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Online Appendix to "Optimal Monetary Policy in the Tariff-Imposing Economy under Imperfect Pass-Through"

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This Online Appendix supplements the main text. Section, equation, figure, and table numbers without a letter prefix, and Appendix A, refer to the main text.

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B Derivation of the SOE Model

The IPT environment of the SOE is presented in Sec. 3.1; this appendix derives the PPT environment, whose producer block is shared by the IPT environment, and the flexible-importer-price markup.

B.1 Perfect Pass-Through Environment

Households and Demand

The Home representative household has lifetime utility given by Eq. (1) in the main text, with $N_t \equiv \int_0^1 N_t(j)\,dj$.

Composite consumption is

$$C_t = \left[(1-\upsilon)^{1/\eta} C_{H,t}^{(\eta-1)/\eta} + \upsilon^{1/\eta} C_{F,t}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}.$$

Under PPT, the *ex-tariff* Home-currency import price satisfies

$$P_{F,t} = \mathcal{E}_t P_{F,t}^*. \tag{B.1}$$

The Home CPI under PPT is

$$P_t^{PPT} \equiv \left[(1-\upsilon) P_{H,t}^{1-\eta} + \upsilon \left(\widetilde{P}_{F,t}^{PPT}\right)^{1-\eta}\right]^{1/(1-\eta)}, \tag{B.2}$$

where P_t^{PPT} denotes the Home Consumer Price Index (CPI) and $\widetilde{P}_{F,t}^{PPT} \equiv (1+\tau_{M,t})\mathcal{E}_t P_{F,t}^*$ denotes the *cum-tariff* import price under PPT. The CES demand system gives

$$C_{H,t} = (1-\upsilon)\left[\mathcal{G}^{PPT}(\tau_{M,t}, S_t)\right]^{\eta} C_t, \tag{B.3}$$

where the CPI–PPI ratio is

$$\mathcal{G}^{PPT}(\tau_{M,t}, S_t) \equiv \frac{P_t^{PPT}}{P_{H,t}}. \tag{B.4}$$

Here S_t denotes the *ex-tariff* relative price of imports, Z_t denotes the *ex-tariff* relative border price of imports, and Ψ_t denotes the LOOP gap, as defined in Sec. 3.1 of the main text.

Under PPT, Eq. (B.1) implies $\Psi_t = 1$. Therefore, $S_t = Z_t$.

Import demand is

$$C_{F,t} = \upsilon\left[\frac{(1+\tau_{M,t})S_t}{\mathcal{G}^{PPT}(\tau_{M,t}, S_t)}\right]^{-\eta} C_t. \tag{B.5}$$

Combining Eq. (B.3) with Eq. (B.5) yields

$$\frac{C_{F,t}}{C_{H,t}} = \frac{\upsilon}{1-\upsilon}\left[(1+\tau_{M,t})S_t\right]^{-\eta}. \tag{B.6}$$

For a given S_t, a higher import tariff reduces $C_{F,t}/C_{H,t}$.

The household's nominal budget constraint is

$$P_{H,t}C_{H,t} + (1+\tau_{M,t})\mathcal{E}_t P_{F,t}^* C_{F,t} + \sum_{h^{t+1}} \xi_{t,t+1}(h^{t+1}|h^t) B_{t+1}(h^{t+1}) = B_t(h^t) + W_t N_t + \mathcal{D}_t + T_t,$$

where the variables are as defined in Sec. 3.1 of the main text. The import-expenditure term contains both the border value $\mathcal{E}_t P_{F,t}^* C_{F,t}$ and the statutory customs payment $\tau_{M,t}\mathcal{E}_t P_{F,t}^* C_{F,t}$. In

the PPT environment, where there is no local-importer subsidy, tariff revenue is rebated to the household as a lump-sum transfer:

$$TR_t = \tau_{M,t}\mathcal{E}_t P^*_{F,t} C_{F,t}, \qquad T_t = TR_t.$$

By CES duality,

$$P_{H,t}C_{H,t} + (1+\tau_{M,t})\mathcal{E}_t P^*_{F,t} C_{F,t} = P_t^{PPT} C_t,$$

so the same budget constraint can equivalently be written as

$$P_t^{PPT} C_t + \sum_{h^{t+1}} \xi_{t,t+1}(h^{t+1}|h^t) B_{t+1}(h^{t+1}) = B_t(h^t) + W_t N_t + \mathcal{D}_t + T_t. \tag{B.7}$$

Maximization of Eq. (1) subject to the budget constraint gives the intratemporal condition

$$\frac{W_t}{P_t^{PPT}} = C_t N_t^{\varphi}, \tag{B.8}$$

and the Euler equation

$$\frac{1}{1+i_t} = \beta \mathrm{E}_t \left[\frac{C_t \mathcal{G}^{PPT}(\tau_{M,t}, S_t)}{C_{t+1}\mathcal{G}^{PPT}(\tau_{M,t+1}, S_{t+1})\,\Pi_{H,t+1}} \right]. \tag{B.9}$$

Here i_t denotes the Home nominal interest rate and $\Pi_{H,t} \equiv P_{H,t}/P_{H,t-1}$ denotes gross Home PPI inflation. The Euler equation follows from $(1+i_t)^{-1} = \mathrm{E}_t Q_{t,t+1}$, where $Q_{t,t+1}$ is the nominal stochastic discount factor defined in Sec. 3.1 of the main text, with P_t^{PPT} in place of P_t, so that $Q_{t,t+1} = \beta(C_t/C_{t+1})(\Pi_{t+1}^{PPT})^{-1}$, and $\Pi_t^{PPT} \equiv P_t^{PPT}/P_{t-1}^{PPT}$ denotes gross CPI inflation. The Foreign household satisfies the analogous Euler equation. The Foreign-currency bond-pricing condition is

$$\frac{1}{1+i_t^*} = \mathrm{E}_t \left[Q_{t,t+1} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right].$$

Log-linearizing this condition together with the Home Euler equation yields Eq. (11) in the main text.

Complete international risk sharing implies

$$S_t = C_t \mathcal{G}^{PPT}(\tau_{M,t}, S_t). \tag{B.10}$$

Production and Pricing Behavior

Each Home firm $j \in [0,1]$ produces $Y_t(j) = A_t N_t(j)$, where $A_t = 1$ throughout the paper. Demand for variety j is $Y_t(j) = [P_{H,t}(j)/P_{H,t}]^{-\varepsilon} Y_t$, where Y_t denotes aggregate Home output and $\varepsilon > 1$ is the elasticity of substitution across producer varieties. Aggregation gives

$$Y_t = \frac{A_t N_t}{\Delta_{H,t}}, \tag{B.11}$$

where $\Delta_{H,t} \equiv \int_0^1 [P_{H,t}(j)/P_{H,t}]^{-\varepsilon} dj$ denotes producer-price dispersion.

Following Monacelli [6], the Home producer's Calvo reset-price condition is

$$\widetilde{p}_{H,t} = \mathcal{M} \frac{\mathcal{Z}_{H,t}}{\mathcal{K}_{H,t}}, \tag{B.12}$$

where $\overline{P}_{H,t}$ denotes the producer reset price, $\widetilde{p}_{H,t} \equiv \overline{P}_{H,t}/P_{H,t}$ is the reset price relative to the current Home PPI, θ is the producer Calvo non-adjustment probability, $MC_t^r \equiv MC_t/P_{H,t}$ is

Home real marginal cost, with nominal marginal cost $MC_t \equiv W_t/A_t$, and $\mathcal{M} \equiv \varepsilon/(\varepsilon-1)$ is the desired producer markup. The auxiliary objects are

$$\begin{aligned} \mathcal{K}_{H,t} &\equiv \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k} \left(\prod_{s=1}^{k} \Pi_{H,t+s} \right)^{\varepsilon}, \\ \mathcal{Z}_{H,t} &\equiv \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k} MC^r_{t+k} \left(\prod_{s=1}^{k} \Pi_{H,t+s} \right)^{1+\varepsilon}. \end{aligned} \tag{B.13}$$

Here $Q_{t,t+k}$ is the nominal stochastic discount factor defined in Sec. 3.1 of the main text, with P_t^{PPT} in place of P_t. Using Eq. (B.8), real marginal cost is

$$MC^r_t = C_t N_t^{\varphi} \mathcal{G}^{PPT}(\tau_{M,t}, S_t). \tag{B.14}$$

The producer Calvo price-index and dispersion laws are

$$\begin{aligned} 1 &= \theta \Pi_{H,t}^{\varepsilon-1} + (1-\theta)\widetilde{p}_{H,t}^{\,1-\varepsilon}, \\ \Delta_{H,t} &= \theta \Pi_{H,t}^{\varepsilon} \Delta_{H,t-1} + (1-\theta)\widetilde{p}_{H,t}^{\,-\varepsilon}. \end{aligned} \tag{B.15}$$

At first order, Home PPI inflation therefore satisfies

$$\pi_{H,t} = \beta \mathrm{E}_t \pi_{H,t+1} + \kappa_H mc^r_t, \tag{B.16}$$

where $\pi_{H,t} \equiv \log \Pi_{H,t}$, $mc^r_t \equiv \log(MC^r_t/MC^r)$, MC^r is the deterministic steady-state value of Home real marginal cost, and $\kappa_H \equiv (1-\theta)(1-\beta\theta)/\theta$.

Market Clearing

Home goods-market clearing is

$$Y_t = (1-\upsilon)\left(\mathcal{G}^{PPT}(\tau_{M,t}, S_t)\right)^{\eta} C_t + \upsilon S_t^{\eta}. \tag{B.17}$$

The first term is Home household demand in Eq. (B.3); the second is Foreign demand for Home goods in the SOE limit.

B.2 Flexible-Importer-Price Markup

With flexible importer prices ($\theta_M = 0$), $\overline{P}_{F,t} = P_{F,t}$ and only the $k=0$ term of Eq. (16) remains, so the reset-price condition becomes

$$1 - \varepsilon_M \frac{P_{F,t} - (1-\tau)\mathcal{E}_t P^*_{F,t}}{P_{F,t} + \tau_{M,t}\mathcal{E}_t P^*_{F,t}} = 0,$$

which can be rearranged as

$$(\varepsilon_M - 1) P_{F,t} = [\varepsilon_M(1-\tau) + \tau_{M,t}] \mathcal{E}_t P^*_{F,t}.$$

Using the fixed importer subsidy $\tau = 1/\varepsilon_M$ and $\Psi_t^{-1} = P_{F,t}/(\mathcal{E}_t P^*_{F,t})$ gives Eq. (17).

C Derivation of the Two-Country Economy Model

This appendix collects the nonlinear private-equilibrium conditions of the two-country economy.

C.1 Perfect Pass-Through Environment

Household preferences are specified in Sec. 4.1, and the consumption baskets in Eq. (27). Under PPT, Eq. (B.1) and

$$P^*_{H,t} = \frac{P_{H,t}}{\mathcal{E}_t} \tag{C.1}$$

imply

$$\Psi_t = 1, \qquad \Psi^*_t = 1. \tag{C.2}$$

Equivalently, the log LOOP gaps used in the welfare analysis satisfy $\psi_t = \psi^*_t = 0$.

The national CPI indices are

$$\begin{aligned} P_t^{PPT} &= \left[\omega_H P_{H,t}^{1-\eta} + \omega_F \{(1+\tau_{M,t})\mathcal{E}_t P^*_{F,t}\}^{1-\eta}\right]^{1/(1-\eta)}, \\ P_t^{*,PPT} &= \left[\omega^*_F (P^*_{F,t})^{1-\eta} + \omega^*_H \left\{(1+\tau^*_{M,t})\frac{P_{H,t}}{\mathcal{E}_t}\right\}^{1-\eta}\right]^{1/(1-\eta)}. \end{aligned} \tag{C.3}$$

Here $\mathcal{G}^{PPT}(\tau_{M,t}, Z_t/\Psi_t)$ uses the two-country Home expenditure weights ω_H and ω_F, with $\Psi_t = 1$ under PPT. The associated bilateral demand system is

$$\begin{aligned} C_{H,t} &= \omega_H \left(\frac{P_{H,t}}{P_t^{PPT}}\right)^{-\eta} C_t, \\ C_{F,t} &= \omega_F \left(\frac{(1+\tau_{M,t})\mathcal{E}_t P^*_{F,t}}{P_t^{PPT}}\right)^{-\eta} C_t, \\ C^*_{F,t} &= \omega^*_F \left(\frac{P^*_{F,t}}{P_t^{*,PPT}}\right)^{-\eta} C^*_t, \\ C^*_{H,t} &= \omega^*_H \left(\frac{(1+\tau^*_{M,t})P_{H,t}/\mathcal{E}_t}{P_t^{*,PPT}}\right)^{-\eta} C^*_t. \end{aligned} \tag{C.4}$$

The Home intratemporal and Euler conditions are Eqs. (B.8) and (B.9), with $\Psi_t = 1$; the Foreign household satisfies the exact starred counterparts. In particular,

$$\frac{1}{1+i^*_t} = \beta \mathrm{E}_t \frac{C^*_t (P_t^{*,PPT}/P^*_{F,t})}{C^*_{t+1}(P_{t+1}^{*,PPT}/P^*_{F,t+1})\Pi^*_{F,t+1}}, \tag{C.5}$$

where $\Pi^*_{F,t} \equiv P^*_{F,t}/P^*_{F,t-1}$ denotes gross Foreign PPI inflation. Complete international risk sharing gives

$$Z_t = \frac{C_t}{C^*_t}\frac{\mathcal{G}^{PPT}(\tau_{M,t}, Z_t/\Psi_t)}{P_t^{*,PPT}/P^*_{F,t}}, \qquad \Psi_t = 1, \tag{C.6}$$

which is the two-country counterpart of Eq. (B.10).

Under PPT, Home production and real marginal cost satisfy

$$Y_t = \frac{N_t}{\Delta_{H,t}}, \qquad MC^r_t = C_t N_t^{\varphi} \mathcal{G}^{PPT}(\tau_{M,t}, Z_t/\Psi_t), \tag{C.7}$$

and producer pricing is governed by Eqs. (B.12)–(B.15). Foreign production, real marginal cost, and producer pricing are given by the corresponding starred conditions.

Because there is no local importer-price dispersion under PPT, bilateral goods-market clearing is

$$Y_t = C_{H,t} + \left(\frac{1-\nu}{\nu}\right) C^*_{H,t}, \qquad Y^*_t = C^*_{F,t} + \left(\frac{\nu}{1-\nu}\right) C_{F,t}. \tag{C.8}$$

Equation (27), Eqs. (C.1)–(C.8), the Home household conditions in Eqs. (B.8)–(B.9) and their Foreign counterparts, and the two producer Calvo blocks characterize the nonlinear two-country PPT private equilibrium for given Home and Foreign monetary-policy paths.

C.2 Imperfect Pass-Through Environment

The IPT environment retains the household optimality conditions, complete-markets risk sharing, and PCP producer blocks of Appendix C.1, with the corresponding IPT price indices.

The Home household-facing *cum-tariff* price of a Foreign variety and its CES index are given by Eqs. (2) and (4), with the country-mass normalization of the importer integrals in Sec. 4.1; the Foreign counterparts are defined analogously.

The CPI indices are therefore

$$\begin{aligned} P^{IPT}_t &= \left[\omega_H P^{1-\eta}_{H,t} + \omega_F (\widetilde{P}^{IPT}_{F,t})^{1-\eta}\right]^{1/(1-\eta)}, \\ P^{*,IPT}_t &= \left[\omega^*_F (P^*_{F,t})^{1-\eta} + \omega^*_H (\widetilde{P}^*_{H,t})^{1-\eta}\right]^{1/(1-\eta)}. \end{aligned} \tag{C.9}$$

Here $\mathcal{G}^{*,IPT}_t(\cdot) \equiv P^{*,IPT}_t / P^*_{F,t}$ denotes the Foreign counterpart of the CPI–PPI ratio $\mathcal{G}^{IPT}_t(\cdot)$ defined in Sec. 3.1. The aggregate bilateral demands are

$$\begin{aligned} C_{H,t} &= \omega_H \left(\frac{P_{H,t}}{P^{IPT}_t}\right)^{-\eta} C_t, \\ C_{F,t} &= \omega_F \left(\frac{\widetilde{P}^{IPT}_{F,t}}{P^{IPT}_t}\right)^{-\eta} C_t, \\ C^*_{F,t} &= \omega^*_F \left(\frac{P^*_{F,t}}{P^{*,IPT}_t}\right)^{-\eta} C^*_t, \\ C^*_{H,t} &= \omega^*_H \left(\frac{\widetilde{P}^*_{H,t}}{P^{*,IPT}_t}\right)^{-\eta} C^*_t. \end{aligned} \tag{C.10}$$

Within the imported-good aggregate, conditional demand is Eq. (3) with the Home-country normalization above, and the Foreign conditional-demand equation is its exact starred counterpart. By CES duality, consumption expenditure in the Home and Foreign household budget constraints equals $P^{IPT}_t C_t$ and $P^{*,IPT}_t C^*_t$, respectively. The statutory duties are remitted on border customs value and rebated lump sum exactly as in Eq. (10) and its Foreign mirror.

The Home intratemporal and Euler equations retain the forms of Eqs. (B.8) and (B.9), with the IPT CPI and $\mathcal{G}^{IPT}_t(\cdot)$ replacing the PPT CPI and $\mathcal{G}^{PPT}(\tau_{M,t}, S_t)$, respectively. The Foreign equations are their exact starred counterparts. Complete international risk sharing implies

$$Z_t = \frac{C_t}{C^*_t} \frac{\mathcal{G}^{IPT}_t(\cdot)}{P^{*,IPT}_t / P^*_{F,t}}. \tag{C.11}$$

The producer blocks remain Eqs. (B.11)–(B.15) and their starred counterparts.

Home local importers use the exact nonlinear pricing problem in Eqs. (14)–(16). Their *ex-tariff* price index satisfies Eq. (12), and the local relative import price obeys Eq. (13). Foreign local importers solve the exact Home–Foreign mirror problem.

Bilateral goods-market clearing is given by Eq. (C.16), derived in Appendix C.3. Together with the household, PCP producer, and LCP importer blocks for both countries, these conditions characterize the nonlinear two-country IPT private equilibrium for given Home and Foreign monetary-policy paths.

C.3 Additional Details of the Two-Country Model

The Home *ex-tariff* relative price of imports can be written as

$$S_t = \frac{P_{F,t}}{\mathcal{E}_t P^*_{H,t}} \frac{1}{\Psi^*_t},$$

where $P_{F,t}/(\mathcal{E}_t P^*_{H,t})$ is the relative price of Home imports to Home exports, measured in Home currency and excluding tariffs. For Home importers, Eq. (17) characterizes the flexible-price benchmark, whereas Eq. (18) governs first-order import-price inflation under sticky prices. Since Foreign importers of Home goods face the corresponding monopolistically competitive pricing problem, their flexible-price condition is

$$(\Psi^*_t)^{-1} = \frac{P^*_{H,t}}{P_{H,t}/\mathcal{E}_t} = 1 + \frac{\tau^*_{M,t}}{\varepsilon_M - 1}, \tag{C.12}$$

and their first-order importer Phillips curve under sticky local-currency prices is

$$\pi^*_{H,t} = \beta \mathrm{E}_t \pi^*_{H,t+1} + \kappa_M \left(\psi^*_t + \frac{\tau^*_{M,t}}{\varepsilon_M - 1} \right). \tag{C.13}$$

Under PPT, the Home relative consumption of imports is determined by

$$\frac{C_{F,t}}{C_{H,t}} = \frac{\omega_F}{\omega_H} \left[(1 + \tau_{M,t}) S_t \right]^{-\eta}. \tag{C.14}$$

The bilateral goods-market conditions follow directly from variety-level clearing. For a Home producer variety j, domestic and export demand imply

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \left[C_{H,t} + \frac{1}{\nu} \int_\nu^1 C^*_{H,t}(\iota) d\iota \right]. \tag{C.15}$$

Aggregating Eq. (C.15) across producer varieties using the producer CES index yields the bilateral goods-market-clearing conditions

$$\begin{aligned} Y_t &= C_{H,t} + \left(\frac{1-\nu}{\nu} \right) C^*_{H,t} \frac{1}{1-\nu} \int_\nu^1 \left(\frac{\widetilde{P}^*_{H,t}(\iota)}{\widetilde{P}^*_{H,t}} \right)^{-\varepsilon_M} d\iota, \\ Y^*_t &= C^*_{F,t} + \left(\frac{\nu}{1-\nu} \right) C_{F,t} \frac{1}{\nu} \int_0^\nu \left(\frac{\widetilde{P}^{IPT}_{F,t}(\iota)}{\widetilde{P}^{IPT}_{F,t}} \right)^{-\varepsilon_M} d\iota, \end{aligned} \tag{C.16}$$

where $C^*_{H,t}(\iota)$ denotes Foreign-country demand for Home goods sold by local importer variety ι, and $\widetilde{P}^*_{H,t}(\iota)$ is the Foreign mirror of the variety-level *cum-tariff* purchaser price in Eq. (2). The corresponding aggregate *cum-tariff* import-price index is

$$\widetilde{P}^*_{H,t} \equiv \left[\frac{1}{1-\nu} \int_\nu^1 \left(\widetilde{P}^*_{H,t}(\iota) \right)^{1-\varepsilon_M} d\iota \right]^{1/(1-\varepsilon_M)}.$$

The corresponding variety-level market-clearing condition for Foreign producers is obtained by interchanging Home and Foreign. Producer-price dispersion affects labor requirements through Eq. (B.11), whereas the two integrals in Eq. (C.16) are not producer-price-dispersion terms: they arise from aggregating physical import quantities across importer varieties at *cum-tariff* purchaser prices.

