

# Optimal Monetary Policy in the Tariff-Imposing Economy under Imperfect Pass-Through Environment: The Return of the ZLB\*

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## Abstract

This paper studies how imperfect nominal exchange-rate pass-through changes the welfare-maximizing monetary response of a tariff-imposing economy. Under Perfect Pass-Through, an import-tariff increase calls for monetary tightening. Under Imperfect Pass-Through, local importers are monopolistically competitive, set sticky local-currency *ex-tariff* prices, and use their pricing power to raise their *ex-tariff* price when the tariff reduces the effective elasticity of demand. This tariff-induced opportunistic price increase raises the importer markup above the level offset by the steady-state subsidy. The resulting inefficiency raises the Home Welfare Relevant (WR) law-of-one-price (LOOP) gap. The Ramsey planner therefore seeks to raise the actual Home LOOP gap toward its higher welfare-relevant value, which requires monetary easing in the small open economy. We refer to this mechanism as the Home Markup Channel.

The same mechanism operates in a two-country economy with endogenous Foreign demand and monetary policy. In addition, a lower Foreign LOOP gap reduces Foreign consumption of Home goods and shifts their allocation inefficiently toward Home households. This Foreign Allocation Channel raises the Home WR LOOP gap further and reinforces the incentive for Home monetary easing, even when the Home currency appreciates in equilibrium because Foreign monetary policy eases more strongly. Reciprocal tariffs strengthen these forces and can make the required accommodation large enough for the Home nominal interest rate to reach the zero lower bound. This result survives under higher substitutability and external habit, while roundabout production reaches the zero lower bound through a stronger incentive to stabilize Home Producer Price Index inflation. No exogenous natural-rate shock is required to generate the ZLB episode.

**Keywords:** Import tariffs; optimal monetary policy; pricing behavior; local-currency pricing; zero lower bound.

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# 1 Introduction

Should a tariff-imposing country adopt monetary tightening or easing? We show that the answer depends critically on the nominal exchange-rate pass-through environment. Under Perfect Pass-Through (PPT), the Ramsey social planner raises the nominal interest rate after an import-tariff increase. Under Imperfect Pass-Through (IPT), the optimal response is instead monetary easing. The difference arises because monopolistically competitive local importers with sticky local-currency prices create an importer-pricing channel that is absent under PPT.

We introduce such local importers into the small-open-economy (SOE) model of Monacelli [39], following the local-currency-pricing (LCP) structure of Monacelli [38]. Home producers use producer-currency pricing (PCP) in both environments. In the IPT environment, local importers set sticky *ex-tariff* prices in Home currency. Their market power allows the import tariff to affect not only the purchaser price paid by households but also the importers' own optimal *ex-tariff* price. Because the tariff reduces the effective elasticity of demand with respect to that price, it creates an opportunity for an opportunistic price increase. The tariff therefore raises the importer markup above the level that is neutralized by the steady-state importer subsidy.

Such strategic price adjustment following tariffs has empirical precedent. Amiti, Redding, and Weinstein [1], Flaaen, Hortaçsu, and Tintelnot [28], and Cavallo, Gopinath, Neiman, and Tang [15] show that recent U.S. tariff episodes enabled domestic producers to raise prices and markups and also raised prices for some closely related goods that were not themselves subject to the tariff, although the evidence for retail margins is more mixed. Thus, a parsimonious and empirically motivated departure from the PPT benchmark—sticky local-currency pricing by monopolistically competitive local importers—is sufficient to generate a distinct monetary-transmission mechanism for tariff shocks.

This tariff-induced increase in the importer markup is inefficient. The Home Welfare Relevant (WR) LOOP Gap consequently rises: a higher Home LOOP gap compresses the importer's *ex-tariff* markup and mitigates the tariff-induced markup distortion. We refer to this mechanism as the Home Markup Channel (HMC). The social planner therefore seeks to raise the actual Home LOOP gap toward its higher welfare-relevant value. In the SOE, doing so requires a nominal depreciation, which is induced by lowering the nominal interest rate. Hence an increase in the import tariff calls for monetary easing under IPT.

The same HMC operates in the two-country economy, but an additional Foreign Allocation Channel (FAC) emerges because Foreign households endogenously allocate expenditure between Home and Foreign goods according to their relative price. Home monetary easing puts depreciation pressure on the Home currency and, relative to a less accommodative Home policy path, lowers the Foreign LOOP gap. A lower Foreign LOOP gap raises the price of Home goods faced by Foreign households, reduces their consumption of Home goods, and shifts the consumption of Home goods inefficiently toward Home households. The Home WR LOOP gap therefore rises further to reduce this inefficient underconsumption of Home goods by Foreign households and the associated inefficient over-allocation of Home goods to Home households. The Home planner consequently has an additional reason to raise the actual Home LOOP gap, so monetary easing remains optimal in the two-country IPT environment even when the equilibrium Home currency appreciates because Foreign monetary policy eases more strongly.

Reciprocal tariffs strengthen this logic. A Foreign import-tariff increase raises the price of Home goods faced by Foreign households and increases the Home WR LOOP gap through the FAC. The case of a high elasticity of substitution between Home and Foreign goods (High Elasticity Case) strengthens this channel because Foreign demand reacts more strongly to the relevant relative-price movement. External habit raises the Home WR LOOP gap through the inherited-consumption terms in the welfare-relevant target. Roundabout production works through a different mechanism: it increases the importance of stabilizing Home Producer Price Index (PPI) inflation. Across these specifications, the required monetary accommodation is large enough for the Home nominal interest rate to reach the zero lower bound (ZLB) under reciprocal tariffs, without adding an exogenous natural-rate shock. In this sense, reciprocal tariff shocks work as an adverse natural-rate shock.

The rest of the paper is organized as follows. Section 2 relates the analysis to the literature on tariffs and optimal monetary policy, pass-through and pricing behavior, international policy interaction, and the ZLB. Section 3 presents the small-open-economy PPT and IPT environments and their Ramsey problems. Section 4 endogenizes the Foreign economy and studies the two-country national-Ramsey game. Section 5 studies reciprocal tariffs and the associated ZLB episodes under the benchmark, High Elasticity, external habit, and roundabout production specifications. Section 6 concludes. The Appendix reports the pricing definitions, Ramsey first-order conditions, and the Benigno–Woodford welfare objects used in the text.

## 2 Related Literature

### 2.1 Tariffs and Monetary Policy

The literature on tariffs and optimal monetary policy can be organized by the side of the tariff shock on which the monetary-policy problem is centered. One group studies the economy whose exports are hit by a foreign tariff. A foreign tariff weakens external demand for that economy’s goods, so the relevant central bank faces a contractionary foreign-demand disturbance. Bergin and Corsetti [8], for example, show in their asymmetric-tariff analysis that cooperative monetary policy calls for expansion in the country whose exports are targeted by the foreign tariff. Hamano, Pappadà, and Punzi [33] likewise study optimal monetary policy after foreign tariff shocks, emphasizing exporter dynamics and labor reallocation. This perspective is important for understanding the monetary response of the country on which a tariff is imposed.

A second, smaller group centers the policy problem on the tariff-imposing economy itself. Monacelli [39] studies import and export tariffs in a small-open-economy New Keynesian model and shows that the effects of import tariffs depend critically on the monetary-policy regime. Bianchi and Coulibaly [11] focus directly on the optimal monetary response of the tariff-imposing country and obtain an expansionary response over a broad set of tariff experiments. Werning, Lorenzoni, and Guerrieri [43] study tariffs on imported intermediate inputs as cost-push shocks and show that optimal policy partially accommodates them. Bergin and Corsetti [9] analyze the exchange-rate stabilization of unilateral Home tariffs and show that the optimal Home monetary response depends on trade elasticities and price flexibility. The present paper belongs to this second group: its central question is how the monetary authority in the tariff-imposing economy should respond to its own import-tariff increase.

An important difference between the present paper and the studies in this second group concerns the treatment of import-price pass-through. Monacelli [39], Bianchi and Coulibaly [11], and Werning, Lorenzoni, and Guerrieri [43] do not focus on imperfect pass-through and instead abstract from the local-importer pricing friction emphasized here. Bergin and Corsetti [9] are closer to the present paper in allowing for local-currency pricing and also consider retaliatory and symmetric tariff-war scenarios. However, they do not introduce monopolistically competitive local importers whose pricing response to tariffs generates an endogenous importer-markup distortion. Thus, the present paper introduces a parsimonious, empirically motivated departure from the PPT benchmark—sticky local-currency pricing by monopolistically competitive local importers—and shows that this pricing friction itself generates a distinct monetary-transmission mechanism. Tariff-induced opportunistic pricing raises the importer markup and the Home WR LOOP gap, thereby creating an endogenous motive for monetary accommodation that is absent from the existing analyses reviewed above.

This modeling difference also has an important policy implication. Under IPT, an import-tariff increase induces opportunistic pricing by local importers, raises the importer markup, and increases the Home WR LOOP gap. This creates scope for reducing welfare costs by raising the actual LOOP gap, which requires monetary accommodation. The conclusion that a more accommodative monetary stance can be desirable is not necessarily inconsistent with the existing studies reviewed above. Even though Monacelli [39] does not argue that an import-tariff shock calls

for outright monetary easing, his finding that CPI inflation targeting exhibits a tightening bias is not necessarily inconsistent with our result. Bergin and Corsetti [9] also find an expansionary optimal monetary stance in their tariff-war analysis, but through a different mechanism. What is distinctive in the present paper is that the welfare cost generated by tariff-induced opportunistic pricing can require substantial monetary accommodation already under a unilateral tariff, and that reciprocal tariffs reinforce this mechanism through the Foreign side of the economy. In a U.S.-size tariff-imposing economy, this reinforcement can make the required accommodation sufficiently strong for the ZLB to bind. Thus, the distinctive policy implication is not simply that monetary accommodation can be desirable under tariffs, but that imperfect pass-through can make the ZLB an endogenous constraint on the optimal monetary response.

## 2.2 Pricing behavior, pass-through, and monetary stabilization

The mechanism is also related to the literature on pricing behavior and exchange-rate pass-through. Betts and Devereux [10] and Devereux and Engel [21] show that local-currency price stickiness weakens the short-run pass-through of exchange-rate movements. Devereux and Engel [22] show that the optimal use of exchange-rate flexibility depends on whether prices are set in producer or local currency, while Corsetti, Dedola, and Leduc [17] emphasize that optimal monetary policy depends on the degree of exchange-rate pass-through into import prices. These results motivate our comparison of PPT and IPT environments. Okano and Eguchi [40] also employ LCP in a small-open-economy policy analysis, although their application concerns fiscal stimulus rather than tariffs.

The welfare interpretation is closely related to work on international relative prices and deviations from the law of one price. De Paoli [20] shows that a national planner in a small open economy has an incentive to use international relative prices rather than mechanically stabilize domestic inflation. Engel [23] shows that under LCP, currency misalignments associated with deviations from the law of one price are inefficient and enter optimal monetary policy. Fujiwara and Wang [29] extend this logic to a noncooperative policy game under LCP. In our tariff problem, the LOOP gap acquires a particularly direct role: a tariff-induced opportunistic increase in the local importer's *ex-tariff* price raises its markup, while a higher LOOP gap compresses that markup. This is the HMC underlying the optimal monetary easing in the IPT environment.

The treatment of the statutory tariff relative to nominal import-price rigidity is also relevant. Chen, Devereux, Shi, and Xu [16] impose the import tax on top of a pre-set price under LCP, whereas Farhi, Gopinath, and Itskhoki [26] place border taxes within an LCP price-setting problem. Our baseline follows the former convention: the statutory tariff is separated from the sticky *ex-tariff* import price. This distinction is consistent with the empirical motivation for separating tariff pass-through from exchange-rate pass-through. Cavallo, Gopinath, Neiman, and Tang [15] document high tariff pass-through at the border together with more limited retail-price adjustment, while Gopinath, Itskhoki, and Rigobon [32] document incomplete exchange-rate pass-through into U.S. import prices for dollar-priced imports. These findings motivate the pass-through distinction, although they are not direct evidence for the specific local-importer structure used here.

## 2.3 International policy interaction and the ZLB

The two-country analysis is related to the literature on international monetary-policy games. Faia and Monacelli [25] formulate welfare-maximizing monetary policy in a two-country New Keynesian economy and compare cooperation with Nash competition between national Ramsey planners. We use their national-Ramsey policy-game logic to ask whether the Home Markup Channel survives when Foreign demand and Foreign monetary policy are jointly endogenous. The two-country extension adds the FAC, through which movements in the Foreign LOOP gap and Foreign tariffs alter the international allocation of Home goods and therefore the Home WR LOOP gap.

Finally, the paper is related to the ZLB literature on international monetary transmission, including Fujiwara et al. [27], Bodenstein, Erceg, and Guerrieri [13], and Billi and Galí [12]. The

role of the ZLB here is different. We do not add a demand or natural-rate shock to create a liquidity trap. Instead, imperfect pass-through makes the welfare-maximizing response to tariffs sufficiently accommodative that reciprocal tariffs can exhaust conventional monetary-policy space. The High Elasticity Case strengthens the FAC, external habit raises the WR LOOP gap through inherited consumption, and roundabout production increases the importance of stabilizing Home PPI inflation. The ZLB analysis therefore follows directly from the same mechanisms that generate optimal monetary easing in the tariff-imposing economy.

### 3 Small Open Economy

The small-open-economy model consists of a representative Home household, monopolistically competitive Home producers, and a monetary authority. The household consumes a constant-elasticity-of-substitution (CES) composite of Home and Foreign goods and trades state-contingent claims with the rest of the world. Home producers use PCP and Calvo pricing in both pass-through environments. The government levies an import tariff and rebates tariff revenue lump sum. No employment subsidy is paid to PCP producers. The difference between the two environments is on the import side. Under PPT there is no independent domestic importer-price block and the LOOP holds for the *ex-tariff* import price. Under IPT, local importers are added. They use LCP and set sticky *ex-tariff* import prices in Home currency.

#### 3.1 PPT Environment

##### 3.1.1 Households and demand

The Home representative household has lifetime utility

$$\mathcal{U} \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right). \quad (1)$$

Here  $C_t$  denotes the Home household's composite consumption and  $N_t \equiv \int_0^1 N_t(j) dj$  denotes hours of labor. The parameter  $\beta \in (0, 1)$  is the discount factor and  $\varphi$  is the inverse Frisch elasticity.

Composite consumption is

$$C_t = \left[ (1-v)^{1/\eta} C_{H,t}^{(\eta-1)/\eta} + v^{1/\eta} C_{F,t}^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}.$$

Here  $C_{H,t}$  and  $C_{F,t}$  denote consumption of Home and Foreign goods, respectively,  $v \in (0, 1)$  is the expenditure weight on Foreign goods, and  $\eta$  is the elasticity of substitution between Home and Foreign goods.

Under PPT, the *ex-tariff* Home-currency import price satisfies

$$P_{F,t} = \mathcal{E}_t P_{F,t}^*. \quad (2)$$

Here  $P_{F,t}$  denotes the *ex-tariff* Home-currency import price,  $P_{F,t}^*$  denotes the Foreign producer-price index in Foreign currency, and  $\mathcal{E}_t$  is the price of Foreign currency in units of Home currency. The Home CPI under the PPT environment is

$$P_t^{PPT} \equiv \left[ (1-v) P_{H,t}^{1-\eta} + v \left( \tilde{P}_{F,t}^{PPT} \right)^{1-\eta} \right]^{1/(1-\eta)}. \quad (3)$$

Here  $P_t^{PPT}$  denotes the Home Consumer Price Index (CPI),  $P_{H,t}$  denotes the Home producer-price index,  $\tau_{M,t}$  denotes the Home import-tariff rate, and  $\tilde{P}_{F,t}^{PPT} \equiv (1 + \tau_{M,t}) \mathcal{E}_t P_{F,t}^*$  denotes the *cum-tariff* import price of Foreign goods under the PPT environment. The CES demand system gives

$$C_{H,t} = (1-v) [\mathcal{G}(\tau_{M,t}, S_t)]^\eta C_t, \quad (4)$$

where the CPI–PPI ratio is

$$\mathcal{G}(\tau_{M,t}, S_t) \equiv \frac{P_t^{PPT}}{P_{H,t}}. \quad (5)$$

where:

$$\begin{aligned} S_t &\equiv \frac{P_{F,t}}{P_{H,t}} \\ &= \frac{Z_t}{\Psi_t}, \end{aligned} \quad (6)$$

denotes the *ex-tariff* relative price of Foreign goods relative to the Home PPI (*ex-tariff* relative price of imports, for short),  $Z_t \equiv \mathcal{E}_t P_{F,t}^* / P_{H,t}$  denotes the *ex-tariff* relative border price of Foreign goods relative to the Home PPI (*ex-tariff* relative border price of imports). Following Monacelli [38], we refer to  $\Psi_t \equiv \mathcal{E}_t P_{F,t}^* / P_{F,t}$  as the LOOP gap.

Under PPT, Eq. (2) implies  $\Psi_t = 1$  and hence  $S_t = Z_t$ . Thus the *ex-tariff* relative price of imports coincides with the *ex-tariff* relative border price of imports.

Import demand is

$$C_{F,t} = v \left[ \frac{(1 + \tau_{M,t}) S_t}{\mathcal{G}(\tau_{M,t}, S_t)} \right]^{-\eta} C_t. \quad (7)$$

Combining Eq. (4) with Eq. (7) yields

$$\frac{C_{F,t}}{C_{H,t}} = \frac{v}{1-v} [(1 + \tau_{M,t}) S_t]^{-\eta}. \quad (8)$$

which suggests that the relative consumption of imports depends on the *ex-tariff* relative price of imports. In addition, once the import tariff is hiked, the relative consumption of imports depends on the *cum-tariff* relative price of imports.

The household's nominal budget constraint is

$$P_{H,t} C_{H,t} + (1 + \tau_{M,t}) \mathcal{E}_t P_{F,t}^* C_{F,t} + \sum_{h^{t+1}} \xi_{t,t+1}(h^{t+1} | h^t) B_{t+1}(h^{t+1}) = B_t(h^t) + W_t N_t + \mathcal{D}_t + T_t.$$

where  $B_{t+1}(h^{t+1})$  is a state-contingent nominal claim purchased at date  $t$ ,  $\xi_{t,t+1}(h^{t+1} | h^t)$  is its date- $t$  state price,  $W_t$  is the nominal wage,  $\mathcal{D}_t$  denotes nominal dividends from firms, and  $T_t$  is the net lump-sum transfer from the government. The import-expenditure term contains both the border value  $\mathcal{E}_t P_{F,t}^* C_{F,t}$  and the statutory customs payment  $\tau_{M,t} \mathcal{E}_t P_{F,t}^* C_{F,t}$ . Hence tariff revenue is

$$TR_t = \tau_{M,t} \mathcal{E}_t P_{F,t}^* C_{F,t}, \quad T_t = TR_t$$

in the PPT environment, where there is no local-importer subsidy. Thus the statutory duty paid as part of household import expenditure is returned to the household as a lump-sum transfer. By CES duality,

$$P_{H,t} C_{H,t} + (1 + \tau_{M,t}) \mathcal{E}_t P_{F,t}^* C_{F,t} = P_t^{PPT} C_t,$$

so the same budget constraint can equivalently be written as

$$P_t^{PPT} C_t + \sum_{h^{t+1}} \xi_{t,t+1}(h^{t+1} | h^t) B_{t+1}(h^{t+1}) = B_t(h^t) + W_t N_t + \mathcal{D}_t + T_t. \quad (9)$$

Maximization of Eq. (1) subject to the budget constraint gives the intratemporal condition

$$\frac{W_t}{P_t^{PPT}} = C_t N_t^\varphi, \quad (10)$$

and the Euler equation

$$\frac{1}{1+i_t} = \beta \mathbb{E}_t \left[ \frac{C_t(\mathcal{G}(\tau_{M,t}, S_t))}{C_{t+1}(\mathcal{G}(\tau_{M,t+1}, S_{t+1}))\Pi_{H,t+1}} \right]. \quad (11)$$

Here  $i_t$  denotes the Home nominal interest rate and  $\Pi_{H,t} \equiv P_{H,t}/P_{H,t-1}$  denotes gross Home PPI inflation. The Euler equation follows from  $\xi_{t,t+1} = \beta(C_t/C_{t+1})(\Pi_{t+1}^{PPT})^{-1}$  and  $(1+i_t)^{-1} = \mathbb{E}_t \xi_{t,t+1}$ , where  $\Pi_t^{PPT} \equiv P_t^{PPT}/P_{t-1}^{PPT}$  denotes gross CPI inflation. The Foreign household satisfies the analogous Euler equation. With  $\xi_{t,t+1}$  denoting the Home-currency nominal state price, the Foreign-currency bond satisfies

$$\frac{1}{1+i_t^*} = \mathbb{E}_t \left[ \xi_{t,t+1} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right],$$

where  $i_t^*$  denotes the Foreign nominal interest rate. We do not impose a separate log-linear UIP equation in the nonlinear system.

Complete international risk sharing implies

$$S_t = C_t \mathcal{G}(\tau_{M,t}, S_t). \quad (12)$$

### 3.1.2 Production and pricing behavior

Each Home firm  $j \in [0, 1]$  produces a differentiated good according to  $Y_t(j) = A_t N_t(j)$ , where  $A_t$  denotes labor productivity. Let  $Y_t$  denote aggregate Home output. Demand for variety  $j$  is  $Y_t(j) = [P_{H,t}(j)/P_{H,t}]^{-\varepsilon} Y_t$ , where  $\varepsilon > 1$  is the elasticity of substitution across Home producer varieties. Aggregation gives

$$Y_t = \frac{A_t N_t}{\Delta_{H,t}}, \quad (13)$$

where  $\Delta_{H,t} \equiv \int_0^1 [P_{H,t}(j)/P_{H,t}]^{-\varepsilon} dj$  denotes producer-price dispersion. We set  $A_t = 1$  throughout the paper.

Home producers face Calvo price rigidity. Following the sequence representation in Monacelli [39], the producer's reset-price condition is

$$\tilde{p}_{H,t} = \mathcal{M} \frac{\mathcal{Z}_{H,t}}{\mathcal{K}_{H,t}}, \quad (14)$$

where  $\tilde{p}_{H,t} \equiv \bar{P}_{H,t}/P_{H,t}$  is the optimal reset price relative to the current Home PPI,  $\theta$  is the producer Calvo non-adjustment probability,  $MC_t^r \equiv MC_t/P_{H,t}$  is Home real marginal cost, and  $\mathcal{M} \equiv \varepsilon/(\varepsilon - 1)$  is the desired producer markup. The auxiliary objects are

$$\begin{aligned} \mathcal{K}_{H,t} &\equiv \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k} \left( \prod_{s=1}^k \Pi_{H,t+s} \right)^{\varepsilon}, \\ \mathcal{Z}_{H,t} &\equiv \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k} MC_{t+k}^r \left( \prod_{s=1}^k \Pi_{H,t+s} \right)^{1+\varepsilon}. \end{aligned} \quad (15)$$

The household nominal stochastic discount factor from  $t$  to  $t+k$ , used in both the producer and importer Calvo pricing problems, is

$$Q_{t,t+k} \equiv \beta^k \frac{C_t(\mathcal{G}(\tau_{M,t}, S_t))}{C_{t+k}(\mathcal{G}(\tau_{M,t+k}, S_{t+k}))} \left( \prod_{s=1}^k \Pi_{H,t+s} \right)^{-1}.$$

Using Eq. (10), real marginal cost is

$$MC_t^r = C_t N_t^{\varphi} \mathcal{G}(\tau_{M,t}, S_t). \quad (16)$$

The producer Calvo price-index and dispersion laws are

$$\begin{aligned} 1 &= \theta \Pi_{H,t}^{\varepsilon-1} + (1-\theta) \tilde{p}_{H,t}^{1-\varepsilon}, \\ \Delta_{H,t} &= \theta \Pi_{H,t}^{\varepsilon} \Delta_{H,t-1} + (1-\theta) \tilde{p}_{H,t}^{\varepsilon}. \end{aligned} \quad (17)$$

At first order, Home PPI inflation therefore satisfies

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa_H mc_t^r, \quad (18)$$

where  $\pi_{H,t} \equiv \log \Pi_{H,t}$ ,  $mc_t^r \equiv \log(MC_t^r/MC^r)$ ,  $MC^r$  is the deterministic steady-state value of Home real marginal cost, and  $\kappa_H \equiv (1-\theta)(1-\beta\theta)/\theta$ .

### 3.1.3 Market clearing

Home goods-market clearing is

$$Y_t = (1-v) (\mathcal{G}(\tau_{M,t}, S_t))^\eta C_t + v S_t^\eta. \quad (19)$$

The first term is Home household demand in Eq. (4); the second is Foreign demand for Home goods in the SOE limit.

## 3.2 IPT Environment

The IPT environment keeps the preferences, CES consumption aggregator, household optimality conditions, complete-risk-sharing condition, Home production technology, and PCP producer-pricing block of the PPT environment. The difference is that local importers set the *ex-tariff* import price in Home currency.

Following Bianchi and Coulibaly [11], we let the statutory tariff enter the household-facing price of imported consumption goods and rebate the resulting customs revenue lump sum to the Home household.<sup>1</sup> In the IPT environment, a local importer stands between the Foreign producer and the Home household. The importer of record purchases the Foreign good at the border value, remits the statutory duty to the Home government, and passes that statutory component through to the Home household on top of the local-currency *ex-tariff* price.

Following Monacelli [38], we introduce monopolistically competitive local importers that purchase Foreign goods at the border and resell differentiated import varieties to Home households at a markup. These importers set prices in Home currency under Calvo pricing. Consequently, the local-currency import price need not move one-for-one with the border price, and the law of one price (LOOP) no longer holds.

Our setting is consistent with empirical evidence. Cavallo et al. [15] find that the 2018–19 U.S. import tariffs were almost fully passed through into the total prices paid by importers at the border, while the response of import prices to exchange rates was much more muted; the effect on retail prices was also more limited, consistent with adjustment in downstream retail margins. Gopinath, Itskhoki, and Rigobon [32] find that the pass-through of nominal exchange-rate movements into U.S. import prices is about 25 percent for dollar-priced imports, even conditional on a price change. Thus, our setting also allows nominal exchange-rate pass-through to be determined independently of tariff pass-through, which is consistent with the evidence in both studies.

### 3.2.1 Households and *cum-tariff* import prices

Similar to the PPT environment, households' utility and optimality conditions have the same form as Eqs. (1) and (10)–(12), with  $P_t^{IPT}$  replacing  $P_t^{PPT}$  wherever the Home CPI appears. The household-facing *cum-tariff* price of Foreign variety  $\iota$  is

$$\tilde{P}_{F,t}^{IPT}(\iota) \equiv P_{F,t}(\iota) + \tau_{M,t} \mathcal{E}_t P_{F,t}^*. \quad (20)$$

<sup>1</sup>Chen et al. [16] likewise impose the import tax on top of a pre-set LCP price. An alternative convention places border taxes inside the LCP price-setting problem, as in Farhi et al. [26]; Erceg et al. [24] emphasize more generally that the macroeconomic effects of trade taxes depend on the assumed tax pass-through.

where  $\tilde{P}_{F,t}^{IPT}(\iota)$  denotes the household-facing *cum-tariff* price of Foreign variety  $\iota$  under the IPT environment, and  $P_{F,t}(\iota)$  denotes the *ex-tariff* Home-currency import price of variety  $\iota$ . Eq. (20) is consistent with the tariff-inclusive household import expenditure and lump-sum-rebate treatment in Bianchi and Coulibaly [11]. It is also consistent with the border-price evidence of Cavallo et al. [15]. The statutory duty is therefore added to the local importer's *ex-tariff* price, rather than multiplying that price. Conditional demand for variety  $\iota$  is

$$C_{F,t}(\iota) = \left( \frac{\tilde{P}_{F,t}^{IPT}(\iota)}{\tilde{P}_{F,t}^{IPT}} \right)^{-\varepsilon_M} C_{F,t}, \quad (21)$$

where  $\varepsilon_M > 1$  is the elasticity of substitution across import varieties and  $\tilde{P}_{F,t}^{IPT}$  denotes the aggregate *cum-tariff* import price index, being the CPI-relevant import-price index under the IPT environment, defined by

$$\tilde{P}_{F,t}^{IPT} \equiv \left[ \int_0^1 \tilde{P}_{F,t}^{IPT}(\iota)^{1-\varepsilon_M} d\iota \right]^{1/(1-\varepsilon_M)} \quad (22)$$

Note that the purchaser price is defined variety by variety in Eq. (20), so the aggregate *cum-tariff* import-price index cannot in general be written as  $(1 + \tau_{M,t})P_{F,t}$  under the imperfect pass-through environment. Equations (21)–(22) imply

$$\int_0^1 \tilde{P}_{F,t}^{IPT}(\iota) C_{F,t}(\iota) d\iota = \tilde{P}_{F,t}^{IPT} C_{F,t}.$$

Home-good demand remains Eq. (4). Aggregate import demand is

$$C_{F,t} = v \left( \frac{\tilde{P}_{F,t}^{IPT}}{P_t^{IPT}} \right)^{-\eta} C_t. \quad (23)$$

The Home CPI under the IPT environment is given by

$$P_t^{IPT} \equiv \left[ (1-v)P_{H,t}^{1-\eta} + v \left( \tilde{P}_{F,t}^{IPT} \right)^{1-\eta} \right]^{1/(1-\eta)}. \quad (24)$$

instead of Eq. (3). The CES consumption aggregator, its expenditure weights, and its elasticity of substitution are identical across the two pass-through environments; only the household-facing *cum-tariff* import price entering that common CES price aggregator differs. Under PPT,  $\tilde{P}_{F,t}^{PPT} = (1 + \tau_{M,t})\mathcal{E}_t P_{F,t}^*$ , whereas under IPT,  $\tilde{P}_{F,t}^{IPT}$  is the aggregate index defined in Eq. (22) from the household-facing variety prices in Eq. (20).

The household's budget constraint is:

$$P_{H,t}C_{H,t} + \int_0^1 [P_{F,t}(\iota) + \tau_{M,t}\mathcal{E}_t P_{F,t}^*] C_{F,t}(\iota) d\iota + \sum_{h^{t+1}} \xi_{t,t+1}(h^{t+1}|h^t) B_{t+1}(h^{t+1}) = B_t(h^t) + W_t N_t + \mathcal{D}_t + T_t.$$

The integral makes the statutory-duty payment explicit on the expenditure side. By Eqs. (20)–(22),

$$\int_0^1 \tilde{P}_{F,t}^{IPT}(\iota) C_{F,t}(\iota) d\iota = \tilde{P}_{F,t}^{IPT} C_{F,t}, \quad P_{H,t}C_{H,t} + \tilde{P}_{F,t}^{IPT} C_{F,t} = P_t^{IPT} C_t.$$

Hence the same budget constraint can equivalently be written as

$$P_t^{IPT} C_t + \sum_{h^{t+1}} \xi_{t,t+1}(h^{t+1}|h^t) B_{t+1}(h^{t+1}) = B_t(h^t) + W_t N_t + \mathcal{D}_t + T_t,$$

which has the same form as Eq. (9), with  $P_t^{IPT}$  replacing  $P_t^{PPT}$ , while  $\mathcal{D}_t$  includes local-importer dividends. The importer of record remits the statutory duty, and customs revenue is

$$TR_t \equiv \tau_{M,t} \mathcal{E}_t P_{F,t}^* \int_0^1 C_{F,t}(\iota) d\iota. \quad (25)$$

The customs revenue in Eq. (25) is exactly the statutory-duty component paid on the left-hand side of the disaggregated budget constraint and is rebated lump sum through  $T_t$ . The government also pays the time-invariant subsidy  $\tau$  on the border acquisition cost, so its net lump-sum transfer is

$$T_t = TR_t - \tau \mathcal{E}_t P_{F,t}^* \int_0^1 C_{F,t}(\iota) d\iota.$$

Here  $\tau$  denotes the subsidy rate paid to local importers; below we set  $\tau = 1/\varepsilon_M$ . The subsidy receipt  $\tau \mathcal{E}_t P_{F,t}^* \int_0^1 C_{F,t}(\iota) d\iota$  is included in importer dividends in  $\mathcal{D}_t$ , while payment of the subsidy  $\tau \mathcal{E}_t P_{F,t}^* \int_0^1 C_{F,t}(\iota) d\iota$  enters  $T_t$  with the opposite sign. Thus the statutory tariff is collected from the importer of record, passed through separately to the purchaser in Eq. (20), and returned to the household as a lump-sum rebate.

### 3.2.2 Production and pricing behavior

The IPT environment contains both Home producers and local importers. The Home producer side is unchanged from the PPT environment: production and producer price setting continue to be governed by Eqs. (13)–(18), i.e., Eqs. (7)–(12). In addition, local importers use LCP and face Calvo rigidity. Their exact nonlinear pricing block is given by Eqs. (21)–(25). The *ex-tariff* import-price index satisfies

$$P_{F,t}^{1-\varepsilon_M} = \theta_M P_{F,t-1}^{1-\varepsilon_M} + (1 - \theta_M) \bar{P}_{F,t}^{1-\varepsilon_M}, \quad (26)$$

Here  $\theta_M$  denotes the importer Calvo non-adjustment probability and  $\bar{P}_{F,t}$  denotes the *ex-tariff* reset price. Because  $P_{F,t}/P_{H,t} = Z_t/\Psi_t$ , the *ex-tariff* relative import price obeys

$$\frac{Z_t}{\Psi_t} = \frac{Z_{t-1}}{\Psi_{t-1}} \frac{\Pi_{F,t}}{\Pi_{H,t}}, \quad (27)$$

where  $\Pi_{F,t} \equiv P_{F,t}/P_{F,t-1}$  denotes gross *ex-tariff* import inflation.

The importer remits  $\tau_{M,t} \mathcal{E}_t P_{F,t}^*$  per imported unit and recovers the same statutory duty from the purchaser. Hence the duty cancels from the importer's net operating cash flow, but it enters the demand faced by a resetting importer. A resetting importer chooses  $\bar{P}_{F,t}$  to solve

$$\max_{\bar{P}_{F,t}} \mathbb{E}_t \sum_{k=0}^{\infty} \theta_M^k Q_{t,t+k} [\bar{P}_{F,t} - (1 - \tau) \mathcal{E}_{t+k} P_{F,t+k}^*] C_{F,t+k|t}, \quad (28)$$

The *ex-tariff* price-setting formulation in Eq. (28) is the same local-currency-pricing importer structure as in Monacelli [38], apart from the steady-state subsidy normalization used here. It is consistent with the low nominal exchange-rate pass-through documented by Gopinath, Itskhoki, and Rigobon [32]. Here  $C_{F,t+k|t}$  denotes demand at date  $t+k$  for an import variety whose *ex-tariff* price was reset at date  $t$  and is given by

$$C_{F,t+k|t} = \left[ \frac{\bar{P}_{F,t} + \tau_{M,t+k} \mathcal{E}_{t+k} P_{F,t+k}^*}{\tilde{P}_{F,t+k}} \right]^{-\varepsilon_M} C_{F,t+k}. \quad (29)$$

The tariff-induced reduction in demand is therefore internalized in the reset-price problem. The exact first-order condition is

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta_M^k Q_{t,t+k} C_{F,t+k|t} \left[ 1 - \varepsilon_M \frac{\bar{P}_{F,t} - (1 - \tau) \mathcal{E}_{t+k} P_{F,t+k}^*}{\bar{P}_{F,t} + \tau_{M,t+k} \mathcal{E}_{t+k} P_{F,t+k}^*} \right] = 0. \quad (30)$$

We set  $\tau = 1/\varepsilon_M$ , which removes the steady-state importer markup and implies  $\Psi = 1$  in the deterministic zero-tariff, zero-inflation steady state.

The mechanism through which the tariff raises the importer's own *ex-tariff* price is especially transparent when importer prices are flexible, so that  $\bar{P}_{F,t} = P_{F,t}$ . From Eqs. (20) and (21), the effective elasticity of demand with respect to the importer's own *ex-tariff* price is

$$\frac{\partial \log C_{F,t}}{\partial \log P_{F,t}} = -\varepsilon_M \frac{P_{F,t}}{P_{F,t} + \tau_{M,t} \mathcal{E}_t P_{F,t}^*}.$$

Because the statutory duty is added to the importer's own price in the purchaser price, a positive tariff makes this demand elasticity smaller in absolute value. Intuitively, once the tariff has already raised the purchaser price, a one-percent increase in  $P_{F,t}$  raises the purchaser price by less than one percent. The associated loss of demand is therefore smaller than it would be without the tariff, giving the importer an incentive to raise its own *ex-tariff* price.

For a simple numerical illustration, suppose the border acquisition price is 1 dollar and the importer's *ex-tariff* price is 1.10 dollars. Before the tariff, the purchaser price is therefore 1.10 dollars. A new one-percent tariff adds 0.01 dollar to the purchaser price, raising it to 1.11 dollars. If the importer then raises its own *ex-tariff* price by one percent, from 1.10 dollars to 1.111 dollars, the purchaser price becomes 1.121 dollars. Relative to the post-tariff purchaser price of 1.11 dollars, however, this is an increase of only  $(1.121 - 1.11)/1.11 \simeq 0.99$  percent, rather than one percent. Hence, for a given demand elasticity  $\varepsilon_M$ , the associated demand decline is approximately  $0.99\varepsilon_M$  percent rather than  $\varepsilon_M$  percent. The importer therefore loses less demand from a given proportional increase in its own price once the tariff is present, strengthening its incentive to raise the *ex-tariff* price. To derive the flexible-price implication, set  $\bar{P}_{F,t} = P_{F,t}$  in Eq. (30). The reset-price condition then becomes

$$1 - \varepsilon_M \frac{P_{F,t} - (1 - \tau) \mathcal{E}_t P_{F,t}^*}{P_{F,t} + \tau_{M,t} \mathcal{E}_t P_{F,t}^*} = 0,$$

which can be rearranged as

$$(\varepsilon_M - 1)P_{F,t} = [\varepsilon_M(1 - \tau) + \tau_{M,t}] \mathcal{E}_t P_{F,t}^*.$$

Using the fixed importer subsidy  $\tau = 1/\varepsilon_M$  gives

$$\Psi_t^{-1} = \frac{P_{F,t}}{\mathcal{E}_t P_{F,t}^*} = 1 + \frac{\tau_{M,t}}{\varepsilon_M - 1}. \quad (31)$$

The inverse LOOP gap  $\Psi_t^{-1}$  corresponds to the importer's *ex-tariff* price markup relative to the border acquisition price;  $\Psi_t$  is the inverse of this markup measure. Equation (31) gives its value under flexible importer prices. The statutory duty does not enter as a net marginal acquisition cost; it lowers the effective own-price elasticity and thereby induces an opportunistic increase in the *ex-tariff* markup. The subsidy is calibrated to eliminate the zero-tariff steady-state importer markup, but it does not adjust with the tariff and therefore does not eliminate the tariff-induced increase in the importer markup. This mechanism is closely related to the classic tax-pass-through result under imperfect competition. Delipalla and Keen [19] compare specific and ad valorem taxation under oligopoly, while Anderson, de Palma, and Kreider [2] show in a differentiated-product Bertrand setting that unit excise taxes can be shifted to consumer prices by more than one-for-one, with the incidence depending on the curvature of demand. Weyl and Fabinger [44] provide

a general treatment of tax pass-through under imperfect competition. The present mechanism has a distinctive international-pricing implementation: the statutory tariff is proportional to the border acquisition price and is added outside the sticky *ex-tariff* import price, so that the tariff lowers the importer's effective own-price elasticity through the purchaser-price denominator. The resulting incentive to raise the *ex-tariff* import price is nevertheless consistent with this imperfect-competition tax-pass-through literature.

