t^* \left(\frac{P_t^*}{P_{F,t}^*}\right) (N_t^*)^{\varphi} \lambda_{zF,t}^H.
\end{aligned}$$

Because the border relative price appears only in the risk-sharing condition, its stationarity condition is simply

$$\lambda_{rs,t}^H = 0.$$

This condition is useful for seeing directly how the implementation differs from one that would treat the border relative price as a separate policy choice.

The Home producer-price-inflation strategy satisfies

$$\begin{aligned}
0 = & -\theta\varepsilon\lambda_{dH,t}^H \Delta_{H,t-1}\Pi_{H,t}^{\varepsilon-1} + \lambda_{qH,t}^H \frac{P_{F,t-1}}{P_{H,t-1}} \frac{\Pi_{F,t}}{\Pi_{H,t}^2} - \theta(\varepsilon-1)\lambda_{pH,t}^H \Pi_{H,t}^{\varepsilon-2} \\
& -\theta(\varepsilon-1)\frac{C_{t-1}(P_{t-1}/P_{H,t-1})}{C_t(P_t/P_{H,t})}\lambda_{kH,t-1}^H \mathcal{K}_{H,t}\Pi_{H,t}^{\varepsilon-2} \\
& -\theta\varepsilon\frac{C_{t-1}(P_{t-1}/P_{H,t-1})}{C_t(P_t/P_{H,t})}\lambda_{zH,t-1}^H \mathcal{Z}_{H,t}\Pi_{H,t}^{\varepsilon-1} + \frac{C_{t-1}(P_{t-1}/P_{H,t-1})}{C_t(P_t/P_{H,t})}\frac{\lambda_{eH,t-1}^H}{\Pi_{H,t}^2}.
\end{aligned}$$

There is no corresponding Home stationarity condition for $\Pi_{F,t}^*$: that complete Foreign PPI-inflation path is the strategy taken as given. By contrast, the two local importer-price inflation rates remain endogenous world-equilibrium variables and satisfy

$$\begin{aligned}
0 = & -\theta_M\varepsilon_M\lambda_{dMH,t}^H \Delta_{F,t-1}\Pi_{F,t}^{\varepsilon_M-1} - \lambda_{qH,t}^H \frac{P_{F,t-1}}{P_{H,t-1}} \frac{1}{\Pi_{H,t}} - \theta_M(\varepsilon_M-1)\lambda_{pMH,t}^H \Pi_{F,t}^{\varepsilon_M-2} \\
& -\theta_M(\varepsilon_M-1)\frac{C_{t-1}(P_{t-1}/P_{H,t-1})}{C_t(P_t/P_{H,t})}\frac{P_{F,t}/P_{H,t}}{P_{F,t-1}/P_{H,t-1}}\lambda_{kMH,t-1}^H \mathcal{K}_{F,t}\Pi_{F,t}^{\varepsilon_M-2} \\
& -\theta_M\varepsilon_M\frac{C_{t-1}(P_{t-1}/P_{H,t-1})}{C_t(P_t/P_{H,t})}\frac{P_{F,t}/P_{H,t}}{P_{F,t-1}/P_{H,t-1}}\lambda_{zMH,t-1}^H \mathcal{Z}_{F,t}\Pi_{F,t}^{\varepsilon_M-1}, \\
0 = & -\theta_M\varepsilon_M\lambda_{dMF,t}^H \Delta_{H,t-1}^*(\Pi_{H,t}^*)^{\varepsilon_M-1} - \lambda_{qF,t}^H \frac{P_{H,t-1}^*}{P_{F,t-1}^*} \frac{1}{\Pi_{F,t}^*} - \theta_M(\varepsilon_M-1)\lambda_{pMF,t}^H (\Pi_{H,t}^*)^{\varepsilon_M-2} \\
& -\theta_M(\varepsilon_M-1)\frac{C_{t-1}^*(P_{t-1}^*/P_{F,t-1}^*)}{C_t^*(P_t^*/P_{F,t}^*)}\frac{P_{H,t}^*/P_{F,t}^*}{P_{H,t-1}^*/P_{F,t-1}^*}\lambda_{kMF,t-1}^H \mathcal{K}_{H,t}^*(\Pi_{H,t}^*)^{\varepsilon_M-2} \\
& -\theta_M\varepsilon_M\frac{C_{t-1}^*(P_{t-1}^*/P_{F,t-1}^*)}{C_t^*(P_t^*/P_{F,t}^*)}\frac{P_{H,t}^*/P_{F,t}^*}{P_{H,t-1}^*/P_{F,t-1}^*}\lambda_{zMF,t-1}^H \mathcal{Z}_{H,t}^*(\Pi_{H,t}^*)^{\varepsilon_M-1}.
\end{aligned}$$