D Social Planner's Problem

The policy problems in this section are the nonlinear commitment problems used to generate the allocations. Welfare cost functions are derived separately in Appendix F.

D.1 Small Open Economy

D.1.1 PPT Environment

In the PPT environment, $\Psi_t = 1$, and the social planner maximizes $\mathcal{U}$ subject to Eqs. (B.4), (B.8), (B.9), (B.10), (B.11), (B.12)–(B.15), (B.1), (B.17), and (20). After the producer reset-price and auxiliary-sequence conditions in Eqs. (B.12)–(B.13) are substituted, the Lagrangian is

$$\begin{aligned}\mathcal{L}^{SOE,PPT} \;=\; & \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \\ & +\lambda_{p,t} \left[1 - \theta \Pi_{H,t}^{\varepsilon-1} - (1-\theta) \widetilde{p}_{H,t}^{\,1-\varepsilon} \right] \\ & +\lambda_{\Delta,t} \left[\Delta_{H,t} - \theta \Pi_{H,t}^{\varepsilon} \Delta_{H,t-1} - (1-\theta) \widetilde{p}_{H,t}^{\,-\varepsilon} \right] \\ & +\lambda_{n,t} \left[N_t - Y_t \Delta_{H,t} \right] \\ & +\lambda_{y,t} \left[Y_t - (1-\upsilon) \mathcal{G}^{PPT}(\tau_{M,t}, Z_t)^{\eta} C_t - \upsilon Z_t^{\eta} \right] \\ & +\lambda_{r,t} \left[Z_t - C_t \mathcal{G}^{PPT}(\tau_{M,t}, Z_t) \right] \\ & +\lambda_{e,t} \left[\frac{1}{1+i_t} - \beta \mathrm{E}_t \frac{C_t \mathcal{G}^{PPT}(\tau_{M,t}, Z_t)}{C_{t+1} \mathcal{G}^{PPT}(\tau_{M,t+1}, Z_{t+1}) \Pi_{H,t+1}} \right] + \mu_{i,t} i_t \Bigg\}. \end{aligned} \tag{D.1}$$

Here $\lambda_{p,t}$, $\lambda_{\Delta,t}$, $\lambda_{n,t}$, $\lambda_{y,t}$, $\lambda_{r,t}$, and $\lambda_{e,t}$ are the multipliers on the price-index, price-dispersion, production, goods-market, risk-sharing, and Euler conditions shown in the same order in Eq. (D.1). The multiplier $\mu_{i,t}$ is the ZLB multiplier defined after Eq. (20).

D.1.2 IPT Environment

The planner's objective is Eq. (1); its exact private-equilibrium constraints are those listed in Sec. 3.2. To give all stationarity conditions in expanded form, we retain the already-defined producer reset price $\widetilde{p}_{H,t}$ and producer sequences $\mathcal{K}_{H,t}$ and $\mathcal{Z}_{H,t}$ in Eqs. (B.12)–(B.13) as independent choice coordinates, instead of substituting them into every Calvo restriction. We also retain the already-defined CPI–PPI ratio $\mathcal{G}_t^{IPT}$ and Home producer price level $P_{H,t}$. Their defining equalities are imposed below, so this is an equivalent representation of the original nonlinear Ramsey problem. The initial producer price level and the pre-shock Calvo cohorts are predetermined.

The original producer sequences in Eq. (B.13) have the exact recursive representation

$$\begin{aligned} \mathcal{K}_{H,t} &= Y_t + \beta\theta \mathrm{E}_t \left[\frac{C_t \mathcal{G}_t^{IPT}}{C_{t+1} \mathcal{G}_{t+1}^{IPT}} \Pi_{H,t+1}^{\varepsilon-1} \mathcal{K}_{H,t+1} \right], \\ \mathcal{Z}_{H,t} &= C_t \mathcal{G}_t^{IPT} N_t^{\varphi} Y_t + \beta\theta \mathrm{E}_t \left[\frac{C_t \mathcal{G}_t^{IPT}}{C_{t+1} \mathcal{G}_{t+1}^{IPT}} \Pi_{H,t+1}^{\varepsilon} \mathcal{Z}_{H,t+1} \right]. \end{aligned} \tag{D.2}$$

These recursions must satisfy the terminal conditions implied by the infinite-sum definitions in Eq. (B.13), which rule out extraneous solutions. The nonlinear Ramsey problem also imposes Eq. (8), written in terms of $\mathcal{G}_t^{IPT}$; the producer price-level identity $P_{H,t} = \Pi_{H,t} P_{H,t-1}$; the *cum-tariff* import-price index in Eq. (4), aggregated over Calvo cohorts of importer prices including pre-shock cohorts; and the *ex-tariff* price-index law in Eq. (12). These restrictions enter the Lagrangian below with the multipliers $\lambda_{g,t}$, $\lambda_{PH,t}$, $\lambda_{I,t}$, and $\lambda_{pF,t}$, respectively. For the Lagrangian below, let $\mathcal{I}_t$ denote the left-hand side of the importer reset-price condition in Eq. (16), with $C_{F,t+k|t}$ defined

in Eq. (15):

$$\mathcal{I}_t = \mathrm{E}_t \sum_{k=0}^{\infty} \theta_M^k Q_{t,t+k} C_{F,t+k|t} \left[1 - \varepsilon_M \frac{\overline{P}_{F,t} - (1-\tau) Z_{t+k} P_{H,t+k}}{\overline{P}_{F,t} + \tau_{M,t+k} Z_{t+k} P_{H,t+k}} \right] = 0, \qquad \tau = 1/\varepsilon_M. \tag{D.3}$$

The fully specified nonlinear Lagrangian, with a distinct multiplier for each exact restriction, is

$$\begin{aligned}
\mathcal{L}^{SOE,IPT} \;=\; & \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \\
& + \lambda_{p,t} \left[1 - \theta \Pi_{H,t}^{\varepsilon-1} - (1-\theta) \widetilde{p}_{H,t}^{\,1-\varepsilon} \right] + \lambda_{\Delta,t} \left[\Delta_{H,t} - \theta \Pi_{H,t}^{\varepsilon} \Delta_{H,t-1} - (1-\theta) \widetilde{p}_{H,t}^{\,-\varepsilon} \right] \\
& + \lambda_{n,t} [N_t - Y_t \Delta_{H,t}] + \lambda_{y,t} \left[Y_t - (1-\upsilon) (\mathcal{G}_t^{IPT})^{\eta} C_t - \upsilon Z_t^{\eta} \right] + \lambda_{r,t} [Z_t - C_t \mathcal{G}_t^{IPT}] \\
& + \lambda_{e,t} \left[\frac{1}{1+i_t} - \beta \mathrm{E}_t \frac{C_t \mathcal{G}_t^{IPT}}{C_{t+1} \mathcal{G}_{t+1}^{IPT} \Pi_{H,t+1}} \right] + \lambda_{s,t} \left[\frac{Z_t}{\Psi_t} - \frac{Z_{t-1}}{\Psi_{t-1}} \frac{\Pi_{F,t}}{\Pi_{H,t}} \right] \\
& + \lambda_{pF,t} \left[1 - \theta_M \Pi_{F,t}^{\varepsilon_M - 1} - (1-\theta_M) \left(\frac{\overline{P}_{F,t} \Psi_t}{Z_t P_{H,t}} \right)^{1-\varepsilon_M} \right] \\
& + \lambda_{I,t} \left[\left(\frac{\widetilde{P}_{F,t}^{IPT}}{P_{H,t}} \right)^{1-\varepsilon_M} - (1-\theta_M) \sum_{j=0}^{\infty} \theta_M^j \left(\frac{\overline{P}_{F,t-j}}{P_{H,t}} + \tau_{M,t} Z_t \right)^{1-\varepsilon_M} \right] + \lambda_{M,t} \mathcal{I}_t \\
& + \lambda_{g,t} \left[(\mathcal{G}_t^{IPT})^{1-\eta} - (1-\upsilon) - \upsilon \left(\frac{\widetilde{P}_{F,t}^{IPT}}{P_{H,t}} \right)^{1-\eta} \right] + \lambda_{PH,t} [P_{H,t} - \Pi_{H,t} P_{H,t-1}] \\
& + \lambda_{h,t} \left[\widetilde{p}_{H,t} - \mathcal{M} \frac{\mathcal{Z}_{H,t}}{\mathcal{K}_{H,t}} \right] \\
& + \lambda_{K,t} \left[\mathcal{K}_{H,t} - Y_t - \beta \theta \mathrm{E}_t \frac{C_t \mathcal{G}_t^{IPT}}{C_{t+1} \mathcal{G}_{t+1}^{IPT}} \Pi_{H,t+1}^{\varepsilon-1} \mathcal{K}_{H,t+1} \right] \\
& + \lambda_{ZH,t} \left[\mathcal{Z}_{H,t} - C_t \mathcal{G}_t^{IPT} N_t^{\varphi} Y_t - \beta \theta \mathrm{E}_t \frac{C_t \mathcal{G}_t^{IPT}}{C_{t+1} \mathcal{G}_{t+1}^{IPT}} \Pi_{H,t+1}^{\varepsilon} \mathcal{Z}_{H,t+1} \right] + \mu_{i,t} i_t \Bigg\}.
\end{aligned} \tag{D.4}$$

Here $\lambda_{K,t}$ and $\lambda_{ZH,t}$ are the multipliers on the recursions in Eq. (D.2), and $\lambda_{h,t}$ is that on the producer reset-price condition in Eq. (B.12). The multipliers $\lambda_{p,t}$, $\lambda_{\Delta,t}$, $\lambda_{n,t}$, $\lambda_{y,t}$, $\lambda_{r,t}$, $\lambda_{e,t}$, and $\lambda_{s,t}$ are those on the price-index, price-dispersion, production, goods-market, risk-sharing, Euler, and relative-price conditions, and $\lambda_{M,t}$ is that on the importer reset-price condition in Eq. (D.3). All equations are nonlinear; the second-order welfare criterion is not the planner's objective.

D.2 Two-Country Economy

We now write the two national problems explicitly. Complete international risk sharing, Eq. (C.6) under PPT and Eq. (C.11) under IPT, is imposed before either national planner differentiates. It is used to eliminate Z_t from the national differentiation problems, with the substitution understood wherever Z_t appears in the expressions below. No separate starred international-relative-price variable is introduced. In this subsection, a superscript H or F on a multiplier identifies the national planner.

D.2.1 PPT Environment

In the PPT environment, after Eq. (C.6) has been substituted, the Home CPI–PPI relation is

$$\left(\mathcal{G}^{PPT} \left(\tau_{M,t}, Z_t/\Psi_t \right) \right)^{1-\eta} \;=\; \omega_H + \omega_F (1+\tau_{M,t})^{1-\eta} \left[\frac{C_t}{C_t^*} \frac{\mathcal{G}^{PPT} \left(\tau_{M,t}, Z_t/\Psi_t \right)}{P_t^{*,PPT}/P_{F,t}^*} \right]^{1-\eta},$$

and Home goods-market clearing is

$$Y_t \;=\; \omega_H \left(\mathcal{G}^{PPT} \left(\tau_{M,t}, Z_t/\Psi_t \right) \right)^{\eta} C_t$$

$$+\frac{1-\nu}{\nu}\omega_H^*\left[\frac{C_t\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)}{C_t^*(1+\tau_{M,t}^*)}\right]^{\eta}C_t^*.$$

The Foreign counterparts are

$$\begin{aligned}\left(\frac{P_t^{*,PPT}}{P_{F,t}^*}\right)^{1-\eta} &= \omega_F^*+\omega_H^*(1+\tau_{M,t}^*)^{1-\eta}\left[\frac{C_t^*}{C_t}\frac{P_t^{*,PPT}/P_{F,t}^*}{\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)}\right]^{1-\eta},\\ Y_t^* &= \omega_F^*\left(\frac{P_t^{*,PPT}}{P_{F,t}^*}\right)^{\eta}C_t^*\\ &\quad+\frac{\nu}{1-\nu}\omega_F\left[\frac{C_t^*(P_t^{*,PPT}/P_{F,t}^*)}{C_t(1+\tau_{M,t})}\right]^{\eta}C_t.\end{aligned}$$

Because the Home planner's constraints include the price-index definitions of both countries (Sec. 4.2), both CPI–PPI relations above enter the Home problem, and $P_t^{*,PPT}/P_{F,t}^*$ is a coordinate of that problem. After the Home reset-price relation has been substituted, the Home national Lagrangian is

$$\begin{aligned}\mathcal{L}_H^{PPT}=\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\Bigg\{&\log C_t-\frac{N_t^{1+\varphi}}{1+\varphi}\\ &+\lambda_{g,t}^H\left[\left(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)\right)^{1-\eta}-\omega_H-\omega_F(1+\tau_{M,t})^{1-\eta}\left[\frac{C_t}{C_t^*}\frac{\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)}{P_t^{*,PPT}/P_{F,t}^*}\right]^{1-\eta}\right]\\ &+\lambda_{gF,t}^H\Bigg[\left(\frac{P_t^{*,PPT}}{P_{F,t}^*}\right)^{1-\eta}-\omega_F^*\\ &\quad-\omega_H^*(1+\tau_{M,t}^*)^{1-\eta}\left[\frac{C_t^*}{C_t}\frac{P_t^{*,PPT}/P_{F,t}^*}{\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)}\right]^{1-\eta}\Bigg]\\ &+\lambda_{y,t}^H\left[Y_t-\omega_H\left(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)\right)^{\eta}C_t-\frac{1-\nu}{\nu}\omega_H^*\left[\frac{C_t\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)}{C_t^*(1+\tau_{M,t}^*)}\right]^{\eta}C_t^*\right]\\ &+\lambda_{n,t}^H\left[N_t-Y_t\Delta_{H,t}\right]\\ &+\lambda_{p,t}^H\left[1-\theta\Pi_{H,t}^{\varepsilon-1}-(1-\theta)\widetilde{p}_{H,t}^{1-\varepsilon}\right]\\ &+\lambda_{\Delta,t}^H\left[\Delta_{H,t}-\theta\Pi_{H,t}^{\varepsilon}\Delta_{H,t-1}-(1-\theta)\widetilde{p}_{H,t}^{-\varepsilon}\right]\\ &+\lambda_{e,t}^H\left[\frac{1}{1+i_t}-\beta\mathrm{E}_t\frac{C_t\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)}{C_{t+1}\mathcal{G}^{PPT}\left(\tau_{M,t+1},Z_{t+1}/\Psi_{t+1}\right)\Pi_{H,t+1}}\right]+\mu_{i,t}^H i_t\Bigg\}.\end{aligned}$$

The multipliers $\lambda_{gF,t}^H$ and $\lambda_{gH,t}^F$ are those of the Home planner on the Foreign CPI–PPI identity and of the Foreign planner on the Home CPI–PPI identity.

Symmetrically, both CPI–PPI relations enter the Foreign problem, and $\mathcal{G}^{PPT}(\tau_{M,t},Z_t/\Psi_t)$ is a coordinate of that problem. The Foreign national Lagrangian is

$$\begin{aligned}\mathcal{L}_F^{PPT} &= \mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\Bigg\{\log C_t^*-\frac{(N_t^*)^{1+\varphi}}{1+\varphi}\\ &\quad+\lambda_{g,t}^F\left[\left(\frac{P_t^{*,PPT}}{P_{F,t}^*}\right)^{1-\eta}-\omega_F^*-\omega_H^*(1+\tau_{M,t}^*)^{1-\eta}\left[\frac{C_t^*}{C_t}\frac{P_t^{*,PPT}/P_{F,t}^*}{\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)}\right]^{1-\eta}\right]\\ &\quad+\lambda_{gH,t}^F\Bigg[\left(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)\right)^{1-\eta}-\omega_H\\ &\qquad-\omega_F(1+\tau_{M,t})^{1-\eta}\left[\frac{C_t}{C_t^*}\frac{\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)}{P_t^{*,PPT}/P_{F,t}^*}\right]^{1-\eta}\Bigg]\end{aligned}$$

$$
\begin{aligned}
&+\lambda^F_{y,t}\left[Y^*_t-\omega^*_F\left(\frac{P^{*,PPT}_t}{P^*_{F,t}}\right)^{\eta}C^*_t-\frac{\nu}{1-\nu}\omega_F\left[\frac{C^*_t(P^{*,PPT}_t/P^*_{F,t})}{C_t(1+\tau_{M,t})}\right]^{\eta}C_t\right]\\
&+\lambda^F_{n,t}\left[N^*_t-Y^*_t\Delta^*_{F,t}\right]\\
&+\lambda^F_{p,t}\left[1-\theta(\Pi^*_{F,t})^{\varepsilon-1}-(1-\theta)(\widetilde{p}^*_{F,t})^{1-\varepsilon}\right]\\
&+\lambda^F_{\Delta,t}\left[\Delta^*_{F,t}-\theta(\Pi^*_{F,t})^{\varepsilon}\Delta^*_{F,t-1}-(1-\theta)(\widetilde{p}^*_{F,t})^{-\varepsilon}\right]\\
&+\lambda^F_{e,t}\left[\frac{1}{1+i^*_t}-\beta\mathrm{E}_t\frac{C^*_t(P^{*,PPT}_t/P^*_{F,t})}{C^*_{t+1}(P^{*,PPT}_{t+1}/P^*_{F,t+1})\Pi^*_{F,t+1}}\right]+\mu^F_{i,t}i^*_t\Bigg\}.
\end{aligned}
$$

D.2.2 IPT Environment

The two national commitment problems use the nonlinear two-country private equilibrium in Appendix C.2. Because the Home planner's constraints include the price-index definitions of both countries, and the planner takes the *ex-tariff* prices set by Foreign importers as given (Sec. 4.2), the Foreign CPI in Eq. (C.9), the Foreign *cum-tariff* prices $\widetilde{P}^*_{H,t}(\iota)/P^*_{F,t}=P^*_{H,t}(\iota)/P^*_{F,t}+\tau^*_{M,t}/Z_t$, and their index $\widetilde{P}^*_{H,t}$ hold in the Home problem, and $\mathcal{G}^{*,IPT}_t(\cdot)$ is a coordinate of that problem. The producer and importer reset-price conditions in Eqs. (B.12) and (16) are represented by their implied reset-price maps in the Lagrangian below.

The Home planner solves

$$
\max_{\{C_t,\mathcal{G}^{IPT}_t(\cdot),\mathcal{G}^{*,IPT}_t(\cdot),N_t,Y_t,\Pi_{H,t},\Delta_{H,t},P_{F,t}/P_{H,t},\Pi_{F,t},i_t\}_{t\geq 0}}\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\left(\log C_t-\frac{N_t^{1+\varphi}}{1+\varphi}\right), \tag{D.5}
$$

subject to the Home household Euler equation, the complete-markets condition in Eq. (C.11), the Home producer technology and Calvo block, the exact additive-duty importer block in Eqs. (2)–(4), (12), and (14)–(16), the local import-price law in Eq. (13), the Home resource constraint in Eq. (C.16), the Home and Foreign CPIs in Eq. (C.9), and

$$
i_t\geq 0. \tag{D.6}
$$

The notation $\mathcal{G}^{IPT}_t(\cdot)$ in the planner's choice set stands for the CPI–PPI ratio constrained by the exact purchaser-price index in Eqs. (2), (4), and (C.9), not the PPT formula $\mathcal{G}^{PPT}(\tau_{M,t},Z_t/\Psi_t)$. The complete cross-sectional distributions $\{P_{F,t}(\iota)\}$ and $\{P^*_{H,t}(\iota)\}$ entering the purchaser-price indices and physical-quantity integrals evolve according to the same Calvo reset/non-reset rule as in Appendix C.2. They are therefore retained inside the exact private-equilibrium constraints and are not replaced by a separate importer-dispersion state.