Thus the IPT environment is subject to the exact importer reset-price condition in Eq. (30), in addition to the producer conditions in Eqs. (13)–(18). At first order, Eq. (30) implies

$$\pi_{F,t} = \beta E_t \pi_{F,t+1} + \kappa_M \left( \psi_t + \frac{\tau_{M,t}}{\varepsilon_M - 1} \right), \quad (32)$$

where  $\pi_{F,t} \equiv \log \Pi_{F,t}$ ,  $\psi_t \equiv \log \Psi_t$ , and  $\kappa_M \equiv (1 - \theta_M)(1 - \beta\theta_M)/\theta_M$ . Equation (32) is used for first-order analytical interpretation, while the nonlinear Ramsey computation retains Eq. (30). Around the zero-tariff steady state,  $\log(1 + \tau_{M,t}) = \tau_{M,t} + O(\tau_{M,t}^2)$ .

The empirical motivation for separating the statutory duty from the sticky *ex-tariff* local import price is discussed in Sec. 2.

Taking logs of Eq. (31) and linearizing around the zero-tariff steady state, the flexible-importer-price benchmark is

$$\psi_t = -\frac{\tau_{M,t}}{\varepsilon_M - 1}.$$

Hence the expression  $\psi_t + \tau_{M,t}/(\varepsilon_M - 1)$  on the right-hand side of Eq. (32) is the actual LOOP gap minus its flexible-price benchmark. Import-price inflation depends on this deviation. Thus the tariff term in Eq. (32) reflects the opportunistic *ex-tariff* price increase implied by Eq. (31), not the statutory duty as a net marginal acquisition cost.

### 3.2.3 Market clearing

Home goods-market clearing remains Eq. (19). The difference from PPT enters through the CPI–PPI ratio in Eq. (5) and the endogenous LOOP gap  $\Psi_t$ .

## 3.3 Ramsey problem

For each pass-through environment, the monetary authority is a social planner that commits at date zero to a complete contingent plan. In either environment, the planner maximizes Eq. (1).

In the PPT environment, the choice variables are  $C_t$ ,  $Z_t$ ,  $N_t$ ,  $Y_t$ ,  $\Pi_{H,t}$ ,  $\tilde{p}_{H,t}$ ,  $\mathcal{K}_{H,t}$ ,  $\mathcal{Z}_{H,t}$ ,  $\Delta_{H,t}$ , and  $i_t$  for  $t \geq 0$ . The constraints are Eqs. (5), (10), (11), and (12) with  $\Psi_t = 1$ , Eqs. (13)–(17), Eq. (2), Eq. (19) with  $\Psi_t = 1$ , and the ZLB conditions in Eq. (33).

In the IPT environment, the choice variables are  $C_t$ ,  $Z_t$ ,  $N_t$ ,  $Y_t$ ,  $\Pi_{H,t}$ ,  $\tilde{p}_{H,t}$ ,  $\mathcal{K}_{H,t}$ ,  $\mathcal{Z}_{H,t}$ ,  $\Delta_{H,t}$ ,  $\Psi_t$ ,  $\Pi_{F,t}$ ,  $\bar{P}_{F,t}$ ,  $P_{F,t}$ ,  $\tilde{P}_{F,t}^{IPT}$ , and  $i_t$  for  $t \geq 0$ . The constraints are Eqs. (5) and (23), Eqs. (10) and (11), Eq. (12), Eqs. (13)–(17), Eqs. (28)–(29), (26), (30), and (27), Eq. (19), and the ZLB condition in Eq. (33).

The feasibility, non-negativity, and complementary-slackness conditions are

$$i_t \geq 0, \quad \mu_{i,t} \geq 0, \quad \mu_{i,t} i_t = 0. \quad (33)$$

Here  $\mu_{i,t}$  denotes the Kuhn–Tucker multiplier on the ZLB constraint. The tariff process is exogenous and tariff revenue is rebated lump sum. The Lagrangians and first-order conditions for the two pass-through environments are reported in On-line Appendix B; the first-order conditions are not repeated in the main text. The numerical policy paths are obtained from these nonlinear Ramsey first-order-condition systems. The second-order welfare cost functions below are reporting and interpretation devices rather than policy objectives.

### 3.4 Calibration and shocks

The benchmark calibration follows Monacelli [39] wherever the model has a direct counterpart. Table 1 summarizes the benchmark parameterization.

| Table 1: Benchmark calibration |       |                                      |
|--------------------------------|-------|--------------------------------------|
| Parameter                      | Value | Interpretation                       |
| $\beta$                        | 0.99  | Discount factor                      |
| $\nu$                          | 0.3   | Openness / Home Bias parameter       |
| $\varphi$                      | 2.2   | Inverse Frisch elasticity            |
| $\eta$                         | 1.5   | Home–Foreign substitution elasticity |
| $\varepsilon$                  | 3.8   | Elasticity across producer varieties |
| $\theta$                       | 0.8   | Producer Calvo parameter             |
| $\varepsilon_M$                | 3.8   | Elasticity across importer varieties |
| $\theta_M$                     | 0.8   | Importer Calvo parameter             |
| $\rho_\tau$                    | 0.7   | Tariff persistence                   |

The tariff process is

$$\tau_{M,t} = \rho_\tau \tau_{M,t-1} + \epsilon_{\tau,t}.$$

We set the one-time innovation  $\epsilon_{\tau,0} = 0.20$ , following Monacelli [39], as our benchmark. Thus the import tariff rises by 20 percentage points on impact. The steady-state tariff is zero. The deterministic quarterly nominal rate is  $1/\beta - 1 = 1.0101$  percent, approximately four percent annualized. Hence a response of  $-101.01$  basis points relative to steady state places the actual nominal rate exactly at the ZLB.

### 3.5 Welfare Cost Function and WR LOOP gap

#### 3.5.1 Welfare Cost Function

To make the planner’s stabilization objectives transparent, we report welfare cost functions based on a second-order approximation to household utility following Benigno and Woodford [6, 7]. In superscripts, PPT and IPT denote the two pass-through environments, respectively. Let  $y_t \equiv \log(Y_t/Y)$  and  $z_t \equiv \log Z_t$ . Suppressing terms that are independent of the Home policy allocation, under PPT,

$$\mathcal{L}_t^{SOE,PPT} \equiv \frac{1}{2} \left[ \hat{\Lambda}_y^{SOE,PPT} (y_t - \hat{y}_t^{w,SOE,PPT})^2 + \hat{\Lambda}_z^{SOE,PPT} (z_t - \hat{z}_t^{w,SOE,PPT})^2 + \Lambda_{\pi,H}^{SOE,PPT} \pi_{H,t}^2 \right],$$

whereas under IPT it can be written as

$$\begin{aligned} \mathcal{L}_t^{SOE,IPT} \equiv & \frac{1}{2} \left[ \hat{\Lambda}_y^{SOE,IPT} (y_t - \hat{y}_t^{w,SOE,IPT})^2 + \hat{\Lambda}_\psi^{SOE,IPT} (\psi_t - \hat{\psi}_t^{w,SOE,IPT})^2 \right. \\ & \left. + \hat{\Lambda}_z^{SOE,IPT} (z_t - \hat{z}_t^{w,SOE,IPT})^2 + \Lambda_{\pi,H}^{SOE,IPT} \pi_{H,t}^2 + \Lambda_{\pi,F}^{SOE,IPT} \pi_{F,t}^2 \right]. \end{aligned}$$

Here the hatted  $\Lambda$  coefficients are the completed-square welfare weights associated with  $y_t$ ,  $z_t$ , and, under IPT,  $\psi_t$  in the expressions above. The corresponding hatted WR reference values are  $\hat{y}_t^{w,SOE,\cdot}$ ,  $\hat{z}_t^{w,SOE,\cdot}$ , and  $\hat{\psi}_t^{w,SOE,IPT}$ . See Appendix D for details on these WR reference values.<sup>2</sup>

<sup>2</sup>The welfare cost function is normalized by steady-state marginal utility of consumption times steady-state consumption,  $U_C(C,N)C$ . Hence it measures welfare losses in units of steady-state consumption: for example,  $\mathcal{L}_t = 0.01$  means, locally, a welfare loss equivalent to a one-percent loss of steady-state consumption. With the logarithmic utility used here,  $U_C(C,N)C = 1$ .

The contrast between the two expressions shows how the open-economy relative-price externality enters the welfare criterion. Under PPT, the LOOP holds identically,  $\Psi_t = 1$ , so the relative-price externality is carried by the WR *ex-tariff* relative border price of imports gap  $z_t - \hat{z}_t^{w,SOE,PPT}$ . Under IPT,  $P_{F,t}/P_{H,t} = Z_t/\Psi_t$ , and the welfare criterion contains both the WR *ex-tariff* relative border price of imports gap  $z_t - \hat{z}_t^{w,SOE,IPT}$  and the WR LOOP-gap deviation  $\psi_t - \hat{\psi}_t^{w,SOE,IPT}$ . A nominal exchange-rate movement can therefore change the *ex-tariff* relative border price  $Z_t$  while, because the local-currency import price  $P_{F,t}$  adjusts only gradually, also generating a movement in the LOOP gap  $\Psi_t$ . The same relative-price externality is thus manifested jointly through  $z_t$  and  $\psi_t$  under IPT. The LOOP-gap movement additionally creates an importer-pricing distortion transmitted through the importer Calvo block.

Complete international risk sharing together with Home goods-market clearing yield

$$z_t = \frac{1}{\eta + (1-\eta)(1-v)^2} y_t - \frac{v(1-v)(\eta-1)}{\eta + (1-\eta)(1-v)^2} \tau_{M,t} + \frac{v(1-v)(\eta-1)}{\eta + (1-\eta)(1-v)^2} \psi_t,$$

which follows from log-linearizing Eqs. (12) and (19) and permits  $z_t$  to be eliminated from this same SOE welfare criterion. Under PPT,  $\Psi_t = 1$ , so the welfare-relevant movement in  $z_t$  is absorbed into the WR output term. The same period welfare cost can therefore be written as

$$\mathcal{L}_t^{SOE,PPT} \equiv \frac{1}{2} \left[ \Lambda_y^{SOE,PPT} (y_t - y_t^{w,SOE,PPT})^2 + \Lambda_{\pi,H}^{SOE,PPT} \pi_{H,t}^2 \right], \quad (34)$$

whereas under IPT the same elimination of  $z_t$  leaves the independently moving LOOP gap generated by sticky local import pricing. The same period welfare cost can be written as

$$\begin{aligned} \mathcal{L}_t^{SOE,IPT} \equiv & \frac{1}{2} \left[ \Lambda_y^{SOE,IPT} (y_t - y_t^{w,SOE,IPT})^2 + \Lambda_{\psi}^{SOE,IPT} (\psi_t - \psi_t^{w,SOE,IPT})^2 \right. \\ & \left. + \Lambda_{\pi,H}^{SOE,IPT} \pi_{H,t}^2 + \Lambda_{\pi,F}^{SOE,IPT} \pi_{F,t}^2 \right]. \end{aligned} \quad (35)$$

The objects  $y_t^{w,SOE,PPT}$  and  $y_t^{w,SOE,IPT}$  are the (logarithmic) WR outputs in the PPT and IPT environments, respectively, and  $y_t - y_t^{w,SOE,PPT}$  and  $y_t - y_t^{w,SOE,IPT}$  are the corresponding WR output gaps. Under the IPT environment,  $\psi_t^{w,SOE,IPT}$  is the (logarithmic) WR LOOP gap. Accordingly,  $\psi_t - \psi_t^{w,SOE,IPT}$ , which measures the deviation of the actual LOOP gap from its welfare-relevant value, is referred to as the WR LOOP-gap deviation. See Appendix D for details on  $y_t^{w,SOE,PPT}$ ,  $y_t^{w,SOE,IPT}$ , and  $\psi_t^{w,SOE,IPT}$ . Thus Eq. (34) contains no separate relative-price term, while Eq. (35) retains the WR LOOP-gap deviation and the import-inflation term. These are the reduced-form traces of the fact that, before the SOE reduction, the relative-price externality is carried by  $z_t$  alone under PPT but jointly by  $z_t$  and  $\psi_t$  under IPT.

### 3.5.2 WR LOOP gap

For the tariff experiment, the definition of the WR LOOP gap in the SOE is

$$\psi_t^{w,SOE,IPT} = \Gamma_{\text{own}}^{SOE} \tau_{M,t}, \quad (36)$$

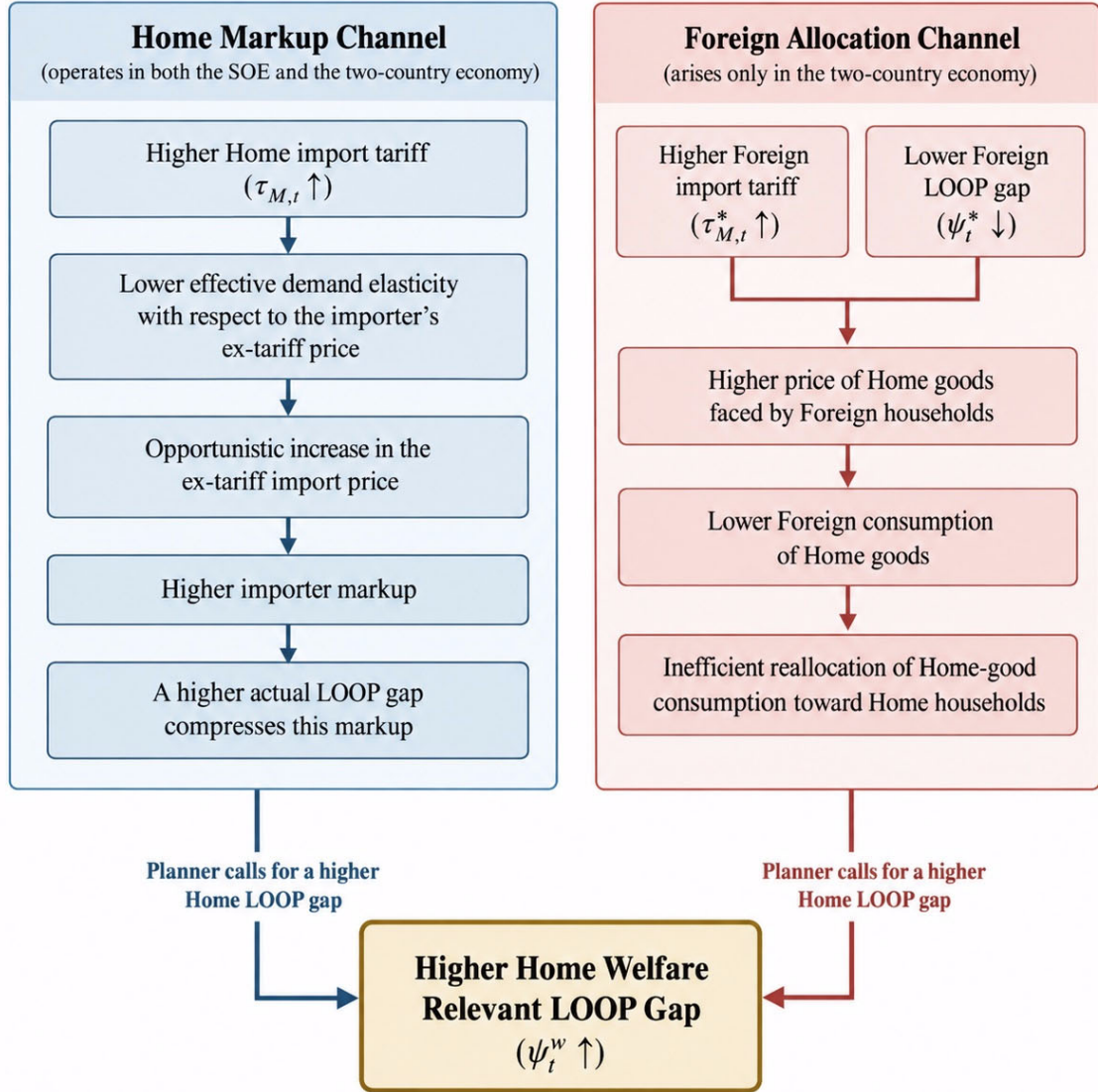
where  $\Gamma_{\text{own}}^{SOE}$  denotes the structural loading of the Home import tariff on the WR LOOP gap in the SOE. See Appendix D for details on  $\Gamma_{\text{own}}^{SOE}$ . Under the benchmark parameterization,  $\Gamma_{\text{own}}^{SOE} = 0.418$ . Equation (31) shows that a tariff raises the private flexible-price importer markup and lowers the corresponding LOOP gap. By contrast, Eq. (36) shows that the WR LOOP gap rises with the tariff: a higher  $\psi_t$  compresses the importer's *ex-tariff* markup.

Why does the tariff shift the WR LOOP gap? In the zero-tariff steady state, the subsidy  $\tau = 1/\varepsilon_M$  removes the importer markup that would otherwise arise from monopolistic competition and implies  $\Psi = 1$ . As discussed in Sec. 3.2.2, however, an increase in the import tariff lowers the effective elasticity of demand with respect to the importer's own *ex-tariff* price. The tariff

therefore creates an opportunity for an opportunistic increase in that price and generates a new tariff-induced importer markup. Since the subsidy remains fixed at its steady-state value rather than adjusting with the tariff, it does not eliminate the tariff-induced increase in the importer markup. Although the resulting importer profits are rebated lump sum to the household, the rebate does not remove the marginal distortion created by the markup. Since  $\Psi_t \equiv \mathcal{E}_t P_{F,t}^* / P_{F,t}$  moves inversely with the importer's *ex-tariff* price relative to the border acquisition price, raising  $\Psi_t$  compresses the tariff-induced markup. The planner therefore has an incentive to raise the WR LOOP gap when the tariff increases, as Eq. (36) shows (HMC, Fig. 1).

Table 2: SOE welfare weights: benchmark values

| Environment | WR Variable                   | Weight                      | Value  |
|-------------|-------------------------------|-----------------------------|--------|
| PPT         | $y_t - y_t^{w,SOE,PPT}$       | $\Lambda_y^{SOE,PPT}$       | 1.678  |
|             | $\pi_{H,t}$                   | $\Lambda_{\pi,H}^{SOE,PPT}$ | 39.873 |
| IPT         | $y_t - y_t^{w,SOE,IPT}$       | $\Lambda_y^{SOE,IPT}$       | 1.678  |
|             | $\psi_t - \psi_t^{w,SOE,IPT}$ | $\Lambda_\psi^{SOE,IPT}$    | 0.497  |
|             | $\pi_{H,t}$                   | $\Lambda_{\pi,H}^{SOE,IPT}$ | 39.873 |
|             | $\pi_{F,t}$                   | $\Lambda_{\pi,F}^{SOE,IPT}$ | 26.568 |



The Home Markup Channel operates in both the SOE and the two-country economy, whereas the Foreign Allocation Channel arises only in the two-country economy.

Figure 1: Determinants of the Home WR LOOP gap

*Note:* The HMC operates in both the SOE and the two-country economy, whereas the FAC arises only in the two-country economy.

Engel [23] interprets deviations from the law of one price under LCP as currency misalignment and a source of welfare loss. In our setting, log-linearizing the definition of the LOOP gap yields

$$\psi_t = e_t + p_{F,t}^* - p_{F,t},$$

with  $e_t \equiv \log \mathcal{E}_t$ ,  $p_{F,t} \equiv \log P_{F,t}$ , and  $p_{F,t}^* \equiv \log P_{F,t}^*$ . Define the WR nominal exchange rate by

$$e_t^{w,SOE,IPT} \equiv p_{F,t} - p_{F,t}^* + \psi_t^{w,SOE,IPT}. \quad (37)$$

It follows immediately that

$$\psi_t - \psi_t^{w,SOE,IPT} = e_t - e_t^{w,SOE,IPT}. \quad (38)$$

Hence, following Engel's currency-misalignment interpretation, reducing the WR LOOP-gap deviation is exactly equivalent to reducing the nominal exchange-rate misalignment, measured by the deviation of the actual nominal exchange rate from the WR nominal exchange rate.

### 3.6 Nominal Interest Rate Responses

Figure 2(a) shows the IRFs following a 20-percentage-point increase in the import tariff. The blue solid lines report the IRFs under the IPT environment, whereas the green dotted lines report those under the PPT environment. On impact, the relative consumption of imports falls from its steady-state value,  $v/(1-v) = 0.3/0.7 \simeq 0.43$ , under both pass-through environments (Panel 8). Under PPT, Eq. (8) shows that the tariff raises the *cum-tariff* relative price of imports and induces expenditure switching toward Home goods. The resulting increase in demand for Home goods raises real marginal cost and thereby puts upward pressure on Home PPI inflation. As Table 2 shows, Home PPI inflation carries the largest quadratic welfare weight. The social planner therefore raises the nominal interest rate under PPT (Panel 1); that is, monetary tightening is optimal in this environment.

Under IPT, stabilizing Home PPI inflation remains important, as Table 2 shows. The planner, however, also faces a tariff-induced importer-markup distortion. As discussed in Secs. 3.1 and 3.5, an increase in the import tariff lowers the effective elasticity of demand with respect to the importer's own *ex-tariff* price, thereby creating an opportunity for an opportunistic price increase. The steady-state importer subsidy,  $\tau = 1/\varepsilon_M$ , eliminates the importer-markup distortion in the zero-tariff steady state, but, because it does not adjust with the tariff, it does not eliminate the tariff-induced increase in the importer markup. Because  $\Psi_t = \mathcal{E}_t P_{F,t}^*/P_{F,t}$  moves inversely with this *ex-tariff* importer markup, mitigating the tariff-induced markup requires a higher LOOP gap (HMC, Fig. 1). Accordingly, Eq. (36) implies that the WR LOOP gap rises with the import tariff (Panel 10). When the local-currency import price adjusts only gradually, nominal depreciation raises the actual LOOP gap. The planner therefore lowers the nominal interest rate to induce depreciation and move the LOOP gap toward its welfare-relevant value (Panel 1). Thus monetary accommodation is chosen under IPT.

At first sight, Panels 9 and 10 of Fig. 2(a) may appear inconsistent: the WR LOOP gap rises, whereas the actual LOOP gap falls. Figure 2(b) resolves this apparent inconsistency by comparing the optimal IPT IRFs with a counterfactual in which the PPT nominal-interest-rate path is imposed on the IPT private equilibrium. The blue solid lines are the optimal IPT IRFs, identical to the blue solid lines in Fig. 2(a), whereas the orange dashed lines are the counterfactual IRFs. Hence the orange dashed line in Panel 1 of Fig. 2(b) is identical to the green dotted line in Panel 1 of Fig. 2(a). The LOOP gap under the optimal IPT policy lies above the counterfactual LOOP gap. Thus monetary accommodation raises the LOOP gap relative to the path that would prevail under PPT monetary policy; it only partially offsets the tariff-induced decline in the LOOP gap rather than reversing it.

The remaining large gap between the actual and WR LOOP gaps reflects the importance of stabilizing Home PPI inflation even under IPT. Further monetary accommodation would induce

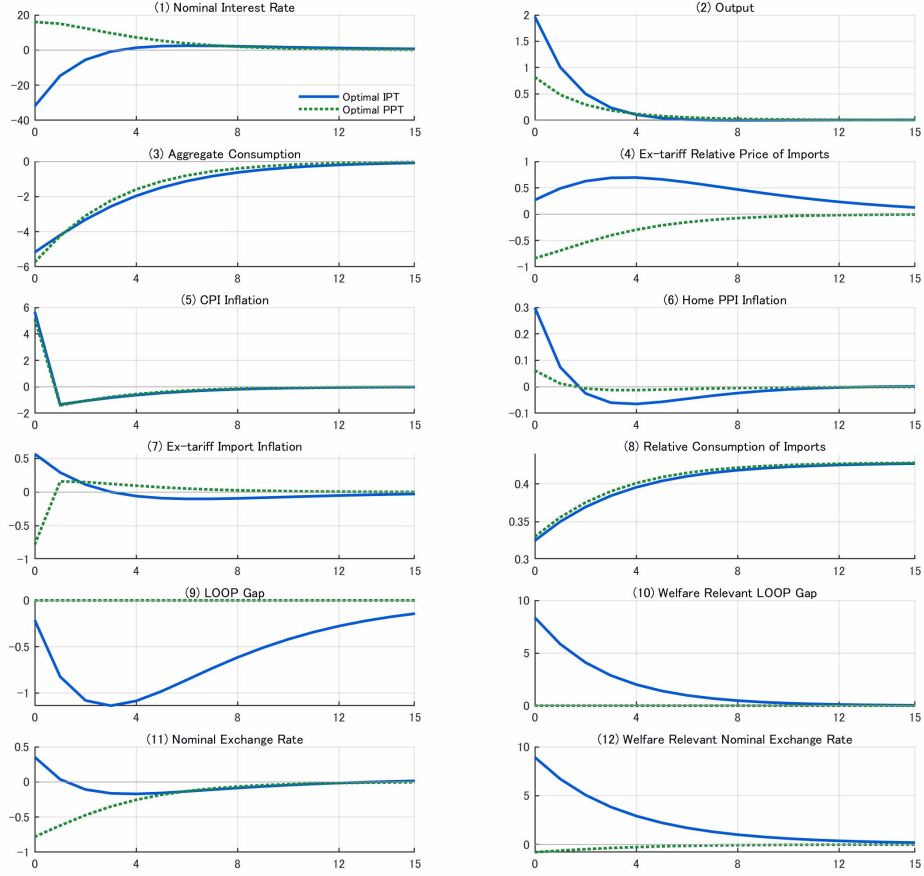


Figure 2: SOE: (a) optimal PPT and IPT responses. The blue solid line reports the optimal IPT allocation and the green dotted line the optimal PPT allocation. Panel 1 reports the deviation of the nominal interest rate from its deterministic steady state in basis points; the other panels report percentage deviations or inflation rates in percentage points. Under PPT,  $e_t^{w,SOE,PPT} = e_t$  because the LOOP holds identically, so the green dotted paths in Panels 11 and 12 coincide.

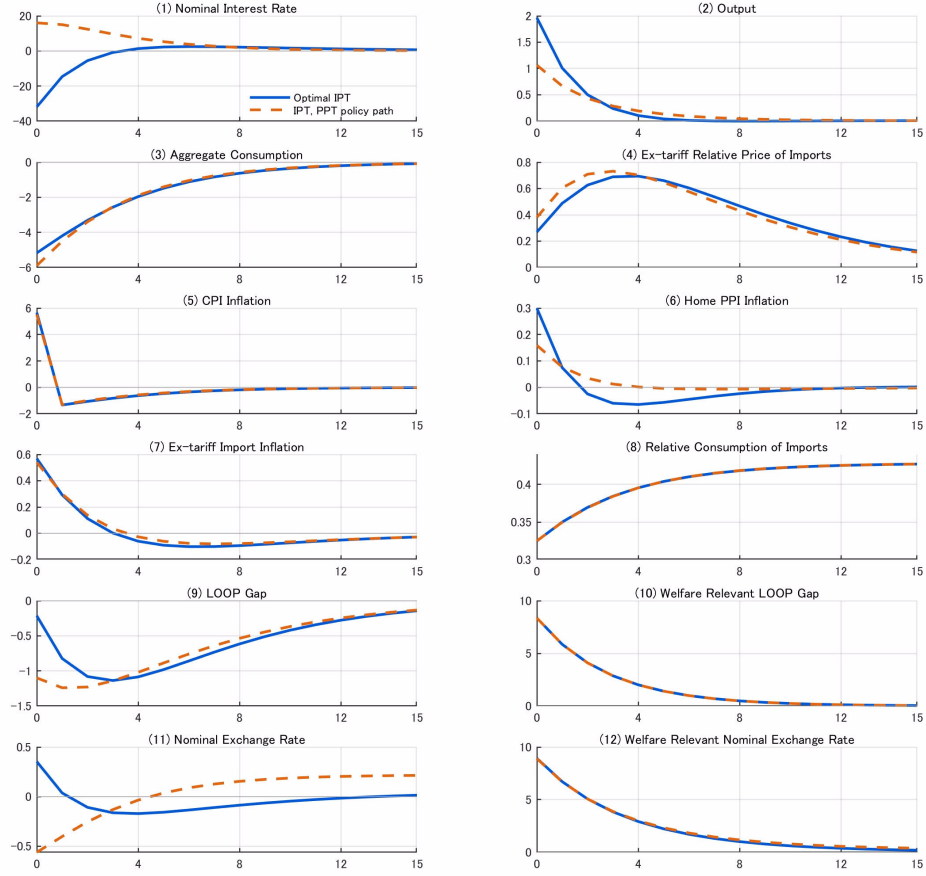


Figure 2: SOE: (b) optimal IPT response and the IPT counterfactual under the PPT nominal-interest-rate path. The blue solid line reports the optimal IPT allocation and the orange dashed line the IPT allocation when the nominal-interest-rate path is fixed at the optimal PPT path.

additional nominal depreciation and raise the LOOP gap, thereby reducing the WR LOOP-gap deviation. At the same time, however, it would put upward pressure on real marginal cost and Home PPI inflation. Given the large welfare weight on Home PPI inflation in Table 2, the planner stops short of eliminating the WR LOOP-gap deviation. Consequently, the actual LOOP gap remains negative even under the optimal IPT policy.

Table 3 evaluates each component of Eq. (35) along the optimal allocation and the nonlinear counterfactual allocation. It also reports the exact total welfare cost obtained by evaluating the original nonlinear lifetime utility in Eq. (1) directly along the same allocations. The entries are discounted sums over the 60-quarter reporting horizon; extending the sum further has a negligible effect because the tariff and the endogenous responses have essentially returned to steady state.

Table 3: SOE IPT environment: discounted welfare-cost components under the optimal and counterfactual PPT interest-rate paths.

| WR Variable                   | PPT-rate CF | Optimal IPT | Difference |
|-------------------------------|-------------|-------------|------------|
| $y_t - y_t^{w,SOE,IPT}$       | 0.0007      | 0.0063      | +0.0056    |
| $\psi_t - \psi_t^{w,SOE,IPT}$ | 0.5053      | 0.4520      | -0.0533    |
| $\pi_{H,t}$                   | 0.0065      | 0.0220      | +0.0155    |
| $\pi_{F,t}$                   | 0.0583      | 0.0637      | +0.0054    |
| Quadratic total               | 0.5708      | 0.5440      | -0.0268    |
| Exact total welfare cost      | 25.7213     | 25.7015     | -0.0198    |

*Note:* All entries are in percent of steady-state consumption. The component rows and “Quadratic total” are discounted sums from the second-order welfare cost function in Eq. (35). “Exact total welfare cost” evaluates the original nonlinear utility in Eq. (1) directly along each allocation, relative to the deterministic steady-state utility path. Because the second-order criterion suppresses terms independent of the endogenous policy allocation, its level is not directly comparable with the exact total welfare-cost level; the relevant comparison is the difference between the optimal and counterfactual allocations. Exact nonlinear welfare has no source-by-source decomposition corresponding to the completed-square rows.

The counterfactual confirms the mechanism. The reduction in the WR LOOP-gap deviation component, 0.0533% in welfare-cost units, is larger than the net reduction in the quadratic welfare criterion because the optimal easing raises the WR output gap and the two sticky-inflation contributions. Even after these offsetting costs, the quadratic welfare criterion falls by 0.0268% of steady-state consumption. The last row of Table 3 shows the exact total welfare cost. While there is a gap between the exact and quadratic total welfare costs, both welfare costs are lower under the optimal IPT allocation than under the counterfactual. Therefore, it can be said that monetary easing reduces welfare costs.

## 4 Two-Country Model

### 4.1 The Model

The small-open-economy model in Sec. 3 takes the Foreign economy as given. We now endogenize that economy and embed the SOE structure in a unit-mass two-country world. Country size is represented by household mass: households are indexed by  $h \in [0, 1]$ , with  $h \in [0, \nu)$  in country  $H$  and  $h \in [\nu, 1]$  in country  $F$ . Because steady-state per-capita output is common across the two countries,  $\nu$  is also Home’s share of steady-state world output and is therefore calibrated to Home’s share of world gross domestic product (GDP). Producer varieties retain the country-specific unit-continuum normalization used in the SOE. To make country-size accounting transparent in the bilateral importer block, Home importer varieties are indexed by  $\iota \in [0, \nu)$  and Foreign importer varieties by  $\iota \in [\nu, 1]$ . Importer CES integrals are normalized by the corresponding country mass,  $\nu$  or  $1 - \nu$ . Hence the country-size ratios in bilateral goods-market clearing convert per-capita demand across countries, while the normalized importer integrals take averages within the importing country. Households are identical within each country, so we continue to write the Home allocation in representative-household form. The Home household retains the preferences in Eq. (1), while

the Foreign representative household has lifetime utility

$$\mathcal{U}^* \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \log C_t^* - \frac{(N_t^*)^{1+\varphi}}{1+\varphi} \right).$$

Here  $C_t^*$  and  $N_t^*$  denote Foreign composite consumption and labor, respectively. The Foreign household maximizes  $\mathcal{U}^*$  subject to the exact starred counterparts of the Home household constraints and optimality conditions.

The two-country extension makes endogenous the Foreign objects that were external to the SOE transmission mechanism. We retain the SOE notation for Home. Country  $F$  now has its own consumption, labor, output, producer prices, Calvo price dispersion, and monetary policy. Each country is specialized in a continuum of differentiated goods produced with the same technology and producer Calvo structure as in Sec. 3.1. Home and Foreign have separate monetary authorities with nominal interest rates  $i_t$  and  $i_t^*$ , respectively, and Home and Foreign governments may levy import tariffs  $\tau_{M,t}$  and  $\tau_{M,t}^*$ , respectively. Tariff revenue is rebated lump sum in the country that collects it. Complete international financial markets link the two national household problems and impose international risk sharing. Hence a Home policy or tariff change can alter the Foreign allocation and, through the common world equilibrium, feed back to Home.

Home bias is introduced by scaling each country's expenditure weight on partner-country goods by the common openness parameter  $v$ . Given the country sizes above, the Home and Foreign expenditure weights are  $\omega_F = v(1 - \nu)$ ,  $\omega_H = 1 - \omega_F$ ,  $\omega_H^* = v\nu$ , and  $\omega_F^* = 1 - \omega_H^*$ . Here  $\omega_F$  is Home's expenditure weight on Foreign goods and  $\omega_H$  is Home's expenditure weight on Home goods. Likewise,  $\omega_H^*$  is Foreign's expenditure weight on Home goods and  $\omega_F^*$  is Foreign's expenditure weight on Foreign goods. Thus  $\omega_F$  and  $\omega_H^*$  are the bilateral import weights, whereas  $\omega_H$  and  $\omega_F^*$  are the domestic-good weights. For  $v < 1$ , the bilateral import weights are smaller than the corresponding partner-country economic-size shares, generating home bias.

These expenditure weights enter the two national consumption baskets:

$$\begin{aligned} C_t &= \left[ \omega_H^{1/\eta} C_{H,t}^{(\eta-1)/\eta} + \omega_F^{1/\eta} C_{F,t}^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}, \\ C_t^* &= \left[ (\omega_F^*)^{1/\eta} (C_{F,t}^*)^{(\eta-1)/\eta} + (\omega_H^*)^{1/\eta} (C_{H,t}^*)^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}. \end{aligned} \quad (39)$$

where  $C_{F,t}^*$  and  $C_{H,t}^*$  denote Foreign consumption of Foreign and Home goods, respectively. Bilateral goods-market clearing therefore replaces the SOE assumption that Foreign demand can be treated as an external object.

In a two-country model, the LOOP also applies explicitly to Home goods. Under PPT it is

$$P_{H,t} = \mathcal{E}_t P_{H,t}^*,$$

where  $P_{H,t}^*$  denotes the Home-goods price in Foreign currency. Define the Foreign LOOP gap by

$$\Psi_t^* \equiv \frac{P_{H,t}/\mathcal{E}_t}{P_{H,t}^*}, \quad \psi_t^* \equiv \log \Psi_t^*.$$

Using  $S_t \equiv P_{F,t}/P_{H,t}$ , the Home *ex-tariff* relative price of imports can therefore be written as

$$S_t = \frac{P_{F,t}}{\mathcal{E}_t P_{H,t}^*} \frac{1}{\Psi_t^*},$$

where  $P_{F,t}/(\mathcal{E}_t P_{H,t}^*)$  is the *ex-tariff* relative consumer price of imports to exports. Under the PPT environment, only PCP producers are present and the LOOP holds in both directions, so  $\Psi_t = \Psi_t^* = 1$ . Under the IPT environment, local importers using LCP operate in both countries, so  $\Psi_t$  and  $\Psi_t^*$  are determined endogenously. Home importers retain the flexible-price markup condition in Eq. (31) and the first-order importer Phillips curve in Eq. (32). Since Foreign

importers of Home goods face the corresponding monopolistically competitive pricing problem, their flexible-price condition is

$$(\Psi_t^*)^{-1} = \frac{P_{H,t}^*}{P_{H,t}/\mathcal{E}_t} = 1 + \frac{\tau_{M,t}^*}{\varepsilon_M - 1}, \quad (40)$$

and their first-order importer Phillips curve under sticky local-currency prices is

$$\pi_{H,t}^* = \beta \mathbb{E}_t \pi_{H,t+1}^* + \kappa_M \left( \psi_t^* + \frac{\tau_{M,t}^*}{\varepsilon_M - 1} \right).$$

where  $\pi_{H,t}^* \equiv \log(P_{H,t}^*/P_{H,t-1}^*)$ . Thus the same tariff-induced opportunistic price increase can arise in either importing country. We denote the Foreign CPI by  $P_t^{*,PPT}$  in the PPT environment and by  $P_t^{*,IPT}$  in the IPT environment. Under PPT, the Home relative consumption of imports is determined by

$$\frac{C_{F,t}}{C_{H,t}} = \frac{\omega_F}{\omega_H} [(1 + \tau_{M,t}) S_t]^{-\eta}. \quad (42)$$

The bilateral goods-market conditions follow directly from variety-level clearing. For a Home producer variety  $j$ , domestic and export demand imply

$$Y_t(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \left[ C_{H,t} + \frac{1}{\nu} \int_{\nu}^1 C_{H,t}^*(\iota) d\iota \right]. \quad (43)$$

Aggregating Eq. (43) with the producer CES index gives the exact bilateral goods-market conditions

$$\begin{aligned} Y_t &= C_{H,t} + \left( \frac{1-\nu}{\nu} \right) C_{H,t}^* \frac{1}{1-\nu} \int_{\nu}^1 \left( \frac{\tilde{P}_{H,t}^*(\iota)}{\tilde{P}_{H,t}^*} \right)^{-\varepsilon_M} d\iota, \\ Y_t^* &= C_{F,t}^* + \left( \frac{\nu}{1-\nu} \right) C_{F,t} \frac{1}{\nu} \int_0^{\nu} \left( \frac{\tilde{P}_{F,t}^{IPT}(\iota)}{\tilde{P}_{F,t}^{IPT}} \right)^{-\varepsilon_M} d\iota. \end{aligned} \quad (44)$$

where  $C_{H,t}^*(\iota)$  denotes Foreign-country demand for Home goods sold by local importer variety  $\iota$ , and  $\tilde{P}_{H,t}^*(\iota)$  is the Foreign mirror of the variety-level *cum-tariff* purchaser price in Eq. (20). The corresponding aggregate *cum-tariff* import-price index is

$$\tilde{P}_{H,t}^* \equiv \left[ \frac{1}{1-\nu} \int_{\nu}^1 \left( \tilde{P}_{H,t}^*(\iota) \right)^{1-\varepsilon_M} d\iota \right]^{1/(1-\varepsilon_M)}.$$

The exact Foreign mirror applies to a Foreign producer variety. Producer-price dispersion affects labor requirements through Eq. (13), whereas the two integrals in Eq. (44) are physical-quantity factors implied by conditional import demand evaluated at *cum-tariff* purchaser prices. They are not producer-price-dispersion terms; they arise from aggregating physical import quantities across importer varieties.