For the Home producer Calvo block, stationarity with respect to the reset price and the two recursive variables gives

$$0 = \lambda_{rH,t}^H + (1-\theta)(\varepsilon-1)\lambda_{pH,t}^H \widetilde{p}_{H,t}^{-\varepsilon} + (1-\theta)\varepsilon\lambda_{dH,t}^H \widetilde{p}_{H,t}^{-\varepsilon-1},$$

$$
\begin{aligned}
0 &= \lambda^H_{kH,t} + \mathcal{M}\frac{\mathcal{Z}_{H,t}}{\mathcal{K}^2_{H,t}}\lambda^H_{rH,t} - \theta\frac{C_{t-1}(P_{t-1}/P_{H,t-1})}{C_t(P_t/P_{H,t})}\Pi^{\varepsilon-1}_{H,t}\lambda^H_{kH,t-1}, \\
0 &= \lambda^H_{zH,t} - \frac{\mathcal{M}}{\mathcal{K}_{H,t}}\lambda^H_{rH,t} - \theta\frac{C_{t-1}(P_{t-1}/P_{H,t-1})}{C_t(P_t/P_{H,t})}\Pi^{\varepsilon}_{H,t}\lambda^H_{zH,t-1}, \\
0 &= -Y_t\lambda^H_{nH,t} + \lambda^H_{dH,t} - \beta\theta\Pi^{\varepsilon}_{H,t+1}\lambda^H_{dH,t+1}.
\end{aligned}
$$

The Foreign producer block has the exact starred analogue of these four conditions, except that Home imposes no stationarity condition with respect to $\Pi^*_{F,t}$ itself.

For the two local importer Calvo blocks, the reset-price conditions are

$$
\begin{aligned}
0 &= \lambda^H_{rMH,t} + (1-\theta_M)(\varepsilon_M-1)\lambda^H_{pMH,t}\widetilde{p}_{F,t}^{-\varepsilon_M} + (1-\theta_M)\varepsilon_M\lambda^H_{dMH,t}\widetilde{p}_{F,t}^{-\varepsilon_M-1}, \\
0 &= \lambda^H_{rMF,t} + (1-\theta_M)(\varepsilon_M-1)\lambda^H_{pMF,t}(\widetilde{p}^*_{H,t})^{-\varepsilon_M} + (1-\theta_M)\varepsilon_M\lambda^H_{dMF,t}(\widetilde{p}^*_{H,t})^{-\varepsilon_M-1}.
\end{aligned}
$$