For later differentiation it is useful to display the Home Lagrangian after both reset-price relations have been substituted. Using the same multiplier notation as in Appendix D.2.1, and adding $\lambda^H_{Mp,t}$ for the Home importer price-index law and $\lambda^H_{q,t}$ for the local relative import-price law, we have

$$
\begin{aligned}
\mathcal{L}^{IPT}_H \;=\; &\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\Bigg\{\log C_t-\frac{N_t^{1+\varphi}}{1+\varphi}\\
&+\lambda^H_{g,t}\left[\mathcal{G}^{IPT}_t(\cdot)^{1-\eta}-\omega_H-\omega_F\left(\frac{\widetilde{P}^{IPT}_{F,t}}{P_{H,t}}\right)^{1-\eta}\right]\\
&+\lambda^H_{gF,t}\left[\mathcal{G}^{*,IPT}_t(\cdot)^{1-\eta}-\omega^*_F-\omega^*_H\left(\frac{\widetilde{P}^*_{H,t}}{P^*_{F,t}}\right)^{1-\eta}\right]\\
&+\lambda^H_{y,t}\Bigg[Y_t-\omega_H\mathcal{G}^{IPT}_t(\cdot)^{\eta}C_t
\end{aligned}
$$

$$
\begin{aligned}
&\quad -\frac{1-\nu}{\nu}\omega_H^*\left(\frac{\widetilde{P}_{H,t}^*}{P_t^{*,IPT}}\right)^{-\eta} C_t^*\frac{1}{1-\nu}\int_\nu^1\left(\frac{\widetilde{P}_{H,t}^*(\iota)}{\widetilde{P}_{H,t}^*}\right)^{-\varepsilon_M} d\iota\Bigg] \\
&+\lambda_{n,t}^H[N_t - Y_t\Delta_{H,t}] \\
&+\lambda_{p,t}^H\left[1-\theta\Pi_{H,t}^{\varepsilon-1}-(1-\theta)\widetilde{p}_{H,t}^{1-\varepsilon}\right] \\
&+\lambda_{\Delta,t}^H\left[\Delta_{H,t}-\theta\Pi_{H,t}^{\varepsilon}\Delta_{H,t-1}-(1-\theta)\widetilde{p}_{H,t}^{-\varepsilon}\right] \\
&+\lambda_{Mp,t}^H\left[1-\theta_M\Pi_{F,t}^{\varepsilon_M-1}-(1-\theta_M)\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)^{1-\varepsilon_M}\right] \\
&+\lambda_{q,t}^H\left[\frac{P_{F,t}}{P_{H,t}}-\frac{P_{F,t-1}}{P_{H,t-1}}\frac{\Pi_{F,t}}{\Pi_{H,t}}\right] \\
&+\lambda_{e,t}^H\left[\frac{1}{1+i_t}-\beta\mathrm{E}_t\frac{C_t\mathcal{G}_t^{IPT}(\cdot)}{C_{t+1}\mathcal{G}_{t+1}^{IPT}(\cdot)\Pi_{H,t+1}}\right]+\mu_{i,t}^H i_t\Bigg\}. \qquad \text{(D.7)}
\end{aligned}
$$

Here $\mathcal{G}_t^{*,IPT}(\cdot)$ also enters through Z_t. The Home and Foreign purchaser-price terms are differentiated after substituting risk sharing. In the Home export-demand term and the Foreign CPI, C_t^* and the Foreign *ex-tariff* prices are held fixed, while the Foreign *cum-tariff* prices and their index change with Z_t. The ratio $\overline{P}_{F,t}/P_{F,t}$ is determined by the exact private importer FOC (16).

Symmetrically, the Foreign planner solves the exact starred problem

$$
\max_{\{C_t^*,\mathcal{G}_t^{*,IPT}(\cdot),\mathcal{G}_t^{IPT}(\cdot),N_t^*,Y_t^*,\Pi_{F,t}^*,\Delta_{F,t}^*,P_{H,t}^*/P_{F,t}^*,\Pi_{H,t}^*,i_t^*\}_{t\geq 0}} \mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\left(\log C_t^* - \frac{(N_t^*)^{1+\varphi}}{1+\varphi}\right), \qquad \text{(D.8)}
$$

subject to the starred counterparts of the Home constraints and $i_t^* \geq 0$. Its Lagrangian is the exact Home–Foreign mirror of Eq. (D.7). The two national commitment problems and the common private equilibrium are solved jointly to satisfy the mutual best-response conditions.

E Social Planner's First Order Conditions

E.1 Small Open Economy

E.1.1 PPT Environment

Following Monacelli [6] (pp. 74–75), we use the following notation for derivatives of the producer reset price and its auxiliary sequences:

$$
\begin{aligned}
\mathcal{K}'_{H,t-j}(x_t) &\equiv \frac{\partial\mathcal{K}_{H,t-j}}{\partial x_t}, \qquad \mathcal{Z}'_{H,t-j}(x_t)\equiv\frac{\partial\mathcal{Z}_{H,t-j}}{\partial x_t}, \\
\widetilde{p}'_{H,t-j}(x_t) &\equiv \frac{\partial\widetilde{p}_{H,t-j}}{\partial x_t} = \mathcal{M}\frac{\mathcal{Z}'_{H,t-j}(x_t)\mathcal{K}_{H,t-j}-\mathcal{K}'_{H,t-j}(x_t)\mathcal{Z}_{H,t-j}}{(\mathcal{K}_{H,t-j})^2}. \qquad \text{(E.1)}
\end{aligned}
$$

The producer reset price $\widetilde{p}_{H,t-j}$ is defined in Eqs. (B.12)–(B.13). The explicit formulas for this derivative are given in Eqs. (E.18)–(E.22), with $\mathcal{G}$ evaluated at the relevant price index. In the SOE–PPT Z_t condition, differentiation of $\mathcal{G}^{PPT}(\tau_{M,t},Z_t)$ additionally uses the chain rule.

The first-order condition for C_t is

$$
\begin{aligned}
0 &= \frac{1}{C_t}-(1-\theta)(1-\varepsilon)\left[\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(C_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(C_t)\right] \\
&\quad +(1-\theta)\varepsilon\left[\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(C_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(C_t)\right]
\end{aligned}
$$

$$
\begin{aligned}
&-(1-\upsilon)\lambda_{y,t}\mathcal{G}^{PPT}(\tau_{M,t},Z_t)^{\eta}-\lambda_{r,t}\mathcal{G}^{PPT}(\tau_{M,t},Z_t)\\
&-\beta\lambda_{e,t}\mathrm{E}_t\left[\frac{\mathcal{G}^{PPT}(\tau_{M,t},Z_t)}{C_{t+1}\mathcal{G}^{PPT}(\tau_{M,t+1},Z_{t+1})\Pi_{H,t+1}}\right]\\
&+\lambda_{e,t-1}\frac{C_{t-1}\mathcal{G}^{PPT}(\tau_{M,t-1},Z_{t-1})}{C_t^2\mathcal{G}^{PPT}(\tau_{M,t},Z_t)\Pi_{H,t}}.
\end{aligned}
$$

The first-order condition for N_t is

$$
\begin{aligned}
0 \;=\; & -N_t^{\varphi}+\lambda_{n,t}\\
&-(1-\theta)(1-\varepsilon)\left[\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(N_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(N_t)\right]\\
&+(1-\theta)\varepsilon\left[\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(N_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(N_t)\right].
\end{aligned}
$$

The first-order condition for Y_t is

$$
\begin{aligned}
0 \;=\; & -\lambda_{n,t}\Delta_{H,t}+\lambda_{y,t}\\
&-(1-\theta)(1-\varepsilon)\left[\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(Y_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(Y_t)\right]\\
&+(1-\theta)\varepsilon\left[\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(Y_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(Y_t)\right].
\end{aligned}
$$

The first-order condition for Z_t is

$$
\begin{aligned}
0 \;=\; & -(1-\theta)(1-\varepsilon)\left[\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(Z_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(Z_t)\right]\\
&+(1-\theta)\varepsilon\left[\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(Z_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(Z_t)\right]\\
&-\lambda_{y,t}\left[(1-\upsilon)\eta\mathcal{G}^{PPT}(\tau_{M,t},Z_t)^{\eta-1}\frac{\partial\mathcal{G}^{PPT}(\tau_{M,t},Z_t)}{\partial Z_t}C_t+\upsilon\eta Z_t^{\eta-1}\right]\\
&+\lambda_{r,t}\left[1-C_t\frac{\partial\mathcal{G}^{PPT}(\tau_{M,t},Z_t)}{\partial Z_t}\right]\\
&-\beta\lambda_{e,t}\mathrm{E}_t\left[\frac{C_t\,\partial\mathcal{G}^{PPT}(\tau_{M,t},Z_t)/\partial Z_t}{C_{t+1}\mathcal{G}^{PPT}(\tau_{M,t+1},Z_{t+1})\Pi_{H,t+1}}\right]\\
&+\lambda_{e,t-1}\frac{C_{t-1}\mathcal{G}^{PPT}(\tau_{M,t-1},Z_{t-1})}{C_t\mathcal{G}^{PPT}(\tau_{M,t},Z_t)^2\Pi_{H,t}}\frac{\partial\mathcal{G}^{PPT}(\tau_{M,t},Z_t)}{\partial Z_t}.
\end{aligned}
$$

The first-order condition for $\Pi_{H,t}$ is

$$
\begin{aligned}
0 \;=\; & -\theta(\varepsilon-1)\lambda_{p,t}\Pi_{H,t}^{\varepsilon-2}-\theta\varepsilon\lambda_{\Delta,t}\Pi_{H,t}^{\varepsilon-1}\Delta_{H,t-1}\\
&-(1-\theta)(1-\varepsilon)\left[\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(\Pi_{H,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(\Pi_{H,t})\right]\\
&+(1-\theta)\varepsilon\left[\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(\Pi_{H,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(\Pi_{H,t})\right]
\end{aligned}
$$

$$+\lambda_{e,t-1}\frac{C_{t-1}\mathcal{G}^{PPT}(\tau_{M,t-1},Z_{t-1})}{C_t\mathcal{G}^{PPT}(\tau_{M,t},Z_t)\Pi_{H,t}^2}.$$

The first-order condition for $\Delta_{H,t}$ is

$$0=\lambda_{\Delta,t}-\lambda_{n,t}Y_t-\beta\theta\mathrm{E}_t[\lambda_{\Delta,t+1}\Pi_{H,t+1}^{\varepsilon}].$$

Finally, the interest-rate condition and the ZLB conditions are

$$-\frac{\lambda_{e,t}}{(1+i_t)^2}+\mu_{i,t}=0,\qquad i_t\geq 0,\qquad \mu_{i,t}\geq 0,\qquad \mu_{i,t}i_t=0.$$

E.1.2 IPT Environment

This subsection gives the stationarity conditions of Eq. (D.4) with respect to *every* independent choice path, retaining the existing producer sequences and reset price as explicit choice coordinates. All restrictions in Eq. (D.4) are imposed exactly. In the importer terms, $Q_{s,t}$ and $C_{F,t|s}$ are the original objects in Eqs. (16) and (15); all occurrences of their derivatives have been evaluated explicitly. Terms proportional to $\mathcal{I}_t$ drop out because Eq. (16) sets $\mathcal{I}_t=0$.

The consumption, labor, output, and producer-price-dispersion FOCs are

$$\begin{aligned}0=&\frac{1}{C_t}-(1-\upsilon)\lambda_{y,t}(\mathcal{G}_t^{IPT})^{\eta}-\lambda_{r,t}\mathcal{G}_t^{IPT}-\beta\lambda_{e,t}\mathrm{E}_t\frac{\mathcal{G}_t^{IPT}}{C_{t+1}\mathcal{G}_{t+1}^{IPT}\Pi_{H,t+1}}+\lambda_{e,t-1}\frac{C_{t-1}\mathcal{G}_{t-1}^{IPT}}{C_t^2\mathcal{G}_t^{IPT}\Pi_{H,t}}\\&-\beta\theta\lambda_{K,t}\mathrm{E}_t\left[\frac{\mathcal{G}_t^{IPT}}{C_{t+1}\mathcal{G}_{t+1}^{IPT}}\Pi_{H,t+1}^{\varepsilon-1}\mathcal{K}_{H,t+1}\right]+\theta\lambda_{K,t-1}\frac{C_{t-1}\mathcal{G}_{t-1}^{IPT}}{C_t^2\mathcal{G}_t^{IPT}}\Pi_{H,t}^{\varepsilon-1}\mathcal{K}_{H,t}\\&-\lambda_{ZH,t}\mathcal{G}_t^{IPT}N_t^{\varphi}Y_t-\beta\theta\lambda_{ZH,t}\mathrm{E}_t\left[\frac{\mathcal{G}_t^{IPT}}{C_{t+1}\mathcal{G}_{t+1}^{IPT}}\Pi_{H,t+1}^{\varepsilon}\mathcal{Z}_{H,t+1}\right]+\theta\lambda_{ZH,t-1}\frac{C_{t-1}\mathcal{G}_{t-1}^{IPT}}{C_t^2\mathcal{G}_t^{IPT}}\Pi_{H,t}^{\varepsilon}\mathcal{Z}_{H,t},\end{aligned}\tag{E.2}$$

$$0=-N_t^{\varphi}+\lambda_{n,t}-\varphi\lambda_{ZH,t}C_t\mathcal{G}_t^{IPT}N_t^{\varphi-1}Y_t,\tag{E.3}$$

$$0=-\lambda_{n,t}\Delta_{H,t}+\lambda_{y,t}-\lambda_{K,t}-\lambda_{ZH,t}C_t\mathcal{G}_t^{IPT}N_t^{\varphi},\tag{E.4}$$

$$0=\lambda_{\Delta,t}-\lambda_{n,t}Y_t-\beta\theta\mathrm{E}_t[\lambda_{\Delta,t+1}\Pi_{H,t+1}^{\varepsilon}].\tag{E.5}$$

The CPI–PPI ratio $\mathcal{G}_t^{IPT}$ has the explicit FOC

$$\begin{aligned}0=&-(1-\upsilon)\eta\lambda_{y,t}(\mathcal{G}_t^{IPT})^{\eta-1}C_t-\lambda_{r,t}C_t+(1-\eta)\lambda_{g,t}(\mathcal{G}_t^{IPT})^{-\eta}-\beta\lambda_{e,t}\mathrm{E}_t\frac{C_t}{C_{t+1}\mathcal{G}_{t+1}^{IPT}\Pi_{H,t+1}}+\lambda_{e,t-1}\frac{C_{t-1}\mathcal{G}_{t-1}^{IPT}}{C_t(\mathcal{G}_t^{IPT})^2\Pi_{H,t}}\\&-\beta\theta\lambda_{K,t}\mathrm{E}_t\left[\frac{C_t}{C_{t+1}\mathcal{G}_{t+1}^{IPT}}\Pi_{H,t+1}^{\varepsilon-1}\mathcal{K}_{H,t+1}\right]+\theta\lambda_{K,t-1}\frac{C_{t-1}\mathcal{G}_{t-1}^{IPT}}{C_t(\mathcal{G}_t^{IPT})^2}\Pi_{H,t}^{\varepsilon-1}\mathcal{K}_{H,t}\\&-\lambda_{ZH,t}C_tN_t^{\varphi}Y_t-\beta\theta\lambda_{ZH,t}\mathrm{E}_t\left[\frac{C_t}{C_{t+1}\mathcal{G}_{t+1}^{IPT}}\Pi_{H,t+1}^{\varepsilon}\mathcal{Z}_{H,t+1}\right]+\theta\lambda_{ZH,t-1}\frac{C_{t-1}\mathcal{G}_{t-1}^{IPT}}{C_t(\mathcal{G}_t^{IPT})^2}\Pi_{H,t}^{\varepsilon}\mathcal{Z}_{H,t}\\&+\frac{\eta-1}{\mathcal{G}_t^{IPT}}\sum_{s=0}^{t}\beta^{s-t}\lambda_{M,s}\theta_M^{t-s}Q_{s,t}C_{F,t|s}\left[1-\varepsilon_M\frac{\overline{P}_{F,s}-(1-\tau)Z_tP_{H,t}}{\overline{P}_{F,s}+\tau_{M,t}Z_tP_{H,t}}\right].\end{aligned}\tag{E.6}$$

The FOCs for the producer auxiliary sequences $\mathcal{K}_{H,t}$ and $\mathcal{Z}_{H,t}$, and the reset price $\widetilde{p}_{H,t}$, are

$$0=\lambda_{K,t}+\lambda_{h,t}\mathcal{M}\frac{\mathcal{Z}_{H,t}}{\mathcal{K}_{H,t}^2}-\theta\lambda_{K,t-1}\frac{C_{t-1}\mathcal{G}_{t-1}^{IPT}}{C_t\mathcal{G}_t^{IPT}}\Pi_{H,t}^{\varepsilon-1},\tag{E.7}$$

$$0=\lambda_{ZH,t}-\lambda_{h,t}\frac{\mathcal{M}}{\mathcal{K}_{H,t}}-\theta\lambda_{ZH,t-1}\frac{C_{t-1}\mathcal{G}_{t-1}^{IPT}}{C_t\mathcal{G}_t^{IPT}}\Pi_{H,t}^{\varepsilon},\tag{E.8}$$

$$0=-(1-\theta)(1-\varepsilon)\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}+(1-\theta)\varepsilon\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}+\lambda_{h,t}.\tag{E.9}$$

The FOC for Home producer-price inflation $\Pi_{H,t}$ is

$$\begin{aligned}0=&-\theta(\varepsilon-1)\lambda_{p,t}\Pi_{H,t}^{\varepsilon-2}-\theta\varepsilon\lambda_{\Delta,t}\Pi_{H,t}^{\varepsilon-1}\Delta_{H,t-1}+\lambda_{s,t}\frac{Z_{t-1}}{\Psi_{t-1}}\frac{\Pi_{F,t}}{\Pi_{H,t}^2}+\lambda_{e,t-1}\frac{C_{t-1}\mathcal{G}_{t-1}^{IPT}}{C_t\mathcal{G}_t^{IPT}\Pi_{H,t}^2}-\lambda_{PH,t}P_{H,t-1}\\&-\theta(\varepsilon-1)\lambda_{K,t-1}\frac{C_{t-1}\mathcal{G}_{t-1}^{IPT}}{C_t\mathcal{G}_t^{IPT}}\Pi_{H,t}^{\varepsilon-2}\mathcal{K}_{H,t}-\theta\varepsilon\lambda_{ZH,t-1}\frac{C_{t-1}\mathcal{G}_{t-1}^{IPT}}{C_t\mathcal{G}_t^{IPT}}\Pi_{H,t}^{\varepsilon-1}\mathcal{Z}_{H,t}.\end{aligned}\tag{E.10}$$

For the producer price level $P_{H,t}$, the FOC is

$$0 = \lambda_{PH,t} - \beta \mathrm{E}_t[\lambda_{PH,t+1}\Pi_{H,t+1}] + \frac{\upsilon(1-\eta)\lambda_{g,t}}{P_{H,t}}\left(\frac{\widetilde{P}^{IPT}_{F,t}}{P_{H,t}}\right)^{1-\eta} + \frac{(1-\theta_M)(1-\varepsilon_M)\lambda_{pF,t}}{P_{H,t}}\left(\frac{\overline{P}_{F,t}\Psi_t}{Z_t P_{H,t}}\right)^{1-\varepsilon_M}$$
$$+\frac{(1-\varepsilon_M)\lambda_{I,t}}{P_{H,t}}\left[-\left(\frac{\widetilde{P}^{IPT}_{F,t}}{P_{H,t}}\right)^{1-\varepsilon_M} + (1-\theta_M)\sum_{j=0}^{\infty}\theta_M^j \frac{\overline{P}_{F,t-j}}{P_{H,t}}\left(\frac{\overline{P}_{F,t-j}}{P_{H,t}} + \tau_{M,t}Z_t\right)^{-\varepsilon_M}\right]$$
$$+\sum_{s=0}^{t}\beta^{s-t}\lambda_{M,s}\theta_M^{t-s}Q_{s,t}C_{F,t|s}\left\{\left[\frac{\eta-1}{P_{H,t}} - \frac{\varepsilon_M\tau_{M,t}Z_t}{\overline{P}_{F,s}+\tau_{M,t}Z_tP_{H,t}}\right]\left[1-\varepsilon_M\frac{\overline{P}_{F,s}-(1-\tau)Z_tP_{H,t}}{\overline{P}_{F,s}+\tau_{M,t}Z_tP_{H,t}}\right]\right.$$
$$\left.+\frac{\varepsilon_M(1-\tau+\tau_{M,t})\overline{P}_{F,s}Z_t}{(\overline{P}_{F,s}+\tau_{M,t}Z_tP_{H,t})^2}\right\}. \tag{E.11}$$