The two-country economy makes explicit the step from the SOE by endogenizing the Foreign economy. A Home tariff first changes Home absorption, the Home LOOP gap, and Home importer marginal cost. Because Foreign demand and production are now endogenous, the same disturbance also changes Foreign consumption, output, producer and importer prices, and the Foreign policy rate. Those Foreign responses alter Home export demand and international relative prices and therefore feed back into the Home allocation. The resulting spillover and spillback are thus generated inside the common two-country equilibrium rather than appended as reduced-form disturbances.

## 4.2 National Ramsey policy under commitment

The baseline two-country policy game follows the national Ramsey approach of Faia and Monacelli [25]. Complete international risk sharing and the relevant goods-market relations are imposed before the national first-order conditions are formed. Hence the international relative price is eliminated before national differentiation and is not a choice variable of either national planner.

For the Ramsey differentiation we substitute the private producer reset-price condition and, in the IPT environment, the exact importer reset-price condition before taking national first-order conditions. In the PPT environment, the Home planner therefore chooses the paths of  $C_t$ ,  $\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)$ ,  $N_t$ ,  $Y_t$ ,  $\Pi_{H,t}$ ,  $\Delta_{H,t}$ , and  $i_t$ , subject to the producer reset-price map and all remaining private-equilibrium restrictions. In the IPT environment,  $S_t$  and  $\Pi_{F,t}$  are additionally endogenous in the Home best response. The complete cross-sectional importer-price distribution continues to obey the exact Calvo updating and purchaser-price definitions in Eqs. (20)–(30); it is not replaced by an aggregate multiplicative-tariff shortcut. In particular,  $Z_t$  is not a Home Ramsey choice variable.

The Foreign planner solves the exact Home–Foreign mirror problem. Its choice set is the starred counterpart of the preceding Home set, with its own local *ex-tariff* import-price ratio and importer inflation added under IPT. No additional international-relative-price variable is introduced into either national choice set.

The Home social planner commits to a complete contingent Home plan and maximizes  $\mathcal{U}$ , taking the complete contingent Foreign plan as given and subject to the common nonlinear two-country private-equilibrium system. The Foreign social planner symmetrically maximizes  $\mathcal{U}^*$ , taking the Home contingent plan as given and subject to the same common private-equilibrium system. The nominal interest rates  $i_t$  and  $i_t^*$  are choice variables and are subject to the ZLB in Home and Foreign, respectively.

The national Ramsey first-order conditions are derived from these nonlinear problems. The resulting nonlinear Home and Foreign Ramsey first-order-condition systems, together with the common nonlinear two-country private-equilibrium system, are solved jointly at their mutual best response. This nonlinear Nash–Ramsey fixed point generates all two-country policy paths reported below. The national Ramsey Lagrangians and first-order conditions, including the ZLB Kuhn–Tucker conditions, are reported in On-line Appendix B. As in the SOE, the second-order welfare loss functions are not solved to obtain the policy allocation; they are introduced only when interpreting the nominal-interest-rate impulse responses.

## 4.3 Calibration and shocks

For the two-country numerical evaluation we retain the benchmark parameterization in Table 1 and set the Home economic-size share to  $\nu = 0.27$  to match the U.S. share of world GDP at current prices in U.S. dollars, using the April 2026 International Monetary Fund (IMF) World Economic Outlook Database [36]. Together with  $\nu = 0.30$ , this implies Home’s import expenditure weight  $\omega_F = \nu(1 - \nu) = 0.219$ . This 21.9 percent share is close to the 22.3 percent average ratio of U.S. imports of goods and services to personal consumption expenditures over 2000–2025 [42]. These values are used in the two-country welfare weights and in the benchmark-parameterization IRFs reported below.

## 4.4 Welfare Cost Function and WR LOOP gap

### 4.4.1 Welfare Cost Function

As in the SOE, the policy allocation is obtained from the nonlinear national Ramsey first-order-condition systems described in Sec. 4.2, not from the second-order welfare representation below. The second-order welfare representation is used only to identify the WR variables and to interpret

the nominal-interest-rate responses. In the PPT environment, the Home period welfare cost is

$$\mathcal{L}_{H,t}^{PPT} \equiv \frac{1}{2} \left[ \Lambda_y^{PPT} (y_t - y_t^{w,PPT})^2 + \Lambda_z^{PPT} (z_t - z_t^{w,PPT})^2 + \Lambda_{\pi,H}^{PPT} \pi_{H,t}^2 \right], \quad (45)$$

whereas in the IPT environment it is

$$\begin{aligned} \mathcal{L}_{H,t}^{IPT} \equiv & \frac{1}{2} \left[ \Lambda_y^{IPT} (y_t - y_t^{w,IPT})^2 + \Lambda_\psi^{IPT} (\psi_t - \psi_t^{w,IPT})^2 + \Lambda_z^{IPT} (z_t - z_t^{w,IPT})^2 \right. \\ & + \Lambda_{\psi^*}^{IPT} (\psi_t^* - \psi_t^{*,w,IPT})^2 + \Lambda_{\pi,H}^{IPT} \pi_{H,t}^2 + \Lambda_{\pi,F}^{IPT} \pi_{F,t}^2 \\ & \left. + \Lambda_{\pi_H^*}^{IPT} (\pi_{H,t}^*)^2 \right]. \end{aligned} \quad (46)$$

In Eqs. (45)–(46),  $y_t^{w,\cdot}$  is the (logarithmic) WR output and  $y_t - y_t^{w,\cdot}$  is the WR output gap. Likewise,  $z_t^{w,\cdot}$  is the (logarithmic) WR *ex-tariff* relative border price of imports and  $z_t - z_t^{w,\cdot}$  is the WR *ex-tariff* relative border price of imports gap. In the IPT environment,  $\psi_t^{w,IPT}$  is the (logarithmic) Home WR LOOP gap and  $\psi_t - \psi_t^{w,IPT}$  is the Home WR LOOP-gap deviation. The object  $\psi_t^{*,w,IPT}$  is the WR reference value attached to the Foreign LOOP gap  $\psi_t^*$  in the *Home* welfare cost function. It is therefore distinct from the Foreign planner’s own WR LOOP gap, denoted below by  $\psi_t^{w*,IPT}$ , which is obtained from the exact Home–Foreign mirror of the national welfare criterion. See Appendices D.2.1 and D.2.2 for details on  $y_t^{w,PPT}$ ,  $z_t^{w,PPT}$ ,  $y_t^{w,IPT}$ ,  $\psi_t^{w,IPT}$ ,  $z_t^{w,IPT}$ , and  $\psi_t^{*,w,IPT}$ .

Under PPT, the LOOP holds and Home welfare contains no LOOP-gap term. Under IPT, the Home tariff creates the same additional importer-markup distortion as in the SOE: it provides an opportunity for an opportunistic increase in importers’ *ex-tariff* prices. The subsidy eliminates the importer-markup distortion in the zero-tariff steady state but does not adjust with the tariff, so the tariff-induced increase in the importer markup remains. Because  $\Psi_t$  moves inversely with the Home importer markup, the Home planner has an incentive to raise the WR LOOP gap to mitigate this additional distortion. In the two-country equilibrium, its exact reference value also depends on the international relative price and the Foreign LOOP gap, as shown in Eq. (47).

The distinction from the SOE is representational as well as economic. In the SOE, complete international risk sharing and Home goods-market clearing permit  $z_t$  to be eliminated from the reduced welfare representation. In the two-country economy,  $z_t$  remains explicit, and movements in Home relative demand feed back into Foreign demand and the common two-country equilibrium. Nevertheless, the LOOP-gap terms have a useful nominal-exchange-rate representation.

Table 4 reports the corresponding benchmark Home welfare weights in the two-country economy. The negative coefficients on the WR *ex-tariff* relative border price of imports gap term,  $\Lambda_z^{PPT} = -0.0920$  and  $\Lambda_z^{IPT} = -0.0978$ , reflect the same strategic relative-price consideration. In the national Ramsey problem, relative-price movements can shift expenditure toward Home goods, support Home output and income, and thereby improve Home welfare. This is the national relative-price externality. The negative sign therefore represents a signed strategic welfare contribution rather than a conventional positive stabilization penalty. Accordingly, we use the term “stabilization objective” for the positive-weight quadratic terms; the negative-weight completed-square term is reported as a signed strategic welfare contribution. This does not make the relative price an independent Ramsey choice variable; it remains an equilibrium outcome that monetary policy can influence strategically.

The numerical weights in Table 4 are evaluations of the exact symbolic welfare coefficients. See Appendices D.2.1 and D.2.2.

#### 4.4.2 WR LOOP gap

The definition of the Home WR LOOP gap is given by:

$$\psi_t^{w,IPT} = \Gamma_{\text{own}} \tau_{M,t} + \Gamma_{\text{cross}} (\tau_{M,t}^* - \psi_t^*) + \Gamma_z z_t. \quad (47)$$

Table 4: Two-country Home welfare weights: benchmark values

| Environment | WR Variable                   | Weight                   | Value  |
|-------------|-------------------------------|--------------------------|--------|
| PPT         | $y_t - y_t^{w,PPT}$           | $\Lambda_y^{PPT}$        | 2.094  |
|             | $z_t - z_t^{w,PPT}$           | $\Lambda_z^{PPT}$        | -0.092 |
|             | $\pi_{H,t}$                   | $\Lambda_{\pi,H}^{PPT}$  | 47.810 |
| IPT         | $y_t - y_t^{w,IPT}$           | $\Lambda_y^{IPT}$        | 2.860  |
|             | $\psi_t - \psi_t^{w,IPT}$     | $\Lambda_\psi^{IPT}$     | 0.406  |
|             | $z_t - z_t^{w,IPT}$           | $\Lambda_z^{IPT}$        | -0.098 |
|             | $\psi_t^* - \psi_t^{*,w,IPT}$ | $\Lambda_{\psi^*}^{IPT}$ | 0.352  |
|             | $\pi_{H,t}$                   | $\Lambda_{\pi,H}^{IPT}$  | 65.313 |
|             | $\pi_{F,t}$                   | $\Lambda_{\pi,F}^{IPT}$  | 21.283 |
|             | $\pi_{H,t}^*$                 | $\Lambda_{\pi,H}^{IPT}$  | 13.519 |

Here  $\Gamma_{\text{own}}$  is the structural loading of the Home tariff,  $\Gamma_{\text{cross}}$  is the structural loading of the Foreign purchaser-price combination  $\tau_{M,t}^* - \psi_t^*$ , and  $\Gamma_z$  is the structural loading of the international relative price  $z_t$ . See Appendix D.2.2 for details on  $\Gamma_{\text{own}}$ ,  $\Gamma_{\text{cross}}$ , and  $\Gamma_z$ . Under the benchmark parameterization,  $\Gamma_{\text{own}} = 0.364$ ,  $\Gamma_{\text{cross}} = 0.060$ , and  $\Gamma_z = -0.016$ . The positive Home-tariff coefficient has the same importer-pricing origin as in the SOE. Equation (31) shows that the Home tariff raises the *ex-tariff* markup that a flexible-price Home importer would charge. The subsidy eliminates the importer-markup distortion in the zero-tariff steady state but, because it does not adjust with the tariff, does not eliminate the additional markup generated by opportunistic pricing. As in Eq. (36), the Home planner therefore has an incentive to raise its own WR LOOP gap to compress the Home importer markup (HMC, Fig. 1).

An increase in the Foreign import tariff  $\tau_{M,t}^*$  raises the price of Home goods faced by Foreign households and lowers their consumption of Home goods. Holding Home output and the other relative prices fixed, a larger share of Home goods is then consumed by Home households. A higher Home LOOP gap  $\psi_t$  lowers the price of Home goods relative to Foreign goods from the perspective of Foreign households and thereby raises their consumption of Home goods. The Home planner therefore calls for a higher Home WR LOOP gap to mitigate the resulting inefficient overconsumption of Home goods in country H (FAC, Fig. 1).

A decline in the Foreign LOOP gap  $\psi_t^*$  works in the same direction. It raises the price of Home goods faced by Foreign households, reduces their consumption of Home goods, and shifts the consumption of Home goods toward Home households. This inefficient overconsumption of Home goods in country H again calls for a higher Home WR LOOP gap. Hence, the Home WR LOOP gap is positively related to the Foreign import tariff and negatively related to the Foreign LOOP gap, as shown in Eq. (47) (FAC, Fig. 1).

Opportunistic import pricing also arises in Foreign. Equation (40) shows that a higher Foreign import tariff raises the *ex-tariff* markup charged by Foreign importers of Home goods, just as a Home tariff does for Home importers.

Accordingly, the Foreign planner has an incentive to raise the Foreign WR LOOP gap to mitigate that additional markup. The exact starred national-welfare mirror, expressed in the same Home international-relative-price coordinate  $z_t$ , can be written compactly as

$$\psi_t^{w*,IPT} = \Gamma_{\text{own}}^* \tau_{M,t}^* + \Gamma_{\text{cross}}^* (\tau_{M,t} - \psi_t) + \Gamma_z^* z_t. \quad (48)$$

where the starred  $\Gamma$  coefficients are the exact Home–Foreign mirrors of the corresponding Home loadings in Eq. (47). Under the benchmark parameterization,  $\Gamma_{\text{own}}^* = 0.328$ ,  $\Gamma_{\text{cross}}^* = 0.024$ , and  $\Gamma_z^* = 0.007$ . Thus the positive own-tariff term has the same opportunistic-importer-pricing interpretation as in the Home problem. The Home tariff and Home LOOP gap enter the Foreign planner’s reference value through the corresponding cross-country consumption-allocation effects: a higher Home tariff reduces Home consumption of Foreign goods and raises the Foreign WR LOOP gap, whereas a higher Home LOOP gap raises Home consumption of Foreign goods and lowers that reference value. The two national WR LOOP values, and hence  $e_t^w$  and  $e_t^{w*}$  in Eq.

(49), therefore generally differ.

Equation (47) also shows that the Foreign import tariff  $\tau_{M,t}^*$  and the Foreign LOOP gap  $\psi_t^*$  enter the Home WR LOOP gap with coefficients of equal absolute value and opposite signs. This equality follows because, in the reduced first-order representation, the two variables enter the Foreign household-facing purchaser-price combination as  $\tau_{M,t}^* - \psi_t^*$ . Hence a one-unit increase in the Foreign import tariff and a one-unit decline in the Foreign LOOP gap have the same marginal effect on the Home WR LOOP gap.

Even in the two-country economy, deviations from the LOOP under the IPT environment can be interpreted as currency misalignment, as discussed in Sec. 3.5. For the Home economy, Eqs. (37) and (38) continue to apply. Analogously, define the Foreign WR nominal exchange rate by

$$e_t^{w*} \equiv p_{H,t} - p_{H,t}^* - \psi_t^{w*,IPT}. \quad (49)$$

The log-linearized definition of the Foreign LOOP gap,  $\psi_t^* = p_{H,t} - e_t - p_{H,t}^*$ , then implies

$$\psi_t^* - \psi_t^{w*,IPT} = -(e_t - e_t^{w*}).$$

Thus the same bilateral nominal exchange rate  $e_t$  is compared with two country-specific WR nominal exchange rates. Holding the other welfare-relevant margins fixed, the Home WR LOOP-gap deviation is reduced as  $e_t$  moves toward  $e_t^w$ , whereas the Foreign WR LOOP-gap deviation is reduced as  $e_t$  moves toward  $e_t^{w*}$ . In general,  $e_t^w$  and  $e_t^{w*}$  do not coincide because the Home and Foreign planners face different national welfare trade-offs. Under PPT, both LOOP gaps vanish identically, so the corresponding WR nominal exchange rates coincide with the actual nominal exchange rate.

## 4.5 Nominal Interest Rate Responses

Figure 3(a) compares the two-country Home responses across the two pass-through environments. The blue solid lines are the IRFs under the IPT environment and the green dotted lines are the IRFs under the PPT environment.

The mechanism under the PPT environment is the same as in the SOE in Sec. 3.6. The Home tariff raises the *cum-tariff* relative price of imports and shifts Home expenditure toward Home goods, as Eq. (42) shows. The resulting rise in Home real marginal cost puts upward pressure on Home PPI inflation  $\pi_{H,t}$ . Because stabilizing  $\pi_{H,t}$  carries a large welfare weight, as shown in Table 4, the Home planner raises the nominal interest rate (Panel 1, Fig. 3(a)). Thus monetary tightening is optimal under PPT.

Under IPT, by contrast, the Home planner lowers the nominal interest rate to raise the Home LOOP gap  $\psi_t$  (Panel 1, Fig. 3(a)). Indeed, the Home WR LOOP gap increases (Panel 10, Fig. 3(a)). As in the SOE, the Home tariff induces an opportunistic increase in Home importers' *ex-tariff* markups. The subsidy eliminates the importer-markup distortion in the zero-tariff steady state but, because it does not adjust with the tariff, does not eliminate this tariff-induced increase in the importer markup. Because the LOOP gap moves inversely with this markup, a higher Home LOOP gap mitigates the additional tariff-induced markup (the HMC in Fig. 1). Nominal depreciation also lowers the Foreign LOOP gap. The resulting increase in the price of Home goods faced by Foreign households reduces their consumption of Home goods, shifting too much consumption of Home goods toward Home households. A higher Home LOOP gap is therefore required: by lowering the relative *ex-tariff* price of imports faced by Home households, it reduces Home consumption of Home goods and mitigates this inefficient reallocation (the FAC in Fig. 1). This channel therefore calls for an additional reduction in the nominal interest rate to raise the Home LOOP gap further. As shown in Table 4, stabilizing Home PPI inflation remains important. However, following the 20% increase in the Home import tariff, the planner also seeks to reduce the Home WR LOOP-gap deviation, although its weight on the welfare cost function is just 0.406. The planner therefore accepts higher Home PPI inflation under IPT than under PPT (Panel 6, Fig. 3(a)).

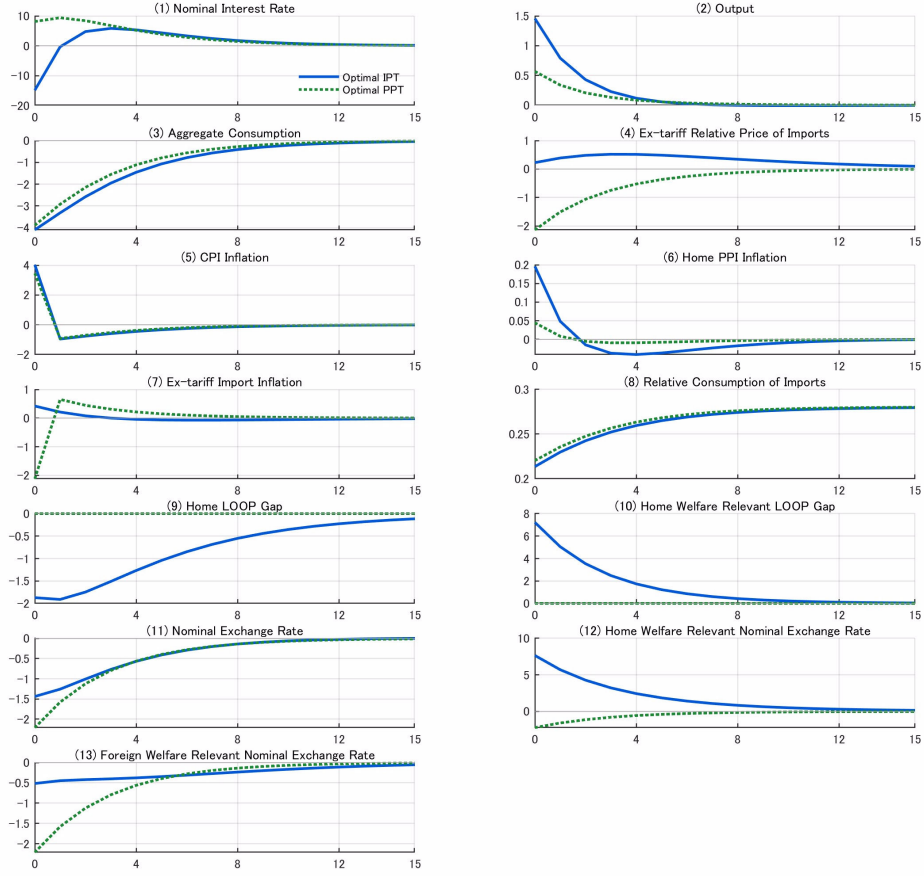


Figure 3: Two-country benchmark parameterization: (a) Home responses under the optimal PPT and IPT allocations. The solid line reports the optimal IPT allocation and the dotted line the optimal PPT allocation. Panel 1 reports deviations of the nominal interest rate from its deterministic steady state in basis points. Under PPT the two WR nominal exchange rates coincide with  $e_t$ .

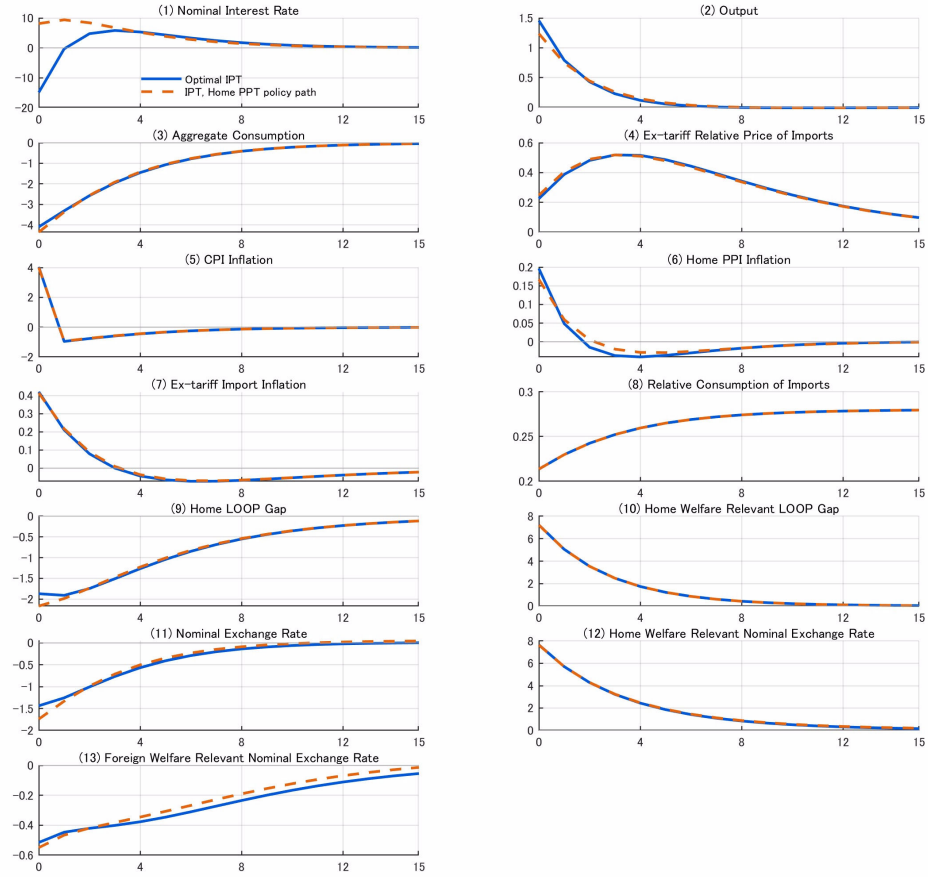


Figure 3: Two-country benchmark parameterization: (c) Home optimal IPT response and the IPT counterfactual under the Home PPT nominal-interest-rate path. The blue solid line reports the optimal IPT allocation and the orange dashed line the counterfactual. In the counterfactual, the Foreign IPT contingent plan is held fixed and the remaining Home private-equilibrium variables are recomputed.

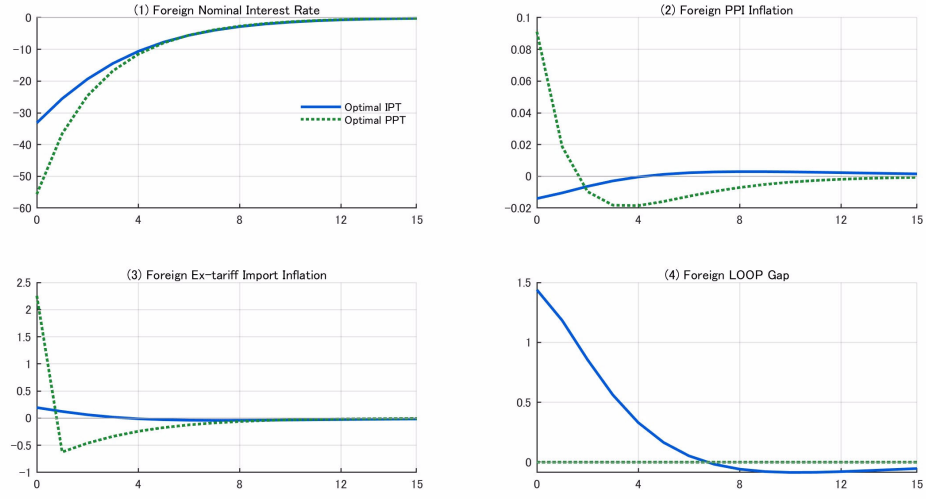


Figure 3: Two-country benchmark parameterization: (b) Foreign responses corresponding to the optimal allocations in Panel (a). The blue solid line reports the Foreign responses associated with the optimal Home IPT allocation and the green dotted line those associated with the optimal PPT allocation.

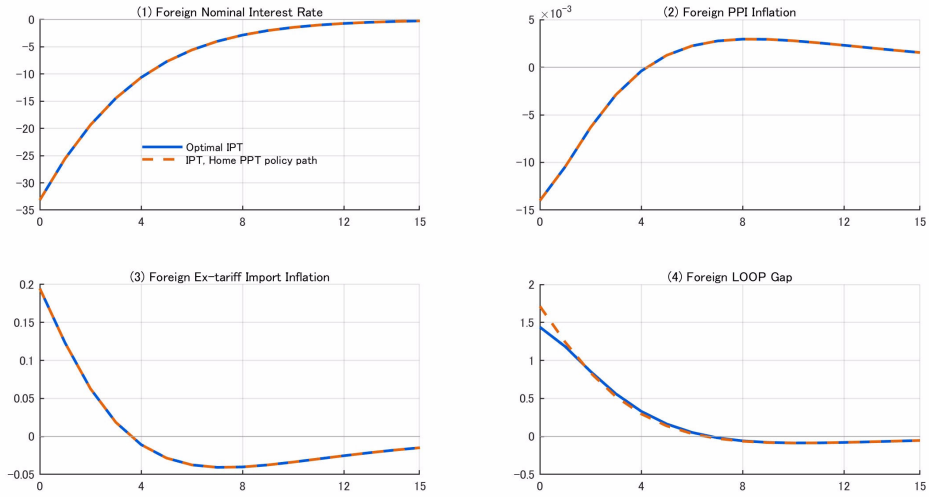


Figure 3: Two-country benchmark parameterization: (d) Foreign responses corresponding to the optimal and counterfactual Home allocations in Panel (c). The blue solid line reports the IPT allocation and the orange dashed line the counterfactual under the Home PPT nominal-interest-rate path. The Foreign contingent plan is held at its IPT path.

In equilibrium, however, the Home currency still appreciates and the Home LOOP gap remains negative (Panels 9 and 11, Fig. 3(a)). The Home tariff lowers Foreign demand and puts downward pressure on Foreign PPI inflation, so the Foreign planner reduces its nominal interest rate even more (Panel 1, Fig. 3(b)). Figure 3(c) identifies the effect of Home monetary accommodation relative to the counterfactual that imposes the Home PPT interest-rate path. Under the optimal IPT policy, the Home nominal exchange rate and the Home LOOP gap lie above their counterfactual paths (Panels 9 and 11, Fig. 3(c)). This counterfactual experiment shows that the Home social planner uses monetary accommodation to raise the Home LOOP gap and reduce the WR LOOP-gap deviation, while placing depreciation pressure on the nominal exchange rate. The equilibrium Home currency nevertheless appreciates because Foreign monetary accommodation is stronger. The Home import tariff also reduces Home demand for imports, so substantial monetary accommodation is required to support Foreign PPI inflation.

Table 5 evaluates the Home welfare cost function in Eq. (46) along the optimal and counterfactual allocations over the 40-quarter reporting horizon. It also reports exact Home total welfare cost obtained by evaluating the original nonlinear lifetime utility in Eq. (1) directly along the same allocations. As discussed in Sec. 4.4, the relative-price row is a signed strategic welfare contribution because its completed-square weight is negative.

Table 5: Two-country IPT environment: discounted Home welfare-cost components under the optimal and counterfactual Home PPT interest-rate paths.

| WR Variable                   | PPT-rate CF | Optimal IPT | Difference |
|-------------------------------|-------------|-------------|------------|
| $y_t - y_t^{w,IPT}$           | 0.0335      | 0.0422      | +0.0088    |
| $\psi_t - \psi_t^{w,IPT}$     | 0.4041      | 0.3931      | -0.0110    |
| $z_t - z_t^{w,IPT}$           | -0.7977     | -0.8116     | -0.0139    |
| $\psi_t^* - \psi_t^{*,w,IPT}$ | 0.1115      | 0.1172      | +0.0056    |
| $\pi_{H,t}$                   | 0.0114      | 0.0153      | +0.0039    |
| $\pi_{F,t}$                   | 0.0272      | 0.0275      | +0.0003    |
| $\pi_{H,t}^*$                 | 0.0045      | 0.0045      | 0.0000     |
| Quadratic total               | -0.2054     | -0.2117     | -0.0063    |
| Exact total welfare cost      | 19.3727     | 19.3154     | -0.0573    |

*Note:* All entries are in percent of steady-state consumption. The component rows and “Quadratic total” are discounted sums from the second-order Home welfare cost function in Eq. (46). “Exact total welfare cost” evaluates the original nonlinear Home utility in Eq. (1) directly along each allocation, relative to the deterministic steady-state utility path. Because the second-order criterion suppresses terms independent of the endogenous Home policy allocation, its level is not directly comparable with the exact total welfare-cost level; the relevant comparison is the difference between the optimal and counterfactual allocations. Exact nonlinear welfare has no source-by-source decomposition corresponding to the completed-square rows.

The counterfactual confirms the LOOP-gap mechanism. Optimal Home easing reduces the Home WR LOOP-gap deviation component by 0.0110% of steady-state consumption and lowers the full signed second-order Home welfare criterion by 0.0063% of steady-state consumption, despite higher costs from some other welfare-relevant components. The last row of Table 5 shows the exact total welfare cost. While there is a gap between the exact and quadratic total welfare costs, both welfare costs are lower under the optimal IPT allocation than under the counterfactual. Therefore, it can be said that monetary easing reduces welfare costs, as in the SOE model.

## 5 Reciprocal Tariffs: The Return of the ZLB

This section studies reciprocal tariffs throughout. The Home import tariff  $\tau_{M,t}$  and the Foreign import tariff  $\tau_{M,t}^*$  both receive a 20-percentage-point innovation, and both tariff processes have persistence 0.7. The two-country IPT environment, country-size parameter  $\nu = 0.27$ , pricing structure, fiscal closure, and national Nash–Ramsey policy game are otherwise those in Sec. 4. We first report the benchmark reciprocal-tariff allocation. We then compare that same reciprocal-tariff experiment with three alternative specifications: Monacelli’s [39] High Elasticity Case, external

habit, and roundabout production. Thus Secs. 5.2, 5.3, and 5.4 use the benchmark reciprocal-tariff allocation in Sec. 5.1 as their reference case.

Tables 6 and 7 collect, respectively, the Home welfare weights and the loadings of the Home WR LOOP gap for the four specifications considered below. The benchmark column corresponds to the benchmark two-country IPT environment in Sec. 4; the remaining columns change only the feature indicated in the column heading.

Table 6: Home welfare weights across reciprocal-tariff specifications

| Variable                    | Benchmark | High Elasticity | External habit | Roundabout production |
|-----------------------------|-----------|-----------------|----------------|-----------------------|
| $y_t - y_t^w$               | 2.860     | 2.221           | 18.593         | 0.962                 |
| $\psi_t - \psi_t^w$         | 0.406     | 1.206           | 0.569          | 0.650                 |
| $z_t - z_t^w$               | -0.098    | -0.480          | 0.883          | 0.087                 |
| $\psi_t^* - \psi_t^{*,w}$   | 0.352     | 1.438           | 0.158          | 0.199                 |
| $c_{t-1} - c_{t-1}^w$       | —         | —               | 0.720          | —                     |
| $c_{t-1}^* - c_{t-1}^{*,w}$ | —         | —               | -0.323         | —                     |
| $\pi_{H,t}$                 | 65.313    | 50.711          | 73.105         | 109.286               |
| $\pi_{F,t}$                 | 21.283    | 33.686          | 25.707         | 34.081                |
| $\pi_{H,t}^*$               | 13.519    | 11.320          | 23.714         | 15.647                |

Note: Specification superscripts on the WR reference values are suppressed in the row labels. The inherited-consumption terms are present only under external habit. Here  $c_t \equiv \log(C_t/C)$  and  $c_t^* \equiv \log(C_t^*/C^*)$ ;  $c_t^w$  and  $c_t^{*,w}$  denote the Home and Foreign WR consumption reference values, respectively, defined in Appendix D.2.3.

Table 7: Loadings of the Home WR LOOP gap across reciprocal-tariff specifications

| Term                                              | Benchmark | High Elasticity | External habit | Roundabout production |
|---------------------------------------------------|-----------|-----------------|----------------|-----------------------|
| $\tau_{M,t} (\Gamma_{\text{own}})$                | 0.364     | 0.053           | 0.657          | 0.453                 |
| $\tau_{M,t}^* - \psi_t^* (\Gamma_{\text{cross}})$ | 0.060     | 0.122           | -0.202         | 0.093                 |
| $z_t (\Gamma_z)$                                  | -0.016    | -0.059          | 0.506          | 0.140                 |
| $c_{t-1} (\Gamma_c)$                              | —         | —               | 0.801          | —                     |
| $c_{t-1}^* (\Gamma_{c^*})$                        | —         | —               | 0.159          | —                     |

Note: Each entry is the coefficient on the term shown in the first column in the corresponding Home WR LOOP gap. Exact symbolic expressions are reported in Appendix D.2.2–D.2.4.

| Table 8: ZLB duration, minimum notional nominal interest rate, and main mechanism |                                 |                                                   |                                                     |                                             |                                                              |   |
|-----------------------------------------------------------------------------------|---------------------------------|---------------------------------------------------|-----------------------------------------------------|---------------------------------------------|--------------------------------------------------------------|---|
| Specification                                                                     | ZLB du-<br>ration<br>(quarters) | Minimum<br>notional<br>rate deviation<br>(points) | Home<br>nominal<br>rate deviation (basis<br>points) | Further Increase in the<br>Home WR LOOP gap | Increase in the<br>Weight for Stabilizing PPI Infla-<br>tion |   |
| Benchmark                                                                         | 1                               | −210.62                                           |                                                     | ○                                           |                                                              |   |
| High Elasticity                                                                   | 3                               | −230.95                                           |                                                     | ○                                           |                                                              |   |
| external habit                                                                    | 2                               | −162.21                                           |                                                     | ○                                           |                                                              |   |
| roundabout pro-<br>duction                                                        | 2                               | −106.95                                           |                                                     |                                             |                                                              | ○ |

Note: The notional nominal interest rate is the unconstrained Home policy rate and is reported as a deviation from its deterministic steady-state value in basis points. ZLB duration counts the quarters in which the actual Home nominal interest rate is at zero.

## 5.1 Benchmark Parameterization

An increase in the Foreign import tariff raises the price of Home goods paid by Foreign households and lowers their consumption of Home goods. As discussed in Sec. 4.4.2, a higher Home LOOP gap mitigates the resulting inefficient allocation of Home goods between the two countries. The required adjustment puts depreciation pressure on the Home currency, making Home goods cheaper for Foreign households and supporting their demand (FAC, Fig. 1).

Figures 4(a) and 4(b) report the exact nonlinear impulse responses. The unilateral allocation remains in these figures only as a diagnostic comparison; the solid reciprocal-tariff allocation is the benchmark for the remaining subsections of this section.

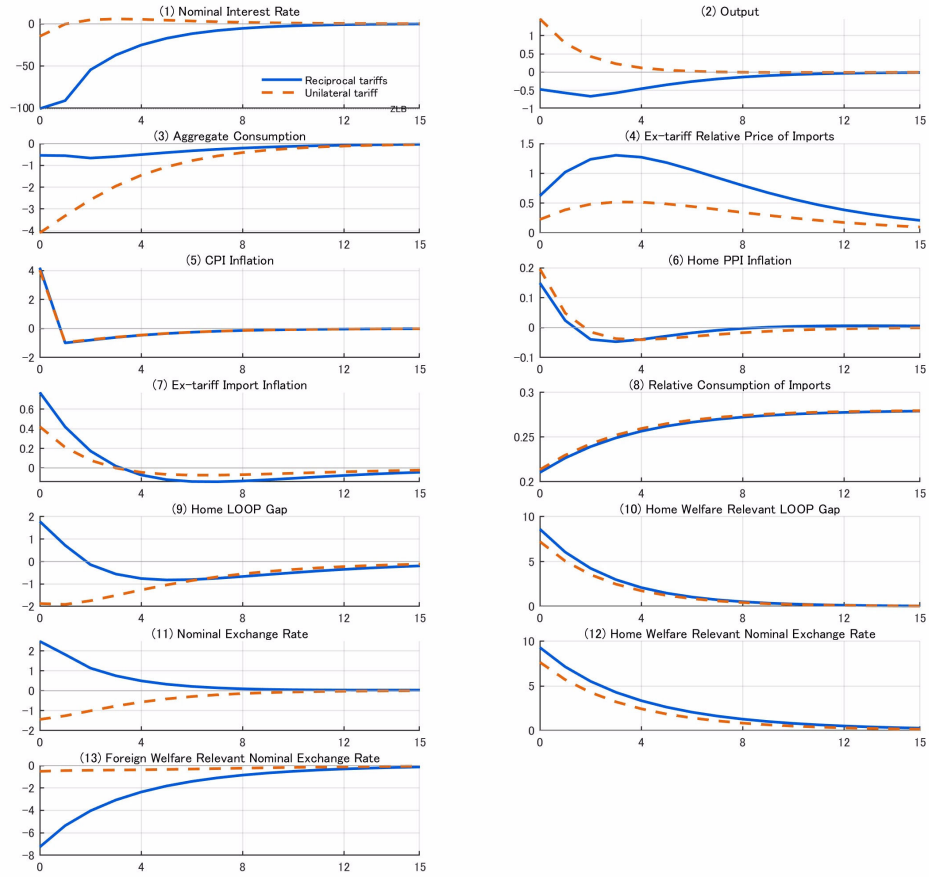


Figure 4: Two-country IPT environment with reciprocal tariff shocks under the benchmark parameterization: (a) Home responses. The solid line reports reciprocal tariffs and the dashed line the unilateral Home tariff allocation. The dotted horizontal line in Panel 1 is the ZLB. Under reciprocal tariffs the Home nominal interest rate is at the ZLB on impact and is positive in the following quarter.