Their $\mathcal{K}$, $\mathcal{Z}$, and dispersion conditions are

$$
\begin{aligned}
0 &= \lambda^H_{kMH,t} + \mathcal{M}_M\frac{\mathcal{Z}_{F,t}}{\mathcal{K}^2_{F,t}}\lambda^H_{rMH,t} - \theta_M\frac{C_{t-1}(P_{t-1}/P_{H,t-1})}{C_t(P_t/P_{H,t})}\frac{P_{F,t}/P_{H,t}}{P_{F,t-1}/P_{H,t-1}}\Pi^{\varepsilon_M-1}_{F,t}\lambda^H_{kMH,t-1}, \\
0 &= \lambda^H_{zMH,t} - \frac{\mathcal{M}_M}{\mathcal{K}_{F,t}}\lambda^H_{rMH,t} - \theta_M\frac{C_{t-1}(P_{t-1}/P_{H,t-1})}{C_t(P_t/P_{H,t})}\frac{P_{F,t}/P_{H,t}}{P_{F,t-1}/P_{H,t-1}}\Pi^{\varepsilon_M}_{F,t}\lambda^H_{zMH,t-1}, \\
0 &= -\frac{\nu}{1-\nu}\omega_F C_t\left(\frac{P_t}{P_{H,t}}\right)^{\eta}\left((1+\tau_{M,t})\frac{P_{F,t}}{P_{H,t}}\right)^{-\eta}\lambda^H_{yF,t} + \lambda^H_{dMH,t} - \beta\theta_M\Pi^{\varepsilon_M}_{F,t+1}\lambda^H_{dMH,t+1}, \\
0 &= \lambda^H_{kMF,t} + \mathcal{M}_M\frac{\mathcal{Z}^*_{H,t}}{(\mathcal{K}^*_{H,t})^2}\lambda^H_{rMF,t} - \theta_M\frac{C^*_{t-1}(P^*_{t-1}/P^*_{F,t-1})}{C^*_t(P^*_t/P^*_{F,t})}\frac{P^*_{H,t}/P^*_{F,t}}{P^*_{H,t-1}/P^*_{F,t-1}}(\Pi^*_{H,t})^{\varepsilon_M-1}\lambda^H_{kMF,t-1}, \\
0 &= \lambda^H_{zMF,t} - \frac{\mathcal{M}_M}{\mathcal{K}^*_{H,t}}\lambda^H_{rMF,t} - \theta_M\frac{C^*_{t-1}(P^*_{t-1}/P^*_{F,t-1})}{C^*_t(P^*_t/P^*_{F,t})}\frac{P^*_{H,t}/P^*_{F,t}}{P^*_{H,t-1}/P^*_{F,t-1}}(\Pi^*_{H,t})^{\varepsilon_M}\lambda^H_{zMF,t-1}, \\
0 &= -\frac{1-\nu}{\nu}\omega^*_H C^*_t\left(\frac{P^*_t}{P^*_{F,t}}\right)^{\eta}\left((1+\tau^*_{M,t})\frac{P^*_{H,t}}{P^*_{F,t}}\right)^{-\eta}\lambda^H_{yH,t} + \lambda^H_{dMF,t} - \beta\theta_M(\Pi^*_{H,t+1})^{\varepsilon_M}\lambda^H_{dMF,t+1}.
\end{aligned}
$$

These equations are the recursive-variable conditions used in the numerical open-loop system. The remaining consumption, CPI–PPI-ratio, and local relative-price conditions are the literal derivatives listed in Eqs. (OB.14)–(OB.15) of the explicit Lagrangian (OB.13); no residual vector or auxiliary derivative operator is introduced.

The Foreign first-order conditions are the exact Home–Foreign mirror of the Home conditions above, with no Foreign first-order condition with respect to $\Pi_{H,t}$ because that is the Home open-loop strategy held fixed in the Foreign best response.

For the nominal rates, the Home conditions are

$$
-\frac{\lambda^H_{eH,t}}{(1+i_t)^2} + \mu^H_{i,t} = 0, \qquad -\frac{\lambda^H_{eF,t}}{(1+i^*_t)^2} = 0,
$$

with

$$
i_t \ge 0, \qquad \mu^H_{i,t} \ge 0, \qquad \mu^H_{i,t} i_t = 0.
$$

The Foreign conditions are the exact mirror,

$$
-\frac{\lambda^F_{eF,t}}{(1+i^*_t)^2} + \mu^F_{i,t} = 0, \qquad -\frac{\lambda^F_{eH,t}}{(1+i_t)^2} = 0, \qquad i^*_t \ge 0, \quad \mu^F_{i,t} \ge 0, \quad \mu^F_{i,t} i^*_t = 0.
$$

Thus a Home deviation holds fixed only the Foreign PPI-inflation strategy; it does not mechanically hold fixed the Foreign allocation or the Foreign nominal rate.

OB.5.2 Second-order Welfare Cost Function

The construction follows Benigno and Woodford [30, 31]. At the deterministic steady state the ZLB is slack. We first evaluate the explicit first-order conditions above at the zero-tariff steady state and solve for the stationary Home multipliers on each of the nonlinear private-equilibrium restrictions in Eq. (OB.13). These stationary multipliers are specific to the open-loop best response; they are not the multipliers from the Faia–Monacelli allocation game.