The relative border-price and LOOP-gap FOCs are

$$0 = -\upsilon\eta\lambda_{y,t}Z_t^{\eta-1} + \lambda_{r,t} + \frac{\lambda_{s,t}}{\Psi_t} - \beta\mathrm{E}_t\left[\frac{\lambda_{s,t+1}\Pi_{F,t+1}}{\Psi_t\Pi_{H,t+1}}\right] + \frac{(1-\theta_M)(1-\varepsilon_M)\lambda_{pF,t}}{Z_t}\left(\frac{\overline{P}_{F,t}\Psi_t}{Z_tP_{H,t}}\right)^{1-\varepsilon_M}$$
$$-(1-\theta_M)(1-\varepsilon_M)\lambda_{I,t}\tau_{M,t}\sum_{j=0}^{\infty}\theta_M^j\left(\frac{\overline{P}_{F,t-j}}{P_{H,t}}+\tau_{M,t}Z_t\right)^{-\varepsilon_M}$$
$$+\sum_{s=0}^{t}\beta^{s-t}\lambda_{M,s}\theta_M^{t-s}Q_{s,t}C_{F,t|s}\left\{-\frac{\varepsilon_M\tau_{M,t}P_{H,t}}{\overline{P}_{F,s}+\tau_{M,t}Z_tP_{H,t}}\left[1-\varepsilon_M\frac{\overline{P}_{F,s}-(1-\tau)Z_tP_{H,t}}{\overline{P}_{F,s}+\tau_{M,t}Z_tP_{H,t}}\right]\right.$$
$$\left.+\frac{\varepsilon_M(1-\tau+\tau_{M,t})\overline{P}_{F,s}P_{H,t}}{(\overline{P}_{F,s}+\tau_{M,t}Z_tP_{H,t})^2}\right\}, \tag{E.12}$$

$$0 = -\lambda_{s,t}\frac{Z_t}{\Psi_t^2} + \beta\mathrm{E}_t\left[\lambda_{s,t+1}\frac{Z_t}{\Psi_t^2}\frac{\Pi_{F,t+1}}{\Pi_{H,t+1}}\right] - \frac{(1-\theta_M)(1-\varepsilon_M)\lambda_{pF,t}}{\Psi_t}\left(\frac{\overline{P}_{F,t}\Psi_t}{Z_tP_{H,t}}\right)^{1-\varepsilon_M}. \tag{E.13}$$

The ex-tariff importer-inflation, importer reset-price, and cum-tariff purchaser-price-index FOCs are

$$0 = -\theta_M(\varepsilon_M-1)\lambda_{pF,t}\Pi_{F,t}^{\varepsilon_M-2} - \lambda_{s,t}\frac{Z_{t-1}/\Psi_{t-1}}{\Pi_{H,t}}, \tag{E.14}$$

$$0 = -\frac{(1-\theta_M)(1-\varepsilon_M)\lambda_{pF,t}}{\overline{P}_{F,t}}\left(\frac{\overline{P}_{F,t}\Psi_t}{Z_tP_{H,t}}\right)^{1-\varepsilon_M} - (1-\theta_M)(1-\varepsilon_M)\sum_{s=t}^{\infty}\beta^{s-t}\mathrm{E}_t\left[\lambda_{I,s}\theta_M^{s-t}\frac{(\overline{P}_{F,t}/P_{H,s}+\tau_{M,s}Z_s)^{-\varepsilon_M}}{P_{H,s}}\right]$$
$$+\lambda_{M,t}\mathrm{E}_t\sum_{k=0}^{\infty}\theta_M^kQ_{t,t+k}C_{F,t+k|t}\left\{-\frac{\varepsilon_M}{\overline{P}_{F,t}+\tau_{M,t+k}Z_{t+k}P_{H,t+k}}\left[1-\varepsilon_M\frac{\overline{P}_{F,t}-(1-\tau)Z_{t+k}P_{H,t+k}}{\overline{P}_{F,t}+\tau_{M,t+k}Z_{t+k}P_{H,t+k}}\right]\right.$$
$$\left.-\frac{\varepsilon_M(1-\tau+\tau_{M,t+k})Z_{t+k}P_{H,t+k}}{(\overline{P}_{F,t}+\tau_{M,t+k}Z_{t+k}P_{H,t+k})^2}\right\}, \tag{E.15}$$

$$0 = -\frac{\upsilon(1-\eta)\lambda_{g,t}}{P_{H,t}}\left(\frac{\widetilde{P}^{IPT}_{F,t}}{P_{H,t}}\right)^{-\eta} + \frac{(1-\varepsilon_M)\lambda_{I,t}}{P_{H,t}}\left(\frac{\widetilde{P}^{IPT}_{F,t}}{P_{H,t}}\right)^{-\varepsilon_M}$$
$$+\frac{\varepsilon_M-\eta}{\widetilde{P}^{IPT}_{F,t}}\sum_{s=0}^{t}\beta^{s-t}\lambda_{M,s}\theta_M^{t-s}Q_{s,t}C_{F,t|s}\left[1-\varepsilon_M\frac{\overline{P}_{F,s}-(1-\tau)Z_tP_{H,t}}{\overline{P}_{F,s}+\tau_{M,t}Z_tP_{H,t}}\right]. \tag{E.16}$$

The nominal-rate condition, together with the exact ZLB complementarity conditions, is

$$0 = -\frac{\lambda_{e,t}}{(1+i_t)^2} + \mu_{i,t}, \qquad i_t \geq 0, \quad \mu_{i,t} \geq 0, \quad \mu_{i,t}i_t = 0. \tag{E.17}$$

The restrictions in Appendix D.1.2, all the FOCs above, and the original terminal and timeless-perspective initial-multiplier conditions jointly characterize this exact SOE–IPT Ramsey problem.

E.2 Two-Country Economy

E.2.1 PPT Environment

For the Home producer's reset price, we use the derivative notation in Eq. (E.1). The derivative of the Foreign producer's reset price is its starred counterpart:

$$(\widetilde{p}^*_{F,t-j})'(x_t) = \mathcal{M}\frac{(\mathcal{Z}^*_{F,t-j})'(x_t)\mathcal{K}^*_{F,t-j} - (\mathcal{K}^*_{F,t-j})'(x_t)\mathcal{Z}^*_{F,t-j}}{(\mathcal{K}^*_{F,t-j})^2}.$$

The Home first-order condition for C_t is

$$
\begin{aligned}
0 \;=\; & \frac{1}{C_t} - (1-\theta)(1-\varepsilon)\left[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(C_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(C_t)\right] \\
& +(1-\theta)\varepsilon\left[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(C_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(C_t)\right] \\
& -\lambda^H_{g,t}\omega_F(1+\tau_{M,t})^{1-\eta}(1-\eta)\frac{1}{C_t}\left[\frac{C_t}{C^*_t}\frac{\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)}{P^{*,PPT}_t/P^*_{F,t}}\right]^{1-\eta} \\
& +\lambda^H_{gF,t}\omega^*_H(1+\tau^*_{M,t})^{1-\eta}(1-\eta)\frac{1}{C_t}\left[\frac{C^*_t}{C_t}\frac{P^{*,PPT}_t/P^*_{F,t}}{\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)}\right]^{1-\eta} \\
& -\lambda^H_{y,t}\omega_H\left(\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)\right)^{\eta} \\
& -\lambda^H_{y,t}\frac{1-\nu}{\nu}\omega^*_H\eta\frac{C^*_t}{C_t}\left[\frac{C_t\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)}{C^*_t(1+\tau^*_{M,t})}\right]^{\eta} \\
& -\beta\lambda^H_{e,t}\mathrm{E}_t\left[\frac{\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)}{C_{t+1}\mathcal{G}^{PPT}\left(\tau_{M,t+1}, Z_{t+1}/\Psi_{t+1}\right)\Pi_{H,t+1}}\right] \\
& +\lambda^H_{e,t-1}\frac{C_{t-1}\mathcal{G}^{PPT}\left(\tau_{M,t-1}, Z_{t-1}/\Psi_{t-1}\right)}{C_t^2\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)\Pi_{H,t}}.
\end{aligned}
$$

The Home first-order condition for $\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)$ is

$$
\begin{aligned}
0 \;=\; & -(1-\theta)(1-\varepsilon)\left[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right))\right] \\
& +(1-\theta)\varepsilon\left[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right))\right] \\
& +(1-\eta)\lambda^H_{g,t}\left(\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)\right)^{-\eta} \\
& -\lambda^H_{g,t}\omega_F(1+\tau_{M,t})^{1-\eta}(1-\eta)\left(\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)\right)^{-1}\left[\frac{C_t}{C^*_t}\frac{\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)}{P^{*,PPT}_t/P^*_{F,t}}\right]^{1-\eta} \\
& +\lambda^H_{gF,t}\omega^*_H(1+\tau^*_{M,t})^{1-\eta}(1-\eta)\left(\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)\right)^{-1}\left[\frac{C^*_t}{C_t}\frac{P^{*,PPT}_t/P^*_{F,t}}{\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)}\right]^{1-\eta} \\
& -\eta\lambda^H_{y,t}\omega_H\left(\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)\right)^{\eta-1}C_t \\
& -\eta\lambda^H_{y,t}\frac{1-\nu}{\nu}\omega^*_H\left(\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)\right)^{-1}C^*_t\left[\frac{C_t\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)}{C^*_t(1+\tau^*_{M,t})}\right]^{\eta} \\
& -\beta\lambda^H_{e,t}\mathrm{E}_t\left[\frac{C_t}{C_{t+1}\mathcal{G}^{PPT}\left(\tau_{M,t+1}, Z_{t+1}/\Psi_{t+1}\right)\Pi_{H,t+1}}\right] \\
& +\lambda^H_{e,t-1}\frac{C_{t-1}\mathcal{G}^{PPT}\left(\tau_{M,t-1}, Z_{t-1}/\Psi_{t-1}\right)}{C_t(\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right))^2\Pi_{H,t}}.
\end{aligned}
$$

The Home planner's FOC for the Foreign CPI–PPI ratio $P^{*,PPT}_t/P^*_{F,t}$ is

$$
\begin{aligned}
0 \;=\; & (1-\eta)\lambda^H_{g,t}\omega_F(1+\tau_{M,t})^{1-\eta}\left(\frac{P^{*,PPT}_t}{P^*_{F,t}}\right)^{-1}\left[\frac{C_t}{C^*_t}\frac{\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)}{P^{*,PPT}_t/P^*_{F,t}}\right]^{1-\eta} \\
& +(1-\eta)\lambda^H_{gF,t}\Bigg[\left(\frac{P^{*,PPT}_t}{P^*_{F,t}}\right)^{-\eta} \\
& \qquad -\omega^*_H(1+\tau^*_{M,t})^{1-\eta}\left(\frac{P^{*,PPT}_t}{P^*_{F,t}}\right)^{-1}\left[\frac{C^*_t}{C_t}\frac{P^{*,PPT}_t/P^*_{F,t}}{\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)}\right]^{1-\eta}\Bigg].
\end{aligned}
$$

The producer reset-price derivatives in these conditions are given by Eqs. (E.18)–(E.22), with $\mathcal{G}_t^{IPT}$ replaced by $\mathcal{G}^{PPT}(\tau_{M,t}, Z_t/\Psi_t)$.

The remaining Home conditions in the PPT environment are

$$
\begin{aligned}
0 \;=\; & -N_t^{\varphi} + \lambda_{n,t}^H - (1-\theta)(1-\varepsilon)\left[\lambda_{p,t}^H \widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(N_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}^H \widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(N_t)\right] \\
& +(1-\theta)\varepsilon\left[\lambda_{\Delta,t}^H \widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(N_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}^H \widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(N_t)\right], \\
0 \;=\; & -\lambda_{n,t}^H\Delta_{H,t} + \lambda_{y,t}^H - (1-\theta)(1-\varepsilon)\left[\lambda_{p,t}^H \widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(Y_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}^H \widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(Y_t)\right] \\
& +(1-\theta)\varepsilon\left[\lambda_{\Delta,t}^H \widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(Y_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}^H \widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(Y_t)\right], \\
0 \;=\; & -\theta(\varepsilon-1)\lambda_{p,t}^H\Pi_{H,t}^{\varepsilon-2} - \theta\varepsilon\lambda_{\Delta,t}^H\Pi_{H,t}^{\varepsilon-1}\Delta_{H,t-1} \\
& -(1-\theta)(1-\varepsilon)\left[\lambda_{p,t}^H \widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(\Pi_{H,t}) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}^H \widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(\Pi_{H,t})\right] \\
& +(1-\theta)\varepsilon\left[\lambda_{\Delta,t}^H \widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(\Pi_{H,t}) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}^H \widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(\Pi_{H,t})\right] \\
& +\lambda_{e,t-1}^H \frac{C_{t-1}\mathcal{G}^{PPT}\left(\tau_{M,t-1}, Z_{t-1}/\Psi_{t-1}\right)}{C_t\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)\Pi_{H,t}^2}, \\
0 \;=\; & \lambda_{\Delta,t}^H - \lambda_{n,t}^H Y_t - \beta\theta \mathrm{E}_t[\lambda_{\Delta,t+1}^H \Pi_{H,t+1}^{\varepsilon}], \\
0 \;=\; & -\frac{\lambda_{e,t}^H}{(1+i_t)^2} + \mu_{i,t}^H.
\end{aligned}
$$

The Home ZLB conditions are

$$
i_t \geq 0, \qquad \mu_{i,t}^H \geq 0, \qquad \mu_{i,t}^H i_t = 0.
$$

The Foreign planner's FOCs for C_t^*, $P_t^{*,PPT}/P_{F,t}^*$, and $\mathcal{G}^{PPT}(\tau_{M,t}, Z_t/\Psi_t)$ are

$$
\begin{aligned}
0 \;=\; & \frac{1}{C_t^*} - (1-\theta)(1-\varepsilon)\left[\lambda_{p,t}^F(\widetilde{p}_{F,t}^*)^{-\varepsilon}(\widetilde{p}_{F,t}^*)'(C_t^*) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}^F(\widetilde{p}_{F,t-j}^*)^{-\varepsilon}(\widetilde{p}_{F,t-j}^*)'(C_t^*)\right] \\
& +(1-\theta)\varepsilon\left[\lambda_{\Delta,t}^F(\widetilde{p}_{F,t}^*)^{-\varepsilon-1}(\widetilde{p}_{F,t}^*)'(C_t^*) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}^F(\widetilde{p}_{F,t-j}^*)^{-\varepsilon-1}(\widetilde{p}_{F,t-j}^*)'(C_t^*)\right] \\
& -\lambda_{g,t}^F\omega_H^*(1+\tau_{M,t}^*)^{1-\eta}(1-\eta)\frac{1}{C_t^*}\left[\frac{C_t^*}{C_t}\frac{P_t^{*,PPT}/P_{F,t}^*}{\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)}\right]^{1-\eta} \\
& +\lambda_{gH,t}^F\omega_F(1+\tau_{M,t})^{1-\eta}(1-\eta)\frac{1}{C_t^*}\left[\frac{C_t}{C_t^*}\frac{\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)}{P_t^{*,PPT}/P_{F,t}^*}\right]^{1-\eta} \\
& -\lambda_{y,t}^F\omega_F^*\left(\frac{P_t^{*,PPT}}{P_{F,t}^*}\right)^{\eta} \\
& -\lambda_{y,t}^F\frac{\nu}{1-\nu}\omega_F\eta\frac{C_t}{C_t^*}\left[\frac{C_t^*(P_t^{*,PPT}/P_{F,t}^*)}{C_t(1+\tau_{M,t})}\right]^{\eta} \\
& -\beta\lambda_{e,t}^F\mathrm{E}_t\left[\frac{P_t^{*,PPT}/P_{F,t}^*}{C_{t+1}^*(P_{t+1}^{*,PPT}/P_{F,t+1}^*)\Pi_{F,t+1}^*}\right] \\
& +\lambda_{e,t-1}^F\frac{C_{t-1}^*(P_{t-1}^{*,PPT}/P_{F,t-1}^*)}{(C_t^*)^2(P_t^{*,PPT}/P_{F,t}^*)\Pi_{F,t}^*},
\end{aligned}
$$

$$
\begin{aligned}
0 \;=\; & -(1-\theta)(1-\varepsilon)\left[\lambda^F_{p,t}(\widetilde{p}^*_{F,t})^{-\varepsilon}(\widetilde{p}^*_{F,t})'(P^{*,PPT}_t/P^*_{F,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{p,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon}(\widetilde{p}^*_{F,t-j})'(P^{*,PPT}_t/P^*_{F,t})\right]\\
& +(1-\theta)\varepsilon\left[\lambda^F_{\Delta,t}(\widetilde{p}^*_{F,t})^{-\varepsilon-1}(\widetilde{p}^*_{F,t})'(P^{*,PPT}_t/P^*_{F,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{\Delta,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon-1}(\widetilde{p}^*_{F,t-j})'(P^{*,PPT}_t/P^*_{F,t})\right]\\
& +(1-\eta)\lambda^F_{g,t}\left(\frac{P^{*,PPT}_t}{P^*_{F,t}}\right)^{-\eta}\\
& -\lambda^F_{g,t}\omega^*_H(1+\tau^*_{M,t})^{1-\eta}(1-\eta)\left(\frac{P^{*,PPT}_t}{P^*_{F,t}}\right)^{-1}\left[\frac{C^*_t}{C_t}\frac{P^{*,PPT}_t/P^*_{F,t}}{\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)}\right]^{1-\eta}\\
& +\lambda^F_{gH,t}\omega_F(1+\tau_{M,t})^{1-\eta}(1-\eta)\left(\frac{P^{*,PPT}_t}{P^*_{F,t}}\right)^{-1}\left[\frac{C_t}{C^*_t}\frac{\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)}{P^{*,PPT}_t/P^*_{F,t}}\right]^{1-\eta}\\
& -\eta\lambda^F_{y,t}\omega^*_F\left(\frac{P^{*,PPT}_t}{P^*_{F,t}}\right)^{\eta-1}C^*_t\\
& -\eta\lambda^F_{y,t}\frac{\nu}{1-\nu}\omega_F\left(\frac{P^{*,PPT}_t}{P^*_{F,t}}\right)^{-1}C_t\left[\frac{C^*_t(P^{*,PPT}_t/P^*_{F,t})}{C_t(1+\tau_{M,t})}\right]^{\eta}\\
& -\beta\lambda^F_{e,t}\mathrm{E}_t\left[\frac{C^*_t}{C^*_{t+1}(P^{*,PPT}_{t+1}/P^*_{F,t+1})\Pi^*_{F,t+1}}\right]\\
& +\lambda^F_{e,t-1}\frac{C^*_{t-1}(P^{*,PPT}_{t-1}/P^*_{F,t-1})}{C^*_t(P^{*,PPT}_t/P^*_{F,t})^2\Pi^*_{F,t}},
\end{aligned}
$$

$$
\begin{aligned}
0 \;=\; & (1-\eta)\lambda^F_{g,t}\omega^*_H(1+\tau^*_{M,t})^{1-\eta}\left(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)\right)^{-1}\left[\frac{C^*_t}{C_t}\frac{P^{*,PPT}_t/P^*_{F,t}}{\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)}\right]^{1-\eta}\\
& +(1-\eta)\lambda^F_{gH,t}\Bigg[\left(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)\right)^{-\eta}\\
& \quad -\omega_F(1+\tau_{M,t})^{1-\eta}\left(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)\right)^{-1}\left[\frac{C_t}{C^*_t}\frac{\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)}{P^{*,PPT}_t/P^*_{F,t}}\right]^{1-\eta}\Bigg].
\end{aligned}
$$

The remaining Foreign planner's FOCs, for N^*_t, Y^*_t, $\Pi^*_{F,t}$, $\Delta^*_{F,t}$, and i^*_t, are

$$
\begin{aligned}
0 \;=\; & -(N^*_t)^{\varphi}+\lambda^F_{n,t}-(1-\theta)(1-\varepsilon)\left[\lambda^F_{p,t}(\widetilde{p}^*_{F,t})^{-\varepsilon}(\widetilde{p}^*_{F,t})'(N^*_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{p,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon}(\widetilde{p}^*_{F,t-j})'(N^*_t)\right]\\
& +(1-\theta)\varepsilon\left[\lambda^F_{\Delta,t}(\widetilde{p}^*_{F,t})^{-\varepsilon-1}(\widetilde{p}^*_{F,t})'(N^*_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{\Delta,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon-1}(\widetilde{p}^*_{F,t-j})'(N^*_t)\right],\\
0 \;=\; & -\lambda^F_{n,t}\Delta^*_{F,t}+\lambda^F_{y,t}-(1-\theta)(1-\varepsilon)\left[\lambda^F_{p,t}(\widetilde{p}^*_{F,t})^{-\varepsilon}(\widetilde{p}^*_{F,t})'(Y^*_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{p,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon}(\widetilde{p}^*_{F,t-j})'(Y^*_t)\right]\\
& +(1-\theta)\varepsilon\left[\lambda^F_{\Delta,t}(\widetilde{p}^*_{F,t})^{-\varepsilon-1}(\widetilde{p}^*_{F,t})'(Y^*_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{\Delta,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon-1}(\widetilde{p}^*_{F,t-j})'(Y^*_t)\right],\\
0 \;=\; & -\theta(\varepsilon-1)\lambda^F_{p,t}(\Pi^*_{F,t})^{\varepsilon-2}-\theta\varepsilon\lambda^F_{\Delta,t}(\Pi^*_{F,t})^{\varepsilon-1}\Delta^*_{F,t-1}\\
& -(1-\theta)(1-\varepsilon)\left[\lambda^F_{p,t}(\widetilde{p}^*_{F,t})^{-\varepsilon}(\widetilde{p}^*_{F,t})'(\Pi^*_{F,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{p,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon}(\widetilde{p}^*_{F,t-j})'(\Pi^*_{F,t})\right]\\
& +(1-\theta)\varepsilon\left[\lambda^F_{\Delta,t}(\widetilde{p}^*_{F,t})^{-\varepsilon-1}(\widetilde{p}^*_{F,t})'(\Pi^*_{F,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{\Delta,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon-1}(\widetilde{p}^*_{F,t-j})'(\Pi^*_{F,t})\right]\\
& +\lambda^F_{e,t-1}\frac{C^*_{t-1}(P^{*,PPT}_{t-1}/P^*_{F,t-1})}{C^*_t(P^{*,PPT}_t/P^*_{F,t})(\Pi^*_{F,t})^2},\\
0 \;=\; & \lambda^F_{\Delta,t}-\lambda^F_{n,t}Y^*_t-\beta\theta\mathrm{E}_t[\lambda^F_{\Delta,t+1}(\Pi^*_{F,t+1})^{\varepsilon}],
\end{aligned}
$$

$$0 = -\frac{\lambda^F_{e,t}}{(1+i^*_t)^2} + \mu^F_{i,t}.$$

The Foreign ZLB conditions are

$$i^*_t \geq 0, \qquad \mu^F_{i,t} \geq 0, \qquad \mu^F_{i,t} i^*_t = 0.$$

E.2.2 IPT Environment

The Home pricing-block derivatives below are calculated from Eq. (D.7) using the exact private reset-price equations. A prime on $\widetilde{p}_{H,t}$ or on $\overline{P}_{F,t}/P_{F,t}$ denotes the total derivative of that reset-price map with respect to the specified Home coordinate; the sums retain current and future terms implied by commitment. The Foreign coordinates held fixed are those specified in Appendix D.2.2: C^*_t and the Foreign *ex-tariff* importer prices.