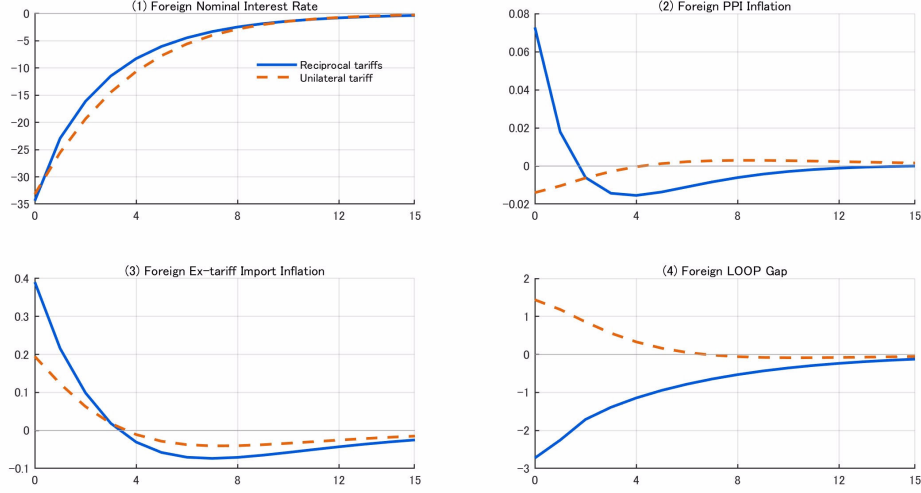


Figure 4: Two-country IPT environment with reciprocal tariff shocks under the benchmark parameterization: (b) Foreign responses. The solid line reports reciprocal tariffs and the dashed line the unilateral Home tariff allocation.

The stronger Home accommodation under reciprocal tariffs follows directly from the Foreign-side mechanism described above. The Foreign tariff raises the Foreign household-facing relative price of Home goods and reduces Foreign demand for Home goods. Consistent with this channel, Home output falls on impact under reciprocal tariffs, whereas it rises under the unilateral tariff (Panel 2, Fig. 4(a)), and Home PPI inflation is lower (Panel 6). At the same time, the Foreign tariff raises the Home WR LOOP gap through  $\tau_{M,t}^*$ , while the endogenous decline in the Foreign LOOP gap  $\psi_t^*$  reinforces this increase because  $\psi_t^*$  enters  $\psi_t^{w,IPT}$  with a negative coefficient. The Home planner therefore has an additional incentive to raise the actual Home LOOP gap toward its WR value. Home monetary accommodation puts depreciation pressure on the Home currency and raises the Home LOOP gap relative to a less accommodative Home path (FAC, Fig. 1). Thus both the contraction in Home export demand and the higher WR LOOP gap call for stronger Home accommodation. The required rate cut exhausts the conventional-policy space on impact. A one-quarter Home ZLB episode therefore occurs (Panel 1, Fig. 4(a)) (Tab. 8).

The Foreign nominal interest rate also falls, but remains strictly positive on impact. Numerically, the Home policy rate is 101.01 basis points below its deterministic steady-state value on impact, exactly exhausting the conventional-policy space, whereas the Foreign rate is 34.40 basis points below steady state.

## 5.2 High Elasticity Case

We next retain the reciprocal tariff shocks of Sec. 5.1 and change only the Home–Foreign substitution elasticity from the benchmark value  $\eta = 1.5$  to  $\eta = 3$ , the High Elasticity Case considered by Monacelli [39]. This experiment therefore isolates the role of stronger expenditure substitution in a two-country reciprocal-tariff environment.

The welfare cost function is the same two-country IPT criterion in Eq. (46), with every coefficient and WR reference value re-evaluated at  $\eta = 3$ . Table 6 reports the resulting welfare weights, and Table 7 reports the corresponding loadings of the Home WR LOOP gap. The central change for the High Elasticity Case is the stronger Foreign-side effect on the Home WR LOOP gap. The loading on  $\tau_{M,t}^* - \psi_t^*$  rises from 0.060 in the benchmark to 0.122 at  $\eta = 3$ . This increase reflects the stronger relative-demand response when the substitution elasticity is higher. The Foreign

purchaser-price combination  $\tau_{M,t}^* - \psi_t^*$  raises the relative price of Home goods faced by Foreign households. When  $\eta$  is larger, a given movement in this combination produces a larger decrease in Foreign consumption of Home goods and therefore a larger inefficient increase in Home-good consumption in country H. The Home planner consequently adjusts the Home WR LOOP gap more strongly. Hence a given Foreign tariff increase or decline in the Foreign LOOP gap raises the Home WR LOOP gap by substantially more (FAC, Fig. 1). In addition, Table 6 shows that  $\Lambda_\psi$  rises from 0.406 to 1.206. As the elasticity of substitution increases, demand becomes more sensitive to relative prices and the expenditure-switching effect becomes stronger. The expenditure switching induced by an import-tariff increase is inefficient. Accordingly, a higher substitution elasticity raises the welfare cost associated with the Home WR LOOP-gap deviation and therefore increases  $\Lambda_\psi$ .

Figures 5(a) and 5(b) compare the High Elasticity Case with the benchmark parameterization. The two lines are identified in the Panel 1 legend and report  $\eta = 3$  and the benchmark  $\eta = 1.5$  case. The comparison therefore holds the reciprocal tariff shocks fixed.

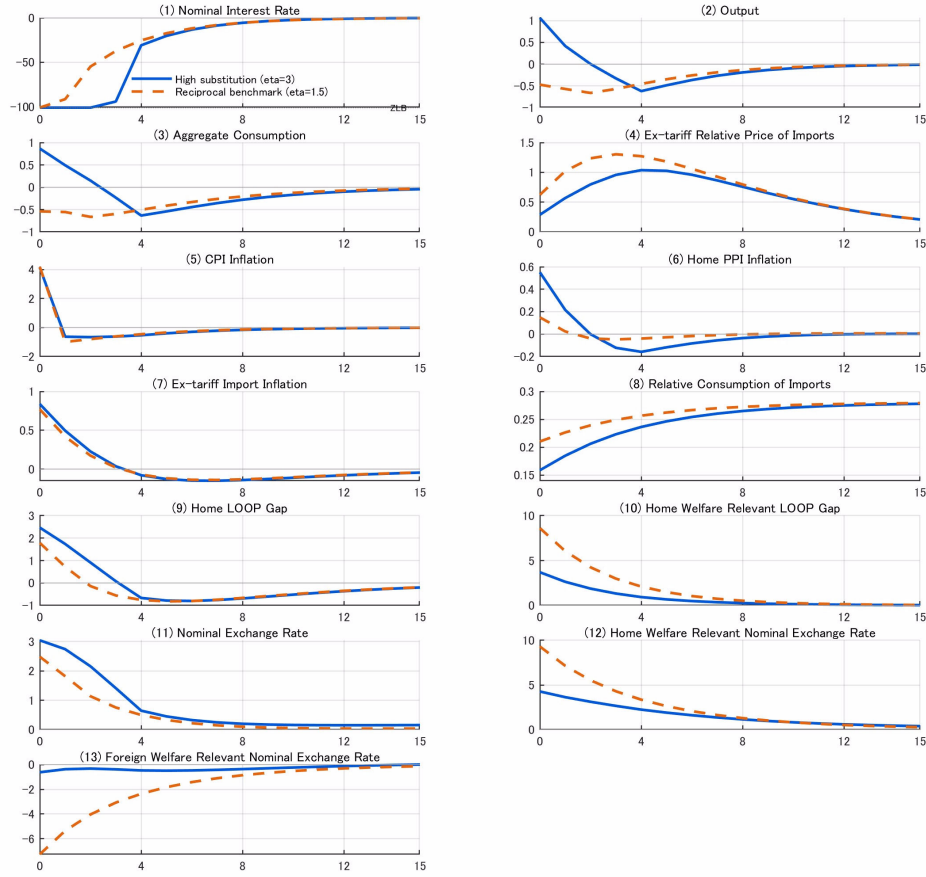


Figure 5: Two-country IPT environment with reciprocal tariff shocks: (a) Home responses in the High Elasticity Case,  $\eta = 3$ , compared with the benchmark parameterization. The two lines are identified in the Panel 1 legend. The horizontal line at  $-101.01$  basis points in Panel 1 is the ZLB.

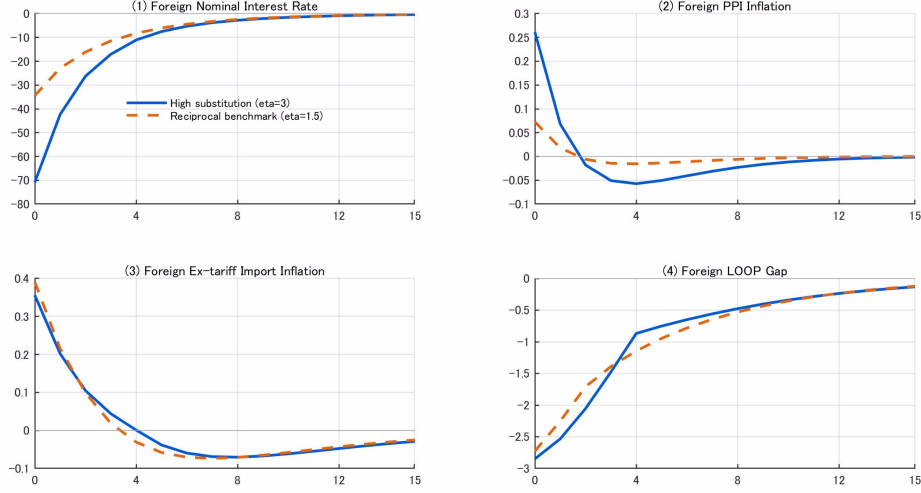


Figure 5: Two-country IPT environment with reciprocal tariff shocks: (b) Foreign responses in the High Elasticity Case,  $\eta = 3$ , compared with the benchmark reciprocal-tariff allocation. The two lines are identified in the Panel 1 legend.

Figure 5(b) shows that the Foreign LOOP gap falls more under  $\eta = 3$  than under the benchmark. Together with the larger Foreign-side loading just described, this raises  $\psi_t^{w,IPT}$  more strongly and calls for a larger increase in the actual Home LOOP gap. The Home planner therefore chooses stronger accommodation, which puts additional depreciation pressure on the Home currency and raises  $\psi_t$  relative to the benchmark path. The larger import-price-inflation weight works against this adjustment, but does not overturn it. The Home nominal interest rate consequently remains at the ZLB for three quarters rather than one (Panel 1, Fig. 5(a)) (Tab. 8).

### 5.3 With External Habit

Finally, we retain the benchmark reciprocal tariff shocks and introduce external habit into both national household preference specifications. For Home,

$$\mathcal{U}^X \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t - \chi C_{t-1}^{\text{agg}}) - \frac{N_t^{1+\varphi}}{1+\varphi} \right], \quad (51)$$

where  $C_t^{\text{agg}}$  denotes aggregate Home consumption and  $\chi$  is the external-habit parameter. The exact starred counterpart applies to Foreign. Each individual household takes lagged aggregate consumption as given, whereas each national Ramsey planner internalizes the effect of aggregate current consumption on its own residents' future habit benchmark. We set  $\chi = 0.71$ , following Smets and Wouters [41]. All pricing, tariff, fiscal, and country-size parameters remain at their benchmark values.

External habit changes the welfare cost function and therefore cannot be interpreted with the no-habit criterion in Eq. (46). The External-Habit column of Table 6 reports all of the resulting welfare weights.

The Home WR LOOP gap also acquires loadings on inherited Home and Foreign consumption. Table 7 reports these loadings together with the tariff, Foreign LOOP-gap, and international relative-price loadings. Their exact symbolic definitions are given in Appendix D.2.3. The positive coefficient on  $c_{t-1}$  in Table 7 can be understood from the cum-tariff relative import price. As the definition of the relative price of imports in Eq. (6), which can be rewritten as  $S_t = \frac{Z_t}{\Psi_t}$ ,

shows, holding  $Z_t$  and the tariff fixed, a higher  $\Psi_t$  lowers the relative price of imports and thereby mitigates the tariff-induced reduction in consumption (because the lower cum-tariff import price reduces the Home CPI and thereby raises the amount of composite consumption attainable from a given nominal consumption expenditure). With external habit, a higher inherited consumption level lowers  $C_t - \chi C_{t-1}^{agg}$  for given current consumption, so more current consumption is required to attain the same utility level. An increase in  $c_{t-1}$  therefore calls for an increase in the Home WR LOOP gap. Appendix D.2.3 gives the symbolic expressions underlying the External-Habit column of Table 7. On-line Appendix B.2.3 reports the complete two-country Ramsey construction.

Figures 6(a) and 6(b) compare  $\chi = 0.71$  with the benchmark parameterization. The tariff innovations and all non-habit structural parameters are identical across the two lines, which are identified in the Panel 1 legend.

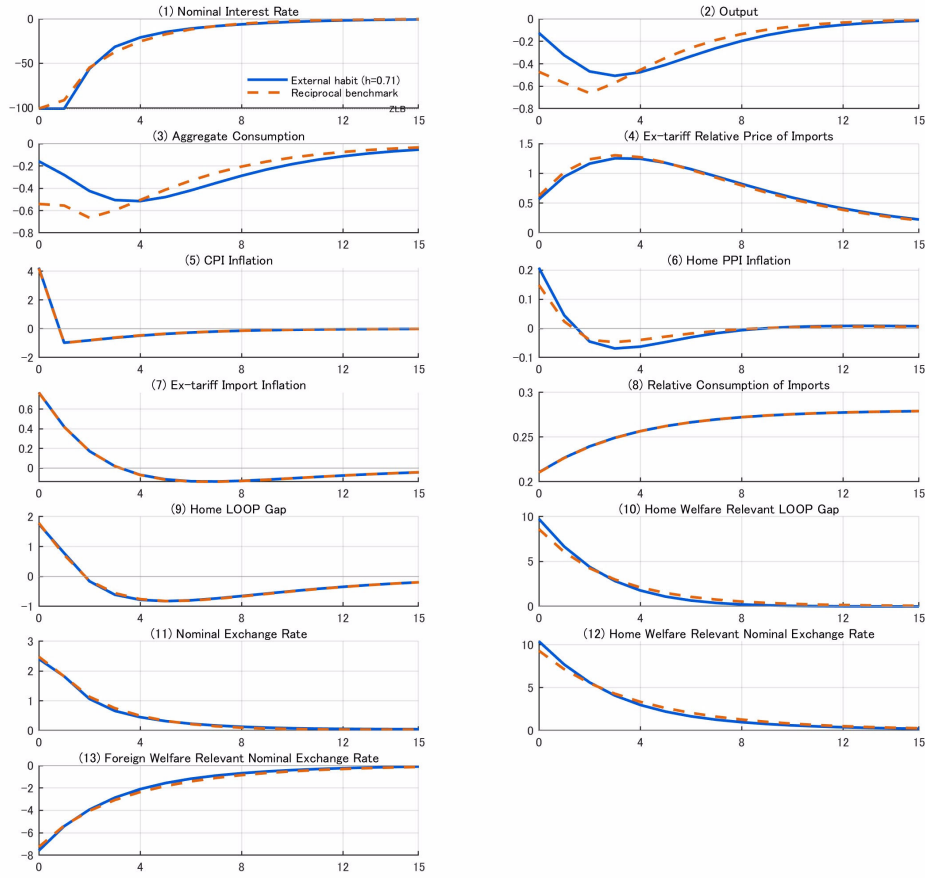


Figure 6: Two-country IPT environment with reciprocal tariff shocks: (a) Home responses with external habit,  $\chi = 0.71$ , compared with the benchmark parameterization. The two lines are identified in the Panel 1 legend. The horizontal line at  $-101.01$  basis points in Panel 1 is the ZLB.

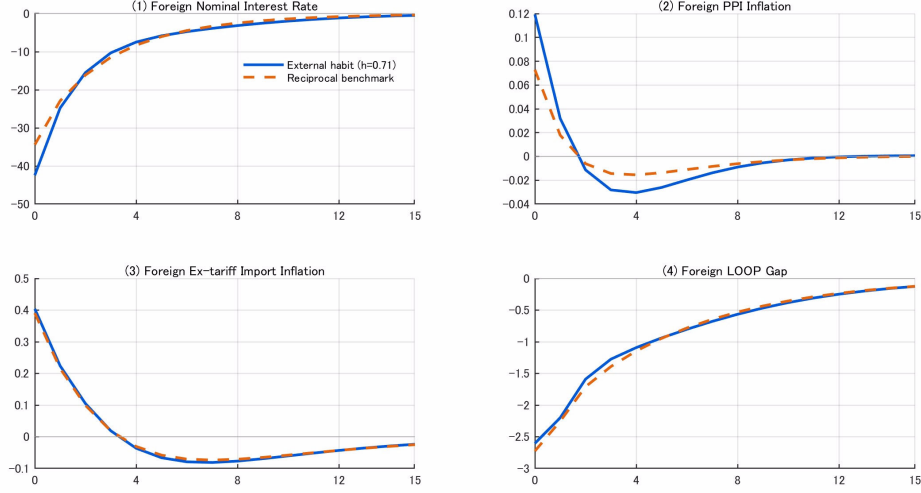


Figure 6: Two-country IPT environment with reciprocal tariff shocks: (b) Foreign responses with external habit,  $\chi = 0.71$ , compared with the benchmark parameterization. The two lines are identified in the Panel 1 legend.

Due to habit formation, higher inherited consumption raises the amount of current consumption required to attain a given utility level and, as the positive loading on  $c_{t-1}$  in Table 7 shows, raises the Home WR LOOP gap. The stronger accommodation needed to raise the actual Home LOOP gap and support current consumption therefore keeps the nominal interest rate at the ZLB longer (Panel 1, Fig. 6(a)) (Tab. 8).

#### 5.4 With Roundabout Production

Finally, we retain the benchmark reciprocal tariff shocks and benchmark preference and pricing parameters and instead change the production structure. Following Basu [5], Home and Foreign producers combine labor with an intermediate-input composite in a roundabout technology. For Home, the aggregate production function is

$$Y_t \Delta_{H,t} = \left( \frac{N_t}{1-\zeta} \right)^{1-\zeta} \left( \frac{\mathcal{I}_t}{\zeta} \right)^\zeta, \quad (52)$$

where  $\mathcal{I}_t$  is the Home intermediate-input composite,  $\zeta$  is the intermediate-input share,  $1-\zeta$  is the labor share, and  $\Delta_{H,t}$  is Home producer-price dispersion. The exact Foreign mirror applies. Huang and Liu [34] regard an intermediate-input share between 0.5 and 0.7 as a reasonable range, so we set  $\zeta = 0.50$ , the lower bound of that range. We solve the exact nonlinear reciprocal-tariff allocation under this production structure and compare it with the benchmark reciprocal-tariff allocation in Sec. 5.1.

The share normalizations in Eq. (52) make the exact unit cost of the roundabout technology equal to  $W_t^{1-\zeta} (P_t^{IPT})^\zeta$ , with the intermediate-input composite purchased at the Home CPI  $P_t^{IPT}$ . To see this, cost minimization for one unit of the left-hand side of Eq. (52) implies  $W_t N_t = (1-\zeta) MC_t$  and  $P_t^{IPT} \mathcal{I}_t = \zeta MC_t$ . Substituting these factor demands back into Eq. (52) gives  $1 = (MC_t/W_t)^{1-\zeta} (MC_t/P_t^{IPT})^\zeta$ , and therefore  $MC_t = W_t^{1-\zeta} (P_t^{IPT})^\zeta$ . Hence cost minimization,

together with the household intratemporal condition in Eq. (10), gives

$$\begin{aligned} MC_t^r &= \frac{P_t^{IPT}}{P_{H,t}} \left( \frac{W_t}{P_t^{IPT}} \right)^{1-\zeta} \\ &= \mathcal{G} \left( \tau_{M,t}, \frac{Z_t}{\Psi_t} \right) (C_t N_t^\varphi)^{1-\zeta}. \end{aligned} \tag{53}$$

Eq. (53) is quite different from Eq. (16) because of the power  $1 - \zeta$  on the marginal rate of substitution. Its log-linearization shows that the consumption–labor component of producer real marginal cost is multiplied by  $1 - \zeta$ , whereas the CPI–PPI ratio continues to enter with unit elasticity. Combining this result with the Home Phillips curve in Eq. (18), and holding consumption, the CPI–PPI ratio, and expected future Home PPI inflation fixed, the response of current Home PPI inflation to labor input is  $\kappa_H(1 - \zeta)\varphi$ . Thus at  $\zeta = 0.50$  the labor-input response of both producer marginal cost and Home PPI inflation is one half of that in the benchmark labor-only technology. Equivalently, through this labor channel, a given deviation of Home PPI inflation corresponds to approximately twice as large a labor-input deviation as in the benchmark economy.

As Table 6 shows, the Home producer-inflation weight rises from its benchmark value of 65.313 to 109.286 under roundabout production. Hence, the Home planner has a stronger incentive to offset this disinflationary pressure.

Figures 7(a) and 7(b) report the comparison. The two lines are identified in the Panel 1 legend and report the roundabout specification and the benchmark parameterization.

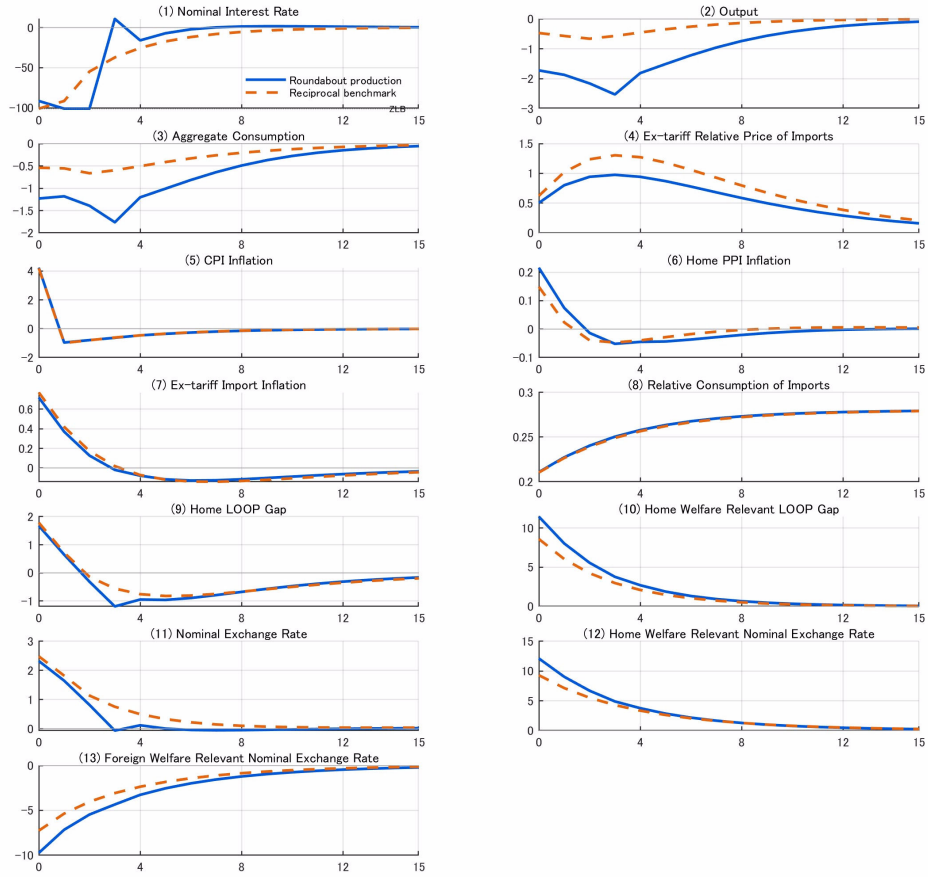


Figure 7: Two-country IPT environment with reciprocal tariff shocks: (a) Home responses with roundabout production,  $\zeta = 0.50$ , compared with the benchmark parameterization. The two lines are identified in the Panel 1 legend. The horizontal line at  $-101.01$  basis points in Panel 1 is the ZLB.

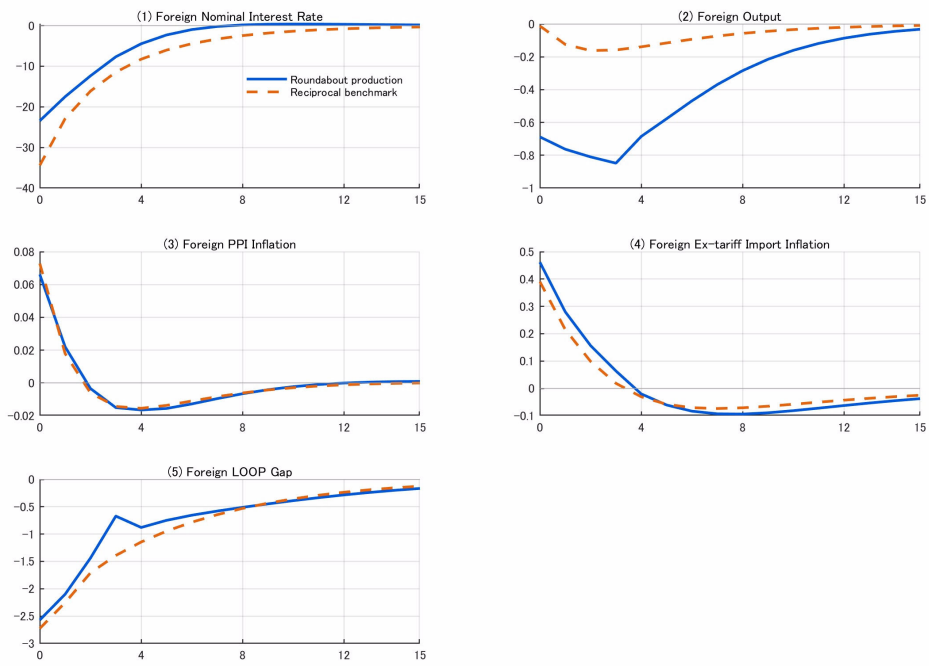


Figure 7: Two-country IPT environment with reciprocal tariff shocks: (b) Foreign responses with roundabout production,  $\zeta = 0.50$ , compared with the benchmark parameterization. The two lines are identified in the Panel 1 legend.

Roundabout production extends ZLB duration. With roundabout production, the required accommodation becomes more persistent. Reciprocal tariffs weaken demand for Home goods and put downward pressure on Home PPI inflation. Because the welfare weight on Home PPI inflation is much larger under roundabout production, 109.286 rather than 65.313, as shown in Table 6, the Home planner has a stronger incentive to offset this disinflationary pressure. Moreover, the flatter labor-to-inflation relation implied by Eq. (53) means that doing so requires a larger labor and output response through this channel. The planner therefore keeps the nominal interest rate low for longer (Panel 1, Fig. 7(a)): the Home policy rate reaches the ZLB in quarters 2 and 3 and becomes positive again in quarter 4. Thus the Home ZLB episode lengthens from one quarter under the benchmark to two quarters under roundabout production, but the binding episode begins one quarter later (Tab. 8). The Foreign rate also falls by less on impact and remains strictly positive throughout.

## 6 Conclusion

This paper studies optimal monetary policy in a tariff-imposing economy when short-run exchange-rate pass-through into local import prices is imperfect. Monopolistically competitive local importers with sticky local-currency prices create scope for an opportunistic increase in *ex-tariff* import prices when the import tariff rises. This opportunistic pricing raises the importer markup above the level neutralized by the steady-state importer subsidy and therefore creates an additional welfare-relevant distortion. The central modeling result is therefore that a parsimonious, empirically motivated departure from the PPT benchmark is sufficient to generate this additional welfare-relevant distortion and the associated motive for monetary accommodation.

The appropriate monetary response is to mitigate that tariff-induced markup distortion. A higher LOOP gap compresses the importer's *ex-tariff* markup, so the tariff increase raises the WR LOOP gap through the HMC. The social planner consequently seeks to raise the actual LOOP gap toward its welfare-relevant value. Monetary easing provides the nominal depreciation pressure required for this adjustment. This mechanism makes monetary accommodation optimal in the IPT environment both in the small open economy and in the two-country economy.

The two-country model adds the FAC. Home monetary easing puts depreciation pressure on the Home currency and lowers the Foreign LOOP gap relative to a less accommodative Home policy path. The resulting increase in the price of Home goods faced by Foreign households reduces their consumption of Home goods and shifts Home-good consumption inefficiently toward Home households. The Home WR LOOP gap therefore rises further, reinforcing the incentive for Home monetary accommodation. Reciprocal tariffs strengthen this channel because the Foreign tariff directly reduces Foreign demand for Home goods.

Motivated by recent U.S. tariff policy, we then study reciprocal tariffs in the two-country economy. The benchmark reciprocal-tariff experiment requires monetary accommodation large enough to reach the ZLB. The High Elasticity Case strengthens the FAC, while external habit raises the WR LOOP gap through inherited consumption. Roundabout production operates differently by increasing the importance of stabilizing Home PPI inflation. Despite these different propagation mechanisms, each case requires monetary easing sufficiently strong for the Home policy rate to reach the ZLB. Thus imperfect pass-through can make the optimal monetary response to tariff increases not merely expansionary but large enough for the lower bound on nominal interest rates to become a central policy constraint.

## Appendix

### A Derivation of the Two-country Economy Model

This Appendix collects the nonlinear private-equilibrium system of the Two-country Economy in one place. All variables and parameters have exactly the definitions used in the main text. No additional state variable, price concept, tariff convention, or auxiliary economic object is introduced here. Country  $H$  has household mass  $\nu$  and country  $F$  has household mass  $1 - \nu$ . The expenditure weights are those in Sec. 4.1, namely  $\omega_F = \nu(1 - \nu)$ ,  $\omega_H = 1 - \omega_F$ ,  $\omega_H^* = \nu\nu$ , and  $\omega_F^* = 1 - \omega_H^*$ .

#### A.1 PPT Environment

The Home and Foreign representative households have the preferences stated in Sec. 4.1. Their consumption baskets are given by Eq. (39). Under the PPT environment, Eq. (2) and

$$P_{H,t}^* = \frac{P_{H,t}}{\mathcal{E}_t} \quad (\text{A.1})$$

are applicable. Hence the two LOOP gaps are equal to one in levels,

$$\Psi_t \equiv \frac{\mathcal{E}_t P_{F,t}^*}{P_{F,t}} = 1, \quad \Psi_t^* \equiv \frac{P_{H,t}/\mathcal{E}_t}{P_{H,t}^*} = 1. \quad (\text{A.2})$$

Accordingly, with the logarithmic definitions used in the welfare analysis,  $\psi_t \equiv \log \Psi_t = 0$  and  $\psi_t^* \equiv \log \Psi_t^* = 0$ . Thus the statement that the LOOP gap is zero applies to its log deviation, whereas the corresponding level is one.

The national CPI indices are

$$\begin{aligned} P_t^{PPT} &= \left[ \omega_H P_{H,t}^{1-\eta} + \omega_F \{ (1 + \tau_{M,t}) \mathcal{E}_t P_{F,t}^* \}^{1-\eta} \right]^{1/(1-\eta)}, \\ P_t^{*,PPT} &= \left[ \omega_F^* (P_{F,t}^*)^{1-\eta} + \omega_H^* \left\{ (1 + \tau_{M,t}^*) \frac{P_{H,t}}{\mathcal{E}_t} \right\}^{1-\eta} \right]^{1/(1-\eta)}. \end{aligned} \quad (\text{A.3})$$

The associated bilateral demand system is

$$\begin{aligned} C_{H,t} &= \omega_H \left( \frac{P_{H,t}}{P_t^{PPT}} \right)^{-\eta} C_t, \\ C_{F,t} &= \omega_F \left( \frac{(1 + \tau_{M,t}) \mathcal{E}_t P_{F,t}^*}{P_t^{PPT}} \right)^{-\eta} C_t, \\ C_{F,t}^* &= \omega_F^* \left( \frac{P_{F,t}^*}{P_t^{*,PPT}} \right)^{-\eta} C_t^*, \\ C_{H,t}^* &= \omega_H^* \left( \frac{(1 + \tau_{M,t}^*) P_{H,t}/\mathcal{E}_t}{P_t^{*,PPT}} \right)^{-\eta} C_t^*. \end{aligned} \quad (\text{A.4})$$

Tariff revenue is rebated lump sum in the importing country, exactly as in the SOE household budget constraint. The Home intratemporal and Euler conditions are Eqs. (10) and (11), with  $\Psi_t = 1$ ; the Foreign household satisfies the exact starred counterparts. In particular,

$$\frac{1}{1 + i_t^*} = \beta E_t \frac{C_t^* (P_t^{*,PPT} / P_{F,t}^*)}{C_{t+1}^* (P_{t+1}^{*,PPT} / P_{F,t+1}^*) \Pi_{F,t+1}^*}. \quad (\text{A.5})$$

Complete international risk sharing gives

$$Z_t = \frac{C_t}{C_t^*} \frac{\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)}{P_t^*, PPT}/P_{F,t}^*, \quad \Psi_t = 1, \quad (\text{A.6})$$

which is the two-country counterpart of Eq. (12).

In each country, production, producer real marginal cost, the producer reset price, and producer-price dispersion are governed by Eqs. (13)–(17) and their exact starred counterparts. Thus Home production satisfies

$$Y_t = \frac{N_t}{\Delta_{H,t}}, \quad MC_t^r = C_t N_t^\varphi \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t), \quad (\text{A.7})$$

and the Home reset-price condition is Eq. (14); Foreign production and producer pricing have the identical starred structure.

Because there is no local importer-price dispersion under PPT, bilateral goods-market clearing is

$$Y_t = C_{H,t} + \left(\frac{1-\nu}{\nu}\right) C_{H,t}^*, \quad Y_t^* = C_{F,t}^* + \left(\frac{\nu}{1-\nu}\right) C_{F,t}. \quad (\text{A.8})$$

Equation (39) and Eqs. (A.1)–(A.8), together with the two producer Calvo blocks and the two monetary-policy conditions, form the nonlinear Two-country PPT private equilibrium used by the national planners.

## A.2 IPT Environment

The IPT environment retains the preferences, national CES consumption baskets, household optimality conditions, complete-markets risk-sharing condition, and PCP producer blocks in Appendix A.1. The difference is that Home and Foreign local importers set sticky *ex-tariff* prices in the currency of the importing country. Thus  $P_{F,t}$  and  $P_{H,t}^*$  are endogenous local importer-price indices. The Home LOOP gap is  $\Psi_t \equiv \mathcal{E}_t P_{F,t}^*/P_{F,t}$  as defined in Sec. 3.2; its exact Foreign mirror is  $\Psi_t^* \equiv (P_{H,t}/\mathcal{E}_t)/P_{H,t}^*$ . These LOOP gaps need not equal one away from the deterministic steady state.

The Home household-facing *cum-tariff* price of a Foreign variety and its CES index are Eqs. (20) and (22). The exact Foreign mirror uses the same additive customs-duty convention and the corresponding country-mass normalization.

The CPI indices are therefore

$$\begin{aligned} P_t^{IPT} &= \left[ \omega_H P_{H,t}^{1-\eta} + \omega_F (\tilde{P}_{F,t}^{IPT})^{1-\eta} \right]^{1/(1-\eta)}, \\ P_t^{*,IPT} &= \left[ \omega_F^* (P_{F,t}^*)^{1-\eta} + \omega_H^* (\tilde{P}_{H,t}^*)^{1-\eta} \right]^{1/(1-\eta)}, \end{aligned} \quad (\text{A.9})$$

and aggregate bilateral demands are

$$\begin{aligned} C_{H,t} &= \omega_H \left( \frac{P_{H,t}}{P_t^{IPT}} \right)^{-\eta} C_t, \\ C_{F,t} &= \omega_F \left( \frac{\tilde{P}_{F,t}^{IPT}}{P_t^{IPT}} \right)^{-\eta} C_t, \\ C_{F,t}^* &= \omega_F^* \left( \frac{P_{F,t}^*}{P_t^{*,IPT}} \right)^{-\eta} C_t^*, \\ C_{H,t}^* &= \omega_H^* \left( \frac{\tilde{P}_{H,t}^*}{P_t^{*,IPT}} \right)^{-\eta} C_t^*. \end{aligned} \quad (\text{A.10})$$

Within the imported-good aggregate, conditional demand is Eq. (21) with the Home-country normalization above, and the Foreign conditional-demand equation is its exact starred counterpart. The Home and Foreign household budget constraints again reduce to  $P_t^{IPT}C_t$  and  $P_t^{*,IPT}C_t^*$  expenditure by CES duality. The statutory duties are remitted on border customs value and rebated lump sum exactly as in Eq. (25) and its Foreign mirror.

The Home intratemporal and Euler equations remain Eqs. (10) and (11), now with endogenous  $\Psi_t$ . The Foreign equations are their exact starred counterparts, and complete international risk sharing remains

$$Z_t = \frac{C_t}{C_t^*} \frac{\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)}{P_t^{*,IPT}/P_{F,t}^*}. \quad (\text{A.11})$$

The producer blocks remain Eqs. (13)–(17) and their starred counterparts.

Home local importers use the exact nonlinear pricing problem in Eqs. (28)–(30). Their *ex-tariff* price index satisfies Eq. (26), and the local relative import price obeys Eq. (27). Foreign local importers solve the exact Home–Foreign mirror problem. In particular, no multiplicative identity of the form  $\tilde{P}_{F,t}^{IPT} = (1 + \tau_{M,t})P_{F,t}$  is imposed when importer prices are sticky; the statutory duty remains additive at the variety level as in Eq. (20).

Bilateral goods-market clearing is given by Eq. (44). Together with the two household blocks, the two PCP producer blocks, and the two LCP importer blocks, the conditions above form the nonlinear Two-country IPT private equilibrium.