The second-order Home welfare criterion is then obtained in the usual Benigno–Woodford way: second-order utility is combined with the second-order expansion of each nonlinear restriction in Eq. (OB.13), weighted by its own stationary multiplier, and discounted lead and lag terms are shifted to the current date. No additional equilibrium-residual notation is needed. The first-order versions of the same model equations eliminate the remaining Calvo and relative-price variables. The policy-dependent real variables that remain are y_t, ψ_t, s_t, and ψ_t^*, together with the three sticky-price inflation rates $\pi_{H,t}$, $\pi_{F,t}$, and $\pi_{H,t}^*$. The Foreign PPI-inflation path $\pi_{F,t}^*$ and the tariff paths are conditioning paths in the Home best response. Terms depending only on those conditioning paths are policy-independent for Home, whereas their cross-products with Home choice variables must be retained.

For the nominal terms it is useful to make the Calvo step explicit. Let $\delta_{H,t} \equiv \Delta_{H,t} - 1$ denote Home producer-price dispersion to second order, and define $\delta_{F,t} \equiv \Delta_{F,t} - 1$ and $\delta_{H,t}^* \equiv \Delta_{H,t}^* - 1$ analogously for the Home and Foreign local importer-price blocks. Combining each Calvo price-index equation with its dispersion recursion gives, around the zero-inflation steady state,

$$\begin{aligned}
\delta_{H,t} &= \theta\delta_{H,t-1} + \frac{\varepsilon\theta}{2(1-\theta)}\pi_{H,t}^2 + O(3), \\
\delta_{F,t} &= \theta_M\delta_{F,t-1} + \frac{\varepsilon_M\theta_M}{2(1-\theta_M)}\pi_{F,t}^2 + O(3), \\
\delta_{H,t}^* &= \theta_M\delta_{H,t-1}^* + \frac{\varepsilon_M\theta_M}{2(1-\theta_M)}(\pi_{H,t}^*)^2 + O(3).
\end{aligned} \tag{OB.16}$$

The first-order components of all three dispersion measures are zero. Hence, when Eq. (OB.16) is substituted into the second-order criterion, the three inflation-square coefficients are determined directly by the stationary Home multipliers on the corresponding dispersion equations:

$$\begin{aligned}
\Lambda_{\pi,H}^{OLN} &\equiv \frac{\varepsilon\theta}{1-\theta}\mu_{H,\Delta_H}^{OLN}, \\
\Lambda_{\pi,F}^{OLN} &\equiv \frac{\varepsilon_M\theta_M}{1-\theta_M}\mu_{H,\Delta_F}^{OLN}, \\
\Lambda_{\pi_H^*}^{OLN} &\equiv \frac{\varepsilon_M\theta_M}{1-\theta_M}\mu_{H,\Delta_H^*}^{OLN}.
\end{aligned} \tag{OB.17}$$

This is the open-loop analogue of the ε/κ and ε_M/κ_M price-dispersion terms in the benchmark Benigno–Woodford representation. The difference is that the stationary multipliers in Eq. (OB.17) solve the explicit open-loop first-order conditions above.

At the benchmark calibration, that system gives

$$\mu_{H,\Delta_H}^{OLN} = 3.31598553, \qquad \mu_{H,\Delta_F}^{OLN} = 0.08880332, \qquad \mu_{H,\Delta_H^*}^{OLN} = 0.72620083. \tag{OB.18}$$

Since $\varepsilon\theta/(1-\theta) = \varepsilon_M\theta_M/(1-\theta_M) = 15.2$, Eqs. (OB.17)–(OB.18) imply

$$\Lambda_{\pi,H}^{OLN} = 50.40298, \qquad \Lambda_{\pi,F}^{OLN} = 1.34981, \qquad \Lambda_{\pi_H^*}^{OLN} = 11.03825.$$

These are the values reported in Table 9.

For the real variables, the first-order versions of the same nonlinear private-equilibrium conditions restrict $y_t, \psi_t, s_t, \psi_t^*$ jointly with the remaining endogenous variables and the conditioning paths. This point is innocuous in evaluating welfare on a feasible open-loop equilibrium path but important for a completed-square decomposition. Because Home's only strategic policy path is $\{\pi_{H,t}\}$, the four displayed real variables are not independent policy coordinates. Eliminating the remaining first-order variables therefore requires an auxiliary reduction normalization. Two algebraically admissible reductions can differ by a linear combination of the first-order equilibrium restrictions. They coincide on the feasible manifold, and hence imply exactly the same welfare and best response, while generating different off-manifold quadratic and linear coefficients.