For the Home producer's reset price, the total derivatives left implicit in Eq. (E.1) have the following closed forms on the perfect-foresight paths of the nonlinear tariff experiment. For each $s \leq t$, direct differentiation of the original sequences in Eq. (B.13), using the exact stochastic-discount factor, gives

$$\widetilde{p}'_{H,s}(C_t) = \frac{\widetilde{p}_{H,s}Y_t}{C_t\mathcal{K}_{H,s}}(\beta\theta)^{t-s}\frac{C_s\mathcal{G}^{IPT}_s}{C_t\mathcal{G}^{IPT}_t}\left(\prod_{u=s+1}^{t}\Pi_{H,u}\right)^{\varepsilon-1}, \tag{E.18}$$

$$\widetilde{p}'_{H,s}(\mathcal{G}^{IPT}_t) = \frac{\widetilde{p}_{H,s}Y_t}{\mathcal{G}^{IPT}_t\mathcal{K}_{H,s}}(\beta\theta)^{t-s}\frac{C_s\mathcal{G}^{IPT}_s}{C_t\mathcal{G}^{IPT}_t}\left(\prod_{u=s+1}^{t}\Pi_{H,u}\right)^{\varepsilon-1}, \tag{E.19}$$

$$\widetilde{p}'_{H,s}(N_t) = \frac{\widetilde{p}_{H,s}}{\mathcal{Z}_{H,s}}(\beta\theta)^{t-s}C_s\mathcal{G}^{IPT}_s\left(\prod_{u=s+1}^{t}\Pi_{H,u}\right)^{\varepsilon}Y_t\varphi N_t^{\varphi-1}, \tag{E.20}$$

$$\widetilde{p}'_{H,s}(Y_t) = \widetilde{p}_{H,s}(\beta\theta)^{t-s}\frac{C_s\mathcal{G}^{IPT}_s}{C_t\mathcal{G}^{IPT}_t}\left(\prod_{u=s+1}^{t}\Pi_{H,u}\right)^{\varepsilon-1}\left[\frac{C_t\mathcal{G}^{IPT}_t N_t^{\varphi}}{\mathcal{Z}_{H,s}}\left(\prod_{u=s+1}^{t}\Pi_{H,u}\right)-\frac{1}{\mathcal{K}_{H,s}}\right], \tag{E.21}$$

$$\widetilde{p}'_{H,s}(\Pi_{H,t}) = \frac{\widetilde{p}_{H,s}}{\Pi_{H,t}}(\beta\theta)^{t-s}\frac{C_s\mathcal{G}^{IPT}_s}{C_t\mathcal{G}^{IPT}_t}\left(\prod_{u=s+1}^{t}\Pi_{H,u}\right)^{\varepsilon-1}\left[\frac{\varepsilon\mathcal{Z}_{H,t}}{\mathcal{Z}_{H,s}}\left(\prod_{u=s+1}^{t}\Pi_{H,u}\right)-(\varepsilon-1)\frac{\mathcal{K}_{H,t}}{\mathcal{K}_{H,s}}\right], \quad s<t. \tag{E.22}$$

The derivative with respect to $\Pi_{H,t}$ vanishes when $s = t$, and all five derivatives vanish for $s > t$. The derivatives with respect to the Home import-price ratio, Home ex-tariff import inflation, Home price dispersion, and the nominal interest rate vanish when other independent Ramsey coordinates are fixed. The producer reset-price derivatives in the following conditions are given by Eqs. (E.18)–(E.22).

Exact importer reset-price and Foreign demand derivatives. The remaining total derivatives in the IPT FOCs can be evaluated directly from the original nonlinear additive-duty problem, without substituting an importer Phillips curve or a quadratic welfare criterion. We give the perfect-foresight expressions used in the tariff simulations. The stochastic counterparts retain conditional expectations in the sums. The indices s and $u \geq s$ denote a reset date and an ensuing date; v indexes a Calvo reset vintage. Write the existing import-price cross section as

$$\begin{aligned}
q_{u|v} &\equiv \frac{\overline{P}_{F,v}}{P_{H,u}} = \frac{\overline{P}_{F,v}}{P_{H,v}}\prod_{k=v+1}^{u}\Pi^{-1}_{H,k}, \qquad L_{v,u} \equiv \prod_{k=v+1}^{u}\Pi^{-1}_{H,k},\\
A_{u|v} &\equiv q_{u|v} + \tau_{M,u}Z_u, \qquad B_{u|v} \equiv q_{u|v} - (1-\tau)Z_u,\\
G_{M,u} &\equiv \frac{\widetilde{P}^{IPT}_{F,u}}{P_{H,u}} = \left[\sum_{v\leq u} w_{u,v}A^{1-\varepsilon_M}_{u|v}\right]^{1/(1-\varepsilon_M)},\\
w_{u,v} &= (1-\theta_M)\theta_M^{u-v} \quad (v \leq u).
\end{aligned} \tag{E.23}$$

The sum includes inherited pre-sample vintages at predetermined prices. In the 140-cohort Dynare implementation, the final cohort collects the residual Calvo mass rather than using the individual-vintage weight specified above. Here $G_{M,u}$ is the purchaser-price index in Eq. (4) divided by $P_{H,u}$.

After removing the common positive factor $C_s\mathcal{G}_s^{IPT}$ from the original reset FOC (which does not alter its roots or total derivatives *at those roots*), Eqs. (15) and (16) imply the following *exact nonlinear* reset-price equation:

$$\begin{aligned} 0 &= F_s \equiv \sum_{u=s}^{\infty} W_{s,u} H_{s,u}, \\ W_{s,u} &\equiv (\beta\theta_M)^{u-s} L_{s,u}\, \omega_F (\mathcal{G}_u^{IPT})^{\eta-1} G_{M,u}^{\varepsilon_M-\eta} A_{u|s}^{-\varepsilon_M}, \\ H_{s,u} &\equiv 1 - \varepsilon_M \frac{B_{u|s}}{A_{u|s}}. \end{aligned} \tag{E.24}$$

The ex-tariff reset price enters in normalized form as $\overline{P}_{F,s}/P_{H,s}$.

For each Home Ramsey choice x_t in Eq. (D.5), complete risk sharing and the evolution of Calvo vintage prices imply the following partial derivatives, holding reset-price ratios fixed:

$$\begin{aligned} \left.\frac{\partial Z_u}{\partial x_t}\right|_{\overline{P}_F/P_H} &= \mathbf{1}_{u=t} Z_u \left(\frac{\mathbf{1}_{x=C}}{C_t} + \frac{\mathbf{1}_{x=\mathcal{G}}}{\mathcal{G}_t^{IPT}} - \frac{\mathbf{1}_{x=\mathcal{G}^*}}{\mathcal{G}_t^{*,IPT}} \right), \\ \left.\frac{\partial q_{u|s}}{\partial x_t}\right|_{\overline{P}_F/P_H} &= -\mathbf{1}_{s<t\le u}\, \mathbf{1}_{x=\Pi_H} \frac{q_{u|s}}{\Pi_{H,t}}. \end{aligned} \tag{E.25}$$

Here $\mathbf{1}_{x=C}$, for example, equals one only for $x_t = C_t$. The right sides vanish for the other independent Ramsey coordinates. A superscript (0) denotes differentiation at fixed reset-price ratios $\{\overline{P}_{F,v}/P_{H,v}\}$. Direct differentiation of the *additive* purchaser-price CES aggregator yields

$$\begin{aligned} G_{M,u,x_t}^{(0)} &= G_{M,u}^{\varepsilon_M} \sum_{v\le u} w_{u,v} A_{u|v}^{-\varepsilon_M} \left[\left.\frac{\partial q_{u|v}}{\partial x_t}\right|_{\overline{P}_F/P_H} + \tau_{M,u} \left.\frac{\partial Z_u}{\partial x_t}\right|_{\overline{P}_F/P_H} \right], \\ G_{M,u,\overline{q}_v} &= \mathbf{1}_{v\le u} G_{M,u}^{\varepsilon_M} w_{u,v} A_{u|v}^{-\varepsilon_M} L_{v,u}, \qquad \overline{q}_v \equiv \frac{\overline{P}_{F,v}}{P_{H,v}}. \end{aligned} \tag{E.26}$$

The expression for $G_{M,u,x_t}^{(0)}$ also includes the inherited initial-price mass, which does not respond to post-sample reset-price choices.

To compute the partial derivatives of F_s in Eq. (E.24), we temporarily treat the purchaser-price indices $G_{M,u}$ and reset-price ratios $\overline{q}_v$ as independent arguments. This gives

$$\begin{aligned} F_{s,\overline{q}_s}^{(0)} &= -\varepsilon_M \sum_{u=s}^{\infty} W_{s,u} \frac{L_{s,u}}{A_{u|s}} \left[H_{s,u} + \frac{A_{u|s} - B_{u|s}}{A_{u|s}} \right], \\ F_{s,G_{M,u}} &= \mathbf{1}_{u\ge s} \frac{\varepsilon_M - \eta}{G_{M,u}} W_{s,u} H_{s,u}, \\ F_{s,x_t}^{(0)} &= \sum_{u=s}^{\infty} W_{s,u} \Bigg\{ H_{s,u} \Bigg[-\mathbf{1}_{s<t\le u} \frac{\mathbf{1}_{x=\Pi_H}}{\Pi_{H,t}} + (\eta-1)\mathbf{1}_{u=t} \frac{\mathbf{1}_{x=\mathcal{G}}}{\mathcal{G}_t^{IPT}} \\ &\qquad - \varepsilon_M \frac{q_{u|s,x_t}^{(0)} + \tau_{M,u} Z_{u,x_t}^{(0)}}{A_{u|s}} \Bigg] \\ &\qquad - \varepsilon_M \left[\frac{q_{u|s,x_t}^{(0)} - (1-\tau) Z_{u,x_t}^{(0)}}{A_{u|s}} - \frac{B_{u|s}}{A_{u|s}^2} (q_{u|s,x_t}^{(0)} + \tau_{M,u} Z_{u,x_t}^{(0)}) \right] \Bigg\}. \end{aligned} \tag{E.27}$$

The $q_{u|s,x_t}^{(0)}$ and $Z_{u,x_t}^{(0)}$ on the right are the *displayed* elementary expressions in Eq. (E.25), not unspecified implicit derivatives.

Because the aggregate purchaser-price index depends on reset prices from multiple vintages, the total reset-price derivative cannot be obtained from $-F_{s,x_t}^{(0)}/F_{s,\overline{q}_s}^{(0)}$ alone. Instead, it is determined by the following vintage-indexed linear system:

$$\begin{aligned} \sum_v J_{sv} \frac{\partial \overline{q}_v}{\partial x_t} &= -F_{s,x_t}^{(0)} - \sum_{u=s}^{\infty} F_{s,G_{M,u}} G_{M,u,x_t}^{(0)}, \\ J_{sv} &= \mathbf{1}_{s=v} F_{s,\overline{q}_s}^{(0)} + \sum_{u\ge\max\{s,v\}} F_{s,G_{M,u}} G_{M,u,\overline{q}_v}, \end{aligned}$$

$$\frac{\partial}{\partial x_t}\left(\frac{\overline{P}_{F,s}}{P_{F,s}}\right) = \frac{1}{S_s}\frac{\partial \overline{q}_s}{\partial x_t} - \mathbf{1}_{x=S}\mathbf{1}_{s=t}\frac{\overline{q}_s}{S_s^2},$$
$$\frac{\partial G_{M,u}}{\partial x_t} = G^{(0)}_{M,u,x_t} + \sum_{v\le u} G_{M,u,\overline{q}_v}\frac{\partial \overline{q}_v}{\partial x_t}. \tag{E.28}$$

In particular, when J is nonsingular, the reset-price derivative in the importer-price-index FOCs is the explicit inverse-Jacobian sum

$$\frac{\partial}{\partial x_t}\left(\frac{\overline{P}_{F,s}}{P_{F,s}}\right) = -\frac{1}{S_s}\sum_v (J^{-1})_{sv}\left[F^{(0)}_{v,x_t} + \sum_{u=v}^{\infty} F_{v,G_{M,u}} G^{(0)}_{M,u,x_t}\right] - \mathbf{1}_{x=S}\mathbf{1}_{s=t}\frac{\overline{q}_s}{S_s^2}.$$

The entries of J and its right-hand side are given by Eqs. (E.25)– (E.27). The finite-cohort version uses the same 140 vintage weights and pre-sample tail as the original Dynare program. In the Home FOCs, every occurrence of $(\overline{P}_{F,s}/P_{F,s})'(x_t)$ is evaluated using this system, and $\partial(\widetilde{P}^{IPT}_{F,t}/P_{H,t})/\partial x_t$ equals $\partial G_{M,t}/\partial x_t$, as given by the final equation of the vintage-indexed system.

The Foreign *cum-tariff* prices in the Home export-demand term and in the Foreign CPI depend on the Home coordinates through Z_t. With C^*_t and the Foreign *ex-tariff* prices fixed,

$$\frac{\partial \log \widetilde{P}^*_{H,t}(\iota)}{\partial x_t} = -\frac{\tau^*_{M,t}/Z_t}{P^*_{H,t}(\iota)/P^*_{F,t} + \tau^*_{M,t}/Z_t} \times\left(\frac{\mathbf{1}_{x=C}}{C_t} + \frac{\mathbf{1}_{x=\mathcal{G}}}{\mathcal{G}^{IPT}_t} - \frac{\mathbf{1}_{x=\mathcal{G}^*}}{\mathcal{G}^{*,IPT}_t}\right),$$

$$\frac{\partial \log \widetilde{P}^*_{H,t}}{\partial x_t} = \frac{1}{1-\nu}\int_\nu^1 \left(\frac{\widetilde{P}^*_{H,t}(\iota)}{\widetilde{P}^*_{H,t}}\right)^{1-\varepsilon_M}\frac{\partial \log \widetilde{P}^*_{H,t}(\iota)}{\partial x_t}\,d\iota,$$

$$\frac{\partial}{\partial x_t}\left[\left(\frac{\widetilde{P}^*_{H,t}}{P^{*,IPT}_t}\right)^{-\eta} C^*_t \frac{1}{1-\nu}\int_\nu^1 \left(\frac{\widetilde{P}^*_{H,t}(\iota)}{\widetilde{P}^*_{H,t}}\right)^{-\varepsilon_M} d\iota\right]$$
$$= \left(\frac{\widetilde{P}^*_{H,t}}{P^{*,IPT}_t}\right)^{-\eta} C^*_t \frac{1}{1-\nu}\int_\nu^1 \left(\frac{\widetilde{P}^*_{H,t}(\iota)}{\widetilde{P}^*_{H,t}}\right)^{-\varepsilon_M}\left[-\varepsilon_M\frac{\partial \log \widetilde{P}^*_{H,t}(\iota)}{\partial x_t} + (\varepsilon_M-\eta)\frac{\partial \log \widetilde{P}^*_{H,t}}{\partial x_t} + \eta\frac{\mathbf{1}_{x=\mathcal{G}^*}}{\mathcal{G}^{*,IPT}_t}\right]d\iota. \tag{E.29}$$

These derivatives vanish for Home choice coordinates other than those indicated above. When $\tau^*_{M,t}=0$, only the term involving $\mathbf{1}_{x=\mathcal{G}^*}$ remains in the export-demand derivative.

The Home consumption FOC is

$$\begin{aligned}
0 = {} & \frac{1}{C_t} - \lambda^H_{g,t}\omega_F(1-\eta)\left(\frac{\widetilde{P}^{IPT}_{F,t}}{P_{H,t}}\right)^{-\eta}\frac{\partial(\widetilde{P}^{IPT}_{F,t}/P_{H,t})}{\partial C_t} - \lambda^H_{y,t}\omega_H \mathcal{G}^{IPT}_t(\cdot)^\eta \\
& -\lambda^H_{y,t}\frac{1-\nu}{\nu}\omega^*_H\frac{\partial}{\partial C_t}\left[\left(\frac{\widetilde{P}^*_{H,t}}{P^{*,IPT}_t}\right)^{-\eta} C^*_t\frac{1}{1-\nu}\int_\nu^1\left(\frac{\widetilde{P}^*_{H,t}(\iota)}{\widetilde{P}^*_{H,t}}\right)^{-\varepsilon_M}d\iota\right] \\
& -\lambda^H_{gF,t}\omega^*_H(1-\eta)\left(\frac{\widetilde{P}^*_{H,t}}{P^*_{F,t}}\right)^{1-\eta}\frac{\partial \log \widetilde{P}^*_{H,t}}{\partial C_t} \\
& -(1-\theta)(1-\varepsilon)\Big[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(C_t) \\
& \quad +\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(C_t)\Big] \\
& +(1-\theta)\varepsilon\Big[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(C_t) \\
& \quad +\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(C_t)\Big] \\
& -(1-\theta_M)(1-\varepsilon_M)\Big[\lambda^H_{Mp,t}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)'(C_t)
\end{aligned}$$

$$+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{Mp,t-j}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)'(C_t)\Big]$$
$$-\beta\lambda^H_{e,t}\mathrm{E}_t\left[\frac{\mathcal{G}^{IPT}_t(\cdot)}{C_{t+1}\mathcal{G}^{IPT}_{t+1}(\cdot)\Pi_{H,t+1}}\right]+\lambda^H_{e,t-1}\frac{C_{t-1}\mathcal{G}^{IPT}_{t-1}(\cdot)}{C_t^2\mathcal{G}^{IPT}_t(\cdot)\Pi_{H,t}}. \tag{E.30}$$

The FOC for the Home CPI–PPI ratio is

$$\begin{aligned}
0 \;=\; & (1-\eta)\lambda^H_{g,t}\mathcal{G}^{IPT}_t(\cdot)^{-\eta}-\lambda^H_{g,t}\omega_F(1-\eta)\left(\frac{\widetilde{P}^{IPT}_{F,t}}{P_{H,t}}\right)^{-\eta}\frac{\partial(\widetilde{P}^{IPT}_{F,t}/P_{H,t})}{\partial\mathcal{G}^{IPT}_t(\cdot)}\\
&-\eta\lambda^H_{y,t}\omega_H\mathcal{G}^{IPT}_t(\cdot)^{\eta-1}C_t\\
&-\lambda^H_{y,t}\frac{1-\nu}{\nu}\omega^*_H\frac{\partial}{\partial\mathcal{G}^{IPT}_t(\cdot)}\left[\left(\frac{\widetilde{P}^*_{H,t}}{P^{*,IPT}_t}\right)^{-\eta}C^*_t\frac{1}{1-\nu}\int_\nu^1\left(\frac{\widetilde{P}^*_{H,t}(\iota)}{\widetilde{P}^*_{H,t}}\right)^{-\varepsilon_M}d\iota\right]\\
&-\lambda^H_{gF,t}\omega^*_H(1-\eta)\left(\frac{\widetilde{P}^*_{H,t}}{P^*_{F,t}}\right)^{1-\eta}\frac{\partial\log\widetilde{P}^*_{H,t}}{\partial\mathcal{G}^{IPT}_t(\cdot)}\\
&-(1-\theta)(1-\varepsilon)\Big[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(\mathcal{G}^{IPT}_t)\\
&\quad+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(\mathcal{G}^{IPT}_t)\Big]\\
&+(1-\theta)\varepsilon\Big[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(\mathcal{G}^{IPT}_t)\\
&\quad+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(\mathcal{G}^{IPT}_t)\Big]\\
&-(1-\theta_M)(1-\varepsilon_M)\Big[\lambda^H_{Mp,t}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)'(\mathcal{G}^{IPT}_t)\\
&\quad+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{Mp,t-j}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)'(\mathcal{G}^{IPT}_t)\Big]\\
&-\beta\lambda^H_{e,t}\mathrm{E}_t\left[\frac{C_t}{C_{t+1}\mathcal{G}^{IPT}_{t+1}(\cdot)\Pi_{H,t+1}}\right]+\lambda^H_{e,t-1}\frac{C_{t-1}\mathcal{G}^{IPT}_{t-1}(\cdot)}{C_t\mathcal{G}^{IPT}_t(\cdot)^2\Pi_{H,t}}.
\end{aligned}\tag{E.31}$$

Here $\widetilde{p}'_{H,t}(\mathcal{G}^{IPT}_t)$ and $(\overline{P}_{F,t}/P_{F,t})'(\mathcal{G}^{IPT}_t)$ mean total derivatives with respect to the displayed CPI–PPI ratio, not derivatives with respect to a newly defined variable.