### A.3 External Habit (IPT Environment Only)

External habit changes only the household preference and the equations that contain marginal utility of consumption; the pricing convention, additive customs-duty treatment, bilateral goods-market clearing, and importer blocks are those in Appendix A.2. Home preference is given by Eq. (51), while Foreign preference is given by:

$$\mathcal{U}^{*,\chi} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t^* - \chi C_{t-1}^{*,\text{agg}}) - \frac{(N_t^*)^{1+\varphi}}{1+\varphi} \right]. \quad (\text{A.12})$$

Each household treats lagged aggregate consumption as external when deriving its private conditions; after symmetry is imposed,  $C_t^{\text{agg}} = C_t$  and  $C_t^{*,\text{agg}} = C_t^*$ . Hence

$$\begin{aligned} \frac{W_t}{P_t^{IPT}} &= (C_t - \chi C_{t-1}) N_t^\varphi, \\ MC_t^{r,h} &= (C_t - \chi C_{t-1}) N_t^\varphi \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t). \end{aligned} \quad (\text{A.13})$$

with the exact starred counterpart. Complete international risk sharing is

$$Z_t = \frac{C_t - \chi C_{t-1}}{C_t^* - \chi C_{t-1}^*} \frac{\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)}{P_t^{*,IPT}/P_{F,t}^*}, \quad (\text{A.14})$$

and the Home Euler equation is

$$\frac{1}{1+i_t} = \beta \mathbb{E}_t \frac{(C_t - \chi C_{t-1}) \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)}{(C_{t+1} - \chi C_t) \mathcal{G}(\tau_{M,t+1}, Z_{t+1}/\Psi_{t+1}) \Pi_{H,t+1}}, \quad (\text{A.15})$$

with the exact Foreign mirror. These equations, together with the unchanged producer and importer blocks and the bilateral market-clearing conditions in Appendix A.2, define the External-Habit private equilibrium.

#### A.4 Roundabout Production (IPT Environment Only)

The roundabout-production experiment retains the household, complete-markets, additive-duty, PCP producer-price, and LCP importer-price blocks in Appendix A.2. It changes the production technology by introducing the intermediate-input composite  $\mathcal{I}_t$  with share  $\zeta$ . The Home production function is given in Eq. (52), with the exact Foreign mirror. The price of the intermediate-input composite is the same Home CPI  $P_t^{IPT}$  used for final consumption, so the Home factor-cost conditions imply

$$\begin{aligned} N_t &= (1 - \zeta) \frac{MC_t^r Y_t \Delta_{H,t}}{C_t N_t^\varphi \mathcal{G}(\tau_{M,t}, Z_t / \Psi_t)}, \\ \mathcal{I}_t &= \zeta \frac{MC_t^r Y_t \Delta_{H,t}}{\mathcal{G}(\tau_{M,t}, Z_t / \Psi_t)}, \end{aligned} \quad (\text{A.16})$$

while Home producer real marginal cost is given in Eq. (53). These expressions are the exact nonlinear counterparts of the first-order factor-demand relations reported in Appendix D.2.4.

The intermediate-input composite has the same Home-Bias CES composition and faces the same purchaser prices as final consumption. Therefore the bilateral resource constraints in Eq. (44) continue to apply with each country's final-consumption absorption replaced by the corresponding total absorption  $C_t + \mathcal{I}_t$  and  $C_t^* + \mathcal{I}_t^*$ . No new relative-price or tariff object is introduced. Together with the exact Foreign mirror of Eqs. (52)–(53), this closes the nonlinear Two-country roundabout-production private equilibrium.

## B Social Planner's Problem

The policy problems in this section are the nonlinear commitment problems used to generate the allocations. welfare cost functions are derived separately in Appendix D; they are not minimized to obtain the policy paths.

### B.1 Small Open Economy

#### B.1.1 PPT Environment

In the PPT environment, the social planner maximizes  $\mathcal{U}$  subject to Eqs. (10), (11), (12), (13), (14)–(17), (2), (19) with  $\Psi_t = 1$ , and (33). After Eqs. (14)–(15) have been substituted, the Lagrangian is

$$\begin{aligned} \mathcal{L}^{SOE, PPT} &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right. \\ &\quad + \lambda_{p,t} \left[ 1 - \theta \Pi_{H,t}^{\varepsilon-1} - (1-\theta) \tilde{p}_{H,t}^{1-\varepsilon} \right] \\ &\quad + \lambda_{\Delta,t} \left[ \Delta_{H,t} - \theta \Pi_{H,t}^{\varepsilon} \Delta_{H,t-1} - (1-\theta) \tilde{p}_{H,t}^{-\varepsilon} \right] \\ &\quad + \lambda_{n,t} [N_t - Y_t \Delta_{H,t}] \\ &\quad + \lambda_{y,t} [Y_t - (1-v) \mathcal{G}(Z_t, \tau_{M,t})^\eta C_t - v Z_t^\eta] \\ &\quad + \lambda_{r,t} [Z_t - C_t \mathcal{G}(Z_t, \tau_{M,t})] \\ &\quad \left. + \lambda_{e,t} \left[ \frac{1}{1+i_t} - \beta \mathbb{E}_t \frac{C_t \mathcal{G}(Z_t, \tau_{M,t})}{C_{t+1} \mathcal{G}(Z_{t+1}, \tau_{M,t+1}) \Pi_{H,t+1}} \right] + \mu_{i,t} i_t \right\}. \end{aligned} \quad (\text{B.1})$$

Here  $\lambda_{p,t}$ ,  $\lambda_{\Delta,t}$ ,  $\lambda_{n,t}$ ,  $\lambda_{y,t}$ ,  $\lambda_{r,t}$ , and  $\lambda_{e,t}$  are the multipliers on the price-index, price-dispersion, production, goods-market, risk-sharing, and Euler conditions shown in the same order in Eq. (B.1). The multiplier  $\mu_{i,t}$  is the ZLB multiplier defined after Eq. (33).

### B.1.2 IPT Environment

In the IPT environment the SOE planner maximizes the same household utility as in Appendix B.1.1, subject to the household, producer, risk-sharing, and goods-market conditions of the SOE together with the exact additive-duty importer block in Eqs. (20)–(30), the importer Calvo price-index law in Eq. (26), the relative-import-price law in Eq. (27), and the ZLB. The planner chooses the same Home allocation and producer-price variables as under PPT and, in addition, the local importer-price variables described in Sec. 3.3. The statutory duty is imposed on border customs value and is not replaced by a multiplicative tariff on the sticky *ex-tariff* import price.

## B.2 Two-country Economy

### B.2.1 PPT Environment

We now write the two national problems explicitly. Complete international risk sharing in Eq. (A.11) is imposed before either national planner differentiates. It is used to eliminate  $Z_t$  from the national differentiation problems and is not a new definition. No separate starred international-relative-price variable is introduced.

In the PPT environment, after Eq. (A.11) has been substituted, the Home CPI–PPI relation is

$$(\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t))^{1-\eta} = \omega_H + \omega_F(1 + \tau_{M,t})^{1-\eta} \left[ \frac{C_t}{C_t^*} \frac{\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)}{P_t^{*,PPT}/P_{F,t}^*} \right]^{1-\eta},$$

and Home goods-market clearing is

$$\begin{aligned} Y_t &= \omega_H (\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t))^\eta C_t \\ &\quad + \frac{1-\nu}{\nu} \omega_H^* \left[ \frac{C_t (\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t))}{C_t^* (1 + \tau_{M,t}^*)} \right]^\eta C_t^*. \end{aligned}$$

The Foreign counterparts, used in the Foreign national problem, are

$$\begin{aligned} \left( \frac{P_t^{*,PPT}}{P_{F,t}^*} \right)^{1-\eta} &= \omega_F^* + \omega_H^* (1 + \tau_{M,t}^*)^{1-\eta} \left[ \frac{C_t^*}{C_t} \frac{P_t^{*,PPT}/P_{F,t}^*}{\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)} \right]^{1-\eta}, \\ Y_t^* &= \omega_F^* \left( \frac{P_t^{*,PPT}}{P_{F,t}^*} \right)^\eta C_t^* \\ &\quad + \frac{\nu}{1-\nu} \omega_F \left[ \frac{C_t^* (P_t^{*,PPT}/P_{F,t}^*)}{C_t (1 + \tau_{M,t})} \right]^\eta C_t. \end{aligned}$$

The Home planner takes the complete Foreign contingent plan as given. After the Home reset-price relation has been substituted, its national Lagrangian is

$$\begin{aligned} \mathcal{L}_H^{PPT} &= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right. \\ &\quad + \lambda_{g,t}^H \left[ (\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t))^{1-\eta} - \omega_H - \omega_F(1 + \tau_{M,t})^{1-\eta} \left[ \frac{C_t}{C_t^*} \frac{\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)}{P_t^{*,PPT}/P_{F,t}^*} \right]^{1-\eta} \right] \\ &\quad + \lambda_{y,t}^H \left[ Y_t - \omega_H (\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t))^\eta C_t - \frac{1-\nu}{\nu} \omega_H^* \left[ \frac{C_t (\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t))}{C_t^* (1 + \tau_{M,t}^*)} \right]^\eta C_t^* \right] \\ &\quad + \lambda_{n,t}^H [N_t - Y_t \Delta_{H,t}] \\ &\quad + \lambda_{p,t}^H [1 - \theta \Pi_{H,t}^{\varepsilon-1} - (1-\theta) \tilde{p}_{H,t}^{1-\varepsilon}] \end{aligned}$$

$$\begin{aligned}
& + \lambda_{\Delta,t}^H \left[ \Delta_{H,t} - \theta \Pi_{H,t}^\varepsilon \Delta_{H,t-1} - (1-\theta) \tilde{p}_{H,t}^{-\varepsilon} \right] \\
& + \lambda_{e,t}^H \left[ \frac{1}{1+i_t} - \beta \mathbb{E}_t \frac{C_t(\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t))}{C_{t+1}(\mathcal{G}(\tau_{M,t+1}, Z_{t+1}/\Psi_{t+1})) \Pi_{H,t+1}} \right] + \mu_{i,t}^H i_t \Big\}.
\end{aligned}$$

In A.3, a superscript  $H$  or  $F$  on a multiplier identifies the national planner, while the subscript  $M$  identifies that planner's local importer LCP local importer-price block.

The Foreign planner takes the complete Home contingent plan as given. Its national Lagrangian is

$$\begin{aligned}
\mathcal{L}_F^{PPT} = & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t^* - \frac{(N_t^*)^{1+\varphi}}{1+\varphi} \right. \\
& + \lambda_{g,t}^F \left[ \left( \frac{P_t^{*,PPT}}{P_{F,t}^*} \right)^{1-\eta} - \omega_F^* - \omega_H^* (1+\tau_{M,t})^{1-\eta} \left[ \frac{C_t^*}{C_t} \frac{P_t^{*,PPT}/P_{F,t}^*}{\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)} \right]^{1-\eta} \right] \\
& + \lambda_{y,t}^F \left[ Y_t^* - \omega_F^* \left( \frac{P_t^{*,PPT}}{P_{F,t}^*} \right)^\eta C_t^* - \frac{\nu}{1-\nu} \omega_F^* \left[ \frac{C_t^* (P_t^{*,PPT}/P_{F,t}^*)}{C_t (1+\tau_{M,t})} \right]^\eta C_t \right] \\
& + \lambda_{n,t}^F [N_t^* - Y_t^* \Delta_{F,t}^*] \\
& + \lambda_{p,t}^F [1 - \theta (\Pi_{F,t}^*)^{\varepsilon-1} - (1-\theta) (\tilde{p}_{F,t}^*)^{1-\varepsilon}] \\
& + \lambda_{\Delta,t}^F [\Delta_{F,t}^* - \theta (\Pi_{F,t}^*)^\varepsilon \Delta_{F,t-1}^* - (1-\theta) (\tilde{p}_{F,t}^*)^{-\varepsilon}] \\
& + \lambda_{e,t}^F \left[ \frac{1}{1+i_t} - \beta \mathbb{E}_t \frac{C_t^* (P_t^{*,PPT}/P_{F,t}^*)}{C_{t+1}^* (P_{t+1}^{*,PPT}/P_{F,t+1}^*) \Pi_{F,t+1}^*} \right] + \mu_{i,t}^F i_t^* \Big\}.
\end{aligned}$$

## B.2.2 IPT Environment

The two national commitment problems use the exact Two-country private equilibrium in A.2. Complete international risk sharing is imposed before national differentiation, as in Sec. 4.2. The producer reset-price condition in Eq. (14) and the exact importer reset-price condition in Eq. (30) are substituted before the planner FOCs are taken. Hence their reset prices are endogenous implicit functions of the national allocation, while the statutory duty remains additive at the variety level.

The Home planner takes the complete contingent Foreign plan as given and solves

$$\max_{\{C_t, \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t), N_t, Y_t, \Pi_{H,t}, \Delta_{H,t}, P_{F,t}/P_{H,t}, \Pi_{F,t}, i_t\}_{t \geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right), \quad (\text{B.2})$$

subject to the Home household Euler equation, the complete-markets condition in Eq. (A.11), the Home producer technology and Calvo block, the exact additive-duty importer block in Eqs. (20)–(30), the local import-price law in Eq. (27), the exact bilateral resource constraints in Eq. (44), and

$$i_t \geq 0. \quad (\text{B.3})$$

The complete cross-sectional distributions  $\{P_{F,t}(\iota)\}$  and  $\{P_{H,t}^*(\iota)\}$  entering the purchaser-price indices and physical- quantity integrals evolve according to the same Calvo reset/non-reset rule as in A.2. They are therefore retained inside the exact private-equilibrium constraints and are not replaced by a separate importer-dispersion state.

For later differentiation it is useful to display the Home Lagrangian after both reset-price relations have been substituted. Using the same multiplier notation as in B.2.1, and adding  $\lambda_{Mp,t}^H$  for the Home importer price-index law and  $\lambda_{q,t}^H$  for the local relative import-price law, we have

$$\mathcal{L}_H^{IPT} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right.$$

$$\begin{aligned}
& +\lambda_{g,t}^H \left[ \mathcal{G}\left(\tau_{M,t}, \frac{Z_t}{\Psi_t}\right)^{1-\eta} - \omega_H - \omega_F \left(\frac{\tilde{P}_{F,t}^{IPT}}{P_{H,t}}\right)^{1-\eta} \right] \\
& +\lambda_{y,t}^H \left[ Y_t - \omega_H \mathcal{G}\left(\tau_{M,t}, \frac{Z_t}{\Psi_t}\right)^\eta C_t \right. \\
& \quad \left. - \frac{1-\nu}{\nu} \omega_H^* \left(\frac{\tilde{P}_{H,t}^*}{P_{F,t}^{*,IPT}}\right)^{-\eta} C_t^* \frac{1}{1-\nu} \int_\nu^1 \left(\frac{\tilde{P}_{H,t}^*(\ell)}{\tilde{P}_{H,t}^*}\right)^{-\varepsilon_M} d\ell \right] \\
& +\lambda_{n,t}^H [N_t - Y_t \Delta_{H,t}] \\
& +\lambda_{p,t}^H \left[ 1 - \theta \Pi_{H,t}^{\varepsilon-1} - (1-\theta) \tilde{p}_{H,t}^{1-\varepsilon} \right] \\
& +\lambda_{\Delta,t}^H \left[ \Delta_{H,t} - \theta \Pi_{H,t}^\varepsilon \Delta_{H,t-1} - (1-\theta) \tilde{p}_{H,t}^{-\varepsilon} \right] \\
& +\lambda_{Mp,t}^H \left[ 1 - \theta_M \Pi_{F,t}^{\varepsilon_M-1} - (1-\theta_M) \left(\frac{\bar{P}_{F,t}}{P_{F,t}}\right)^{1-\varepsilon_M} \right] \\
& +\lambda_{q,t}^H \left[ \frac{P_{F,t}}{P_{H,t}} - \frac{P_{F,t-1}}{P_{H,t-1}} \frac{\Pi_{F,t}}{\Pi_{H,t}} \right] \\
& +\lambda_{e,t}^H \left[ \frac{1}{1+i_t} - \beta E_t \frac{C_t \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)}{C_{t+1} \mathcal{G}(\tau_{M,t+1}, Z_{t+1}/\Psi_{t+1}) \Pi_{H,t+1}} \right] + \mu_{i,t}^H i_t \Big\}. \quad (\text{B.4})
\end{aligned}$$

All occurrences of  $Z_t$  and  $\mathcal{E}_t$  inside the purchaser-price and reset-price objects in Eq. (B.4) are eliminated with the complete-markets condition before differentiation. In particular,  $\bar{P}_{F,t}/P_{F,t}$  is the reset-price ratio implied by the exact private importer FOC (30); no new importer auxiliary variable is introduced here.

The Foreign planner takes the complete contingent Home plan as given and solves the exact starred problem

$$\max_{\{C_t^*, P_t^{*,IPT}/P_{F,t}^*, N_t^*, Y_t^*, \Pi_{F,t}^*, \Delta_{F,t}^*, P_{H,t}^*/P_{F,t}^*, \Pi_{H,t}^*, i_t^*\}_{t \geq 0}} E_0 \sum_{t=0}^{\infty} \beta^t \left( \log C_t^* - \frac{(N_t^*)^{1+\varphi}}{1+\varphi} \right), \quad (\text{B.5})$$

subject to the starred A.2 equilibrium restrictions and  $i_t^* \geq 0$ . Its Lagrangian is the exact Home–Foreign mirror of Eq. (B.4). The two national commitment problems and the common private equilibrium are solved jointly at their mutual best response.

### B.2.3 External Habit (IPT Environment Only)

The external-habit planners have the same price-setting and market-clearing constraints as in B.2.2, but the household objective, risk sharing, Euler equations, and the stochastic discount factors in the producer and importer reset-price maps are those in A.3. The Home problem is therefore

$$\max_{\{C_t, \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t), N_t, Y_t, \Pi_{H,t}, \Delta_{H,t}, P_{F,t}/P_{H,t}, \Pi_{F,t}, i_t\}_{t \geq 0}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t - \chi C_{t-1}) - \frac{N_t^{1+\varphi}}{1+\varphi} \right], \quad (\text{B.6})$$

subject to Eqs. (A.13)–(A.15), the exact producer and additive-duty importer blocks of A.2, Eq. (44), and  $i_t \geq 0$ . The planner internalizes the effect of current aggregate consumption on next period’s habit benchmark, although the private household treats lagged aggregate consumption as external.

After the habit-specific reset-price maps are substituted, the Home Lagrangian is

$$\mathcal{L}_H^{IPT,h} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(C_t - \chi C_{t-1}) - \frac{N_t^{1+\varphi}}{1+\varphi} \right.$$

$$\begin{aligned}
& +\lambda_{g,t}^{H,h} \left[ \mathcal{G} \left( \tau_{M,t}, \frac{Z_t}{\Psi_t} \right)^{1-\eta} - \omega_H - \omega_F \left( \frac{\tilde{P}_{F,t}^{IPT}}{P_{H,t}} \right)^{1-\eta} \right] + \lambda_{y,t}^{H,h} \left[ Y_t - \omega_H \mathcal{G} \left( \tau_{M,t}, \frac{Z_t}{\Psi_t} \right)^\eta C_t \right. \\
& \quad \left. - \frac{1-\nu}{\nu} \omega_H^* \left( \frac{\tilde{P}_{H,t}^*}{P_t^{*,IPT}} \right)^{-\eta} C_t^* \frac{1}{1-\nu} \int_\nu^1 \left( \frac{\tilde{P}_{H,t}^*(\iota)}{\tilde{P}_{H,t}^*} \right)^{-\varepsilon_M} d\iota \right] \\
& +\lambda_{n,t}^{H,h} [N_t - Y_t \Delta_{H,t}] + \lambda_{p,t}^{H,h} [1 - \theta \Pi_{H,t}^{\varepsilon-1} - (1-\theta) \tilde{p}_{H,t}^{1-\varepsilon}] \\
& +\lambda_{\Delta,t}^{H,h} [\Delta_{H,t} - \theta \Pi_{H,t}^\varepsilon \Delta_{H,t-1} - (1-\theta) \tilde{p}_{H,t}^{\varepsilon-1}] \\
& +\lambda_{Mp,t}^{H,h} \left[ 1 - \theta_M \Pi_{F,t}^{\varepsilon_M-1} - (1-\theta_M) \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)^{1-\varepsilon_M} \right] + \lambda_{q,t}^{H,h} \left[ \frac{P_{F,t}}{P_{H,t}} - \frac{P_{F,t-1}}{P_{H,t-1}} \frac{\Pi_{F,t}}{\Pi_{H,t}} \right] \\
& +\lambda_{e,t}^{H,h} \left[ \frac{1}{1+i_t} - \beta E_t \frac{(C_t - \chi C_{t-1}) \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)}{(C_{t+1} - \chi C_t) \mathcal{G}(\tau_{M,t+1}, Z_{t+1}/\Psi_{t+1}) \Pi_{H,t+1}} \right] + \mu_{i,t}^{H,h} i_t \Big\}. \quad (B.7)
\end{aligned}$$

The Foreign planner takes the complete contingent Home plan as given and solves

$$\max_{\{C_t^*, P_t^{*,IPT}/P_{F,t}^*, N_t^*, Y_t^*, \Pi_{F,t}^*, \Delta_{F,t}^*, P_{H,t}^*/P_{F,t}^*, \Pi_{H,t}^*, i_t^*\}_{t \geq 0}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t^* - \chi C_{t-1}^*) - \frac{(N_t^*)^{1+\varphi}}{1+\varphi} \right], \quad (B.8)$$

subject to the exact starred A.3 private-equilibrium restrictions and  $i_t^* \geq 0$ . Its Lagrangian is the exact starred counterpart of Eq. (B.7); in particular, the Foreign planner internalizes the effect of  $C_t^*$  on the date- $t+1$  aggregate Foreign habit benchmark.

### B.2.4 Roundabout Production (IPT Environment Only)

The roundabout-production planners retain the no-habit utilities and the additive-duty IPT price-setting system in B.2.2. Total absorption is now  $C_t + \mathcal{I}_t$  at Home and  $C_t^* + \mathcal{I}_t^*$  abroad, and the production side is the exact A.4 technology and factor-demand system. The Home planner solves

$$\max_{\{C_t, \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t), N_t, \mathcal{I}_t, Y_t, M C_t^*, \Pi_{H,t}, \Delta_{H,t}, P_{F,t}/P_{H,t}, \Pi_{F,t}, i_t\}_{t \geq 0}} E_0 \sum_{t=0}^{\infty} \beta^t \left( \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right), \quad (B.9)$$

subject to the A.2 household, complete-markets, producer-price, additive-duty importer-price, and ZLB restrictions, the A.4 production and factor-demand restrictions, and the A.4 bilateral resource constraints with total absorption.

After the producer and importer reset-price maps are substituted, a complete Home Lagrangian is

$$\begin{aligned}
\mathcal{L}_H^{IPT,RA} = & E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right. \\
& +\lambda_{g,t}^{H,RA} \left[ \mathcal{G} \left( \tau_{M,t}, \frac{Z_t}{\Psi_t} \right)^{1-\eta} - \omega_H - \omega_F \left( \frac{\tilde{P}_{F,t}^{IPT}}{P_{H,t}} \right)^{1-\eta} \right] \\
& +\lambda_{y,t}^{H,RA} \left[ Y_t - \omega_H \mathcal{G} \left( \tau_{M,t}, \frac{Z_t}{\Psi_t} \right)^\eta (C_t + \mathcal{I}_t) \right. \\
& \quad \left. - \frac{1-\nu}{\nu} \omega_H^* \left( \frac{\tilde{P}_{H,t}^*}{P_t^{*,IPT}} \right)^{-\eta} (C_t^* + \mathcal{I}_t^*) \frac{1}{1-\nu} \int_\nu^1 \left( \frac{\tilde{P}_{H,t}^*(\iota)}{\tilde{P}_{H,t}^*} \right)^{-\varepsilon_M} d\iota \right] \\
& \left. +\lambda_{T,t}^{H,RA} \left[ Y_t \Delta_{H,t} - \left( \frac{N_t}{1-\zeta} \right)^{1-\zeta} \left( \frac{\mathcal{I}_t}{\zeta} \right)^\zeta \right] \right\}
\end{aligned}$$

$$\begin{aligned}
& + \lambda_{N,t}^{H,RA} \left[ N_t - (1 - \zeta) \frac{MC_t^* Y_t \Delta_{H,t}}{C_t N_t^\varphi \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)} \right] + \lambda_{I,t}^{H,RA} \left[ \mathcal{I}_t - \zeta \frac{MC_t^* Y_t \Delta_{H,t}}{\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)} \right] \\
& + \lambda_{p,t}^{H,RA} [1 - \theta \Pi_{H,t}^{\varepsilon-1} - (1 - \theta) \tilde{p}_{H,t}^{1-\varepsilon}] + \lambda_{\Delta,t}^{H,RA} [\Delta_{H,t} - \theta \Pi_{H,t}^\varepsilon \Delta_{H,t-1} - (1 - \theta) \tilde{p}_{H,t}^{\varepsilon-1}] \\
& + \lambda_{Mp,t}^{H,RA} \left[ 1 - \theta_M \Pi_{F,t}^{\varepsilon_M-1} - (1 - \theta_M) \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)^{1-\varepsilon_M} \right] + \lambda_{q,t}^{H,RA} \left[ \frac{P_{F,t}}{P_{H,t}} - \frac{P_{F,t-1}}{P_{H,t-1}} \frac{\Pi_{F,t}}{\Pi_{H,t}} \right] \\
& + \lambda_{e,t}^{H,RA} \left[ \frac{1}{1 + i_t} - \beta \mathbf{E}_t \frac{C_t \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)}{C_{t+1} \mathcal{G}(\tau_{M,t+1}, Z_{t+1}/\Psi_{t+1}) \Pi_{H,t+1}} \right] + \mu_{i,t}^{H,RA} i_t \Big\}. \quad (\text{B.10})
\end{aligned}$$

The explicit marginal-cost identity in Eq. (53) is implied by the technology and two factor-demand restrictions in this Lagrangian and therefore is not imposed a second time. The Foreign planner takes the complete contingent Home plan as given and solves

$$\max_{\{C_t^*, P_t^*, I^{PT}/P_{F,t}^*, N_t^*, \mathcal{I}_t^*, Y_t^*, MC_t^*, \Pi_{F,t}^*, \Delta_{F,t}^*, P_{H,t}^*/P_{F,t}^*, \Pi_{H,t}^*, i_t^*\}_{t \geq 0}} \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \log C_t^* - \frac{(N_t^*)^{1+\varphi}}{1+\varphi} \right), \quad (\text{B.11})$$

subject to the exact starred A.2 and A.4 restrictions and  $i_t^* \geq 0$ . Its Lagrangian is obtained from Eq. (B.10) by the exact Home–Foreign mirror transformation, including the Foreign technology and both Foreign factor-demand restrictions. The two national Roundabout commitment problems and the common private equilibrium are solved at their mutual best response.

## C FONC's for Social Planner

### C.1 Small Open Economy

#### C.1.1 PPT Environment

As on pp. 74–75 of Monacelli [39], it is useful only for taking derivatives to write

$$\begin{aligned}
\mathcal{K}'_{H,t-j}(x_t) & \equiv \frac{\partial \mathcal{K}_{H,t-j}}{\partial x_t}, \quad \mathcal{Z}'_{H,t-j}(x_t) \equiv \frac{\partial \mathcal{Z}_{H,t-j}}{\partial x_t}, \\
\tilde{p}'_{H,t-j}(x_t) & \equiv \frac{\partial \tilde{p}_{H,t-j}}{\partial x_t} = \mathcal{M} \frac{\mathcal{Z}'_{H,t-j}(x_t) \mathcal{K}_{H,t-j} - \mathcal{K}'_{H,t-j}(x_t) \mathcal{Z}_{H,t-j}}{(\mathcal{K}_{H,t-j})^2}. \quad (\text{C.1})
\end{aligned}$$

This is derivative notation for the reset price already defined in Eqs. (14)–(15); it introduces no new economic variable.

The first-order condition for  $C_t$  is

$$\begin{aligned}
0 = & \frac{1}{C_t} - (1 - \theta)(1 - \varepsilon) \left[ \lambda_{p,t} \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}'_{H,t}(C_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j} \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}'_{H,t-j}(C_t) \right] \\
& + (1 - \theta) \varepsilon \left[ \lambda_{\Delta,t} \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}'_{H,t}(C_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j} \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}'_{H,t-j}(C_t) \right] \\
& - (1 - v) \lambda_{y,t} \mathcal{G}(Z_t, \tau_{M,t})^\eta - \lambda_{r,t} \mathcal{G}(Z_t, \tau_{M,t}) \\
& - \beta \lambda_{e,t} \mathbf{E}_t \left[ \frac{\mathcal{G}(Z_{t+1}, \tau_{M,t+1})}{C_{t+1} \mathcal{G}(Z_{t+1}, \tau_{M,t+1}) \Pi_{H,t+1}} \right] \\
& + \lambda_{e,t-1} \frac{C_{t-1} \mathcal{G}(Z_{t-1}, \tau_{M,t-1})}{C_t^2 \mathcal{G}(Z_t, \tau_{M,t}) \Pi_{H,t}}.
\end{aligned}$$

The first-order condition for  $N_t$  is

$$0 = -N_t^\varphi + \lambda_{n,t}$$

$$\begin{aligned}
& -(1-\theta)(1-\varepsilon) \left[ \lambda_{p,t} \tilde{p}_{H,t}^{\varepsilon} \tilde{p}_{H,t}^{\varepsilon} (N_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j} \tilde{p}_{H,t-j}^{\varepsilon} \tilde{p}_{H,t-j}^{\varepsilon} (N_t) \right] \\
& + (1-\theta)\varepsilon \left[ \lambda_{\Delta,t} \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t}^{\varepsilon} (N_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j} \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j}^{\varepsilon} (N_t) \right].
\end{aligned}$$

The first-order condition for  $Y_t$  is

$$\begin{aligned}
0 &= -\lambda_{n,t} \Delta_{H,t} + \lambda_{y,t} \\
& -(1-\theta)(1-\varepsilon) \left[ \lambda_{p,t} \tilde{p}_{H,t}^{\varepsilon} \tilde{p}_{H,t}^{\varepsilon} (Y_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j} \tilde{p}_{H,t-j}^{\varepsilon} \tilde{p}_{H,t-j}^{\varepsilon} (Y_t) \right] \\
& + (1-\theta)\varepsilon \left[ \lambda_{\Delta,t} \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t}^{\varepsilon} (Y_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j} \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j}^{\varepsilon} (Y_t) \right].
\end{aligned}$$

The first-order condition for  $Z_t$  is

$$\begin{aligned}
0 &= -(1-\theta)(1-\varepsilon) \left[ \lambda_{p,t} \tilde{p}_{H,t}^{\varepsilon} \tilde{p}_{H,t}^{\varepsilon} (Z_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j} \tilde{p}_{H,t-j}^{\varepsilon} \tilde{p}_{H,t-j}^{\varepsilon} (Z_t) \right] \\
& + (1-\theta)\varepsilon \left[ \lambda_{\Delta,t} \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t}^{\varepsilon} (Z_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j} \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j}^{\varepsilon} (Z_t) \right] \\
& - \lambda_{y,t} \left[ (1-v)\eta \mathcal{G}(Z_t, \tau_{M,t})^{\eta-1} \frac{\partial \mathcal{G}(Z_t, \tau_{M,t})}{\partial Z_t} C_t + v\eta Z_t^{\eta-1} \right] \\
& + \lambda_{r,t} \left[ 1 - C_t \frac{\partial \mathcal{G}(Z_t, \tau_{M,t})}{\partial Z_t} \right] \\
& - \beta \lambda_{e,t} E_t \left[ \frac{C_t \partial \mathcal{G}(Z_t, \tau_{M,t}) / \partial Z_t}{C_{t+1} \mathcal{G}(Z_{t+1}, \tau_{M,t+1}) \Pi_{H,t+1}} \right] \\
& + \lambda_{e,t-1} \frac{C_{t-1} \mathcal{G}(Z_{t-1}, \tau_{M,t-1})}{C_t \mathcal{G}(Z_t, \tau_{M,t})^2 \Pi_{H,t}} \frac{\partial \mathcal{G}(Z_t, \tau_{M,t})}{\partial Z_t}.
\end{aligned}$$

The first-order condition for  $\Pi_{H,t}$  is

$$\begin{aligned}
0 &= -\theta(\varepsilon-1) \lambda_{p,t} \Pi_{H,t}^{\varepsilon-2} - \theta \varepsilon \lambda_{\Delta,t} \Pi_{H,t}^{\varepsilon-1} \Delta_{H,t-1} \\
& -(1-\theta)(1-\varepsilon) \left[ \lambda_{p,t} \tilde{p}_{H,t}^{\varepsilon} \tilde{p}_{H,t}^{\varepsilon} (\Pi_{H,t}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j} \tilde{p}_{H,t-j}^{\varepsilon} \tilde{p}_{H,t-j}^{\varepsilon} (\Pi_{H,t}) \right] \\
& + (1-\theta)\varepsilon \left[ \lambda_{\Delta,t} \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t}^{\varepsilon} (\Pi_{H,t}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j} \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j}^{\varepsilon} (\Pi_{H,t}) \right] \\
& + \lambda_{e,t-1} \frac{C_{t-1} \mathcal{G}(Z_{t-1}, \tau_{M,t-1})}{C_t \mathcal{G}(Z_t, \tau_{M,t}) \Pi_{H,t}^2}.
\end{aligned}$$

The first-order condition for  $\Delta_{H,t}$  is

$$0 = \lambda_{\Delta,t} - \lambda_{n,t} Y_t - \beta \theta \lambda_{\Delta,t+1} \Pi_{H,t+1}^{\varepsilon}.$$

Finally, the interest-rate condition and the ZLB conditions are

$$-\frac{\lambda_{e,t}}{(1+i_t)^2} + \mu_{i,t} = 0, \quad i_t \geq 0, \quad \mu_{i,t} \geq 0, \quad \mu_{i,t} i_t = 0.$$

### C.1.2 IPT Environment

In the IPT environment, the producer block is unchanged and the planner additionally imposes the customs-duty importer block in Eqs. (20)–(22) and Eqs. (28)–(30), together with the local relative-import-price law in Eq. (27). The household budget constraint remains the one stated in Sec. 3, with consumption expenditure  $P_t C_t$ , and tariff revenue is given by Eq. (25). The Ramsey first-order conditions used for computation are obtained by differentiating this constrained problem directly. We do not replace the nonlinear importer reset-price condition by the linearized Phillips curve in Eq. (32).