For any particular reduction normalization, write the resulting real quadratic form as

$$\mathcal{L}_{H,t}^{OLN,real} = \frac{1}{2}\sum_{u} \Lambda_u^{(0),OLN} u_t^2 - \sum_{u<v} C_{uv}^{(0),OLN} u_t v_t - \sum_{u} h_{u,t}^{(0),OLN} u_t + \text{s.o.t.i.p.}, \quad u \in \{y, \psi, s, \psi^*\}, \tag{OB.19}$$

and applying the sequential completion in Eqs. (OB.2)–(OB.3) yields the conditional (Logarithmic) Welfare Relevant Output $y_t^{w,OLN}$, Home Welfare Relevant LOOP Gap $\psi_t^{w,OLN}$, Welfare Relevant Ex-tariff Relative Price of Imports $s_t^{w,OLN}$, and Foreign Welfare Relevant LOOP Gap $\psi_t^{*,w,OLN}$ for that normalization. Terms depending only on the taken-as-given Foreign PPI strategy and exogenous tariffs are policy-independent for Home and enter s.o.t.i.p. after the relevant cross terms have been retained in the real Welfare Relevant variables. To make the individual Welfare Relevant variables reproducible, the paper fixes the following reporting normalization. Retain y_t, ψ_t, s_t, and ψ_t^*; eliminate the remaining real variables with the first-order static allocation, price-index, goods-market, labor, risk-sharing, and relative-price identities; do not use the Euler equations or Phillips curves to eliminate any of these four variables; and complete squares first in y, then in ψ, then in s, and finally in ψ^*. Under this normalization the four real sequential-completion pivots are

$$\Lambda_y^{OLN} = 3.72631579, \quad \Lambda_\psi^{OLN} = 1.53443497, \quad \Lambda_s^{OLN} = 0.70019236, \quad \Lambda_{\psi^*}^{OLN} = -0.47228824. \tag{OB.20}$$

The associated real Welfare Relevant variables are

$$\begin{aligned}
y_t^{w,OLN} &= 0.42416145\,\psi_t + 0.56836388\,s_t + 0.20999546\,\psi_t^* + 0.07649457\,\tau_{M,t} - 0.15311094\,\tau_{M,t}^*,\\
\psi_t^{w,OLN} &= -0.64464456\,s_t - 0.14585235\,\psi_t^* + 0.13773264\,\tau_{M,t} + 0.25402169\,\tau_{M,t}^*,\\
s_t^{w,OLN} &= -0.20958734\,\psi_t^* - 0.32924666\,\tau_{M,t} + 0.31158857\,\tau_{M,t}^*,\\
\psi_t^{*,w,OLN} &= 0.04672744\,\tau_{M,t} + 0.02829915\,\tau_{M,t}^*.
\end{aligned} \tag{OB.21}$$

Alternative admissible reductions can change these off-manifold Welfare Relevant variables by terms proportional to first-order equilibrium restrictions, but they do not change welfare on a feasible path or the open-loop best response. All reported OLN Welfare Relevant variables and Fig. 7(a) therefore use the fixed normalization in Eqs. (OB.20)–(OB.21). The three inflation weights in Eq. (OB.17) remain uniquely pinned down by the Calvo dispersion multipliers.

For any fixed reduction normalization, the resulting Home open-loop Nash period loss can therefore be written as

$$\begin{aligned}
\mathcal{L}_{H,t}^{OLN} = \frac{1}{2}\Big[&\Lambda_y^{OLN}(y_t - y_t^{w,OLN})^2 + \Lambda_\psi^{OLN}(\psi_t - \psi_t^{w,OLN})^2 + \Lambda_s^{OLN}(s_t - s_t^{w,OLN})^2\\
&+\Lambda_{\psi^*}^{OLN}(\psi_t^* - \psi_t^{*,w,OLN})^2 + \Lambda_{\pi,H}^{OLN}\pi_{H,t}^2 + \Lambda_{\pi,F}^{OLN}\pi_{F,t}^2 + \Lambda_{\pi_H^*}^{OLN}(\pi_{H,t}^*)^2\Big].
\end{aligned}$$

This is Eq. (44). The Foreign welfare cost is obtained from its own stationary first-order conditions with the Home PPI strategy taken as given. Thus both the first-order conditions and the Welfare Cost Function correspond to the same open-loop strategy space used in the numerical experiment.