The Home planner's FOC for the Foreign CPI–PPI ratio is

$$\begin{aligned}
0 \;=\; & (1-\eta)\lambda^H_{gF,t}\mathcal{G}^{*,IPT}_t(\cdot)^{-\eta}-\lambda^H_{g,t}\omega_F(1-\eta)\left(\frac{\widetilde{P}^{IPT}_{F,t}}{P_{H,t}}\right)^{-\eta}\frac{\partial(\widetilde{P}^{IPT}_{F,t}/P_{H,t})}{\partial\mathcal{G}^{*,IPT}_t(\cdot)}\\
&-\lambda^H_{y,t}\frac{1-\nu}{\nu}\omega^*_H\frac{\partial}{\partial\mathcal{G}^{*,IPT}_t(\cdot)}\left[\left(\frac{\widetilde{P}^*_{H,t}}{P^{*,IPT}_t}\right)^{-\eta}C^*_t\frac{1}{1-\nu}\int_\nu^1\left(\frac{\widetilde{P}^*_{H,t}(\iota)}{\widetilde{P}^*_{H,t}}\right)^{-\varepsilon_M}d\iota\right]\\
&-\lambda^H_{gF,t}\omega^*_H(1-\eta)\left(\frac{\widetilde{P}^*_{H,t}}{P^*_{F,t}}\right)^{1-\eta}\frac{\partial\log\widetilde{P}^*_{H,t}}{\partial\mathcal{G}^{*,IPT}_t(\cdot)}\\
&-(1-\theta_M)(1-\varepsilon_M)\Big[\lambda^H_{Mp,t}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)'(\mathcal{G}^{*,IPT}_t)\\
&\quad+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{Mp,t-j}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)'(\mathcal{G}^{*,IPT}_t)\Big].
\end{aligned}\tag{E.32}$$

The ratio $\mathcal{G}^{*,IPT}_t(\cdot)$ enters the Home importer block only through Z_t; it does not directly enter the Home producer reset-price condition when other independent Ramsey coordinates are held fixed.

The labor and output FOCs are

$$\begin{aligned}
0 \;=\; & -N_t^{\varphi}+\lambda^H_{n,t}-(1-\theta)(1-\varepsilon)\Big[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(N_t)\\
&\quad+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(N_t)\Big]
\end{aligned}$$

$$
\begin{aligned}
&+(1-\theta)\varepsilon\Big[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(N_t)\\
&\quad+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(N_t)\Big], \qquad \text{(E.33)}\\
0 \;=\; &-\lambda^H_{n,t}\Delta_{H,t}+\lambda^H_{y,t}-(1-\theta)(1-\varepsilon)\Big[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(Y_t)\\
&\quad+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(Y_t)\Big]\\
&+(1-\theta)\varepsilon\Big[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(Y_t)\\
&\quad+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(Y_t)\Big]. \qquad \text{(E.34)}
\end{aligned}
$$

The FOC for the local *ex-tariff* import-price ratio $P_{F,t}/P_{H,t}$ is

$$
\begin{aligned}
0 \;=\; &-\lambda^H_{g,t}\omega_F(1-\eta)\left(\frac{\widetilde{P}^{IPT}_{F,t}}{P_{H,t}}\right)^{-\eta}\frac{\partial(\widetilde{P}^{IPT}_{F,t}/P_{H,t})}{\partial(P_{F,t}/P_{H,t})}\\
&-(1-\theta_M)(1-\varepsilon_M)\Big[\lambda^H_{Mp,t}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)'(P_{F,t}/P_{H,t})\\
&\quad+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{Mp,t-j}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)'(P_{F,t}/P_{H,t})\Big]\\
&+\lambda^H_{q,t}-\beta\mathrm{E}_t\left[\lambda^H_{q,t+1}\frac{\Pi_{F,t+1}}{\Pi_{H,t+1}}\right]. \qquad \text{(E.35)}
\end{aligned}
$$

The FOCs for Home PPI inflation $\Pi_{H,t}$ and *ex-tariff* import-price inflation $\Pi_{F,t}$ are

$$
\begin{aligned}
0 \;=\; &-\theta(\varepsilon-1)\lambda^H_{p,t}\Pi^{\varepsilon-2}_{H,t}-\theta\varepsilon\lambda^H_{\Delta,t}\Pi^{\varepsilon-1}_{H,t}\Delta_{H,t-1}\\
&-(1-\theta)(1-\varepsilon)\Big[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(\Pi_{H,t})\\
&\quad+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(\Pi_{H,t})\Big]\\
&+(1-\theta)\varepsilon\Big[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(\Pi_{H,t})\\
&\quad+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(\Pi_{H,t})\Big]\\
&-(1-\theta_M)(1-\varepsilon_M)\Big[\lambda^H_{Mp,t}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)'(\Pi_{H,t})\\
&\quad+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{Mp,t-j}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)'(\Pi_{H,t})\Big]\\
&+\lambda^H_{q,t}\frac{P_{F,t-1}}{P_{H,t-1}}\frac{\Pi_{F,t}}{\Pi^2_{H,t}}+\lambda^H_{e,t-1}\frac{C_{t-1}\mathcal{G}^{IPT}_{t-1}(\cdot)}{C_t\mathcal{G}^{IPT}_t(\cdot)\Pi^2_{H,t}}, \qquad \text{(E.36)}\\
0 \;=\; &-\theta_M(\varepsilon_M-1)\lambda^H_{Mp,t}\Pi^{\varepsilon_M-2}_{F,t}\\
&-(1-\theta_M)(1-\varepsilon_M)\Big[\lambda^H_{Mp,t}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)'(\Pi_{F,t})\\
&\quad+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{Mp,t-j}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)'(\Pi_{F,t})\Big]-\lambda^H_{q,t}\frac{P_{F,t-1}}{P_{H,t-1}}\frac{1}{\Pi_{H,t}}. \qquad \text{(E.37)}
\end{aligned}
$$

The FOCs for producer-price dispersion $\Delta_{H,t}$ and the nominal interest rate i_t, including the ZLB complementarity conditions, are

$$
\begin{aligned}
0 \;&=\; \lambda^H_{\Delta,t}-\lambda^H_{n,t}Y_t-\beta\theta\mathrm{E}_t[\lambda^H_{\Delta,t+1}\Pi^{\varepsilon}_{H,t+1}], \qquad \text{(E.38)}\\
0 \;&=\; -\frac{\lambda^H_{e,t}}{(1+i_t)^2}+\mu^H_{i,t}, \qquad i_t\geq 0, \quad \mu^H_{i,t}\geq 0, \quad \mu^H_{i,t}i_t=0. \qquad \text{(E.39)}
\end{aligned}
$$

The exact purchaser-price cross section in Appendix C.2 is retained directly; importer-price dispersion is therefore not a separate state and has no separate FOC. The Foreign FOCs, including the FOC for $\mathcal{G}_t^{IPT}(\cdot)$ with multiplier $\lambda_{gH,t}^F$, are obtained from Eqs. (E.30)–(E.39) by the exact Home–Foreign mirror transformation, with Z_t replaced by Z_t^{-1}, and satisfy

$$-\frac{\lambda_{e,t}^F}{(1+i_t^*)^2}+\mu_{i,t}^F=0, \qquad i_t^*\geq 0, \quad \mu_{i,t}^F\geq 0, \quad \mu_{i,t}^F i_t^*=0. \tag{E.40}$$

Together with the common private equilibrium in Appendix C.2, these conditions are the two national best-response systems used in the IPT simulations.

Numerical implementation. The numerical implementation solves the original nonlinear national Ramsey first-order conditions jointly using Dynare's `perfect_foresight_solver(lmmcp)`. The IPT computations use 140 importer-price cohorts and a transition horizon of 110 periods. Under the timeless perspective, lagged Ramsey multipliers are initialized at their deterministic steady-state values. The replication package documents the numerical variables, cohort equations, and input files.

F Welfare Cost Functions

This appendix derives the welfare cost functions in Secs. 3.4 and 4.4 from a second-order approximation to household utility. Because no employment subsidy is paid, the steady state is distorted, and the second-order expansion of utility contains policy-dependent linear terms. Following Benigno and Woodford [1, 2], we eliminate these terms with second-order approximations of the equilibrium conditions. Appendix F.1 treats the SOE and Appendix F.3 the two-country economy. After the steady state is described, each proceeds in five steps:

1. the second-order approximation of period utility;
2. the second-order approximation of producer- and importer-price dispersion;
3. the second-order approximation of the equilibrium conditions;
4. the elimination of the linear terms by undetermined coefficients; and
5. the welfare cost function and the welfare-relevant variables.

Notation and normalization. A variable without a time subscript denotes its deterministic steady-state value, lower-case variables denote log deviations from the zero-tariff, zero-inflation steady state, and $O(3)$ collects terms of third and higher order. t.i.p. denotes terms independent of policy, including given initial-cohort boundary terms, and s.o.t.i.p. denotes second-order terms independent of the national planner's allocation. Utility is normalized by $U_C C$, which equals one under logarithmic utility, so the welfare cost is measured in units of steady-state consumption. To first order, $\log(1+\tau_{M,t})=\tau_{M,t}$, so the tariff rates enter directly.

F.1 Small Open Economy

This subsection derives the welfare cost functions in Eqs. (21) and (22) and the weights in Table 2.

F.1.1 Steady State

The point of expansion is the original *distorted* zero-tariff deterministic steady state. Let $\mathcal{G}$ denote the common steady-state value of $\mathcal{G}^{PPT}$ and $\mathcal{G}_t^{IPT}$; Z, Y, and C are those of Z_t, Y_t, and C_t. The original nonlinear equilibrium restrictions give

$$\mathcal{G}^{1-\eta}=(1-\upsilon)+\upsilon Z^{1-\eta}, \qquad C=Z/\mathcal{G}, \qquad Y=(1-\upsilon)Z\mathcal{G}^{\eta-1}+\upsilon Z^{\eta}, \qquad ZY^{\varphi}=\mathcal{M}^{-1}, \tag{F.1}$$

where $\mathcal{M}^{-1} = (\varepsilon-1)/\varepsilon$ is the inverse of the desired producer markup. Thus neither $Z = 1$ nor $\mathcal{G} = 1$ may be imposed in coefficient calculations. The quantities in Eq. (F.1) are determined entirely by the original structural parameters, not separately calibrated. For the first-order reduction of the exact goods-market and risk-sharing conditions, define the following steady-state structural coefficients:

$$\begin{aligned} \alpha_F &\equiv \frac{\upsilon Z^{1-\eta}}{(1-\upsilon)+\upsilon Z^{1-\eta}}, \\ p_* &\equiv \frac{Y}{(1-\upsilon)Z\mathcal{G}^{\eta-1}[1+(\eta-1)\alpha_F]+\eta\upsilon Z^{\eta}}, \\ k_* &\equiv \frac{(\eta-1)(1-\upsilon)Z\mathcal{G}^{\eta-1}\alpha_F}{(1-\upsilon)Z\mathcal{G}^{\eta-1}[1+(\eta-1)\alpha_F]+\eta\upsilon Z^{\eta}}. \end{aligned} \tag{F.2}$$

Here α_F is the steady-state expenditure share on imports, which differs from the openness parameter υ unless $Z^{1-\eta} = 1$. Implicit differentiation of the exact goods-market and risk-sharing conditions gives, to first order,

$$z_t = p_* y_t + k_*(\psi_t - \tau_{M,t}), \qquad \psi_t = 0 \quad \text{under PPT.} \tag{F.3}$$

F.1.2 Second-Order Approximation of Utility

The period utility is $\log C_t - N_t^{1+\varphi}/(1+\varphi)$ (Eq. (1)). Complete risk sharing, Eq. (9) or its PPT counterpart, gives $C_t = Z_t/\mathcal{G}_t$ exactly, and the production function, Eq. (B.11) with $A_t = 1$, gives $N_t = Y_t\Delta_{H,t}$. Hence

$$\begin{aligned} \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} &= \log Z + z_t - \log\mathcal{G}_t - \frac{Y^{1+\varphi}}{1+\varphi}e^{(1+\varphi)(y_t+\log\Delta_{H,t})} \\ &= z_t - g_t - Y^{1+\varphi}\left[y_t + \frac{1+\varphi}{2}y_t^2 + \log\Delta_{H,t}\right] + \text{t.i.p.} + O(3), \end{aligned} \tag{F.4}$$

where $g_t \equiv \log(\mathcal{G}_t/\mathcal{G})$, with $\mathcal{G}_t$ the CPI–PPI ratio under PPT or IPT, as appropriate, and $\log\Delta_{H,t}$ is of second order. The linear terms $z_t - g_t - Y^{1+\varphi}y_t$ do not vanish at the distorted steady state; they are eliminated in Appendix F.1.5.

F.1.3 Second-Order Approximation of Price Dispersion

Producer and importer prices follow Calvo rules. Consider a reset-price condition of the form $\mathrm{E}_t\sum_{k\geq 0}(\theta\beta)^k\mathcal{F}(u_{t,k}, X_{t+k}) = 0$, where $u_{t,k} \equiv \log(\overline{P}_t/P_{t+k})$ is the price of a seller that reset at t relative to the index at $t+k$, X_{t+k} collects date-$(t+k)$ variables, and the index satisfies $\int(P_t(j)/P_t)^{1-\varepsilon}dj = 1$. Multiplying this reset-price condition by a constant coefficient Θ_R and re-indexing by calendar date $s = t+k$ gives

$$\sum_t \beta^t\Theta_R\mathrm{E}_t\sum_k(\theta\beta)^k\mathcal{F}(u_{t,k},X_{t+k}) = \frac{\Theta_R}{1-\theta}\sum_s\beta^s\int\mathcal{F}(\log(P_s(j)/P_s),X_s)\,dj + \text{t.i.p.} \tag{F.5}$$

The factor $1/(1-\theta)$ in Eq. (F.5) arises because the cohort that last reset at $s-k$ has mass $(1-\theta)\theta^k$. Let $V_s \equiv \int[\log(P_s(j)/P_s)]^2dj$. The price-index normalization implies $\int\log(P_s(j)/P_s)\,dj = (\varepsilon-1)V_s/2 + O(3)$. The dispersion index satisfies $\log\int(P_s(j)/P_s)^{-\varepsilon}dj = \varepsilon V_s/2 + O(3)$. Finally, the Calvo law of motion gives $\sum_s\beta^s V_s = \kappa^{-1}\sum_s\beta^s\pi_s^2 + \text{t.i.p.}$, where $\kappa = (1-\theta)(1-\beta\theta)/\theta$. Since products of $\int\log(P_s(j)/P_s)\,dj$ with X_s are third order,

$$\begin{aligned} &\frac{\Theta_R}{1-\theta}\sum_s\beta^s\int\mathcal{F}(\log(P_s(j)/P_s),X_s)\,dj \\ &\quad= \frac{\Theta_R}{1-\theta}\sum_s\beta^s\left[\mathcal{F}(0,X_s) + \frac{1}{2}\{(\varepsilon-1)\mathcal{F}_u + \mathcal{F}_{uu}\}V_s\right] + \text{t.i.p.} + O(3). \end{aligned} \tag{F.6}$$

For Home producers, the price-index normalization therefore gives $\log \Delta_{H,t} = \varepsilon V_{H,t}/2 + O(3)$, and the Calvo law of motion turns the discounted dispersion sum into $\kappa_H^{-1} \sum_s \beta^s \pi_{H,s}^2$, with κ_H as in Eq. (B.16). For Home importers, with $\log(P_{F,t}(\iota)/P_{F,t})$ denoting the cohort log-price deviation, the normalization of the *ex-tariff* CES price index implies

$$\int_0^1 \left(\frac{P_{F,t}(\iota)}{P_{F,t}}\right)^{-\varepsilon_M} d\iota = 1 + \frac{\varepsilon_M}{2} \int_0^1 \left[\log \frac{P_{F,t}(\iota)}{P_{F,t}}\right]^2 d\iota + O(3).$$

Thus, the importer restriction weighted by the fixed *ex-tariff* reset price has a dispersion coefficient of $\varepsilon_M/2$. The Calvo law of motion converts its discounted dispersion sum into $\kappa_M^{-1} \sum_s \beta^s \pi_{F,s}^2$, with κ_M as in Eq. (18). These manipulations hold up to the usual inherited-cohort initial boundary terms. Price dispersion therefore enters the welfare cost only through the squared inflation rates; their weights are given in Appendix F.1.6 once the multipliers are known.

F.1.4 Second-Order Approximation of the Equilibrium Conditions

Price index. The import tariff is the *additive* duty in Eq. (2). At zero importer-price dispersion, the purchaser-price ratio is

$$\frac{\widetilde{P}_{F,t}^{IPT}}{P_{H,t}} = Z(e^{z_t - \psi_t} + \tau_{M,t} e^{z_t}),$$

and interactions of the first-order tariff with second-order importer-price dispersion contribute only from third order. Writing $(1+\tau_{M,t})Ze^{z_t-\psi_t}$ instead would omit the second-order term $\tau_{M,t}\psi_t$ from its logarithm. Hence, abstracting from importer-price dispersion, Eqs. (2) and (8) imply

$$\begin{aligned}
\left(\frac{P_t^{IPT}}{P_{H,t}}\right)^{1-\eta} &= (1-\upsilon) + \upsilon \left[Z(e^{z_t-\psi_t} + \tau_{M,t}e^{z_t})\right]^{1-\eta}, \\
\left(\frac{P_t^{PPT}}{P_{H,t}}\right)^{1-\eta} &= (1-\upsilon) + \upsilon \left[(1+\tau_{M,t})Ze^{z_t}\right]^{1-\eta}.
\end{aligned} \tag{F.7}$$

To differentiate these conditions, let $\widetilde{p}_{F,t} \equiv \log(\widetilde{P}_{F,t}/P_{H,t})$ be the logarithm of the *cum-tariff* purchaser-price ratio. In derivatives evaluated at the steady state, use indices $a, b \in \{y, z, \psi, \tau_M\}$; under PPT the ψ coordinate is absent. A subscript a or ab on a function denotes its first or second partial derivative at the zero-tariff steady state, and $\delta_{az} = 1$ if $a = z$ and zero otherwise. The price index and the purchaser price imply

$$g_a = \alpha_F \widetilde{p}_{F,a}, \qquad g_{ab} = \alpha_F \widetilde{p}_{F,ab} + (1-\eta)\alpha_F(1-\alpha_F)\widetilde{p}_{F,a}\widetilde{p}_{F,b}. \tag{F.8}$$

The only nonzero purchaser-price derivatives are

$$\begin{array}{c|l}
\text{PPT} & \widetilde{p}_{F,z} = 1, \quad \widetilde{p}_{F,\tau_M} = 1, \quad \widetilde{p}_{F,\tau_M\tau_M} = -1 \\
\text{IPT} & \widetilde{p}_{F,z} = 1, \quad \widetilde{p}_{F,\psi} = -1, \quad \widetilde{p}_{F,\tau_M} = 1, \quad \widetilde{p}_{F,\psi\tau_M} = 1, \quad \widetilde{p}_{F,\tau_M\tau_M} = -1.
\end{array} \tag{F.9}$$

In particular, the mixed derivative $\widetilde{p}_{F,\psi\tau_M} = 1$ is a direct consequence of the *additive* tariff and would be lost under a multiplicative purchaser-price convention.