The ZLB condition remains

$$-\frac{\lambda_{e,t}}{(1+i_t)^2} + \mu_{i,t} = 0, \quad i_t \geq 0, \quad \mu_{i,t} \geq 0, \quad \mu_{i,t} i_t = 0.$$

## C.2 Two-country Economy

### C.2.1 PPT Environment

For the Home reset price the derivative notation is that in Eq. (C.1). The Foreign reset price uses the exact starred counterpart,

$$(\tilde{p}_{F,t-j}^*)'(x_t) = \mathcal{M} \frac{(Z_{F,t-j}^*)'(x_t) \mathcal{K}_{F,t-j}^* - (\mathcal{K}_{F,t-j}^*)'(x_t) Z_{F,t-j}^*}{(\mathcal{K}_{F,t-j}^*)^2}.$$

The Home first-order condition for  $C_t$  is

$$\begin{aligned} 0 = & \frac{1}{C_t} - (1-\theta)(1-\varepsilon) \left[ \lambda_{p,t}^H \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t} (C_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^H \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j} (C_t) \right] \\ & + (1-\theta)\varepsilon \left[ \lambda_{\Delta,t}^H \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t} (C_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^H \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j} (C_t) \right] \\ & - \lambda_{g,t}^H \omega_F (1+\tau_{M,t})^{1-\eta} (1-\eta) \frac{1}{C_t} \left[ \frac{C_t}{C_t^*} \frac{\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)}{P_t^*, PPT/P_{F,t}^*} \right]^{1-\eta} \\ & - \lambda_{y,t}^H \omega_H (\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t))^\eta \\ & - \lambda_{y,t}^H \frac{1-\nu}{\nu} \omega_H^* \eta \frac{C_t^*}{C_t} \left[ \frac{C_t (\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t))}{C_t^* (1+\tau_{M,t}^*)} \right]^\eta \\ & - \beta \lambda_{e,t}^H \mathbf{E}_t \left[ \frac{\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)}{C_{t+1} (\mathcal{G}(\tau_{M,t+1}, Z_{t+1}/\Psi_{t+1})) \Pi_{H,t+1}} \right] \\ & + \lambda_{e,t-1}^H \frac{C_{t-1} (\mathcal{G}(\tau_{M,t-1}, Z_{t-1}/\Psi_{t-1}))}{C_t^2 (\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)) \Pi_{H,t}}. \end{aligned}$$

The Home first-order condition for  $\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)$  is

$$\begin{aligned} 0 = & -(1-\theta)(1-\varepsilon) \left[ \lambda_{p,t}^H \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t} (\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^H \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j} (\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)) \right] \\ & + (1-\theta)\varepsilon \left[ \lambda_{\Delta,t}^H \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t} (\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^H \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j} (\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)) \right] \\ & + (1-\eta) \lambda_{g,t}^H (\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t))^{-\eta} \end{aligned}$$

$$\begin{aligned}
& -\lambda_{g,t}^H \omega_F (1 + \tau_{M,t})^{1-\eta} (1 - \eta) (\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t))^{-1} \left[ \frac{C_t}{C_t^*} \frac{\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)}{P_t^{*,PPT}/P_{F,t}^*} \right]^{1-\eta} \\
& -\eta \lambda_{y,t}^H \omega_H (\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t))^{\eta-1} C_t \\
& -\eta \lambda_{y,t}^H \frac{1-\nu}{\nu} \omega_H^* (\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t))^{-1} C_t^* \left[ \frac{C_t (\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t))}{C_t^* (1 + \tau_{M,t})} \right]^\eta \\
& -\beta \lambda_{e,t}^H \mathbb{E}_t \left[ \frac{C_t}{C_{t+1} (\mathcal{G}(\tau_{M,t+1}, Z_{t+1}/\Psi_{t+1})) \Pi_{H,t+1}} \right] \\
& + \lambda_{e,t-1}^H \frac{C_{t-1} (\mathcal{G}(\tau_{M,t-1}, Z_{t-1}/\Psi_{t-1}))}{C_t (\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t))^2 \Pi_{H,t}}.
\end{aligned}$$

The remaining Home conditions in the PPT environment are

$$\begin{aligned}
0 &= -N_t^\varphi + \lambda_{n,t}^H - (1 - \theta)(1 - \varepsilon) \left[ \lambda_{p,t}^H \tilde{p}_{H,t}^{-\varepsilon} \tilde{p}'_{H,t}(N_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^H \tilde{p}_{H,t-j}^{-\varepsilon} \tilde{p}'_{H,t-j}(N_t) \right] \\
&+ (1 - \theta) \varepsilon \left[ \lambda_{\Delta,t}^H \tilde{p}_{H,t}^{-\varepsilon-1} \tilde{p}'_{H,t}(N_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^H \tilde{p}_{H,t-j}^{-\varepsilon-1} \tilde{p}'_{H,t-j}(N_t) \right], \\
0 &= -\lambda_{n,t}^H \Delta_{H,t} + \lambda_{y,t}^H - (1 - \theta)(1 - \varepsilon) \left[ \lambda_{p,t}^H \tilde{p}_{H,t}^{-\varepsilon} \tilde{p}'_{H,t}(Y_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^H \tilde{p}_{H,t-j}^{-\varepsilon} \tilde{p}'_{H,t-j}(Y_t) \right] \\
&+ (1 - \theta) \varepsilon \left[ \lambda_{\Delta,t}^H \tilde{p}_{H,t}^{-\varepsilon-1} \tilde{p}'_{H,t}(Y_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^H \tilde{p}_{H,t-j}^{-\varepsilon-1} \tilde{p}'_{H,t-j}(Y_t) \right], \\
0 &= -\theta(\varepsilon - 1) \lambda_{p,t}^H \Pi_{H,t}^{\varepsilon-2} - \theta \varepsilon \lambda_{\Delta,t}^H \Pi_{H,t}^{\varepsilon-1} \Delta_{H,t-1} \\
&- (1 - \theta)(1 - \varepsilon) \left[ \lambda_{p,t}^H \tilde{p}_{H,t}^{-\varepsilon} \tilde{p}'_{H,t}(\Pi_{H,t}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^H \tilde{p}_{H,t-j}^{-\varepsilon} \tilde{p}'_{H,t-j}(\Pi_{H,t}) \right] \\
&+ (1 - \theta) \varepsilon \left[ \lambda_{\Delta,t}^H \tilde{p}_{H,t}^{-\varepsilon-1} \tilde{p}'_{H,t}(\Pi_{H,t}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^H \tilde{p}_{H,t-j}^{-\varepsilon-1} \tilde{p}'_{H,t-j}(\Pi_{H,t}) \right] \\
&+ \lambda_{e,t-1}^H \frac{C_{t-1} (\mathcal{G}(\tau_{M,t-1}, Z_{t-1}/\Psi_{t-1}))}{C_t (\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)) \Pi_{H,t}^2}, \\
0 &= \lambda_{\Delta,t}^H - \lambda_{n,t}^H Y_t - \beta \theta \lambda_{\Delta,t+1}^H \Pi_{H,t+1}^\varepsilon, \\
0 &= -\frac{\lambda_{e,t}^H}{(1 + i_t)^2} + \mu_{i,t}^H.
\end{aligned}$$

The Home ZLB conditions are

$$i_t \geq 0, \quad \mu_{i,t}^H \geq 0, \quad \mu_{i,t}^H i_t = 0.$$

The Foreign first-order conditions in the PPT environment are written explicitly as

$$\begin{aligned}
0 &= \frac{1}{C_t^*} - (1 - \theta)(1 - \varepsilon) \left[ \lambda_{p,t}^F (\tilde{p}_{F,t}^*)^{-\varepsilon} (\tilde{p}_{F,t}^*)' (C_t^*) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^F (\tilde{p}_{F,t-j}^*)^{-\varepsilon} (\tilde{p}_{F,t-j}^*)' (C_t^*) \right] \\
&+ (1 - \theta) \varepsilon \left[ \lambda_{\Delta,t}^F (\tilde{p}_{F,t}^*)^{-\varepsilon-1} (\tilde{p}_{F,t}^*)' (C_t^*) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^F (\tilde{p}_{F,t-j}^*)^{-\varepsilon-1} (\tilde{p}_{F,t-j}^*)' (C_t^*) \right]
\end{aligned}$$

$$\begin{aligned}
& -\lambda_{g,t}^F \omega_H^* (1 + \tau_{M,t}^*)^{1-\eta} (1 - \eta) \frac{1}{C_t^*} \left[ \frac{C_t^*}{C_t} \frac{P_t^{*,PPT}/P_{F,t}^*}{\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)} \right]^{1-\eta} \\
& -\lambda_{y,t}^F \omega_F^* \left( \frac{P_t^{*,PPT}}{P_{F,t}^*} \right)^\eta \\
& -\lambda_{y,t}^F \frac{\nu}{1-\nu} \omega_F \eta \frac{C_t}{C_t^*} \left[ \frac{C_t^* (P_t^{*,PPT}/P_{F,t}^*)}{C_t (1 + \tau_{M,t})} \right]^\eta \\
& -\beta \lambda_{e,t}^F \mathbf{E}_t \left[ \frac{P_t^{*,PPT}/P_{F,t}^*}{C_{t+1}^* (P_{t+1}^{*,PPT}/P_{F,t+1}^*) \Pi_{F,t+1}^*} \right] \\
& +\lambda_{e,t-1}^F \frac{C_{t-1}^* (P_{t-1}^{*,PPT}/P_{F,t-1}^*)}{(C_t^*)^2 (P_t^{*,PPT}/P_{F,t}^*) \Pi_{F,t}^*}, \\
0 = & -(1 - \theta)(1 - \varepsilon) \left[ \lambda_{p,t}^F (\tilde{p}_{F,t}^*)^{-\varepsilon} (\tilde{p}_{F,t}^*)' (P_t^{*,PPT}/P_{F,t}^*) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^F (\tilde{p}_{F,t-j}^*)^{-\varepsilon} (\tilde{p}_{F,t-j}^*)' (P_t^{*,PPT}/P_{F,t}^*) \right] \\
& + (1 - \theta) \varepsilon \left[ \lambda_{\Delta,t}^F (\tilde{p}_{F,t}^*)^{-\varepsilon-1} (\tilde{p}_{F,t}^*)' (P_t^{*,PPT}/P_{F,t}^*) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^F (\tilde{p}_{F,t-j}^*)^{-\varepsilon-1} (\tilde{p}_{F,t-j}^*)' (P_t^{*,PPT}/P_{F,t}^*) \right] \\
& + (1 - \eta) \lambda_{g,t}^F \left( \frac{P_t^{*,PPT}}{P_{F,t}^*} \right)^{-\eta} \\
& -\lambda_{g,t}^F \omega_H^* (1 + \tau_{M,t}^*)^{1-\eta} (1 - \eta) \left( \frac{P_t^{*,PPT}}{P_{F,t}^*} \right)^{-1} \left[ \frac{C_t^*}{C_t} \frac{P_t^{*,PPT}/P_{F,t}^*}{\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)} \right]^{1-\eta} \\
& -\eta \lambda_{y,t}^F \omega_F^* \left( \frac{P_t^{*,PPT}}{P_{F,t}^*} \right)^{\eta-1} C_t^* \\
& -\eta \lambda_{y,t}^F \frac{\nu}{1-\nu} \omega_F \left( \frac{P_t^{*,PPT}}{P_{F,t}^*} \right)^{-1} C_t \left[ \frac{C_t^* (P_t^{*,PPT}/P_{F,t}^*)}{C_t (1 + \tau_{M,t})} \right]^\eta \\
& -\beta \lambda_{e,t}^F \mathbf{E}_t \left[ \frac{C_t^*}{C_{t+1}^* (P_{t+1}^{*,PPT}/P_{F,t+1}^*) \Pi_{F,t+1}^*} \right] \\
& +\lambda_{e,t-1}^F \frac{C_{t-1}^* (P_{t-1}^{*,PPT}/P_{F,t-1}^*)}{C_t^* (P_t^{*,PPT}/P_{F,t}^*)^2 \Pi_{F,t}^*},
\end{aligned}$$

together with

$$\begin{aligned}
0 = & -(N_t^*)^\varphi + \lambda_{n,t}^F - (1 - \theta)(1 - \varepsilon) \left[ \lambda_{p,t}^F (\tilde{p}_{F,t}^*)^{-\varepsilon} (\tilde{p}_{F,t}^*)' (N_t^*) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^F (\tilde{p}_{F,t-j}^*)^{-\varepsilon} (\tilde{p}_{F,t-j}^*)' (N_t^*) \right] \\
& + (1 - \theta) \varepsilon \left[ \lambda_{\Delta,t}^F (\tilde{p}_{F,t}^*)^{-\varepsilon-1} (\tilde{p}_{F,t}^*)' (N_t^*) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^F (\tilde{p}_{F,t-j}^*)^{-\varepsilon-1} (\tilde{p}_{F,t-j}^*)' (N_t^*) \right], \\
0 = & -\lambda_{n,t}^F \Delta_{F,t}^* + \lambda_{y,t}^F - (1 - \theta)(1 - \varepsilon) \left[ \lambda_{p,t}^F (\tilde{p}_{F,t}^*)^{-\varepsilon} (\tilde{p}_{F,t}^*)' (Y_t^*) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^F (\tilde{p}_{F,t-j}^*)^{-\varepsilon} (\tilde{p}_{F,t-j}^*)' (Y_t^*) \right] \\
& + (1 - \theta) \varepsilon \left[ \lambda_{\Delta,t}^F (\tilde{p}_{F,t}^*)^{-\varepsilon-1} (\tilde{p}_{F,t}^*)' (Y_t^*) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^F (\tilde{p}_{F,t-j}^*)^{-\varepsilon-1} (\tilde{p}_{F,t-j}^*)' (Y_t^*) \right],
\end{aligned}$$

$$\begin{aligned}
0 &= -\theta(\varepsilon - 1)\lambda_{p,t}^F(\Pi_{F,t}^*)^{\varepsilon-2} - \theta\varepsilon\lambda_{\Delta,t}^F(\Pi_{F,t}^*)^{\varepsilon-1}\Delta_{F,t-1}^* \\
&\quad - (1-\theta)(1-\varepsilon) \left[ \lambda_{p,t}^F(\tilde{p}_{F,t}^*)^{-\varepsilon}(\tilde{p}_{F,t}^*)'(\Pi_{F,t}^*) + \sum_{j=1}^{\infty} \beta^{-j}\lambda_{p,t-j}^F(\tilde{p}_{F,t-j}^*)^{-\varepsilon}(\tilde{p}_{F,t-j}^*)'(\Pi_{F,t}^*) \right] \\
&\quad + (1-\theta)\varepsilon \left[ \lambda_{\Delta,t}^F(\tilde{p}_{F,t}^*)^{-\varepsilon-1}(\tilde{p}_{F,t}^*)'(\Pi_{F,t}^*) + \sum_{j=1}^{\infty} \beta^{-j}\lambda_{\Delta,t-j}^F(\tilde{p}_{F,t-j}^*)^{-\varepsilon-1}(\tilde{p}_{F,t-j}^*)'(\Pi_{F,t}^*) \right] \\
&\quad + \lambda_{e,t-1}^F \frac{C_{t-1}^*(P_{t-1}^{*,PPT}/P_{F,t-1}^*)}{C_t^*(P_t^{*,PPT}/P_{F,t}^*)(\Pi_{F,t}^*)^2}, \\
0 &= \lambda_{\Delta,t}^F - \lambda_{n,t}^F Y_t^* - \beta\theta\lambda_{\Delta,t+1}^F(\Pi_{F,t+1}^*)^\varepsilon, \\
0 &= -\frac{\lambda_{e,t}^F}{(1+i_t^*)^2} + \mu_{i,t}^F.
\end{aligned}$$

The Foreign ZLB conditions are

$$i_t^* \geq 0, \quad \mu_{i,t}^F \geq 0, \quad \mu_{i,t}^F i_t^* = 0.$$

### C.2.2 IPT Environment

We differentiate the exact intertemporal Lagrangian (B.4) after complete risk sharing and the two private reset-price conditions have been substituted. A prime on  $\tilde{p}_{H,t}$  or on  $\bar{P}_{F,t}/P_{F,t}$  below denotes the total derivative of that exact reset-price map with respect to the indicated Ramsey choice; hence all current and future terms implied by commitment are retained. The cross-sectional purchaser-price integrals are differentiated directly and are not replaced by an importer-dispersion variable.

The Home consumption FOC is

$$\begin{aligned}
0 &= \frac{1}{C_t} - \lambda_{g,t}^H \omega_F (1-\eta) \left( \frac{\tilde{P}_{F,t}^{IPT}}{P_{H,t}} \right)^{-\eta} \frac{\partial(\tilde{P}_{F,t}^{IPT}/P_{H,t})}{\partial C_t} - \lambda_{y,t}^H \omega_H \mathcal{G} \left( \tau_{M,t}, \frac{Z_t}{\Psi_t} \right)^\eta \\
&\quad - \lambda_{y,t}^H \frac{1-\nu}{\nu} \omega_H^* \frac{\partial}{\partial C_t} \left[ \left( \frac{\tilde{P}_{H,t}^*}{P_t^{*,IPT}} \right)^{-\eta} C_t^* \frac{1}{1-\nu} \int_\nu^1 \left( \frac{\tilde{P}_{H,t}^*(\iota)}{\tilde{P}_{H,t}^*} \right)^{-\varepsilon_M} d\iota \right] \\
&\quad - (1-\theta)(1-\varepsilon) \left[ \lambda_{p,t}^H \tilde{p}_{H,t}^{-\varepsilon} \tilde{p}_{H,t}'(C_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^H \tilde{p}_{H,t-j}^{-\varepsilon} \tilde{p}_{H,t-j}'(C_t) \right] \\
&\quad + (1-\theta)\varepsilon \left[ \lambda_{\Delta,t}^H \tilde{p}_{H,t}^{-\varepsilon-1} \tilde{p}_{H,t}'(C_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^H \tilde{p}_{H,t-j}^{-\varepsilon-1} \tilde{p}_{H,t-j}'(C_t) \right] \\
&\quad - (1-\theta_M)(1-\varepsilon_M) \left[ \lambda_{Mp,t}^H \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)'(C_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{Mp,t-j}^H \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)'(C_t) \right] \\
&\quad - \beta \lambda_{e,t}^H E_t \left[ \frac{\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)}{C_{t+1} \mathcal{G}(\tau_{M,t+1}, Z_{t+1}/\Psi_{t+1}) \Pi_{H,t+1}} \right] + \lambda_{e,t-1}^H \frac{C_{t-1} \mathcal{G}(\tau_{M,t-1}, Z_{t-1}/\Psi_{t-1})}{C_t^2 \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t) \Pi_{H,t}}. \tag{C.2}
\end{aligned}$$

The FOC for the Home CPI-PPI ratio is

$$\begin{aligned}
0 &= (1-\eta)\lambda_{g,t}^H \mathcal{G} \left( \tau_{M,t}, \frac{Z_t}{\Psi_t} \right)^{-\eta} - \lambda_{g,t}^H \omega_F (1-\eta) \left( \frac{\tilde{P}_{F,t}^{IPT}}{P_{H,t}} \right)^{-\eta} \frac{\partial(\tilde{P}_{F,t}^{IPT}/P_{H,t})}{\partial \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)} \\
&\quad - \eta \lambda_{y,t}^H \omega_H \mathcal{G} \left( \tau_{M,t}, \frac{Z_t}{\Psi_t} \right)^{\eta-1} C_t - \lambda_{y,t}^H \frac{1-\nu}{\nu} \omega_H^* \frac{\partial}{\partial \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)} \left[ \left( \frac{\tilde{P}_{H,t}^*}{P_t^{*,IPT}} \right)^{-\eta} C_t^* \frac{1}{1-\nu} \int_\nu^1 \left( \frac{\tilde{P}_{H,t}^*(\iota)}{\tilde{P}_{H,t}^*} \right)^{-\varepsilon_M} d\iota \right]
\end{aligned}$$

$$\begin{aligned}
& -(1-\theta)(1-\varepsilon) \left[ \lambda_{p,t}^H \tilde{p}_{H,t}^{-\varepsilon} \tilde{p}'_{H,t}(\mathcal{G}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^H \tilde{p}_{H,t-j}^{-\varepsilon} \tilde{p}'_{H,t-j}(\mathcal{G}) \right] \\
& + (1-\theta)\varepsilon \left[ \lambda_{\Delta,t}^H \tilde{p}_{H,t}^{-\varepsilon-1} \tilde{p}'_{H,t}(\mathcal{G}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^H \tilde{p}_{H,t-j}^{-\varepsilon-1} \tilde{p}'_{H,t-j}(\mathcal{G}) \right] \\
& - (1-\theta_M)(1-\varepsilon_M) \left[ \lambda_{Mp,t}^H \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)'(\mathcal{G}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{Mp,t-j}^H \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)'(\mathcal{G}) \right] \\
& - \beta \lambda_{e,t}^H E_t \left[ \frac{C_t}{C_{t+1} \mathcal{G}(\tau_{M,t+1}, Z_{t+1}/\Psi_{t+1}) \Pi_{H,t+1}} \right] + \lambda_{e,t-1}^H \frac{C_{t-1} \mathcal{G}(\tau_{M,t-1}, Z_{t-1}/\Psi_{t-1})}{C_t \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)^2 \Pi_{H,t}}. \tag{C.3}
\end{aligned}$$

Here  $\tilde{p}'_{H,t}(\mathcal{G})$  and  $(\bar{P}_{F,t}/P_{F,t})'(\mathcal{G})$  mean total derivatives with respect to the displayed CPI-PPI ratio, not derivatives with respect to a newly defined variable.

The labor and output FOCs are

$$\begin{aligned}
0 &= -N_t^\varphi + \lambda_{n,t}^H - (1-\theta)(1-\varepsilon) \left[ \lambda_{p,t}^H \tilde{p}_{H,t}^{-\varepsilon} \tilde{p}'_{H,t}(N_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^H \tilde{p}_{H,t-j}^{-\varepsilon} \tilde{p}'_{H,t-j}(N_t) \right] \\
&+ (1-\theta)\varepsilon \left[ \lambda_{\Delta,t}^H \tilde{p}_{H,t}^{-\varepsilon-1} \tilde{p}'_{H,t}(N_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^H \tilde{p}_{H,t-j}^{-\varepsilon-1} \tilde{p}'_{H,t-j}(N_t) \right], \tag{C.4}
\end{aligned}$$

$$\begin{aligned}
0 &= -\lambda_{n,t}^H \Delta_{H,t} + \lambda_{y,t}^H - (1-\theta)(1-\varepsilon) \left[ \lambda_{p,t}^H \tilde{p}_{H,t}^{-\varepsilon} \tilde{p}'_{H,t}(Y_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^H \tilde{p}_{H,t-j}^{-\varepsilon} \tilde{p}'_{H,t-j}(Y_t) \right] \\
&+ (1-\theta)\varepsilon \left[ \lambda_{\Delta,t}^H \tilde{p}_{H,t}^{-\varepsilon-1} \tilde{p}'_{H,t}(Y_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^H \tilde{p}_{H,t-j}^{-\varepsilon-1} \tilde{p}'_{H,t-j}(Y_t) \right]. \tag{C.5}
\end{aligned}$$

For the local *ex-tariff* import-price ratio,

$$\begin{aligned}
0 &= -\lambda_{g,t}^H \omega_F (1-\eta) \left( \frac{\tilde{P}_{F,t}^{IPT}}{P_{H,t}} \right)^{-\eta} \frac{\partial(\tilde{P}_{F,t}^{IPT}/P_{H,t})}{\partial(P_{F,t}/P_{H,t})} \\
&- \lambda_{y,t}^H \frac{1-\nu}{\nu} \omega_H^* \frac{\partial}{\partial(P_{F,t}/P_{H,t})} \left[ \left( \frac{\tilde{P}_{H,t}^*}{P_t^{*,IPT}} \right)^{-\eta} C_t^* \frac{1}{1-\nu} \int_{\nu}^1 \left( \frac{\tilde{P}_{H,t}^*(\iota)}{\tilde{P}_{H,t}^*} \right)^{-\varepsilon_M} d\iota \right] \\
&- (1-\theta_M)(1-\varepsilon_M) \left[ \lambda_{Mp,t}^H \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)'(P_{F,t}/P_{H,t}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{Mp,t-j}^H \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)'(P_{F,t-j}/P_{H,t-j}) \right] \\
&+ \lambda_{q,t}^H - \beta \lambda_{q,t+1}^H \frac{\Pi_{F,t+1}}{\Pi_{H,t+1}}.
\end{aligned}$$

The two inflation FOCs are

$$\begin{aligned}
0 &= -\theta(\varepsilon-1) \lambda_{p,t}^H \Pi_{H,t}^{\varepsilon-2} - \theta \varepsilon \lambda_{\Delta,t}^H \Pi_{H,t}^{\varepsilon-1} \Delta_{H,t-1} \\
&- (1-\theta)(1-\varepsilon) \left[ \lambda_{p,t}^H \tilde{p}_{H,t}^{-\varepsilon} \tilde{p}'_{H,t}(\Pi_{H,t}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^H \tilde{p}_{H,t-j}^{-\varepsilon} \tilde{p}'_{H,t-j}(\Pi_{H,t}) \right] \\
&+ (1-\theta)\varepsilon \left[ \lambda_{\Delta,t}^H \tilde{p}_{H,t}^{-\varepsilon-1} \tilde{p}'_{H,t}(\Pi_{H,t}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^H \tilde{p}_{H,t-j}^{-\varepsilon-1} \tilde{p}'_{H,t-j}(\Pi_{H,t}) \right]
\end{aligned}$$

$$\begin{aligned}
& -(1 - \theta_M)(1 - \varepsilon_M) \left[ \lambda_{Mp,t}^H \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)' (\Pi_{H,t}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{Mp,t-j}^H \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)' (\Pi_{H,t}) \right] \\
& + \lambda_{q,t}^H \frac{P_{F,t-1}}{P_{H,t-1}} \frac{\Pi_{F,t}}{\Pi_{H,t}^2} + \lambda_{e,t-1}^H \frac{C_{t-1} \mathcal{G}(\tau_{M,t-1}, Z_{t-1}/\Psi_{t-1})}{C_t \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t) \Pi_{H,t}^2}, \\
0 = & -\theta_M(\varepsilon_M - 1) \lambda_{Mp,t}^H \Pi_{F,t}^{\varepsilon_M - 2} \\
& -(1 - \theta_M)(1 - \varepsilon_M) \left[ \lambda_{Mp,t}^H \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)' (\Pi_{F,t}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{Mp,t-j}^H \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)' (\Pi_{F,t}) \right]
\end{aligned}$$

The producer-dispersion and nominal-rate conditions are

$$0 = \lambda_{\Delta,t}^H - \lambda_{n,t}^H Y_t - \beta \theta \lambda_{\Delta,t+1}^H \Pi_{H,t+1}^\varepsilon, \quad (\text{C.9})$$

$$0 = -\frac{\lambda_{e,t}^H}{(1 + i_t)^2} + \mu_{i,t}^H, \quad i_t \geq 0, \quad \mu_{i,t}^H \geq 0, \quad \mu_{i,t}^H i_t = 0. \quad (\text{C.10})$$

There is no FOC with respect to a separate importer-dispersion state: in the current micro-importer model the exact purchaser-price cross section in A.2 is retained directly. The Foreign FOCs are obtained from Eqs. (C.2)–(C.10) by the exact Home–Foreign mirror transformation and satisfy

$$-\frac{\lambda_{e,t}^F}{(1 + i_t^*)^2} + \mu_{i,t}^F = 0, \quad i_t^* \geq 0, \quad \mu_{i,t}^F \geq 0, \quad \mu_{i,t}^F i_t^* = 0. \quad (\text{C.11})$$

Together with the common A.2 private equilibrium, these conditions are the two national best-response systems used in the IPT simulations.

### C.2.3 External Habit (IPT Environment Only)

The FOCs in this subsection are obtained by differentiating the explicit habit Lagrangian in Eq. (B.7). The producer and importer reset-price maps are the A.3 habit-specific maps, so every total derivative of  $\tilde{p}_{H,t}$  and  $\bar{P}_{F,t}/P_{F,t}$  below contains the habit-adjusted stochastic discount factor. In addition, the Home planner internalizes the effect of current aggregate consumption on next period's habit benchmark. Thus the private Euler equation remains Eq. (A.15), while the planner's direct utility derivative contains both the current- and future-habit terms.

The Home habit consumption FOC is

$$\begin{aligned}
0 = & \frac{1}{C_t - \chi C_{t-1}} - \beta \frac{\chi}{C_{t+1} - \chi C_t} - \lambda_{g,t}^{H,h} \omega_F (1 - \eta) \left( \frac{\tilde{P}_{F,t}^{IPT}}{P_{H,t}} \right)^{-\eta} \frac{\partial(\tilde{P}_{F,t}^{IPT}/P_{H,t})}{\partial C_t} - \lambda_{y,t}^{H,h} \omega_H \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)^\eta \\
& - \lambda_{y,t}^{H,h} \frac{1 - \nu}{\nu} \omega_H^* \frac{\partial}{\partial C_t} \left[ \left( \frac{\tilde{P}_{H,t}^*}{P_t^{*,IPT}} \right)^{-\eta} C_t^* \frac{1}{1 - \nu} \int_\nu^1 \left( \frac{\tilde{P}_{H,t}^*(\iota)}{\tilde{P}_{H,t}^*} \right)^{-\varepsilon_M} d\iota \right] \\
& -(1 - \theta)(1 - \varepsilon) \left[ \lambda_{p,t}^{H,h} \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t}(C_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^{H,h} \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j}(C_t) \right] \\
& +(1 - \theta)\varepsilon \left[ \lambda_{\Delta,t}^{H,h} \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t}(C_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^{H,h} \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j}(C_t) \right] \\
& -(1 - \theta_M)(1 - \varepsilon_M) \left[ \lambda_{Mp,t}^{H,h} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)' (C_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{Mp,t-j}^{H,h} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)' (C_t) \right] \\
& - \beta \lambda_{e,t}^{H,h} \mathbb{E}_t \left[ \frac{\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)}{(C_{t+1} - \chi C_t) \mathcal{G}(\tau_{M,t+1}, Z_{t+1}/\Psi_{t+1}) \Pi_{H,t+1}} + \frac{\chi(C_t - \chi C_{t-1}) \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)}{(C_{t+1} - \chi C_t)^2 \mathcal{G}(\tau_{M,t+1}, Z_{t+1}/\Psi_{t+1}) \Pi_{H,t+1}} \right] + \lambda_{e,t}^{H,h}
\end{aligned}$$

The Home CPI-PPI-ratio FOC is

$$\begin{aligned}
0 = & (1 - \eta) \lambda_{g,t}^{H,h} \mathcal{G} \left( \tau_{M,t}, \frac{Z_t}{\Psi_t} \right)^{-\eta} - \lambda_{g,t}^{H,h} \omega_F (1 - \eta) \left( \frac{\tilde{P}_{F,t}^{IPT}}{P_{H,t}} \right)^{-\eta} \frac{\partial(\tilde{P}_{F,t}^{IPT}/P_{H,t})}{\partial \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)} \\
& - \eta \lambda_{y,t}^{H,h} \omega_H \mathcal{G} \left( \tau_{M,t}, \frac{Z_t}{\Psi_t} \right)^{\eta-1} C_t - \lambda_{y,t}^{H,h} \frac{1-\nu}{\nu} \omega_H^* \frac{\partial}{\partial \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)} \left[ \left( \frac{\tilde{P}_{H,t}^*}{P_t^{*,IPT}} \right)^{-\eta} C_t^* \frac{1}{1-\nu} \int_{\nu}^1 \left( \frac{\tilde{P}_{H,t}^*(\iota)}{\tilde{P}_{H,t}^*} \right)^{-\varepsilon_M} d\iota \right. \\
& \left. - (1 - \theta)(1 - \varepsilon) \left[ \lambda_{p,t}^{H,h} \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t}'(\mathcal{G}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^{H,h} \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j}'(\mathcal{G}) \right] \right. \\
& \left. + (1 - \theta)\varepsilon \left[ \lambda_{\Delta,t}^{H,h} \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t}'(\mathcal{G}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^{H,h} \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j}'(\mathcal{G}) \right] \right. \\
& \left. - (1 - \theta_M)(1 - \varepsilon_M) \left[ \lambda_{Mp,t}^{H,h} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)'(\mathcal{G}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{Mp,t-j}^{H,h} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)'(\mathcal{G}) \right] \right. \\
& \left. - \beta \lambda_{e,t}^{H,h} \mathbf{E}_t \left[ \frac{C_t - \chi C_{t-1}}{(C_{t+1} - \chi C_t) \mathcal{G}(\tau_{M,t+1}, Z_{t+1}/\Psi_{t+1}) \Pi_{H,t+1}} \right] + \lambda_{e,t-1}^{H,h} \frac{(C_{t-1} - \chi C_{t-2}) \mathcal{G}(\tau_{M,t-1}, Z_{t-1}/\Psi_{t-1})}{(C_t - \chi C_{t-1}) \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)^2 \Pi_{H,t}} \right]. \quad (C)
\end{aligned}$$

Here  $\tilde{p}_{H,t}'(\mathcal{G})$  and  $(\bar{P}_{F,t}/P_{F,t})'(\mathcal{G})$  mean total derivatives with respect to the displayed CPI-PPI ratio, not derivatives with respect to a newly defined variable.

The Home labor and output FOCs are

$$\begin{aligned}
0 = & -N_t^\varphi + \lambda_{n,t}^{H,h} - (1 - \theta)(1 - \varepsilon) \left[ \lambda_{p,t}^{H,h} \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t}'(N_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^{H,h} \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j}'(N_t) \right] \\
& + (1 - \theta)\varepsilon \left[ \lambda_{\Delta,t}^{H,h} \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t}'(N_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^{H,h} \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j}'(N_t) \right], \quad (C.14)
\end{aligned}$$

$$\begin{aligned}
0 = & -\lambda_{n,t}^{H,h} \Delta_{H,t} + \lambda_{y,t}^{H,h} - (1 - \theta)(1 - \varepsilon) \left[ \lambda_{p,t}^{H,h} \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t}'(Y_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^{H,h} \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j}'(Y_t) \right] \\
& + (1 - \theta)\varepsilon \left[ \lambda_{\Delta,t}^{H,h} \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t}'(Y_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^{H,h} \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j}'(Y_t) \right]. \quad (C.15)
\end{aligned}$$

For the Home local *ex-tariff* import-price ratio,

$$\begin{aligned}
0 = & -\lambda_{g,t}^{H,h} \omega_F (1 - \eta) \left( \frac{\tilde{P}_{F,t}^{IPT}}{P_{H,t}} \right)^{-\eta} \frac{\partial(\tilde{P}_{F,t}^{IPT}/P_{H,t})}{\partial(P_{F,t}/P_{H,t})} \\
& - \lambda_{y,t}^{H,h} \frac{1-\nu}{\nu} \omega_H^* \frac{\partial}{\partial(P_{F,t}/P_{H,t})} \left[ \left( \frac{\tilde{P}_{H,t}^*}{P_t^{*,IPT}} \right)^{-\eta} C_t^* \frac{1}{1-\nu} \int_{\nu}^1 \left( \frac{\tilde{P}_{H,t}^*(\iota)}{\tilde{P}_{H,t}^*} \right)^{-\varepsilon_M} d\iota \right] \\
& - (1 - \theta_M)(1 - \varepsilon_M) \left[ \lambda_{Mp,t}^{H,h} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)'(P_{F,t}/P_{H,t}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{Mp,t-j}^{H,h} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)'(P_{F,t}/P_{H,t}) \right. \\
& \left. + \lambda_{q,t}^{H,h} - \beta \lambda_{q,t+1}^{H,h} \frac{\Pi_{F,t+1}}{\Pi_{H,t+1}} \right].
\end{aligned}$$

The Home producer- and importer-price inflation FOCs are

$$0 = -\theta(\varepsilon - 1) \lambda_{p,t}^{H,h} \Pi_{H,t}^{\varepsilon-2} - \theta \varepsilon \lambda_{\Delta,t}^{H,h} \Pi_{H,t}^{\varepsilon-1} \Delta_{H,t-1}$$

$$\begin{aligned}
& -(1-\theta)(1-\varepsilon) \left[ \lambda_{p,t}^{H,h} \tilde{p}_{H,t}^{\sim\varepsilon} \tilde{p}_{H,t}(\Pi_{H,t}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^{H,h} \tilde{p}_{H,t-j}^{\sim\varepsilon} \tilde{p}_{H,t-j}(\Pi_{H,t}) \right] \\
& + (1-\theta)\varepsilon \left[ \lambda_{\Delta,t}^{H,h} \tilde{p}_{H,t}^{\sim\varepsilon-1} \tilde{p}_{H,t}(\Pi_{H,t}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^{H,h} \tilde{p}_{H,t-j}^{\sim\varepsilon-1} \tilde{p}_{H,t-j}(\Pi_{H,t}) \right] \\
& - (1-\theta_M)(1-\varepsilon_M) \left[ \lambda_{Mp,t}^{H,h} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)' (\Pi_{H,t}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{Mp,t-j}^{H,h} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)' (\Pi_{H,t}) \right] \\
& + \lambda_{q,t}^{H,h} \frac{P_{F,t-1}}{P_{H,t-1}} \frac{\Pi_{F,t}}{\Pi_{H,t}^2} + \lambda_{e,t}^{H,h} \frac{(C_{t-1} - \chi C_{t-2}) \mathcal{G}(\tau_{M,t-1}, Z_{t-1}/\Psi_{t-1})}{(C_t - \chi C_{t-1}) \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t) \Pi_{H,t}^2}, \\
0 = & -\theta_M(\varepsilon_M - 1) \lambda_{Mp,t}^{H,h} \Pi_{F,t}^{\varepsilon_M-2} \\
& - (1-\theta_M)(1-\varepsilon_M) \left[ \lambda_{Mp,t}^{H,h} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)' (\Pi_{F,t}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{Mp,t-j}^{H,h} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)' (\Pi_{F,t}) \right]
\end{aligned}$$

The Home producer-dispersion and nominal-rate conditions are

$$0 = \lambda_{\Delta,t}^{H,h} - \lambda_{n,t}^{H,h} Y_t - \beta \theta \lambda_{\Delta,t+1}^{H,h} \Pi_{H,t+1}^\varepsilon, \quad (\text{C.19})$$

$$0 = -\frac{\lambda_{e,t}^{H,h}}{(1+i_t)^2} + \mu_{i,t}^{H,h}, \quad i_t \geq 0, \quad \mu_{i,t}^{H,h} \geq 0, \quad \mu_{i,t}^{H,h} i_t = 0. \quad (\text{C.20})$$

There is again no separate importer-dispersion FOC: the exact purchaser-price cross section is retained as in A.2. The Foreign FOCs are the exact starred mirror of Eqs. (C.12)–(C.20), with  $1/(C_t^* - \chi C_{t-1}^*) - \beta \chi / (C_{t+1}^* - \chi C_t^*)$  as the direct Foreign planner utility derivative and with the Foreign habit-adjusted reset-price maps. The Foreign rate satisfies

$$-\frac{\lambda_{e,t}^{F,h}}{(1+i_t^*)^2} + \mu_{i,t}^{F,h} = 0, \quad i_t^* \geq 0, \quad \mu_{i,t}^{F,h} \geq 0, \quad \mu_{i,t}^{F,h} i_t^* = 0. \quad (\text{C.21})$$

Hence C.2.3 lists the FOCs of the Social Planner's Problem in B.2.3 rather than inferring them from the quadratic welfare criterion.

#### C.2.4 Roundabout Production (IPT Environment Only)

The FOCs below are obtained directly from the Roundabout Lagrangian (B.10). Relative to C.2.2, Home and Foreign goods demand use total absorption  $C_t + \mathcal{I}_t$  and  $C_t^* + \mathcal{I}_t^*$ , and the production block consists of the A.4 technology and two factor-demand restrictions. The producer and importer reset-price maps remain the exact A.2 maps, with producer real marginal cost determined jointly with the Roundabout factor demands.

The Home consumption FOC is

$$\begin{aligned}
0 = & \frac{1}{C_t} - \lambda_{g,t}^{H,RA} \omega_F (1-\eta) \left( \frac{\tilde{P}_{F,t}^{IPT}}{P_{H,t}} \right)^{-\eta} \frac{\partial(\tilde{P}_{F,t}^{IPT}/P_{H,t})}{\partial C_t} - \lambda_{y,t}^{H,RA} \omega_H \mathcal{G}\left(\tau_{M,t}, \frac{Z_t}{\Psi_t}\right)^\eta \\
& - \lambda_{y,t}^{H,RA} \frac{1-\nu}{\nu} \omega_H^* \frac{\partial}{\partial C_t} \left[ \left( \frac{\tilde{P}_{H,t}^*}{P_t^*} \right)^{-\eta} (C_t^* + \mathcal{I}_t^*) \frac{1}{1-\nu} \int_\nu^1 \left( \frac{\tilde{P}_{H,t}^*(\iota)}{\tilde{P}_{H,t}^*} \right)^{-\varepsilon_M} d\iota \right] \\
& - (1-\theta)(1-\varepsilon) \left[ \lambda_{p,t}^{H,RA} \tilde{p}_{H,t}^{\sim\varepsilon} \tilde{p}_{H,t}(C_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^{H,RA} \tilde{p}_{H,t-j}^{\sim\varepsilon} \tilde{p}_{H,t-j}(C_t) \right]
\end{aligned}$$

$$\begin{aligned}
& +(1-\theta)\varepsilon \left[ \lambda_{\Delta,t}^{H,RA} \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t}'(C_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^{H,RA} \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j}'(C_t) \right] \\
& -(1-\theta_M)(1-\varepsilon_M) \left[ \lambda_{Mp,t}^{H,RA} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)'(C_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{Mp,t-j}^{H,RA} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)'(C_t) \right] \\
& +(1-\zeta)\lambda_{N,t}^{H,RA} \frac{MC_t^r Y_t \Delta_{H,t}}{C_t^2 N_t^\varphi \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)} \\
& -\beta \lambda_{e,t}^{H,RA} \mathbf{E}_t \left[ \frac{\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)}{C_{t+1} \mathcal{G}(\tau_{M,t+1}, Z_{t+1}/\Psi_{t+1}) \Pi_{H,t+1}} \right] + \lambda_{e,t-1}^{H,RA} \frac{C_{t-1} \mathcal{G}(\tau_{M,t-1}, Z_{t-1}/\Psi_{t-1})}{C_t^2 \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t) \Pi_{H,t}}. \tag{C.22}
\end{aligned}$$

The Home CPI-PPI-ratio FOC is

$$\begin{aligned}
0 = & (1-\eta)\lambda_{g,t}^{H,RA} \mathcal{G}\left(\tau_{M,t}, \frac{Z_t}{\Psi_t}\right)^{-\eta} - \lambda_{g,t}^{H,RA} \omega_F (1-\eta) \left( \frac{\tilde{P}_{F,t}^{IPT}}{P_{H,t}} \right)^{-\eta} \frac{\partial(\tilde{P}_{F,t}^{IPT}/P_{H,t})}{\partial \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)} \\
& -\eta \lambda_{y,t}^{H,RA} \omega_H \mathcal{G}\left(\tau_{M,t}, \frac{Z_t}{\Psi_t}\right)^{\eta-1} (C_t + \mathcal{I}_t) - \lambda_{y,t}^{H,RA} \frac{1-\nu}{\nu} \omega_H^* \frac{\partial}{\partial \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)} \left[ \left( \frac{\tilde{P}_{H,t}^*}{P_t^{*,IPT}} \right)^{-\eta} (C_t^* + \mathcal{I}_t^*) \frac{1}{1-\nu} \int_\nu \right] \\
& -(1-\theta)(1-\varepsilon) \left[ \lambda_{p,t}^{H,RA} \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t}'(\mathcal{G}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^{H,RA} \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j}'(\mathcal{G}) \right] \\
& +(1-\theta)\varepsilon \left[ \lambda_{\Delta,t}^{H,RA} \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t}'(\mathcal{G}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^{H,RA} \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j}'(\mathcal{G}) \right] \\
& -(1-\theta_M)(1-\varepsilon_M) \left[ \lambda_{Mp,t}^{H,RA} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)'(\mathcal{G}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{Mp,t-j}^{H,RA} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)'(\mathcal{G}) \right] \\
& +(1-\zeta)\lambda_{N,t}^{H,RA} \frac{MC_t^r Y_t \Delta_{H,t}}{C_t N_t^\varphi \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)^2} + \zeta \lambda_{I,t}^{H,RA} \frac{MC_t^r Y_t \Delta_{H,t}}{\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)^2} \\
& -\beta \lambda_{e,t}^{H,RA} \mathbf{E}_t \left[ \frac{C_t}{C_{t+1} \mathcal{G}(\tau_{M,t+1}, Z_{t+1}/\Psi_{t+1}) \Pi_{H,t+1}} \right] + \lambda_{e,t-1}^{H,RA} \frac{C_{t-1} \mathcal{G}(\tau_{M,t-1}, Z_{t-1}/\Psi_{t-1})}{C_t \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)^2 \Pi_{H,t}}.
\end{aligned}$$

Here  $\tilde{p}_{H,t}(\mathcal{G})$  and  $(\bar{P}_{F,t}/P_{F,t})'(\mathcal{G})$  mean total derivatives with respect to the displayed CPI-PPI ratio, not derivatives with respect to a newly defined variable.

The Home labor, intermediate-input, producer-real-marginal-cost, and output FOCs are

$$\begin{aligned}
0 = & -N_t^\varphi - \lambda_{T,t}^{H,RA} (1-\zeta) \frac{(N_t/(1-\zeta))^{1-\zeta} (\mathcal{I}_t/\zeta)^\zeta}{N_t} \\
& + \lambda_{N,t}^{H,RA} \left[ 1 + (1-\zeta) \varphi \frac{MC_t^r Y_t \Delta_{H,t}}{C_t N_t^{\varphi+1} \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)} \right] \\
& -(1-\theta)(1-\varepsilon) \left[ \lambda_{p,t}^{H,RA} \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t}'(N_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^{H,RA} \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j}'(N_t) \right] \\
& +(1-\theta)\varepsilon \left[ \lambda_{\Delta,t}^{H,RA} \tilde{p}_{H,t}^{\varepsilon-1} \tilde{p}_{H,t}'(N_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^{H,RA} \tilde{p}_{H,t-j}^{\varepsilon-1} \tilde{p}_{H,t-j}'(N_t) \right], \tag{C.24}
\end{aligned}$$

$$0 = -\lambda_{y,t}^{H,RA} \omega_H \mathcal{G}\left(\tau_{M,t}, \frac{Z_t}{\Psi_t}\right)^\eta - \lambda_{T,t}^{H,RA} \zeta \frac{(N_t/(1-\zeta))^{1-\zeta} (\mathcal{I}_t/\zeta)^\zeta}{\mathcal{I}_t} + \lambda_{I,t}^{H,RA}, \tag{C.25}$$

$$0 = -(1-\zeta)\lambda_{N,t}^{H,RA} \frac{Y_t \Delta_{H,t}}{C_t N_t^\varphi \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)} - \zeta \lambda_{I,t}^{H,RA} \frac{Y_t \Delta_{H,t}}{\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)}$$

$$\begin{aligned}
& -(1-\theta)(1-\varepsilon) \left[ \lambda_{p,t}^{H,RA} \tilde{p}_{H,t}^{\sim\varepsilon} \tilde{p}_{H,t}(MC_t^r) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^{H,RA} \tilde{p}_{H,t-j}^{\sim\varepsilon} \tilde{p}_{H,t-j}(MC_t^r) \right] \\
& + (1-\theta)\varepsilon \left[ \lambda_{\Delta,t}^{H,RA} \tilde{p}_{H,t}^{\sim\varepsilon-1} \tilde{p}_{H,t}(MC_t^r) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^{H,RA} \tilde{p}_{H,t-j}^{\sim\varepsilon-1} \tilde{p}_{H,t-j}(MC_t^r) \right], \quad (C.26)
\end{aligned}$$

$$\begin{aligned}
0 = & \lambda_{y,t}^{H,RA} + \lambda_{T,t}^{H,RA} \Delta_{H,t} - (1-\zeta) \lambda_{N,t}^{H,RA} \frac{MC_t^r \Delta_{H,t}}{C_t N_t^\varphi \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)} - \zeta \lambda_{I,t}^{H,RA} \frac{MC_t^r \Delta_{H,t}}{\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)} \\
& - (1-\theta)(1-\varepsilon) \left[ \lambda_{p,t}^{H,RA} \tilde{p}_{H,t}^{\sim\varepsilon} \tilde{p}_{H,t}(Y_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^{H,RA} \tilde{p}_{H,t-j}^{\sim\varepsilon} \tilde{p}_{H,t-j}(Y_t) \right] \\
& + (1-\theta)\varepsilon \left[ \lambda_{\Delta,t}^{H,RA} \tilde{p}_{H,t}^{\sim\varepsilon-1} \tilde{p}_{H,t}(Y_t) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^{H,RA} \tilde{p}_{H,t-j}^{\sim\varepsilon-1} \tilde{p}_{H,t-j}(Y_t) \right]. \quad (C.27)
\end{aligned}$$

For the Home local *ex-tariff* import-price ratio,

$$\begin{aligned}
0 = & -\lambda_{g,t}^{H,RA} \omega_F (1-\eta) \left( \frac{\tilde{P}_{F,t}^{IPT}}{P_{H,t}} \right)^{-\eta} \frac{\partial(\tilde{P}_{F,t}^{IPT}/P_{H,t})}{\partial(P_{F,t}/P_{H,t})} \\
& - \lambda_{y,t}^{H,RA} \frac{1-\nu}{\nu} \omega_H^* \frac{\partial}{\partial(P_{F,t}/P_{H,t})} \left[ \left( \frac{\tilde{P}_{H,t}^*}{P_t^{*,IPT}} \right)^{-\eta} (C_t^* + \mathcal{I}_t^*) \frac{1}{1-\nu} \int_\nu^1 \left( \frac{\tilde{P}_{H,t}^*(\iota)}{\tilde{P}_{H,t}^*} \right)^{-\varepsilon_M} d\iota \right] \\
& - (1-\theta_M)(1-\varepsilon_M) \left[ \lambda_{Mp,t}^{H,RA} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)' (P_{F,t}/P_{H,t}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{Mp,t-j}^{H,RA} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)' \right] \\
& + \lambda_{q,t}^{H,RA} - \beta \lambda_{q,t+1}^{H,RA} \frac{\Pi_{F,t+1}}{\Pi_{H,t+1}}.
\end{aligned}$$

The Home producer- and importer-price inflation FOCs are

$$\begin{aligned}
0 = & -\theta(\varepsilon-1) \lambda_{p,t}^{H,RA} \Pi_{H,t}^{\varepsilon-2} - \theta \varepsilon \lambda_{\Delta,t}^{H,RA} \Pi_{H,t}^{\varepsilon-1} \Delta_{H,t-1} \\
& - (1-\theta)(1-\varepsilon) \left[ \lambda_{p,t}^{H,RA} \tilde{p}_{H,t}^{\sim\varepsilon} \tilde{p}_{H,t}(\Pi_{H,t}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{p,t-j}^{H,RA} \tilde{p}_{H,t-j}^{\sim\varepsilon} \tilde{p}_{H,t-j}(\Pi_{H,t}) \right] \\
& + (1-\theta)\varepsilon \left[ \lambda_{\Delta,t}^{H,RA} \tilde{p}_{H,t}^{\sim\varepsilon-1} \tilde{p}_{H,t}(\Pi_{H,t}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{\Delta,t-j}^{H,RA} \tilde{p}_{H,t-j}^{\sim\varepsilon-1} \tilde{p}_{H,t-j}(\Pi_{H,t}) \right] \\
& - (1-\theta_M)(1-\varepsilon_M) \left[ \lambda_{Mp,t}^{H,RA} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)' (\Pi_{H,t}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{Mp,t-j}^{H,RA} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)' (\Pi_{H,t}) \right] \\
& + \lambda_{q,t}^{H,RA} \frac{P_{F,t-1}}{P_{H,t-1}} \frac{\Pi_{F,t}}{\Pi_{H,t}^2} + \lambda_{e,t-1}^{H,RA} \frac{C_{t-1} \mathcal{G}(\tau_{M,t-1}, Z_{t-1}/\Psi_{t-1})}{C_t \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t) \Pi_{H,t}^2}, \\
0 = & -\theta_M(\varepsilon_M-1) \lambda_{Mp,t}^{H,RA} \Pi_{F,t}^{\varepsilon_M-2} \\
& - (1-\theta_M)(1-\varepsilon_M) \left[ \lambda_{Mp,t}^{H,RA} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t}}{P_{F,t}} \right)' (\Pi_{F,t}) + \sum_{j=1}^{\infty} \beta^{-j} \lambda_{Mp,t-j}^{H,RA} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)^{-\varepsilon_M} \left( \frac{\bar{P}_{F,t-j}}{P_{F,t-j}} \right)' (\Pi_{F,t}) \right].
\end{aligned}$$

The Home producer-dispersion and nominal-rate conditions are

$$0 = \lambda_{\Delta,t}^{H,RA} + \lambda_{T,t}^{H,RA} Y_t - (1-\zeta) \lambda_{N,t}^{H,RA} \frac{MC_t^r Y_t}{C_t N_t^\varphi \mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)} - \zeta \lambda_{I,t}^{H,RA} \frac{MC_t^r Y_t}{\mathcal{G}(\tau_{M,t}, Z_t/\Psi_t)} - \beta \theta \lambda_{\Delta,t+1}^{H,RA} \Pi_{H,t+1}^{\varepsilon-1} \quad (C.31)$$

$$0 = -\frac{\lambda_{e,t}^{H,RA}}{(1+i_t)^2} + \mu_{i,t}^{H,RA}, \quad i_t \geq 0, \quad \mu_{i,t}^{H,RA} \geq 0, \quad \mu_{i,t}^{H,RA} i_t = 0. \quad (\text{C.32})$$

There is no separate importer-dispersion FOC in C.2.4. The Foreign FOCs are the exact starred mirror of the displayed Home system, with  $\mathcal{I}_t^*$ ,  $MC_t^{r*}$ , and the Foreign technology and factor-demand multipliers, and with

$$-\frac{\lambda_{e,t}^{F,RA}}{(1+i_t^*)^2} + \mu_{i,t}^{F,RA} = 0, \quad i_t^* \geq 0, \quad \mu_{i,t}^{F,RA} \geq 0, \quad \mu_{i,t}^{F,RA} i_t^* = 0. \quad (\text{C.33})$$

Thus C.2.4 contains the FOCs of the Social Planner's Problem in B.2.4 rather than a reference to the benchmark problem alone.

## D Welfare Cost Function

The second-order criteria in this section are derived around the distorted deterministic steady state using the Benigno–Woodford linear–quadratic method. Policy-dependent first-order terms are eliminated with the first-order equilibrium restrictions before the remaining second-order terms are reduced and completed into squares. The nonlinear Ramsey allocations in the main text are generated by Appendices B and C, not by minimizing these quadratic criteria.

**Structural-parameter closure.** All welfare cost function coefficients in this section are functions of the primitive structural parameters already defined in the main text. For the benchmark SOE and Two-country economies these are

$$\beta, \quad \varphi, \quad \eta, \quad \varepsilon, \quad \varepsilon_M, \quad \theta, \quad \theta_M, \quad v, \quad \nu, \quad (\text{D.1})$$

with  $\chi$  added only in D.2.3 and  $\zeta$  added only in D.2.4. The bilateral expenditure weights are not additional parameters:

$$\omega_F = v(1 - \nu), \quad \omega_H = 1 - \omega_F, \quad \omega_H^* = \nu\nu, \quad \omega_F^* = 1 - \omega_H^*. \quad (\text{D.2})$$

Likewise,

$$\mathcal{M} = \frac{\varepsilon}{\varepsilon - 1}, \quad L = \mathcal{M}^{-1} = \frac{\varepsilon - 1}{\varepsilon}, \quad \kappa_H = \frac{(1 - \theta)(1 - \beta\theta)}{\theta}, \quad \kappa_M = \frac{(1 - \theta_M)(1 - \beta\theta_M)}{\theta_M}. \quad (\text{D.3})$$

The steady-state importer subsidy is not an additional structural parameter: throughout the IPT environment it is fixed at  $\tau = 1/\varepsilon_M$ , as in Sec. 3.1. The tariff rates  $\tau_{M,t}$  and  $\tau_{M,t}^*$  and their persistence parameters determine the exogenous conditioning paths, but the shock persistence does not enter any per-period welfare weight. Hence the tariff processes are not free parameters of the welfare cost function. Every  $\Theta$  coefficient below is determined by the displayed Benigno–Woodford first-order coefficient-matching system; every real quadratic coefficient is an entry of the Hessian and tariff-loading matrices of the corresponding second-order augmented welfare polynomial; and every  $\Lambda$  is obtained by the displayed sequential Schur completion. Thus no welfare weight or WR reference-value coefficient is an independently calibrated parameter.

For the sequential square completion used below, retain the algebraic representation already used in the previous derivation. If a reduced policy-dependent quadratic loss is written as

$$\mathcal{L}_t = \frac{1}{2} \sum_{k=1}^n \Lambda_k^{(0)} z_{k,t}^2 - \sum_{k < j} C_{kj}^{(0)} z_{k,t} z_{j,t} - \sum_{k=1}^n h_{k,t}^{(0)} z_{k,t} + \text{s.o.t.i.p.}, \quad (\text{D.4})$$

then at step  $k$  the completed-square reference value is

$$z_{k,t}^w = \frac{h_{k,t}^{(k-1)} + \sum_{j=k+1}^n C_{kj}^{(k-1)} z_{j,t}}{\Lambda_k^{(k-1)}}. \quad (\text{D.5})$$

For  $j, l > k$ , the remaining coefficients are updated according to

$$\begin{aligned} \Lambda_j^{(k)} &= \Lambda_j^{(k-1)} - \frac{[C_{kj}^{(k-1)}]^2}{\Lambda_k^{(k-1)}}, \\ C_{jl}^{(k)} &= C_{jl}^{(k-1)} + \frac{C_{kj}^{(k-1)} C_{kl}^{(k-1)}}{\Lambda_k^{(k-1)}}, \\ h_{j,t}^{(k)} &= h_{j,t}^{(k-1)} + \frac{h_{k,t}^{(k-1)} C_{kj}^{(k-1)}}{\Lambda_k^{(k-1)}}. \end{aligned} \quad (\text{D.6})$$

These are algebraic completion formulas only; they do not introduce additional private-equilibrium variables.

## D.1 Small Open Economy

**Representation with the *ex-tariff* relative border price retained.** The two unnumbered welfare representations at the beginning of Sec. 3.5 retain the WR *Ex-tariff* Relative Border Price of Imports Gap explicitly. The period welfare cost itself remains  $\mathcal{L}_t^{SOE}$ .

### D.1.1 PPT Environment

For PPT, write the real part of the second-order welfare polynomial, up to terms independent of the endogenous allocation, as

$$\begin{aligned} \mathcal{L}_{t,\text{real}}^{SOE,PPT} &= \frac{1}{2} \left[ -2\hat{C}_{yy}^{SOE,PPT} y_t^2 - 2\hat{C}_{zz}^{SOE,PPT} z_t^2 \right] - \hat{C}_{yz}^{SOE,PPT} y_t z_t \\ &\quad - \hat{C}_{ym}^{SOE,PPT} y_t \tau_{M,t} - \hat{C}_{zm}^{SOE,PPT} z_t \tau_{M,t} + \text{s.o.t.i.p.}, \end{aligned} \quad (\text{D.7})$$

where s.o.t.i.p. denotes terms independent of the endogenous allocation to the order considered. The coefficients  $\hat{C}_{yy}^{SOE,PPT}$ ,  $\hat{C}_{zz}^{SOE,PPT}$ ,  $\hat{C}_{yz}^{SOE,PPT}$ ,  $\hat{C}_{ym}^{SOE,PPT}$ , and  $\hat{C}_{zm}^{SOE,PPT}$  are the literal coefficients on the corresponding quadratic or tariff-interaction terms in this welfare polynomial. They are not free coefficients. Equivalently, they are the corresponding Hessian and tariff-loading derivatives of the negative Benigno–Woodford augmented second-order real welfare contribution generated by the exact B.1.1 constraints and the stationary multipliers in Eq. (D.15). Since the constraints and those multipliers are functions only of Eqs. (D.1)–(D.3), each hatted coefficient is a structural function of the primitive parameters.

The first-order approximations of Eqs. (12) and (19) imply

$$y_t = [\eta + (1 - \eta)(1 - v)^2] z_t + v(1 - v)(\eta - 1)(\tau_{M,t} - \psi_t). \quad (\text{D.8})$$

Under PPT,  $\psi_t = 0$ . Using Eq. (D.8) to remove  $z_t$  from the output–relative-price cross term before completing squares gives

$$\begin{aligned} \hat{\Lambda}_y^{SOE,PPT} &\equiv -2\hat{C}_{yy}^{SOE,PPT} - \frac{2\hat{C}_{yz}^{SOE,PPT}}{\eta + (1 - \eta)(1 - v)^2}, \\ \hat{y}_t^{w,SOE,PPT} &\equiv \frac{1}{\hat{\Lambda}_y^{SOE,PPT}} \left[ \hat{C}_{ym}^{SOE,PPT} - \frac{v(1 - v)(\eta - 1)}{\eta + (1 - \eta)(1 - v)^2} \hat{C}_{yz}^{SOE,PPT} \right] \tau_{M,t}, \end{aligned}$$

$$\begin{aligned}
\hat{\Lambda}_z^{SOE,PPT} &\equiv -2\hat{C}_{zz}^{SOE,PPT}, \\
\hat{z}_t^{w,SOE,PPT} &\equiv \frac{\hat{C}_{zm}^{SOE,PPT}}{\hat{\Lambda}_z^{SOE,PPT}}\tau_{M,t}.
\end{aligned} \tag{D.9}$$

Thus the WR output appearing in the first unnumbered welfare representation in Sec. 3.5 contains no  $z_t$ .

### D.1.2 IPT Environment

For IPT, the corresponding real polynomial is

$$\begin{aligned}
\mathcal{L}_{t,\text{real}}^{SOE,IPT} &= \frac{1}{2} \left[ -2\hat{C}_{yy}^{SOE,IPT} y_t^2 - 2\hat{C}_{\psi\psi}^{SOE,IPT} \psi_t^2 - 2\hat{C}_{zz}^{SOE,IPT} z_t^2 \right. \\
&\quad - \hat{C}_{y\psi}^{SOE,IPT} y_t \psi_t - \hat{C}_{yz}^{SOE,IPT} y_t z_t - \hat{C}_{\psi z}^{SOE,IPT} \psi_t z_t \\
&\quad \left. - \hat{C}_{ym}^{SOE,IPT} y_t \tau_{M,t} - \hat{C}_{\psi m}^{SOE,IPT} \psi_t \tau_{M,t} - \hat{C}_{zm}^{SOE,IPT} z_t \tau_{M,t} + \text{s.o.t.i.p.} \right] \tag{D.10}
\end{aligned}$$

The hatted  $C$  coefficients again denote the literal coefficients on the corresponding terms in the welfare polynomial. Equivalently, they are entries of the Hessian and tariff-loading derivatives of the negative Benigno–Woodford augmented second-order real welfare contribution generated by B.1.2 and the stationary multipliers in Eqs. (D.15)–(D.16). They are therefore structural functions rather than additional parameters. Applying Eq. (D.8) to the  $y_t z_t$  term before sequentially completing the  $y_t$ ,  $\psi_t$ , and  $z_t$  squares gives

$$\begin{aligned}
\hat{\Lambda}_y^{SOE,IPT} &\equiv -2\hat{C}_{yy}^{SOE,IPT} - \frac{2\hat{C}_{yz}^{SOE,IPT}}{\eta + (1-\eta)(1-v)^2}, \\
\hat{y}_t^{w,SOE,IPT} &\equiv \frac{1}{\hat{\Lambda}_y^{SOE,IPT}} \left\{ \left[ \hat{C}_{ym}^{SOE,IPT} - \frac{v(1-v)(\eta-1)}{\eta + (1-\eta)(1-v)^2} \hat{C}_{yz}^{SOE,IPT} \right] \tau_{M,t} \right. \\
&\quad \left. + \left[ \hat{C}_{y\psi}^{SOE,IPT} + \frac{v(1-v)(\eta-1)}{\eta + (1-\eta)(1-v)^2} \hat{C}_{yz}^{SOE,IPT} \right] \psi_t \right\}, \\
\hat{\Lambda}_\psi^{SOE,IPT} &\equiv -2\hat{C}_{\psi\psi}^{SOE,IPT} - \frac{1}{\hat{\Lambda}_y^{SOE,IPT}} \left[ \hat{C}_{y\psi}^{SOE,IPT} + \frac{v(1-v)(\eta-1)}{\eta + (1-\eta)(1-v)^2} \hat{C}_{yz}^{SOE,IPT} \right]^2, \\
\hat{\psi}_t^{w,SOE,IPT} &\equiv \frac{1}{\hat{\Lambda}_\psi^{SOE,IPT}} \left\{ \left[ \hat{C}_{\psi m}^{SOE,IPT} + \frac{1}{\hat{\Lambda}_y^{SOE,IPT}} \left( \hat{C}_{y\psi}^{SOE,IPT} + \frac{v(1-v)(\eta-1)}{\eta + (1-\eta)(1-v)^2} \hat{C}_{yz}^{SOE,IPT} \right) \right. \right. \\
&\quad \left. \left. \times \left( \hat{C}_{ym}^{SOE,IPT} - \frac{v(1-v)(\eta-1)}{\eta + (1-\eta)(1-v)^2} \hat{C}_{yz}^{SOE,IPT} \right) \right] \tau_{M,t} + \hat{C}_{\psi z}^{SOE,IPT} z_t \right\}, \\
\hat{\Lambda}_z^{SOE,IPT} &\equiv -2\hat{C}_{zz}^{SOE,IPT} - \frac{(\hat{C}_{\psi z}^{SOE,IPT})^2}{\hat{\Lambda}_\psi^{SOE,IPT}}, \\
\hat{z}_t^{w,SOE,IPT} &\equiv \frac{1}{\hat{\Lambda}_z^{SOE,IPT}} \left\{ \hat{C}_{zm}^{SOE,IPT} + \frac{\hat{C}_{\psi z}^{SOE,IPT}}{\hat{\Lambda}_\psi^{SOE,IPT}} \left[ \hat{C}_{\psi m}^{SOE,IPT} + \frac{1}{\hat{\Lambda}_y^{SOE,IPT}} \left( \hat{C}_{y\psi}^{SOE,IPT} + \frac{v(1-v)(\eta-1)}{\eta + (1-\eta)(1-v)^2} \hat{C}_{yz}^{SOE,IPT} \right) \right. \right. \\
&\quad \left. \left. \times \left( \hat{C}_{ym}^{SOE,IPT} - \frac{v(1-v)(\eta-1)}{\eta + (1-\eta)(1-v)^2} \hat{C}_{yz}^{SOE,IPT} \right) \right] \right\} \tau_{M,t}.
\end{aligned}$$

The WR output in Eq. (D.11) therefore depends on the tariff and the LOOP gap, but not on  $z_t$ .

**Representation after eliminating the *ex-tariff* relative border price.** The WR variables used in the numbered SOE welfare-cost equations in Sec. 3.5 are defined here. Around the zero-tariff steady state, the first-order relation  $\log(1 + \tau_{M,t}) = \tau_{M,t}$  applies. We therefore use the tariff

rate  $\tau_{M,t}$  directly and introduce no auxiliary tariff variable. In the present SOE experiments there is no export-tariff disturbance. After the first-order elimination of  $z_t$ , the unhatted coefficients  $C_{yy}^{SOE,PPT}$ ,  $C_{ym}^{SOE,PPT}$ ,  $C_{yy}^{IPT}$ ,  $C_{y\psi}^{IPT}$ ,  $C_{ym}^{IPT}$ ,  $C_{\psi\psi}^{IPT}$ , and  $C_{\psi m}^{IPT}$  are the corresponding entries of the reduced structural Hessian and tariff-loading vector. Hence they are algebraic transforms of the hatted structural coefficients above, not additional parameters. Thus

$$y_t^{w,SOE,PPT} \equiv \frac{C_{ym}^{SOE,PPT}}{\Lambda_y^{SOE,PPT}} \tau_{M,t}, \quad (D.12)$$

$$y_t^{w,SOE,IPT} \equiv \frac{C_{ym}^{IPT}}{\Lambda_y^{SOE,IPT}} \tau_{M,t} + \frac{C_{y\psi}^{IPT}}{\Lambda_y^{SOE,IPT}} \psi_t. \quad (D.13)$$

After completing the output square, collecting the remaining terms in  $\psi_t$  gives the WR LOOP gap directly as

$$\psi_t^{w,SOE,IPT} \equiv \frac{C_{\psi m}^{IPT} + \frac{C_{y\psi}^{IPT} C_{ym}^{IPT}}{\Lambda_y^{SOE,IPT}}}{\Lambda_\psi^{SOE,IPT}} \tau_{M,t}. \quad (D.14)$$

Here  $C_{ym}^{SOE,PPT}$ ,  $C_{ym}^{IPT}$ ,  $C_{\psi m}^{IPT}$ , and  $C_{y\psi}^{IPT}$  are the corresponding cross coefficients in the second-order welfare expansion; the coefficient notation is retained from the underlying symbolic welfare derivation. Equation (D.14) depends only on the exogenous tariff innovation and therefore is the reference value entering the LOOP-gap square in Eq. (35). The compact main-text coefficient  $\Gamma_{own}^{SOE}$  is defined as the entire structural coefficient multiplying  $\tau_{M,t}$  on the right-hand side of Eq. (D.14).

The coefficients  $\Theta_n$ ,  $\Theta_H^{AS}$ , and  $\Theta_M^{AS}$  in Table 2 are the undetermined coefficients used to eliminate the first-order terms in the Benigno–Woodford expansion. For the PPT environment, temporary coefficients  $\Theta_\chi$ ,  $\Theta_Y$ ,  $\Theta_n$ ,  $\Theta_{mc,H}$ , and  $\Theta_H^{AS}$  are attached to the corresponding first-order equilibrium restrictions. In the IPT environment,  $\Theta_{mc,M}$  and  $\Theta_M^{AS}$  are additionally attached to the importer marginal-cost identity and the importer aggregate-supply restriction.

For the SOE steady state used in the welfare expansion,  $d_H = 1 - v$ ,  $\lambda_M = v$ ,  $L = \mathcal{M}^{-1}$ , where  $\mathcal{M} = \varepsilon/(\varepsilon - 1)$ , and

$$\mathcal{D}_S \equiv 1 + \eta\varphi + d_H\varphi(\eta - 1)(\lambda_M - 1).$$

Coefficient matching gives

$$\begin{aligned} \Theta_Y = \Theta_n &= \frac{L + \varphi(1 - \lambda_M)}{\mathcal{D}_S}, \\ \Theta_\chi &= \frac{Ld_H(\eta - 1) + \eta\varphi + 1}{\mathcal{D}_S}, \\ \Theta_{mc,H} &= \frac{(1 - \lambda_M) - L\eta - Ld_H(\eta - 1)(\lambda_M - 1)}{\mathcal{D}_S}, \\ \Theta_H^{AS} &= -\Theta_{mc,H}. \end{aligned} \quad (D.15)$$

The producer coefficients are unchanged in the IPT environment. The importer coefficients satisfy

$$\Theta_{mc,M} = \lambda_M \Theta_\chi, \quad \Theta_M^{AS} = -\lambda_M \Theta_\chi. \quad (D.16)$$

Thus the three coefficients entering the SOE inflation weights are defined by Eqs. (D.15) and (D.16). The exact SOE weight definitions are

$$\Lambda_y^{SOE,PPT} \equiv -2C_{yy}^{SOE,PPT}, \quad \Lambda_{\pi,H}^{SOE,PPT} \equiv \frac{\varepsilon}{\kappa_H} (\Theta_n - \Theta_H^{AS}),$$

$$\begin{aligned}
\Lambda_y^{SOE,IPT} &\equiv -2C_{yy}^{IPT}, & \Lambda_\psi^{SOE,IPT} &\equiv -2C_{\psi\psi}^{IPT} - \frac{(C_{y\psi}^{IPT})^2}{\Lambda_y^{SOE,IPT}}, \\
\Lambda_{\pi,H}^{SOE,IPT} &\equiv \frac{\varepsilon}{\kappa_H}(\Theta_n - \Theta_H^{AS}), & \Lambda_{\pi,F}^{SOE,IPT} &\equiv -\frac{\varepsilon_M}{\kappa_M}\Theta_M^{AS}.
\end{aligned} \tag{D.17}$$

The second-order welfare criteria are expansions around the distorted deterministic steady state. The linear-quadratic construction follows Benigno and Woodford [6, 7]: first-order equilibrium restrictions are used to eliminate terms that would otherwise make a naive second-order utility expansion incorrect around a distorted steady state, and price-dispersion laws generate the inflation-square terms. The WR variables in Eqs. (34), (35), (45), (46) are obtained by exact sequential completion of squares. They are reference values in the welfare criterion, not new private-equilibrium variables.

The numerical allocations reported in the main text are not solutions to the quadratic approximations. They are obtained from the nonlinear Ramsey first-order-condition systems; OccBin is used only to implement the occasionally binding ZLB in the numerical solution. The loss functions are used to interpret those allocations and to identify which nominal and real distortions are welfare-relevant.

## D.2 Two-country Economy

For completeness, this subsection records the two real WR variables appearing in Eq. (45) and the four real WR variables appearing in Eq. (46). Throughout the first-order representation used to construct these WR variables, we use  $\log(1 + \tau_{M,t}) = \tau_{M,t}$  and  $\log(1 + \tau_{M,t}^*) = \tau_{M,t}^*$ , without introducing separate tariff variables.

### D.2.1 PPT Environment

The PPT undetermined coefficients are obtained from the AS-inclusive Benigno–Woodford coefficient matching for the national Home planner. No coefficient from the IPT environment is reused. Attaching  $\Theta_S^{PPT}$ ,  $\Theta_\chi^{PPT}$ ,  $\Theta_Y^{PPT}$ ,  $\Theta_n^{PPT}$ ,  $\Theta_{mc,H}^{PPT}$ , and  $\Theta_H^{AS,PPT}$  to the first-order international-relative-price, Home CPI, Home goods-market, labor, producer-marginal-cost, and producer aggregate-supply restrictions gives

$$\begin{aligned}
0 &= 1 - \frac{(1-v)(\eta-1)}{\eta}\Theta_S^{PPT} - (\omega_H + \eta\omega_F)\Theta_Y^{PPT} - \Theta_{mc,H}^{PPT}, \\
0 &= \Theta_\chi^{PPT} - \eta\Theta_Y^{PPT} - \Theta_{mc,H}^{PPT}, \\
0 &= \Theta_S^{PPT} - \omega_F\Theta_\chi^{PPT}, \\
0 &= -\frac{1}{\eta}\Theta_S^{PPT} + \Theta_Y^{PPT} - \Theta_n^{PPT}, \\
0 &= -L + \Theta_n^{PPT} - \varphi\Theta_{mc,H}^{PPT}, \\
0 &= \Theta_{mc,H}^{PPT} + \Theta_H^{AS,PPT}.
\end{aligned} \tag{D.18}$$

Solving this system gives

$$\begin{aligned}
\Theta_\chi^{PPT} &= \frac{1 + \varphi\eta + \omega_H(\eta-1)L}{\left[1 + \frac{(1-v)(\eta-1)}{\eta}\omega_F\right](1 + \varphi\eta) - \omega_H(\eta-1)(\varphi + \omega_F/\eta)}, \\
\Theta_Y^{PPT} &= \frac{\left[1 + \frac{(1-v)(\eta-1)}{\eta}\omega_F\right]L + \varphi + \omega_F/\eta}{\left[1 + \frac{(1-v)(\eta-1)}{\eta}\omega_F\right](1 + \varphi\eta) - \omega_H(\eta-1)(\varphi + \omega_F/\eta)}, \\
\Theta_S^{PPT} &= \omega_F\Theta_\chi^{PPT},
\end{aligned}$$

$$\begin{aligned}
\Theta_n^{PPT} &= \Theta_Y^{PPT} - \frac{\omega_F}{\eta} \Theta_\chi^{PPT}, \\
\Theta_{mc,H}^{PPT} &= \Theta_\chi^{PPT} - \eta \Theta_Y^{PPT}, \\
\Theta_H^{AS,PPT} &= \eta \Theta_Y^{PPT} - \Theta_\chi^{PPT}.
\end{aligned} \tag{D.19}$$

Every object in Eqs. (D.18)–(D.19) is therefore an explicit function of  $v, \nu, \eta, \varphi$ , and  $\varepsilon$  through  $\omega_H, \omega_F$ , and  $L$ . Under the benchmark calibration, inserting these coefficients into the inflation-weight formula below reproduces  $\Lambda_{\pi,H}^{PPT} = 47.81085$  before rounding.

The PCP WR variables can be obtained directly from the reduced second-order Home-welfare polynomial. Let  $C_{ym}^{PPT}, C_{ym^*}^{PPT}, C_{y^*m}^{PPT}$ , and  $C_{y^*m^*}^{PPT}$  denote, consistently with the coefficient notation in Sec. 4.4, the literal coefficients multiplying  $y_t \tau_{M,t}, y_t \tau_{M,t}^*, y_t^* \tau_{M,t},$  and  $y_t^* \tau_{M,t}^*$ , respectively. These coefficients, and  $C_{yy}^{PPT}, C_{yy^*}^{PPT}$ , and  $C_{y^*y^*}^{PPT}$  below, are the Hessian and tariff-loading entries of the Benigno–Woodford augmented welfare constructed from B.2.1 and C.2.1 at the stationary distorted steady state. Thus the  $C^{PPT}$  coefficients are structural functions of Eqs. (D.1)–(D.3), not additional parameters. Before eliminating Foreign output, the real quadratic terms in the Home period loss are

$$\begin{aligned}
\frac{1}{2} \big\{ & -2C_{yy}^{PPT} y_t^2 - 2C_{y^*y^*}^{PPT} (y_t^*)^2 - 2C_{yy^*}^{PPT} y_t y_t^* \\
& -2(C_{ym}^{PPT} \tau_{M,t} + C_{ym^*}^{PPT} \tau_{M,t}^*) y_t \\
& -2(C_{y^*m}^{PPT} \tau_{M,t} + C_{y^*m^*}^{PPT} \tau_{M,t}^*) y_t^* \big\} + \text{s.o.t.i.p.}
\end{aligned} \tag{D.20}$$

where s.o.t.i.p. collects terms independent of the endogenous allocation to the order considered. The first-order world goods-market conditions and complete international risk sharing imply, with  $D_S \equiv \eta + (1 - \eta)(1 - \nu)^2$ ,

$$\begin{aligned}
D_S z_t &= y_t - y_t^* - v[\eta \omega_H - (1 - \nu)(1 - \nu)] \tau_{M,t} \\
&+ v[\eta \omega_F^* - (1 - \nu)\nu] \tau_{M,t}^*,
\end{aligned} \tag{D.21}$$

Substitution of Eq. (D.21) into Eq. (D.20) gives an output–relative-price cross term proportional to  $-2D_S[-2C_{y^*y^*}^{PPT} - C_{yy^*}^{PPT}]y_t z_t$ . The symbolic Benigno–Woodford coefficients satisfy

$$C_{yy^*}^{PPT} = -2C_{y^*y^*}^{PPT},$$

so this cross term cancels exactly. Completing the remaining  $y_t$  and  $z_t$  squares yields the final WR variables entering Eq. (45):

$$y_t^{w,PPT} \equiv \frac{C_{ym}^{PPT} + C_{ym^*}^{PPT}}{\Lambda_y^{PPT}} \tau_{M,t} + \frac{C_{ym^*}^{PPT} + C_{y^*m^*}^{PPT}}{\Lambda_y^{PPT}} \tau_{M,t}^*. \tag{D.22}$$

$$\begin{aligned}
z_t^{w,PPT} \equiv \frac{1}{D_S} \bigg\{ & \left[ -v\{\eta \omega_H - (1 - \nu)(1 - \nu)\} + \frac{C_{ym^*}^{PPT}}{2C_{y^*y^*}^{PPT}} \right] \tau_{M,t} \\
& + \left[ v\{\eta \omega_F^* - (1 - \nu)\nu\} + \frac{C_{ym^*}^{PPT}}{2C_{y^*y^*}^{PPT}} \right] \tau_{M,t}^* \bigg\}.
\end{aligned} \tag{D.23}$$

The corresponding PPT weight definitions are

$$\Lambda_y^{PPT} \equiv -2(C_{yy}^{PPT} - C_{y^*y^*}^{PPT}), \quad \Lambda_z^{PPT} \equiv -2D_S^2 C_{y^*y^*}^{PPT}, \quad \Lambda_{\pi,H}^{PPT} \equiv \frac{\varepsilon}{\kappa_H} (\Theta_n^{PPT} - \Theta_H^{AS,PPT}). \tag{D.24}$$

Equation (D.22) defines the (logarithmic) WR output, while Eq. (D.23) defines the (logarithmic) WR *ex-tariff* relative border price of imports. These are the final, rather than intermediate, PCP WR variables. For the unilateral Home-tariff experiment,  $\tau_{M,t}^* = 0$ .