Risk sharing and goods-market clearing. Combining the exact risk-sharing and Home goods-market-clearing conditions for each environment yields

$$y_t = \log\left[(1-\upsilon)Ze^{z_t}\mathcal{G}_t^{\eta-1} + \upsilon(Ze^{z_t})^\eta\right] - \log Y, \tag{F.10}$$

where consumption has been eliminated by $c_t = z_t - g_t$. Denote the right-hand side of Eq. (F.10) minus y_t by $\mathcal{R}$. Its derivatives are

$$\mathcal{R}_a = -\delta_{ay} + \frac{(1-\upsilon)Z\mathcal{G}^{\eta-1} + \eta\upsilon Z^\eta}{Y}\delta_{az} + (\eta-1)\frac{(1-\upsilon)Z\mathcal{G}^{\eta-1}}{Y} g_a,$$

$$\mathcal{R}_{ab} \quad = \quad (\eta-1)\frac{(1-\upsilon)Z\mathcal{G}^{\eta-1}}{Y}g_{ab} + (\eta-1)^2\frac{(1-\upsilon)\upsilon Z^{1+\eta}\mathcal{G}^{\eta-1}}{Y^2}(g_a-\delta_{az})(g_b-\delta_{bz}). \quad \text{(F.11)}$$

At the steady state, $\mathcal{R}_z = 1/p_*$ and $\mathcal{R}_y = -1$; in IPT, $\mathcal{R}_\psi = -k_*/p_*$ and $\mathcal{R}_{\tau_M} = k_*/p_*$. Consequently the implicit first derivatives are $z_y = p_*$, $z_\psi = k_*$ and $z_{\tau_M} = -k_*$ (the z_ψ derivative is absent under PPT). More importantly, for i, j in (y, τ_M) under PPT and in (y, ψ, τ_M) under IPT, the *second* derivatives are

$$z_{ij} = -\frac{\mathcal{R}_{ij} + \mathcal{R}_{iz}z_j + \mathcal{R}_{jz}z_i + \mathcal{R}_{zz}z_iz_j}{\mathcal{R}_z}. \quad \text{(F.12)}$$

These second derivatives must be retained when differentiating welfare; first-order substitution alone does not reproduce the numerical weights.

Aggregate-supply restrictions. Attaching a constant multiplier to a Calvo reset-price condition and using Eq. (F.6), the condition contributes, period by period, its zero-dispersion term plus the dispersion term of Appendix F.1.3. For Home producers, the reset-price-weighted aggregate-supply restriction at zero dispersion is $(Y/Z)e^{y_t-z_t}(e^{\varphi y_t+z_t}-1) = 0$, where $e^{\varphi y_t+z_t}$ is real marginal cost relative to its steady-state value. For Home importers, the *ex-tariff reset price* $\overline{P}_{F,t}$ in Eq. (16) is fixed across all future terms of a given cohort. Multiplying that condition by this fixed reset price and summing across resetting cohorts gives the present-value importer aggregate-supply restriction. Weighting it by the future *cum-tariff* purchaser price instead would introduce time-varying tariff terms. At zero price dispersion, the importer restriction, normalized by the steady-state import share, is proportional to

$$\left(\frac{P_t^{IPT}}{P_{H,t}}\right)^{\eta-1}\left(\frac{\widetilde{P}_{F,t}^{IPT}}{P_{H,t}}\right)^{-\eta}\frac{P_{F,t}}{P_{H,t}}\frac{\tau_{M,t}+(\varepsilon_M-1)(1-e^{-\psi_t})}{(\varepsilon_M-1)(e^{-\psi_t}+\tau_{M,t})},$$

where $P_{F,t}/P_{H,t} = Ze^{z_t-\psi_t}$ is the *ex-tariff*, not the *cum-tariff*, price.

Production. The production function gives $n_t = y_t + \log\Delta_{H,t}$, so $n_t = y_t$ to first order.

F.1.5 Eliminating the Linear Terms

The linear terms in Eq. (F.4) are eliminated by attaching stationary multipliers to the first-order versions of the equilibrium conditions. Under PPT, Θ_g, Θ_Y, Θ_n, $\Theta_{mc,H}$, and Θ_H^{AS} are attached to the CPI identity, goods-market clearing, the labor identity, the producer real-marginal-cost identity, and the producer aggregate-supply restriction. Under IPT, Θ_M^{AS} is additionally attached to the importer aggregate-supply restriction. These multipliers are the undetermined coefficients of Benigno and Woodford; they equal the steady-state Ramsey multipliers on the corresponding constraints (Appendix H.1).

Consumption is eliminated by the exact SOE risk-sharing identity $c_t = z_t - g_t$. At first order, goods-market clearing is $y_t = [(1-\upsilon)Z\mathcal{G}^{\eta-1}/Y]\{z_t + (\eta-1)g_t\} + (\upsilon Z^\eta/Y)\eta z_t$, the CPI identity is $g_t = \alpha_F(z_t - \psi_t + \tau_{M,t})$, labor input satisfies $n_t = y_t$, and the producer real-marginal-cost deviation is $mc_{H,t} = z_t + \varphi n_t$. With the producer aggregate-supply multiplier normalized as in Eq. (F.15) below, setting to zero the coefficients on $g_t, z_t, y_t, n_t, mc_{H,t}$ and, under IPT, ψ_t yields

$$\begin{aligned}
0 &= -1 + \Theta_g - (\eta-1)[(1-\upsilon)Z\mathcal{G}^{\eta-1}/Y]\Theta_Y,\\
0 &= 1 - \alpha_F\Theta_g - [\{(1-\upsilon)Z\mathcal{G}^{\eta-1} + \eta\upsilon Z^\eta\}/Y]\Theta_Y - \Theta_{mc,H},\\
0 &= \Theta_Y - \Theta_n,\\
0 &= -Y^{1+\varphi} + \Theta_n - \varphi\Theta_{mc,H},\\
0 &= \Theta_{mc,H} + (Y/Z)\Theta_H^{AS},\\
0 &= \alpha_F\Theta_g + C^{1-\eta}\Theta_M^{AS} \qquad \text{(IPT only).}
\end{aligned} \quad \text{(F.13)}$$

All these multipliers attach to existing equilibrium identities, and their normalizations are fixed by the coefficient-matching equations, which imply

$$\begin{aligned}
\Theta_Y = \Theta_n &= Y^{1+\varphi} - \varphi(Y/Z)\Theta_H^{AS}, \\
\Theta_{mc,H} &= -(Y/Z)\Theta_H^{AS}, \\
\Theta_g &= 1 + (\eta-1)[(1-\upsilon)Z\mathcal{G}^{\eta-1}/Y]\Theta_Y, \\
\Theta_M^{AS} &= -\alpha_F\Theta_g/C^{1-\eta} \qquad \text{(IPT only)}.
\end{aligned} \tag{F.14}$$

Substituting these expressions into the second and last equations of Eq. (F.13) gives the closed forms for the multipliers on the producer and importer aggregate-supply restrictions,

$$\begin{aligned}
\Theta_H^{AS} &= \frac{Y^{1+\varphi} - (1-\alpha_F)p_*}{(Y/Z)(\varphi + p_*)}, \\
\Theta_M^{AS} &= -\frac{1}{C^{1-\eta}}\left[\alpha_F + k_*\frac{Y^{1+\varphi} + \varphi(1-\alpha_F)}{\varphi + p_*}\right], \qquad \text{(IPT only)}.
\end{aligned} \tag{F.15}$$

The producer coefficient is the same under PPT and IPT. The PPT system omits the importer condition and its multiplier.

F.1.6 Welfare Cost Function

Adding the multiplier terms to Eq. (F.4), the augmented real period welfare contribution under IPT is

$$\begin{aligned}
\mathcal{W} &= \log Z + z_t - \log\frac{P_t^{IPT}}{P_{H,t}} - \frac{Y^{1+\varphi}}{1+\varphi}e^{(1+\varphi)y_t} + \Theta_H^{AS}\frac{Y}{Z}e^{y_t - z_t}(e^{\varphi y_t + z_t} - 1) \\
&\quad + \Theta_M^{AS}\left(\frac{P_t^{IPT}}{P_{H,t}}\right)^{\eta-1}\left(\frac{\widetilde{P}_{F,t}^{IPT}}{P_{H,t}}\right)^{-\eta} Z e^{z_t - \psi_t}\frac{\tau_{M,t} + (\varepsilon_M - 1)(1 - e^{-\psi_t})}{(\varepsilon_M - 1)(e^{-\psi_t} + \tau_{M,t})},
\end{aligned} \tag{F.16}$$

where the dispersion terms are collected separately below. Under PPT, the importer term is omitted and the PPT price index in Eq. (F.7) is used. The exact identities in Eqs. (F.7)–(F.10) eliminate C_t and then implicitly eliminate z_t, including its *second* derivatives. The first derivatives of the resulting expression in y_t and ψ_t are zero at the steady state, which checks the multipliers in Appendix F.1.5.

The partial derivatives of $\mathcal{W}$, holding y, z, ψ, τ_M independently fixed, are as follows. In both environments,

$$\begin{aligned}
\mathcal{W}_y &= -Y^{1+\varphi} + \varphi(Y/Z)\Theta_H^{AS}, \\
\mathcal{W}_z &= 1 - \alpha_F + (Y/Z)\Theta_H^{AS}, \\
\mathcal{W}_{yy} &= -(1+\varphi)Y^{1+\varphi} + \varphi(\varphi+2)(Y/Z)\Theta_H^{AS}, \\
\mathcal{W}_{yz} &= (Y/Z)\Theta_H^{AS}, \\
\mathcal{W}_{zz} &= -g_{zz} - (Y/Z)\Theta_H^{AS}, \\
\mathcal{W}_{y\tau_M} &= 0, \qquad \mathcal{W}_{y\psi} = 0 \quad \text{(IPT only)}.
\end{aligned} \tag{F.17}$$

Under PPT the remaining derivatives are

$$\mathcal{W}_{\tau_M} = -\alpha_F, \qquad \mathcal{W}_{z\tau_M} = -g_{z\tau_M}, \qquad \mathcal{W}_{\tau_M\tau_M} = -g_{\tau_M\tau_M}. \tag{F.18}$$

Under IPT the additional first derivatives are

$$\mathcal{W}_\psi = \alpha_F + C^{1-\eta}\Theta_M^{AS}, \qquad \mathcal{W}_{\tau_M} = -\alpha_F + \frac{C^{1-\eta}\Theta_M^{AS}}{\varepsilon_M - 1}. \tag{F.19}$$

The remaining independent second derivatives under IPT are

$$\mathcal{W}_{z\psi} = -g_{z\psi} + (C^{1-\eta}\Theta_M^{AS})(\eta-1)(\alpha_F - 1),$$

$$
\begin{aligned}
\mathcal{W}_{z\tau_M} &= -g_{z\tau_M} + \frac{(C^{1-\eta}\Theta_M^{AS})(\eta-1)(\alpha_F-1)}{\varepsilon_M-1},\\
\mathcal{W}_{\psi\psi} &= -g_{\psi\psi} + (C^{1-\eta}\Theta_M^{AS})[1+2(\eta-1)(1-\alpha_F)],\\
\mathcal{W}_{\psi\tau_M} &= -g_{\psi\tau_M} + (C^{1-\eta}\Theta_M^{AS})\left[-\frac{\varepsilon_M-2}{\varepsilon_M-1} + \frac{(\eta-1)(1-\alpha_F)}{\varepsilon_M-1} + (\eta-1)\alpha_F - \eta\right],\\
\mathcal{W}_{\tau_M\tau_M} &= -g_{\tau_M\tau_M} + \frac{2C^{1-\eta}\Theta_M^{AS}}{\varepsilon_M-1}[(\eta-1)\alpha_F - \eta - 1].
\end{aligned}
\tag{F.20}
$$

The importer derivatives follow by applying the product rule to the ex-tariff-reset-price-weighted term in the displayed $\mathcal{W}$; they retain both the duty–LOOP interaction and the steady-state importer subsidy $\tau = 1/\varepsilon_M$.

After z_t is eliminated, the total second derivative is

$$
\left.\frac{d^2\mathcal{W}}{di\,dj}\right|_0 = \mathcal{W}_{ij} + \mathcal{W}_{iz}z_j + \mathcal{W}_{zj}z_i + \mathcal{W}_{zz}z_iz_j + \mathcal{W}_z z_{ij},
\tag{F.21}
$$

where every quantity on the right is evaluated at the steady state. Write the real part of the period welfare cost as $\frac{1}{2}v_t'Qv_t - v_t'B\tau_{M,t} + \text{t.i.p.} + O(3)$, where $v_t = y_t$ under PPT and $v_t = (y_t, \psi_t)'$ under IPT. For policy coordinates i, j this expression enters Q_{ij} with a minus sign; for a policy coordinate i and tariff $j = \tau_M$ it enters B_i with a plus sign. Eqs. (F.8)–(F.21), together with the multipliers in Eq. (F.14), determine every entry of the SOE Q and B from the structural parameters. Both the first and the second derivatives of the implicitly determined z_t must be retained; using only the log-linear market-clearing identity omits genuine second-order terms.

Explicit coefficients. The steady state in Eq. (F.1) reduces to the scalar equation $ZY^{\varphi} = \mathcal{M}^{-1}$ in Z, with $\mathcal{G}$ and Y given by the first and third relations; we solve it numerically. Given Z, every coefficient below is an explicit function of the structural parameters. Since $Y = (1-\upsilon)Z\mathcal{G}^{\eta-1} + \upsilon Z^{\eta}$, the coefficients in Eq. (F.2) satisfy

$$
p_* = \frac{1}{1+(\eta-1)\{\upsilon Z^{\eta} + \alpha_F(1-\upsilon)Z\mathcal{G}^{\eta-1}\}/Y}, \qquad k_* = (\eta-1)\frac{(1-\upsilon)Z\mathcal{G}^{\eta-1}}{Y}\alpha_F p_*.
$$

By Eqs. (F.14) and (F.15), the multipliers that enter the welfare derivatives are

$$
\begin{aligned}
&\frac{Y}{Z}\Theta_H^{AS} = \frac{Y^{1+\varphi} - (1-\alpha_F)p_*}{\varphi + p_*}, \qquad \Theta_Y = \Theta_n = p_*\mathcal{W}_z,\\
&\Theta_g = 1 + \frac{k_*}{\alpha_F}\mathcal{W}_z, \qquad C^{1-\eta}\Theta_M^{AS} = -\alpha_F - k_*\mathcal{W}_z,
\end{aligned}
$$

where $\mathcal{W}_z = 1 - \alpha_F + (Y/Z)\Theta_H^{AS} = [Y^{1+\varphi} + \varphi(1-\alpha_F)]/(\varphi + p_*)$ is the derivative in Eq. (F.17). Substituting Eq. (F.9) into Eq. (F.8) gives $g_{zz} = g_{z\tau_M} = -(\eta-1)\alpha_F(1-\alpha_F)$ in both environments and, under IPT, $g_{\psi\psi} = -g_{z\psi} = -(\eta-1)\alpha_F(1-\alpha_F)$ and $g_{\psi\tau_M} = \alpha_F + (\eta-1)\alpha_F(1-\alpha_F)$. Hence the derivatives in Eqs. (F.17)–(F.20) become

$$
\begin{aligned}
\mathcal{W}_{zz} &= (\eta-1)\alpha_F(1-\alpha_F) - (Y/Z)\Theta_H^{AS},\\
\mathcal{W}_{z\tau_M} &= (\eta-1)\alpha_F(1-\alpha_F) \quad \text{(PPT)}, \qquad \mathcal{W}_{z\tau_M} = (\eta-1)(1-\alpha_F)\left[\alpha_F - \frac{C^{1-\eta}\Theta_M^{AS}}{\varepsilon_M-1}\right] \quad \text{(IPT)},\\
\mathcal{W}_{z\psi} &= (\eta-1)(1-\alpha_F)k_*\mathcal{W}_z,\\
\mathcal{W}_{\psi\psi} &= (\eta-1)\alpha_F(1-\alpha_F) + C^{1-\eta}\Theta_M^{AS}[1+2(\eta-1)(1-\alpha_F)],\\
\mathcal{W}_{\psi\tau_M} &= -\alpha_F - (\eta-1)\alpha_F(1-\alpha_F) - C^{1-\eta}\Theta_M^{AS}\left[2 - \frac{1}{\varepsilon_M-1} + (\eta-1)(1-\alpha_F)\frac{\varepsilon_M-2}{\varepsilon_M-1}\right].
\end{aligned}
\tag{F.22}
$$

The second derivatives of z_t in Eq. (F.12) are

$$
z_{yy} = (\eta-1)k_*p_*^2(1-\alpha_F)\left[1 - \frac{\upsilon Z^{\eta}}{Y}\frac{1-\alpha_F}{\alpha_F}\right],
$$

$$
\begin{aligned}
z_{y\psi} &= -z_{y\tau_M} = -(\eta-1)k_* p_*(1-\alpha_F)\left[1-k_*+\frac{\upsilon Z^\eta}{Y}\frac{\alpha_F+(1-\alpha_F)k_*}{\alpha_F}\right], \\
z_{\psi\psi} &= -k_* - z_{\psi\tau_M} = (\eta-1)k_*\left[(1-\alpha_F)(1-k_*)^2-\frac{\upsilon Z^\eta}{Y}\frac{\{\alpha_F+(1-\alpha_F)k_*\}^2}{\alpha_F}\right], \qquad \text{(F.23)}
\end{aligned}
$$

where the derivatives involving ψ_t arise only under IPT, and z_{yy} and $z_{y\tau_M}$ are the same under PPT. The term $-k_*$ in $z_{\psi\tau_M}$ comes from the additive duty in Eq. (F.7). With $z_y = p_*$, $z_\psi = k_*$, and $z_{\tau_M} = -k_*$, Eq. (F.21) gives

$$
\begin{aligned}
Q_{11}^{SOE,PPT} = Q_{11}^{SOE,IPT} &= -\mathcal{W}_{yy} - 2p_*\mathcal{W}_{yz} - p_*^2\mathcal{W}_{zz} - \mathcal{W}_z z_{yy}, \\
Q_{12}^{SOE,IPT} &= -k_*\mathcal{W}_{yz} - p_*\mathcal{W}_{z\psi} - p_*k_*\mathcal{W}_{zz} - \mathcal{W}_z z_{y\psi}, \\
Q_{22}^{SOE,IPT} &= -\mathcal{W}_{\psi\psi} - 2k_*\mathcal{W}_{z\psi} - k_*^2\mathcal{W}_{zz} - \mathcal{W}_z z_{\psi\psi}, \\
B_1^{SOE,PPT},\ B_1^{SOE,IPT} &= -k_*\mathcal{W}_{yz} + p_*\mathcal{W}_{z\tau_M} - p_*k_*\mathcal{W}_{zz} - \mathcal{W}_z z_{y\psi}, \\
B_2^{SOE,IPT} &= \mathcal{W}_{\psi\tau_M} - k_*\mathcal{W}_{z\psi} + k_*\mathcal{W}_{z\tau_M} - k_*^2\mathcal{W}_{zz} - \mathcal{W}_z(k_* + z_{\psi\psi}), \qquad \text{(F.24)}
\end{aligned}
$$

where $B_1^{SOE,PPT}$ and $B_1^{SOE,IPT}$ use the PPT and IPT values of $\mathcal{W}_{z\tau_M}$, respectively. Under the baseline parameterization,

$$
\begin{aligned}
Q^{SOE,PPT} &= \begin{pmatrix}1.6542172374\end{pmatrix}, \quad B^{SOE,PPT} = \begin{pmatrix}0.0991577246\end{pmatrix}, \\
Q^{SOE,IPT} &= \begin{pmatrix} 1.6542172374 & -0.0036494190 \\ -0.0036494190 & 0.5062890263 \end{pmatrix}, \\
B^{SOE,IPT} &= \begin{pmatrix}0.1358745616 \\ 0.2128527417\end{pmatrix}. \qquad \text{(F.25)}
\end{aligned}
$$

Completing the squares, with the output square first, gives the welfare-relevant variables in Eqs. (21) and (22):

$$
y_t^{w,SOE,PPT} \equiv \frac{B_1^{SOE,PPT}}{Q_{11}^{SOE,PPT}}\tau_{M,t}, \qquad \text{(F.26)}
$$

$$
y_t^{w,SOE,IPT} \equiv \frac{B_1^{SOE,IPT}}{Q_{11}^{SOE,IPT}}\tau_{M,t} - \frac{Q_{12}^{SOE,IPT}}{Q_{11}^{SOE,IPT}}\psi_t, \qquad \text{(F.27)}
$$

and

$$
\psi_t^{w,SOE,IPT} \equiv \frac{B_2^{SOE,IPT} - Q_{12}^{SOE,IPT}B_1^{SOE,IPT}/Q_{11}^{SOE,IPT}}{\Lambda_\psi^{SOE,IPT}}\tau_{M,t} \equiv \Gamma_{\text{own}}^{SOE}\tau_{M,t}, \qquad \text{(F.28)}
$$

with the real weights $\Lambda_y^{SOE,PPT} = Q_{11}^{SOE,PPT}$, $\Lambda_y^{SOE,IPT} = Q_{11}^{SOE,IPT}$, and

$$
\Lambda_\psi^{SOE,IPT} \equiv Q_{22}^{SOE,IPT} - \frac{(Q_{12}^{SOE,IPT})^2}{Q_{11}^{SOE,IPT}}. \qquad \text{(F.29)}
$$

The reference value in Eq. (F.28) depends only on the tariff and is the one entering the LOOP-gap square in Eq. (22); the main-text coefficient $\Gamma_{\text{own}}^{SOE}$ in Eq. (23) is the entire coefficient on $\tau_{M,t}$. Under the baseline parameterization, $y_t^{w,SOE,PPT} = (0.0991577246/1.6542172374)\tau_{M,t}$, $\Lambda_\psi^{SOE,IPT} = 0.5062809752$, and

$$
\begin{aligned}
\Gamma_{\text{own}}^{SOE} &= \frac{0.2128527417}{0.5062809752} + \frac{0.0036494190 \cdot 0.1358745616}{1.6542172374 \cdot 0.5062809752} \\
&= 0.4204241362 + 0.0005920764 = 0.4210162126.
\end{aligned}
$$

The tariff–LOOP component $B_2^{SOE,IPT}/\Lambda_\psi^{SOE,IPT}$ accounts for most of $\Gamma_{\text{own}}^{SOE}$; the output-interaction term contributes the remainder. Separating $\mathcal{W}_{\psi\tau_M}$ from the other terms of $B_2^{SOE,IPT}$ in Eq. (F.24) splits the tariff–LOOP component further:

$$
\Gamma_{\text{own}}^{SOE} = \frac{\mathcal{W}_{\psi\tau_M}}{\Lambda_\psi^{SOE,IPT}} + \frac{B_2^{SOE,IPT} - \mathcal{W}_{\psi\tau_M}}{\Lambda_\psi^{SOE,IPT}} - \frac{Q_{12}^{SOE,IPT}B_1^{SOE,IPT}}{Q_{11}^{SOE,IPT}\Lambda_\psi^{SOE,IPT}} = 0.5557 - 0.1353 + 0.0006.
$$

The first term is the tariff–LOOP interaction in the purchaser price and in the importer restriction at a given border price. The second is the effect of the adjustment of the *ex-tariff* border price required by goods-market clearing and risk sharing, including its second derivatives. The third is the output–LOOP cross term.