### D.2.2 IPT Environment

The benchmark no-habit structural quadratic in this subsection is obtained exactly by setting  $\chi = 0$  in the fully explicit D.2.3 construction, deleting the two lagged-consumption coordinates, and using Eqs. (D.34) and (D.39). Consequently every  $\widehat{C}_{uv}^{IPT}$  used below is an entry of that explicit structural Hessian or its tariff-loading columns; none is a free welfare parameter. In particular, if  $Q^{IPT}$  and  $B^{IPT}$  denote the first four rows and columns, and the corresponding two tariff-loading columns, of  $Q^h$  and  $B^h$  in Eq. (D.40) evaluated at  $\chi = 0$ , then all  $b^H$  coefficients below are generated mechanically by Eqs. (D.5)–(D.6). For example,

$$b_{ym}^H = \frac{B_{11}^{IPT}}{Q_{11}^{IPT}}, \quad b_{ym^*}^H = \frac{B_{12}^{IPT}}{Q_{11}^{IPT}}, \quad (\text{D.25})$$

and the remaining  $b^H$  are the corresponding second-, third-, and fourth-step Schur-completion loadings. They are therefore structural functions, not additional calibrated parameters. Use complete risk sharing and the first-order world equilibrium to eliminate Foreign output. Before collecting the reduced welfare coefficients, express all local-relative-price terms in the manuscript coordinate  $z_t = \log(\mathcal{E}_t P_{F,t}^*/P_{H,t})$ . In particular, the identity stated in Sec. 4.1,  $\log(P_{F,t}/P_{H,t}) = z_t - \psi_t$ , is imposed before the real quadratic form is completed. The resulting real Home coordinates are therefore  $y_t$ ,  $\psi_t$ ,  $z_t$ , and  $\psi_t^*$ . The exact sequential completion of this manuscript-coordinate two-country Home-welfare criterion gives

$$y_t^{w,IPT} = b_{ym}^H \tau_{M,t} + b_{ym^*}^H \tau_{M,t}^*, \quad (\text{D.26})$$

$$\psi_t^{w,IPT} = b_{\psi z}^H z_t + b_{\psi \psi^*}^H \psi_t^* + b_{\psi m}^H \tau_{M,t} + b_{\psi m^*}^H \tau_{M,t}^*, \quad (\text{D.27})$$

$$\begin{aligned} z_t^{w,IPT} &= b_{z\psi^*}^H \psi_t^* + b_{zm}^H \tau_{M,t} + b_{zm^*}^H \tau_{M,t}^*, \\ \psi_t^{*,w,IPT} &= b_{\psi^* m}^H \tau_{M,t} + b_{\psi^* m^*}^H \tau_{M,t}^*. \end{aligned} \quad (\text{D.28})$$

For the compact main-text notation in Eq. (47), define  $\Gamma_z \equiv b_{\psi z}^H$ ,  $\Gamma_{\text{own}} \equiv b_{\psi m}^H$ , and  $\Gamma_{\text{cross}} \equiv b_{\psi m^*}^H = -b_{\psi^* \psi}^H$ . The last equality collects the Foreign tariff and Foreign LOOP gap into the purchaser-price combination  $\tau_{M,t}^* - \psi_t^*$  already implied by the first-order Foreign relative price. The corresponding starred  $\Gamma$  loadings in Eq. (48) are obtained by the exact Home–Foreign mirror of this construction. The corresponding real-weight definitions are

$$\begin{aligned} \Lambda_y^{IPT} &\equiv -2\widehat{C}_{yy}^{IPT}, & \Lambda_\psi^{IPT} &\equiv -2\widehat{C}_{\psi\psi}^{IPT}, \\ \Lambda_z^{IPT} &\equiv -2\widehat{C}_{zz}^{IPT} - \frac{(\widehat{C}_{\psi z}^{IPT})^2}{\Lambda_\psi^{IPT}}, \\ \Lambda_{\psi^*}^{IPT} &\equiv -2\widehat{C}_{\psi^*\psi^*}^{IPT} - \frac{(\widehat{C}_{\psi\psi^*}^{IPT})^2}{\Lambda_\psi^{IPT}} \\ &\quad - \frac{1}{\Lambda_z^{IPT}} \left[ \widehat{C}_{z\psi^*}^{IPT} + \frac{\widehat{C}_{\psi z}^{IPT} \widehat{C}_{\psi\psi^*}^{IPT}}{\Lambda_\psi^{IPT}} \right]^2. \end{aligned} \quad (\text{D.29})$$

Evaluating Eq. (D.34) at  $\chi = 0$  and suppressing the superscript  $h$ , the three sticky-inflation weights are

$$\Lambda_{\pi,H}^{IPT} = \frac{\varepsilon}{\kappa_H} (\Theta_n - \Theta_H^{AS}), \quad \Lambda_{\pi,F}^{IPT} = -\frac{\varepsilon_M}{\kappa_M} \Theta_M^{AS}, \quad \Lambda_{\pi^*H}^{IPT} = \frac{\varepsilon_M}{\kappa_M} \Theta_Y \omega_F. \quad (\text{D.30})$$

Here the hatted  $C$  coefficients are the literal coefficients of the reduced second-order Home-welfare polynomial after the preceding coordinate transformation; the sequential completion is performed in the order  $y, \psi, s, \psi^*$  using Eqs. (D.5)–(D.6). The coefficients  $b^H$  are the exact completed-square coefficients obtained from that same manuscript-coordinate polynomial and are functions only of the structural parameters. In this representation,  $y_t^{w,IPT}$  is the (logarithmic) WR output,  $\psi_t^{w,IPT}$

is the (logarithmic) Home WR LOOP gap, and  $z_t^{w,IPT}$  is the (logarithmic) WR *Ex-tariff* Relative Border Price of Imports;  $\psi_t^{*,w,IPT}$  is the corresponding Foreign WR LOOP gap. Their associated differences in Eq. (46) use the Gap/Deviation terminology defined in Sec. 4.4. For the unilateral Home-tariff experiment,  $\tau_{M,t}^* = 0$ . In particular, Eq. (D.27) makes explicit that the Home WR LOOP gap depends not only on the Home tariff but also on the endogenous international relative price  $z_t$  and the Foreign LOOP gap  $\psi_t^*$ . These WR variables are obtained by completed squares and are not additional private-equilibrium variables.

**High Elasticity Case.** The High Elasticity Case has exactly the same welfare cost function as the two-country IPT environment above, evaluated at  $\eta = 3$ . No separate derivation is required. Numerically, the Home welfare weights are  $\Lambda_y^{(3)} = 2.221$ ,  $\Lambda_\psi^{(3)} = 1.206$ ,  $\Lambda_z^{(3)} = -0.480$ ,  $\Lambda_{\psi^*}^{(3)} = 1.438$ ,  $\Lambda_{\pi,H}^{(3)} = 50.711$ ,  $\Lambda_{\pi,F}^{(3)} = 33.686$ , and  $\Lambda_{\pi_H^*}^{(3)} = 11.320$ .

**Benchmark Reciprocal Tariff Case.** The Benchmark Reciprocal Tariff Case also has exactly the same welfare cost function as the two-country IPT environment above. The structural parameters and welfare weights are unchanged; reciprocity only makes both tariff paths,  $\tau_{M,t}$  and  $\tau_{M,t}^*$ , nonzero in the same WR variables. Hence no separate welfare cost function derivation is required.

### D.2.3 External Habit (IPT Environment Only)

Let  $c_t \equiv \log(C_t/C)$  and  $c_t^* \equiv \log(C_t^*/C^*)$ . At the zero-tariff, zero-inflation deterministic steady state,

$$C = Y = N = \left[ \frac{1}{\mathcal{M}(1-\chi)} \right]^{1/(1+\varphi)}, \quad L \equiv \mathcal{M}^{-1} = \frac{\varepsilon-1}{\varepsilon}, \quad N^{1+\varphi} = \frac{L}{1-\chi}. \quad (\text{D.31})$$

where, throughout Appendices D.2.3 and D.2.4, a variable without a time subscript denotes its deterministic steady-state value. The direct second-order expansion of the habit term is

$$\begin{aligned} \log(C_t - \chi C_{t-1}) &= \log[(1-\chi)C] + \frac{c_t - \chi c_{t-1}}{1-\chi} + \frac{c_t^2 - \chi c_{t-1}^2}{2(1-\chi)} \\ &\quad - \frac{(c_t - \chi c_{t-1})^2}{2(1-\chi)^2} + O(3), \end{aligned} \quad (\text{D.32})$$

with the exact starred counterpart. Hence inherited consumption is an endogenous component of the second-order criterion and cannot be assigned to s.o.t.i.p. after the impact period.

Following Benigno and Woodford [6, 7], the policy-dependent first-order terms are eliminated before the quadratic criterion is interpreted. Discounted summation of the linear habit term makes the coefficient on current consumption equal to  $(1-\beta\chi)/(1-\chi)$ . Attaching the same undetermined coefficients as in the benchmark two-country IPT environment gives the habit-specific coefficient-matching system

$$\begin{aligned} 0 &= \frac{1-\beta\chi}{1-\chi} (1 - \Theta_{mc,H}^h - \Theta_{mc,M}^h) - \omega_H \Theta_Y^h, \\ 0 &= \Theta_\chi^h - \eta \omega_H \Theta_Y^h - \Theta_{mc,H}^h - \Theta_{mc,M}^h, \\ 0 &= -\frac{L}{1-\chi} + \Theta_n^h - \varphi \Theta_{mc,H}^h, \\ 0 &= \Theta_Y^h - \Theta_n^h, \\ 0 &= -\omega_F \Theta_\chi^h + \Theta_{mc,M}^h, \\ 0 &= \Theta_{mc,H}^h + \Theta_H^{AS,h}, \\ 0 &= \Theta_{mc,M}^h + \Theta_M^{AS,h}. \end{aligned} \quad (\text{D.33})$$

Solving Eq. (D.33) by the method of undetermined coefficients gives

$$\begin{aligned}
\Theta_Y^h = \Theta_n^h &= \frac{\frac{1-\beta\chi}{1-\chi} \left(1 + \frac{L}{(1-\chi)\omega_H\varphi}\right)}{\omega_H + \frac{1-\beta\chi}{1-\chi} \left(\omega_F\eta + \frac{1}{\omega_H\varphi}\right)}, \\
\Theta_{mc,H}^h &= \frac{\Theta_Y^h - L/(1-\chi)}{\varphi}, \quad \Theta_\chi^h = \eta\Theta_Y^h + \frac{\Theta_{mc,H}^h}{\omega_H}, \\
\Theta_{mc,M}^h &= \omega_F\Theta_\chi^h, \quad \Theta_H^{AS,h} = -\Theta_{mc,H}^h, \quad \Theta_M^{AS,h} = -\Theta_{mc,M}^h.
\end{aligned} \tag{D.34}$$

These expressions reduce exactly to the benchmark two-country coefficients when  $\chi = 0$ .

For the reduction to the manuscript coordinates, the first-order CPI-PPI deviations are

$$\log \mathcal{G}(\tau_{M,t}, S_t) = \omega_F(z_t - \psi_t + \tau_{M,t}), \quad \log \mathcal{G}^*(\tau_{M,t}^*, S_t) = \omega_H^*(-z_t - \psi_t^* + \tau_{M,t}^*). \tag{D.35}$$

The first-order risk-sharing and two-country goods-market conditions then give

$$\begin{aligned}
c_t &= y_t + \omega_F \left[ (1-\chi)(z_t - \log \mathcal{G}(\tau_{M,t}, S_t) + \log \mathcal{G}^*(\tau_{M,t}^*, S_t)) + \chi(c_{t-1} - c_{t-1}^*) \right] \\
&\quad - \eta \left[ \omega_H \log \mathcal{G}(\tau_{M,t}, S_t) + \omega_F (\log \mathcal{G}^*(\tau_{M,t}^*, S_t) + z_t + \psi_t^* - \tau_{M,t}^*) \right],
\end{aligned} \tag{D.36}$$

$$c_t^* = c_t - (1-\chi)(z_t - \log \mathcal{G}(\tau_{M,t}, S_t) + \log \mathcal{G}^*(\tau_{M,t}^*, S_t)) - \chi(c_{t-1} - c_{t-1}^*). \tag{D.37}$$

Thus  $c_t$  and  $c_t^*$  are explicit linear functions of  $(y_t, \psi_t, z_t, \psi_t^*, c_{t-1}, c_{t-1}^*, \tau_{M,t}, \tau_{M,t}^*)$ . The first-order Home producer real marginal cost is

$$mc_{H,t}^{h,(1)} = \frac{c_t - \chi c_{t-1}}{1-\chi} + \varphi y_t + \log \mathcal{G}(\tau_{M,t}, S_t). \tag{D.38}$$

After substituting Eq. (D.34) into the second-order equilibrium restrictions, forward-integrating the producer and importer Calvo aggregate-supply blocks, and using Eqs. (D.35)–(D.38), the policy-dependent real part of the Benigno–Woodford augmented Home welfare contribution is

$$\begin{aligned}
\mathcal{W}_{H,t}^{h,(2),real} &= -\frac{\chi(1-\Theta_{mc,H}^h - \omega_F\Theta_\chi^h)}{2(1-\chi)^2} (c_t - c_{t-1})^2 - \frac{\chi\omega_F\Theta_\chi^h}{2(1-\chi)^2} (c_t^* - c_{t-1}^*)^2 \\
&\quad - \frac{1+\varphi}{2} \frac{L}{1-\chi} y_t^2 - \frac{\omega_H\omega_F(1-\eta)}{2} \Theta_\chi^h (z_t - \psi_t + \tau_{M,t})^2 \\
&\quad - \frac{\omega_H\omega_F}{2} \Theta_Y^h \left[ (c_t + \eta \log \mathcal{G}(\tau_{M,t}, S_t)) - \{c_t^* - \eta(-z_t - \psi_t^* + \tau_{M,t}^* - \log \mathcal{G}^*(\tau_{M,t}^*, S_t))\} \right]^2 \\
&\quad + \Theta_{mc,H}^h \left[ mc_{H,t}^{h,(1)} \left( \frac{c_t - \chi c_{t-1}}{1-\chi} + \log \mathcal{G}(\tau_{M,t}, S_t) - y_t \right) - \frac{1}{2} (mc_{H,t}^{h,(1)})^2 \right] \\
&\quad - \frac{\omega_F\Theta_\chi^h}{\varepsilon_M - 1} \left[ \{(\varepsilon_M - 1)\psi_t + \tau_{M,t}\} \right. \\
&\quad \times \left\{ c_t - \eta(z_t - \psi_t + \tau_{M,t} - \log \mathcal{G}(\tau_{M,t}, S_t)) + (z_t - \psi_t) - \frac{c_t - \chi c_{t-1}}{1-\chi} - \log \mathcal{G}(\tau_{M,t}, S_t) \right\} \\
&\quad \left. + \frac{\varepsilon_M - 1}{2} \psi_t^2 + \tau_{M,t} \psi_t - \frac{\varepsilon_M + 1}{2} \tau_{M,t}^2 \right] + \text{s.o.t.i.p.}
\end{aligned} \tag{D.39}$$

The three Calvo dispersion contributions are added separately below. Equation (D.39) contains only model variables and the solved coefficients in Eq. (D.34), so the reduced quadratic coefficients can be reproduced directly from it.

Order the real coordinates as

$$z_{1,t}^h = y_t, \quad z_{2,t}^h = \psi_t, \quad z_{3,t}^h = z_t, \quad z_{4,t}^h = \psi_t^*, \quad z_{5,t}^h = c_{t-1}, \quad z_{6,t}^h = c_{t-1}^*.$$

Define the entries of the reduced real Hessian and the two tariff-loading columns directly from Eq. (D.39) by

$$\begin{aligned} Q_{ij}^h &\equiv -\frac{\partial^2 \mathcal{W}_{H,t}^{h,(2),real}}{\partial z_{i,t}^h \partial z_{j,t}^h}, \quad i, j = 1, \dots, 6, \\ B_{i1}^h &\equiv \frac{\partial^2 \mathcal{W}_{H,t}^{h,(2),real}}{\partial z_{i,t}^h \partial \tau_{M,t}}, \quad B_{i2}^h \equiv \frac{\partial^2 \mathcal{W}_{H,t}^{h,(2),real}}{\partial z_{i,t}^h \partial \tau_{M,t}^*}. \end{aligned} \quad (D.40)$$

Hence, up to pure tariff terms,

$$\mathcal{L}_{H,t}^{h,real} = \frac{1}{2} (z_t^h)' Q^h z_t^h - (z_t^h)' B^h (\tau_{M,t}, \tau_{M,t}^*)' + \text{s.o.t.i.p.} \quad (D.41)$$

and  $Q^h$  is symmetric.

The six real coefficients reported for the External-Habit specification in Table 6 are the successive Schur pivots of  $Q^h$  in the order  $y, \psi, s, \psi^*, c^-, c^{*-}$ . If  $Q_{1:k,1:k}^h$  denotes the leading  $k \times k$  block, their exact symbolic values are

$$\begin{aligned} \Lambda_y^h &= Q_{11}^h, \\ \Lambda_\psi^h &= \frac{\det Q_{1:2,1:2}^h}{Q_{11}^h}, \\ \Lambda_z^h &= \frac{\det Q_{1:3,1:3}^h}{\det Q_{1:2,1:2}^h}, \\ \Lambda_{\psi^*}^h &= \frac{\det Q_{1:4,1:4}^h}{\det Q_{1:3,1:3}^h}, \\ \Lambda_{c^-}^h &= \frac{\det Q_{1:5,1:5}^h}{\det Q_{1:4,1:4}^h}, \\ \Lambda_{c^{*-}}^h &= \frac{\det Q^h}{\det Q_{1:5,1:5}^h}. \end{aligned} \quad (D.42)$$

Equivalently, these are obtained by applying the sequential updates in Eqs. (D.5)–(D.6) with  $n = 6$ . In particular, after completing the output square, the exact symbolic Home WR LOOP gap is

$$\begin{aligned} \psi_t^{w,h} &= \frac{1}{\Lambda_\psi^h} \left[ \left( B_{21}^h - \frac{Q_{21}^h B_{11}^h}{Q_{11}^h} \right) \tau_{M,t} + \left( B_{22}^h - \frac{Q_{21}^h B_{12}^h}{Q_{11}^h} \right) \tau_{M,t}^* \right. \\ &\quad - \left( Q_{23}^h - \frac{Q_{21}^h Q_{13}^h}{Q_{11}^h} \right) z_t - \left( Q_{24}^h - \frac{Q_{21}^h Q_{14}^h}{Q_{11}^h} \right) \psi_t^* \\ &\quad \left. - \left( Q_{25}^h - \frac{Q_{21}^h Q_{15}^h}{Q_{11}^h} \right) c_{t-1} - \left( Q_{26}^h - \frac{Q_{21}^h Q_{16}^h}{Q_{11}^h} \right) c_{t-1}^* \right]. \end{aligned} \quad (D.43)$$

For the loadings reported in Table 7,  $\Gamma_{\text{own}}^h$ ,  $\Gamma_{\text{cross}}^h$ ,  $\Gamma_z^h$ ,  $\Gamma_c^h$ , and  $\Gamma_{c^*}^h$  are defined as the structural coefficients on  $\tau_{M,t}$ ,  $\tau_{M,t}^* - \psi_t^*$ ,  $z_t$ ,  $c_{t-1}$ , and  $c_{t-1}^*$ , respectively, in Eq. (D.43). The remaining five WR reference values follow from the same six-step completion and therefore require no additional equilibrium definitions.

The sticky-inflation coefficients are

$$\Lambda_{\pi,H}^h = \frac{\varepsilon}{\kappa_H} (\Theta_n^h - \Theta_H^{AS,h}), \quad \Lambda_{\pi,F}^h = -\frac{\varepsilon_M}{\kappa_M} \Theta_M^{AS,h}, \quad \Lambda_{\pi^*,H}^h = \frac{\varepsilon_M}{\kappa_M} \Theta_Y^h \omega_F. \quad (D.44)$$

Evaluating the symbolic expressions at  $\chi = 0.71$  and the benchmark parameters in Table 1, with  $\nu = 0.27$ , gives

$$\begin{aligned}\Theta_Y^h = \Theta_n^h &= 1.481777, & \Theta_{mc,H}^h &= -0.481390, & \Theta_\chi^h &= 1.606290, \\ \Theta_{mc,M}^h &= 0.351778, & \Theta_H^{AS,h} &= 0.481390, & \Theta_M^{AS,h} &= -0.351778,\end{aligned}\quad (\text{D.45})$$

and

$$\begin{aligned}(\Lambda_y^h, \Lambda_\psi^h, \Lambda_z^h) &= (18.593099, 0.569299, 0.882957), \\ (\Lambda_{\psi^*}^h, \Lambda_{c^-}^h, \Lambda_{c^*-}^h) &= (0.158126, 0.719937, -0.323476), \\ (\Lambda_{\pi,H}^h, \Lambda_{\pi,F}^h, \Lambda_{\pi_H^*}^h) &= (73.105259, 25.706820, 23.714139).\end{aligned}\quad (\text{D.46})$$

Equation (D.43) then gives

$$\psi_t^{w,h} = 0.505604 z_t + 0.201654 \psi_t^* + 0.800624 c_{t-1} + 0.159227 c_{t-1}^* + 0.656684 \tau_{M,t} - 0.201654 \tau_{M,t}^*, \quad (\text{D.47})$$

which gives the unrounded loadings corresponding to the External-Habit column of Table 7. As a nesting check, setting  $\chi = 0$  removes the two inherited-consumption rows and columns and reproduces  $(\Lambda_y^{IPT}, \Lambda_\psi^{IPT}, \Lambda_z^{IPT}, \Lambda_{\psi^*}^{IPT}) = (2.860011, 0.406343, -0.097828, 0.352099)$  and Eq. (47). Thus the habit-specific coefficients reported in Table 6 are obtained from the same Benigno–Woodford construction as the benchmark criterion rather than from a primitive utility Hessian.

The resulting Home period loss combines these completed-square real terms with the three sticky-inflation terms; its numerical weights are reported in Table 6. The complete nonlinear national Ramsey construction is reported in On-line Appendix B.2.3.

#### D.2.4 Roundabout Production (IPT Environment Only)

**Benigno–Woodford coefficient matching.** The roundabout-production experiment retains the two-country IPT environment and changes only the Home and Foreign production structure by introducing the intermediate-input share  $\zeta$ . Let

$$L \equiv \mathcal{M}^{-1} = \frac{\varepsilon - 1}{\varepsilon}, \quad \sigma_C \equiv \frac{C}{Y} = 1 - \zeta L, \quad \sigma_I \equiv \frac{\mathcal{I}}{Y} = \zeta L, \quad (\text{D.48})$$

where  $\mathcal{I}_t$  is the intermediate-input composite and  $\sigma_C + \sigma_I = 1$ . The steady-state labor-disutility coefficient is

$$D_{RA} \equiv N^{1+\varphi} = \frac{(1 - \zeta)L}{1 - \zeta L}. \quad (\text{D.49})$$

The first-order factor-demand relations are

$$n_t = \frac{y_t - \zeta c_t}{1 + \zeta \varphi}, \quad \iota_t^I = \frac{(1 - \zeta)c_t + (1 + \varphi)y_t}{1 + \zeta \varphi}, \quad (\text{D.50})$$

with the exact Foreign mirror. Here  $\iota_t^I$  is the log deviation of the existing intermediate-input composite  $\mathcal{I}_t$ .

Attach the Benigno–Woodford undetermined coefficients  $\Theta_\chi^{RA}$ ,  $\Theta_Y^{RA}$ ,  $\Theta_N^{RA}$ ,  $\Theta_I^{RA}$ ,  $\Theta_{mc,H}^{RA}$ ,  $\Theta_{mc,M}^{RA}$ ,  $\Theta_H^{AS,RA}$ , and  $\Theta_M^{AS,RA}$  to the first-order CPI, gross-output, labor-demand, intermediate-input-demand, producer-marginal-cost, importer-marginal-cost, and two aggregate-supply restrictions. Matching the policy-dependent first-order terms gives

$$\begin{aligned}0 &= 1 - \omega_H \sigma_C \Theta_Y^{RA} + \zeta \Theta_N^{RA} - (1 - \zeta) \Theta_I^{RA} - (1 - \zeta) \Theta_{mc,H}^{RA} - \Theta_{mc,M}^{RA}, \\ 0 &= \Theta_\chi^{RA} - \eta \omega_H \Theta_Y^{RA} - \Theta_{mc,H}^{RA} - \Theta_{mc,M}^{RA},\end{aligned}$$

$$\begin{aligned}
0 &= -D_{RA} + (1 + \zeta\varphi)\Theta_N^{RA} - (1 - \zeta)\varphi\Theta_I^{RA} - (1 - \zeta)\varphi\Theta_{mc,H}^{RA}, \\
0 &= \Theta_Y^{RA} - \Theta_N^{RA} - \Theta_I^{RA}, \\
0 &= -\omega_H\sigma_I\Theta_Y^{RA} + \Theta_I^{RA}, \\
0 &= -\omega_F\Theta_\chi^{RA} + \Theta_{mc,M}^{RA}, \\
0 &= \Theta_{mc,H}^{RA} + \Theta_H^{AS,RA}, \quad 0 = \Theta_{mc,M}^{RA} + \Theta_M^{AS,RA}.
\end{aligned} \tag{D.51}$$

Solving Eq. (D.51) gives the closed-form coefficients

$$\begin{aligned}
\Theta_Y^{RA} &= \frac{1 + \frac{1 - \zeta + \omega_F/\omega_H}{(1 - \zeta)\varphi} D_{RA}}{\omega_H(1 - \zeta L) - \left[ \zeta - \omega_F\eta - \omega_H\zeta L - \frac{1 - \zeta + \omega_F/\omega_H}{(1 - \zeta)\varphi} \{1 + \zeta\varphi - \omega_H\zeta L(1 + \varphi)\} \right]}, \\
\Theta_{mc,H}^{RA} &= \frac{\{1 + \zeta\varphi - \omega_H\zeta L(1 + \varphi)\}\Theta_Y^{RA} - D_{RA}}{(1 - \zeta)\varphi}, \\
\Theta_\chi^{RA} &= \eta\Theta_Y^{RA} + \frac{\Theta_{mc,H}^{RA}}{\omega_H}, \\
\Theta_I^{RA} &= \omega_H\zeta L\Theta_Y^{RA}, \quad \Theta_N^{RA} = (1 - \omega_H\zeta L)\Theta_Y^{RA}, \\
\Theta_{mc,M}^{RA} &= \omega_F\Theta_\chi^{RA}, \quad \Theta_H^{AS,RA} = -\Theta_{mc,H}^{RA}, \quad \Theta_M^{AS,RA} = -\Theta_{mc,M}^{RA}.
\end{aligned} \tag{D.52}$$

No calibrated structural value is used in Eqs. (D.51)–(D.52).

**First-order two-country reduction.** For the manuscript price coordinates define the first-order purchaser-relative price deviations already implied by the model as

$$x_{H,t} = z_t - \psi_t + \tau_{M,t}, \quad x_{F,t} = -z_t - \psi_t^* + \tau_{M,t}^*, \tag{D.53}$$

and therefore

$$\log \mathcal{G}(\tau_{M,t}, S_t) = \omega_F x_{H,t}, \quad \log \mathcal{G}^*(\tau_{M,t}^*, S_t) = \omega_H^* x_{F,t}, \quad c_t - c_t^* = z_t - \log \mathcal{G}(\tau_{M,t}, S_t) + \log \mathcal{G}^*(\tau_{M,t}^*, S_t). \tag{D.54}$$

Using Eq. (D.50), the first-order log deviation of gross absorption is

$$d_t^A = \left[ \sigma_C + \sigma_I \frac{1 - \zeta}{1 + \zeta\varphi} \right] c_t + \sigma_I \frac{1 + \varphi}{1 + \zeta\varphi} y_t, \tag{D.55}$$

with the exact Foreign mirror. Here  $d_t^A$  is only the log deviation of the existing gross-absorption object  $C_t + \mathcal{I}_t$ .

The two first-order goods-market conditions and Eq. (D.54) give

$$\begin{aligned}
&\left[ \sigma_C + \sigma_I \frac{1 - \zeta}{1 + \zeta\varphi} \right] c_t + \omega_F \sigma_I \frac{1 + \varphi}{1 + \zeta\varphi} y_t^* \\
&= \left( 1 - \omega_H \sigma_I \frac{1 + \varphi}{1 + \zeta\varphi} \right) y_t + \omega_F \left[ \sigma_C + \sigma_I \frac{1 - \zeta}{1 + \zeta\varphi} \right] (z_t - \log \mathcal{G}(\tau_{M,t}, S_t) + \log \mathcal{G}^*(\tau_{M,t}^*, S_t)) \\
&\quad - \eta \omega_H \omega_F x_{H,t} + \eta \omega_F \omega_H^* x_{F,t}, \\
&\left[ \sigma_C + \sigma_I \frac{1 - \zeta}{1 + \zeta\varphi} \right] c_t - \left( 1 - \omega_F^* \sigma_I \frac{1 + \varphi}{1 + \zeta\varphi} \right) y_t^* \\
&= \omega_F^* \left[ \sigma_C + \sigma_I \frac{1 - \zeta}{1 + \zeta\varphi} \right] (z_t - \log \mathcal{G}(\tau_{M,t}, S_t) + \log \mathcal{G}^*(\tau_{M,t}^*, S_t)) - \omega_H^* \sigma_I \frac{1 + \varphi}{1 + \zeta\varphi} y_t \\
&\quad + \eta \omega_H^* \omega_H x_{H,t} - \eta \omega_F^* \omega_H^* x_{F,t}.
\end{aligned} \tag{D.56}$$

with

$$c_t^* = c_t - (z_t - \log \mathcal{G}(\tau_{M,t}, S_t) + \log \mathcal{G}^*(\tau_{M,t}^*, S_t)). \quad (\text{D.57})$$

Equations (D.53)–(D.57) therefore express  $c_t, c_t^*, y_t^*, n_t, n_t^*, \iota_t^I$ , and  $\iota_t^{I*}$  as linear functions of  $(y_t, \psi_t, z_t, \psi_t^*, \tau_{M,t}, \tau_{M,t}^*)$ . At  $\zeta = 0.50$  the Home-consumption equation implied by this system is

$$c_t = 0.609756 y_t - 0.308423 z_t + 0.336727 \psi_t - 0.332477 \psi_t^* - 0.336727 \tau_{M,t} + 0.332477 \tau_{M,t}^*. \quad (\text{D.58})$$

**Second-order augmented welfare and reduced Hessian.** The second-order expansion of gross absorption is

$$\log \frac{C_t + \mathcal{I}_t}{C + \mathcal{I}} = d_t^A + \frac{1}{2} \sigma_C \sigma_I (c_t - \iota_t^I)^2 + O(3), \quad (\text{D.59})$$

and the first-order Home producer real marginal cost is

$$mc_{H,t}^{RA,(1)} = (1 - \zeta)(c_t + \varphi n_t) + \log \mathcal{G}(\tau_{M,t}, S_t). \quad (\text{D.60})$$

After substituting Eq. (D.52) into the full second-order equilibrium restrictions, forward-integrating the producer and importer Calvo aggregate-supply blocks, and imposing Eqs. (D.53)–(D.57), the policy-dependent real part of the Benigno–Woodford augmented Home welfare contribution is

$$\begin{aligned} \mathcal{W}_{H,t}^{RA,(2),real} = & -\frac{1+\varphi}{2} D_{RA} n_t^2 - \frac{\omega_H \omega_F (1-\eta)}{2} \Theta_\chi^{RA} x_{H,t}^2 \\ & - \frac{\omega_H \omega_F}{2} \Theta_Y^{RA} \left[ d_t^A + \eta \log \mathcal{G}(\tau_{M,t}, S_t) - \{d_t^{A*} - \eta(x_{F,t} - \log \mathcal{G}^*(\tau_{M,t}^*, S_t))\} \right]^2 \\ & - \frac{\Theta_Y^{RA} \sigma_C \sigma_I}{2} [\omega_H (c_t - \iota_t^I)^2 + \omega_F (c_t^* - \iota_t^{I*})^2] \\ & + \Theta_{mc,H}^{RA} \left[ mc_{H,t}^{RA,(1)} (c_t + \log \mathcal{G}(\tau_{M,t}, S_t) - y_t) - \frac{1}{2} (mc_{H,t}^{RA,(1)})^2 \right] \\ & - \frac{\omega_F \Theta_\chi^{RA}}{\varepsilon_M - 1} \left[ \{(\varepsilon_M - 1) \psi_t + \tau_{M,t}\} \left\{ d_t^A - \eta(x_{H,t} - \log \mathcal{G}(\tau_{M,t}, S_t)) + (z_t - \psi_t) - c_t - \log \mathcal{G}(\tau_{M,t}, S_t) \right\} \right. \\ & \quad \left. + \frac{\varepsilon_M - 1}{2} \psi_t^2 + \tau_{M,t} \psi_t - \frac{\varepsilon_M + 1}{2} \tau_{M,t}^2 \right] + \text{s.o.t.i.p.} \end{aligned} \quad (\text{D.61})$$

All objects on the right-hand side of Eq. (D.61) are either structural parameters or variables already defined by the model and the first-order reduction above. The quadratic coefficients can therefore be reproduced directly from this equation.

Order the real coordinates as

$$z_{1,t}^{RA} = y_t, \quad z_{2,t}^{RA} = \psi_t, \quad z_{3,t}^{RA} = z_t, \quad z_{4,t}^{RA} = \psi_t^*.$$

Define the reduced real Hessian and the two tariff-loading columns by

$$\begin{aligned} Q_{ij}^{RA} & \equiv -\frac{\partial^2 \mathcal{W}_{H,t}^{RA,(2),real}}{\partial z_{i,t}^{RA} \partial z_{j,t}^{RA}}, \quad i, j = 1, \dots, 4, \\ B_{i1}^{RA} & \equiv \frac{\partial^2 \mathcal{W}_{H,t}^{RA,(2),real}}{\partial z_{i,t}^{RA} \partial \tau_{M,t}}, \quad B_{i2}^{RA} \equiv \frac{\partial^2 \mathcal{W}_{H,t}^{RA,(2),real}}{\partial z_{i,t}^{RA} \partial \tau_{M,t}^*}. \end{aligned} \quad (\text{D.62})$$

Thus, up to pure tariff terms,

$$\mathcal{L}_{H,t}^{RA,real} = \frac{1}{2}(z_t^{RA})' Q^{RA} z_t^{RA} - (z_t^{RA})' B^{RA} (\tau_{M,t}, \tau_{M,t}^*)' + \text{s.o.t.i.p.} \quad (\text{D.63})$$

and  $Q^{RA}$  is symmetric.

The four real coefficients reported for roundabout production in Table 6 are the successive Schur pivots of  $Q^{RA}$  in the order  $y, \psi, s, \psi^*$ . Hence

$$\begin{aligned} \Lambda_y^{RA} &= Q_{11}^{RA}, \\ \Lambda_\psi^{RA} &= \frac{\det Q_{1:2,1:2}^{RA}}{Q_{11}^{RA}}, \\ \Lambda_z^{RA} &= \frac{\det Q_{1:3,1:3}^{RA}}{\det Q_{1:2,1:2}^{RA}}, \\ \Lambda_{\psi^*}^{RA} &= \frac{\det Q^{RA}}{\det Q_{1:3,1:3}^{RA}}. \end{aligned} \quad (\text{D.64})$$

After completing the output square, the exact symbolic Home WR LOOP gap is

$$\begin{aligned} \psi_t^{w,RA} &= \frac{1}{\Lambda_\psi^{RA}} \left[ \left( B_{21}^{RA} - \frac{Q_{21}^{RA} B_{11}^{RA}}{Q_{11}^{RA}} \right) \tau_{M,t} + \left( B_{22}^{RA} - \frac{Q_{21}^{RA} B_{12}^{RA}}{Q_{11}^{RA}} \right) \tau_{M,t}^* \right. \\ &\quad \left. - \left( Q_{23}^{RA} - \frac{Q_{21}^{RA} Q_{13}^{RA}}{Q_{11}^{RA}} \right) z_t - \left( Q_{24}^{RA} - \frac{Q_{21}^{RA} Q_{14}^{RA}}{Q_{11}^{RA}} \right) \psi_t^* \right]. \end{aligned} \quad (\text{D.65})$$

For the loadings reported in Table 7,  $\Gamma_{\text{own}}^{RA}$ ,  $\Gamma_{\text{cross}}^{RA}$ , and  $\Gamma_z^{RA}$  are defined as the structural coefficients on  $\tau_{M,t}$ ,  $\tau_{M,t}^* - \psi_t^*$ , and  $z_t$ , respectively, in Eq. (D.65). The remaining three WR reference values follow from the same four-step sequential completion.

The sticky-inflation coefficients are

$$\Lambda_{\pi,H}^{RA} = \frac{\varepsilon}{\kappa_H} (\Theta_Y^{RA} + \Theta_{mc,H}^{RA}), \quad \Lambda_{\pi,F}^{RA} = \frac{\varepsilon_M}{\kappa_M} \omega_F \Theta_\chi^{RA}, \quad \Lambda_{\pi_H^*}^{RA} = \frac{\varepsilon_M}{\kappa_M} \omega_F \Theta_Y^{RA}. \quad (\text{D.66})$$

Evaluating the symbolic expressions at  $\zeta = 0.50$  and the benchmark parameters in Table 1, with  $\nu = 0.27$ , gives

$$\Theta_Y^{RA} = 0.977681, \quad \Theta_{mc,H}^{RA} = 0.517809, \quad \Theta_\chi^{RA} = 2.129530. \quad (\text{D.67})$$

The reduced Hessian and tariff-loading matrix are then

$$Q^{RA} = \begin{pmatrix} 0.961881 & -0.005532 & 0.092926 & 0.129069 \\ -0.005532 & 0.650135 & -0.091517 & 0.059579 \\ 0.092926 & -0.091517 & 0.108569 & 0.023443 \\ 0.129069 & 0.059579 & 0.023443 & 0.226085 \end{pmatrix}, \quad (\text{D.68})$$

$$B^{RA} = \begin{pmatrix} -0.252528 & 0.129069 \\ 0.296242 & 0.059579 \\ 0.100845 & 0.023443 \\ 0.000510 & 0.226085 \end{pmatrix}. \quad (\text{D.69})$$

Equations (D.64) and (D.66) therefore give

$$\begin{aligned} (\Lambda_y^{RA}, \Lambda_\psi^{RA}, \Lambda_z^{RA}, \Lambda_{\psi^*}^{RA}) &= (0.961881, 0.650103, 0.086858, 0.198829), \\ (\Lambda_{\pi,H}^{RA}, \Lambda_{\pi,F}^{RA}, \Lambda_{\pi_H^*}^{RA}) &= (109.285839, 34.080668, 15.646664). \end{aligned} \quad (\text{D.70})$$

Equation (D.65) gives

$$\psi_t^{w,RA} = 0.139951 z_t - 0.092787 \psi_t^* + 0.453450 \tau_{M,t} + 0.092787 \tau_{M,t}^*. \quad (\text{D.71})$$

which gives the unrounded loadings corresponding to the roundabout-production column of Table 7. As a nesting check, setting  $\zeta = 0$  eliminates the intermediate-input terms and reproduces

$$(\Lambda_y^{IPT}, \Lambda_\psi^{IPT}, \Lambda_z^{IPT}, \Lambda_{\psi^*}^{IPT}) = (2.860011, 0.406343, -0.097828, 0.352099),$$

together with Eq. (47). Thus the roundabout-production criterion is obtained from the same Benigno–Woodford construction as the benchmark two-country criterion.

The resulting Home period loss combines these completed-square real terms with the three sticky-inflation terms; its numerical weights are reported in Table 6.

**Parameter-closure audit.** The preceding definitions exhaust the coefficient symbols used in D.1–D.2.4. Section by section, D.1.1 depends only on  $(\beta, \varphi, \eta, \varepsilon, \theta, \nu)$ ; D.1.2 adds only  $(\varepsilon_M, \theta_M)$ ; D.2.1 and D.2.2 additionally require the country- size parameter  $\nu$ ; D.2.3 adds only the external-habit parameter  $\chi$ ; and D.2.4 adds only the Roundabout share  $\zeta$ . Thus the only primitive structural parameters entering the per-period welfare cost functions are those in Eq. (D.1), together with  $\chi$  in D.2.3 and  $\zeta$  in D.2.4. The expenditure shares, desired markup, Calvo slopes, steady-state Roundabout shares, and importer subsidy are all explicit functions of those primitives. The  $\Theta$  coefficients are solved from displayed linear systems; the  $Q$ ,  $B$ ,  $C$ , and  $\hat{C}$  coefficients are Hessian or tariff-loading entries of explicit augmented second-order welfare expressions; and the  $\Lambda$  and  $b$  coefficients are Schur-completion transforms of those entries. The tariff innovations and shock persistence determine conditioning paths only. Thus no welfare weight, WR reference-value loading, or auxiliary coefficient in D is an independently calibrated parameter.

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