Because $\Lambda_y^{SOE,IPT} > 0$ and $\Lambda_\psi^{SOE,IPT} > 0$, $Q^{SOE,IPT}$ is positive definite, and the quadratic form in (y_t, ψ_t) attains its unique unconstrained minimum at $(Q^{SOE,IPT})^{-1} B^{SOE,IPT} \tau_{M,t}$. The welfare-relevant LOOP gap, $\Gamma_{\text{own}}^{SOE} \tau_{M,t}$, equals the ψ-coordinate of this minimizer. If the squares are completed in the reverse order, the target for ψ_t depends on y_t; evaluated at the minimizing y_t, it takes the same value. Hence $\Gamma_{\text{own}}^{SOE}$, and in particular its sign, does not depend on the order of completion.

The dispersion terms give the inflation weights. A dispersion increase $\Delta_{H,t} - 1$ enters the augmented welfare contribution with coefficient $-Y^{1+\varphi} + (1+\varphi)(Y/Z)\Theta_H^{AS}$, and the importer dispersion term enters with coefficient $C^{1-\eta}\Theta_M^{AS}$. By Appendix F.1.3,

$$\begin{aligned}
\Lambda_{\pi,H}^{SOE,PPT} &= \Lambda_{\pi,H}^{SOE,IPT} = \frac{\varepsilon}{\kappa_H}\left[Y^{1+\varphi} - (1+\varphi)(Y/Z)\Theta_H^{AS}\right] \\
&= \frac{\varepsilon}{\kappa_H}\frac{Y^{1+\varphi}(p_*-1) + (1+\varphi)(1-\alpha_F)p_*}{\varphi + p_*}, \\
\Lambda_{\pi,F}^{SOE,IPT} &= -\frac{\varepsilon_M}{\kappa_M} C^{1-\eta}\Theta_M^{AS} = \frac{\varepsilon_M}{\kappa_M}(\alpha_F + k_* \mathcal{W}_z).
\end{aligned} \tag{F.30}$$

The corresponding baseline values are

$$\begin{aligned}
\Lambda_y^{SOE,IPT} &= 1.6542172374, \qquad \Lambda_\psi^{SOE,IPT} = 0.5062809752, \\
\Lambda_{\pi,H}^{SOE,IPT} &= 39.3775970086, \qquad \Lambda_{\pi,F}^{SOE,IPT} = 27.2946757381.
\end{aligned}$$

For the high-substitution specification $\eta = 3$, holding all other primitive parameters fixed, the corresponding weights are $\Lambda_y^{SOE,IPT} = 0.7493034093$, $\Lambda_\psi^{SOE,IPT} = 1.2406311889$, $\Lambda_{\pi,H}^{SOE,IPT} = 19.3033398990$, and $\Lambda_{\pi,F}^{SOE,IPT} = 37.2065139728$, with $\Gamma_{\text{own}}^{SOE} = 0.2927934031$. These coefficients were checked by computing the exact steady state in Eq. (F.1), eliminating z_t by implicit differentiation, and comparing the resulting Hessian with finite-difference calculations; the Calvo inflation-square factor was verified separately on the nonlinear reset-price index. Collecting the terms gives Eqs. (21) and (22) in the main text.

F.2 Unreduced SOE Welfare Cost Functions

This subsection reports the SOE welfare cost functions without eliminating the *ex-tariff* relative border price z_t. Suppressing terms that are independent of the Home policy allocation, under PPT,

$$\mathcal{L}_t^{SOE,PPT} \equiv \frac{1}{2}\left[\widehat{\Lambda}_y^{SOE,PPT}(y_t - \widehat{y}_t^{w,SOE,PPT})^2 + \widehat{\Lambda}_z^{SOE,PPT}(z_t - \widehat{z}_t^{w,SOE,PPT})^2 + \Lambda_{\pi,H}^{SOE,PPT}\pi_{H,t}^2\right],$$

whereas under IPT it can be written as

$$\begin{aligned}
\mathcal{L}_t^{SOE,IPT} \equiv \frac{1}{2}\Big[&\widehat{\Lambda}_y^{SOE,IPT}(y_t - \widehat{y}_t^{w,SOE,IPT})^2 + \widehat{\Lambda}_\psi^{SOE,IPT}(\psi_t - \widehat{\psi}_t^{w,SOE,IPT})^2 \\
&+\widehat{\Lambda}_z^{SOE,IPT}(z_t - \widehat{z}_t^{w,SOE,IPT})^2 + \Lambda_{\pi,H}^{SOE,IPT}\pi_{H,t}^2 + \Lambda_{\pi,F}^{SOE,IPT}\pi_{F,t}^2\Big].
\end{aligned}$$

Here the hatted Λ coefficients are the completed-square welfare weights associated with y_t, z_t, and, under IPT, ψ_t in the expressions above. The corresponding hatted welfare-relevant reference values are $\widehat{y}_t^{w,SOE,\cdot}$, $\widehat{z}_t^{w,SOE,\cdot}$, and $\widehat{\psi}_t^{w,SOE,IPT}$. Their derivation follows.

The contrast between the two expressions shows how the relative-price externality discussed in Sec. 4.4 enters the SOE welfare criterion. Under PPT, the LOOP holds, so the externality is carried by the deviation of z_t from its welfare-relevant value, $z_t - \widehat{z}_t^{w,SOE,PPT}$. Under IPT, the

welfare criterion contains this deviation, $z_t - \widehat{z}_t^{w,SOE,IPT}$, together with the welfare-relevant LOOP-gap deviation, $\psi_t - \widehat{\psi}_t^{w,SOE,IPT}$. Because the *ex-tariff* import price $P_{F,t}$ adjusts only gradually, a nominal exchange-rate movement changes both Z_t and Ψ_t, so the externality operates jointly through z_t and ψ_t.

These unreduced representations retain z_t rather than eliminate it. As in the Benigno–Woodford treatment of the national problem, attach the stationary multiplier $-p_*\mathcal{W}_z$ to the exact Home goods-market residual $\mathcal{R} = 0$ defined after Eq. (F.10). Thus the real contribution is $\mathcal{W} - p_*\mathcal{W}_z\mathcal{R}$, which agrees with $\mathcal{W}$ on the equilibrium manifold. Since $\mathcal{W}_y = -p_*\mathcal{W}_z$, $\mathcal{W}_\psi = -k_*\mathcal{W}_z$ (under IPT), and $\mathcal{R}_y = -1$, $\mathcal{R}_z = 1/p_*$, and $\mathcal{R}_\psi = -k_*/p_*$, the augmented contribution has no linear terms in the real coordinates y_t, z_t, and ψ_t. Under PPT, write its real part, up to terms independent of the allocation, as

$$\begin{aligned}\mathcal{L}_{t,\text{real}}^{SOE,PPT} &= \frac{1}{2}\left[-2\widehat{C}_{yy}^{SOE,PPT}y_t^2 - 2\widehat{C}_{zz}^{SOE,PPT}z_t^2\right] - \widehat{C}_{yz}^{SOE,PPT}y_t z_t \\ &\quad -\widehat{C}_{ym}^{SOE,PPT}y_t\tau_{M,t} - \widehat{C}_{zm}^{SOE,PPT}z_t\tau_{M,t} + \text{s.o.t.i.p.},\end{aligned} \tag{F.31}$$

where, for $a, b \in \{y, z, \psi\}$ (with ψ absent under PPT), $\widehat{C}_{aa} = (\mathcal{W}_{aa} - p_*\mathcal{W}_z\mathcal{R}_{aa})/2$, $\widehat{C}_{ab} = \mathcal{W}_{ab} - p_*\mathcal{W}_z\mathcal{R}_{ab}$ for distinct real coordinates, and $\widehat{C}_{am} = \mathcal{W}_{a\tau_M} - p_*\mathcal{W}_z\mathcal{R}_{a\tau_M}$. The derivatives of $\mathcal{W}$ and $\mathcal{R}$ are given in Appendix F.1.6 and Eq. (F.11), respectively. Because $\mathcal{R}_{ya} = 0$ for every coordinate a, $\widehat{C}_{ym} = \widehat{C}_{y\psi} = 0$ remains valid. The multiplier changes only the five coefficients $\widehat{C}_{zz}$, $\widehat{C}_{z\psi}$, $\widehat{C}_{\psi\psi}$, $\widehat{C}_{zm}$, and $\widehat{C}_{\psi m}$ (with the ψ coefficients absent under PPT). Completing the y_t square first, then the z_t square, gives

$$\begin{aligned}\widehat{\Lambda}_y^{SOE,PPT} &= -2\widehat{C}_{yy}^{SOE,PPT}, \\ \widehat{y}_t^{w,SOE,PPT} &= \frac{\widehat{C}_{yz}^{SOE,PPT}z_t + \widehat{C}_{ym}^{SOE,PPT}\tau_{M,t}}{\widehat{\Lambda}_y^{SOE,PPT}}, \\ \widehat{\Lambda}_z^{SOE,PPT} &= -2\widehat{C}_{zz}^{SOE,PPT} - \frac{(\widehat{C}_{yz}^{SOE,PPT})^2}{\widehat{\Lambda}_y^{SOE,PPT}}, \\ \widehat{z}_t^{w,SOE,PPT} &= \frac{\widehat{C}_{zm}^{SOE,PPT} + \widehat{C}_{yz}^{SOE,PPT}\widehat{C}_{ym}^{SOE,PPT}/\widehat{\Lambda}_y^{SOE,PPT}}{\widehat{\Lambda}_z^{SOE,PPT}}\tau_{M,t}.\end{aligned} \tag{F.32}$$

These are square-completion identities; they do not treat y_t and z_t as independently feasible. Under IPT, the corresponding real polynomial is

$$\begin{aligned}\mathcal{L}_{t,\text{real}}^{SOE,IPT} &= \frac{1}{2}\left[-2\widehat{C}_{yy}^{SOE,IPT}y_t^2 - 2\widehat{C}_{\psi\psi}^{SOE,IPT}\psi_t^2 - 2\widehat{C}_{zz}^{SOE,IPT}z_t^2\right] \\ &\quad -\widehat{C}_{y\psi}^{SOE,IPT}y_t\psi_t - \widehat{C}_{yz}^{SOE,IPT}y_t z_t - \widehat{C}_{\psi z}^{SOE,IPT}\psi_t z_t \\ &\quad -\widehat{C}_{ym}^{SOE,IPT}y_t\tau_{M,t} - \widehat{C}_{\psi m}^{SOE,IPT}\psi_t\tau_{M,t} - \widehat{C}_{zm}^{SOE,IPT}z_t\tau_{M,t} + \text{s.o.t.i.p.}\end{aligned} \tag{F.33}$$

and completion in the order (y_t, ψ_t, z_t) gives

$$\begin{aligned}\widehat{\Lambda}_y^{SOE,IPT} &= -2\widehat{C}_{yy}^{SOE,IPT}, \\ \widehat{y}_t^{w,SOE,IPT} &= \frac{\widehat{C}_{y\psi}^{SOE,IPT}\psi_t + \widehat{C}_{yz}^{SOE,IPT}z_t + \widehat{C}_{ym}^{SOE,IPT}\tau_{M,t}}{\widehat{\Lambda}_y^{SOE,IPT}}, \\ \widehat{\Lambda}_\psi^{SOE,IPT} &= -2\widehat{C}_{\psi\psi}^{SOE,IPT} - \frac{(\widehat{C}_{y\psi}^{SOE,IPT})^2}{\widehat{\Lambda}_y^{SOE,IPT}}, \\ \widehat{\psi}_t^{w,SOE,IPT} &= \frac{1}{\widehat{\Lambda}_\psi^{SOE,IPT}}\left[\left(\widehat{C}_{\psi z}^{SOE,IPT} + \frac{\widehat{C}_{y\psi}^{SOE,IPT}\widehat{C}_{yz}^{SOE,IPT}}{\widehat{\Lambda}_y^{SOE,IPT}}\right)z_t\right. \\ &\qquad \left. + \left(\widehat{C}_{\psi m}^{SOE,IPT} + \frac{\widehat{C}_{y\psi}^{SOE,IPT}\widehat{C}_{ym}^{SOE,IPT}}{\widehat{\Lambda}_y^{SOE,IPT}}\right)\tau_{M,t}\right], \\ \widehat{\Lambda}_z^{SOE,IPT} &= -2\widehat{C}_{zz}^{SOE,IPT} - \frac{(\widehat{C}_{yz}^{SOE,IPT})^2}{\widehat{\Lambda}_y^{SOE,IPT}} - \frac{\left(\widehat{C}_{\psi z}^{SOE,IPT} + \widehat{C}_{y\psi}^{SOE,IPT}\widehat{C}_{yz}^{SOE,IPT}/\widehat{\Lambda}_y^{SOE,IPT}\right)^2}{\widehat{\Lambda}_\psi^{SOE,IPT}},\end{aligned}$$

$$\widehat{z}_t^{w,SOE,IPT} = \frac{\tau_{M,t}}{\widehat{\Lambda}_z^{SOE,IPT}}\Bigg[\widehat{C}_{zm}^{SOE,IPT} + \frac{\widehat{C}_{yz}^{SOE,IPT}\widehat{C}_{ym}^{SOE,IPT}}{\widehat{\Lambda}_y^{SOE,IPT}} + \frac{\widehat{C}_{\psi z}^{SOE,IPT} + \widehat{C}_{y\psi}^{SOE,IPT}\widehat{C}_{yz}^{SOE,IPT}/\widehat{\Lambda}_y^{SOE,IPT}}{\widehat{\Lambda}_\psi^{SOE,IPT}}\left(\widehat{C}_{\psi m}^{SOE,IPT} + \frac{\widehat{C}_{y\psi}^{SOE,IPT}\widehat{C}_{ym}^{SOE,IPT}}{\widehat{\Lambda}_y^{SOE,IPT}}\right)\Bigg] \tag{F.34}$$

In particular, the unreduced $\widehat{y}_t^{w,SOE,IPT}$ can depend on z_t, whereas the reduced welfare-relevant output in Eq. (F.27) does not.

The coefficients $\widehat{C}$ are explicit functions of the derivatives in Eqs. (F.11), (F.17), and (F.22). Since $\mathcal{W}_{y\tau_M} = \mathcal{W}_{y\psi} = \mathcal{R}_{y\tau_M} = \mathcal{R}_{y\psi} = 0$, Eqs. (F.32) and (F.34) reduce to

$$\begin{aligned} \widehat{\Lambda}_y^{SOE,PPT} &= \widehat{\Lambda}_y^{SOE,IPT} = -\mathcal{W}_{yy}, \qquad \widehat{y}_t^{w,SOE,PPT} = \widehat{y}_t^{w,SOE,IPT} = \frac{\mathcal{W}_{yz}}{-\mathcal{W}_{yy}} z_t, \\ \widehat{\Lambda}_z^{SOE,PPT} &= -\mathcal{W}_{zz} + p_*\mathcal{W}_z\mathcal{R}_{zz} - \frac{\mathcal{W}_{yz}^2}{-\mathcal{W}_{yy}}, \widehat{z}_t^{w,SOE,PPT} = \frac{\mathcal{W}_{z\tau_M} - p_*\mathcal{W}_z\mathcal{R}_{z\tau_M}}{\widehat{\Lambda}_z^{SOE,PPT}}\tau_{M,t}, \end{aligned} \tag{F.35}$$

and, under IPT,

$$\begin{aligned} \widehat{\Lambda}_\psi^{SOE,IPT} &= -\mathcal{W}_{\psi\psi} + p_*\mathcal{W}_z\mathcal{R}_{\psi\psi}, \\ \widehat{\psi}_t^{w,SOE,IPT} &= \frac{(\mathcal{W}_{z\psi} - p_*\mathcal{W}_z\mathcal{R}_{z\psi})z_t}{\widehat{\Lambda}_\psi^{SOE,IPT}} + \frac{(\mathcal{W}_{\psi\tau_M} - p_*\mathcal{W}_z\mathcal{R}_{\psi\tau_M})\tau_{M,t}}{\widehat{\Lambda}_\psi^{SOE,IPT}}, \\ \widehat{\Lambda}_z^{SOE,IPT} &= -\mathcal{W}_{zz} + p_*\mathcal{W}_z\mathcal{R}_{zz} - \frac{\mathcal{W}_{yz}^2}{-\mathcal{W}_{yy}} \\ &\quad - \frac{(\mathcal{W}_{z\psi} - p_*\mathcal{W}_z\mathcal{R}_{z\psi})^2}{\widehat{\Lambda}_\psi^{SOE,IPT}}, \\ \widehat{z}_t^{w,SOE,IPT} &= \frac{\tau_{M,t}}{\widehat{\Lambda}_z^{SOE,IPT}}\Bigg[\mathcal{W}_{z\tau_M} - p_*\mathcal{W}_z\mathcal{R}_{z\tau_M} \\ &\quad + \frac{(\mathcal{W}_{z\psi} - p_*\mathcal{W}_z\mathcal{R}_{z\psi})(\mathcal{W}_{\psi\tau_M} - p_*\mathcal{W}_z\mathcal{R}_{\psi\tau_M})}{\widehat{\Lambda}_\psi^{SOE,IPT}}\Bigg], \end{aligned} \tag{F.36}$$

where $\mathcal{W}_{z\tau_M}$ takes its PPT or IPT value in Eq. (F.22). At the baseline calibration, $\widehat{\Lambda}_z^{SOE,PPT} = -0.0590220689$ and $\widehat{\Lambda}_z^{SOE,IPT} = -0.0591082287$. Thus the unreduced quadratic is not positive definite over unrestricted (y_t, z_t, ψ_t). On the goods-market-clearing manifold, substituting the first-order relation $z_t = p_* y_t + k_*(\psi_t - \tau_{M,t})$ into the augmented quadratic recovers exactly the reduced Q and B of Eq. (F.24) and the positive-definite reduced welfare cost functions. The multiplier on the exact restriction accounts for the second-order terms in z_t without requiring its second-order substitution in this unreduced representation.

F.3 Two-Country Economy

This subsection derives the Home welfare cost functions in Eqs. (28) and (29), their welfare-relevant variables, and every weight in Table 3 from the structural parameters in Table 1. The steps are those of Appendix F.1, applied to the Home national problem; the Foreign national criterion is obtained by interchanging the country labels, replacing ν by $1-\nu$, and using $-z_t$ as the Foreign *ex-tariff* border-price coordinate.

F.3.1 Steady State, Notation, and the National Problem

At the zero-tariff, zero-inflation deterministic steady state, $Z = \Psi = \Psi^* = 1$, $P/P_H = P^*/P_F^* = 1$, and $C = C^* = Y = Y^* = N = N^*$. The producer reset condition, Eq. (B.12), gives $MC^r = \mathcal{M}^{-1}$, and Eq. (B.14) then gives $N^{1+\varphi} = \mathcal{M}^{-1}$. Because $\{(1-\nu)/\nu\}\omega_H^* = \omega_F$, Home exports absorb the share ω_F of Home output and Home households absorb the share ω_H. Hence, $z_t = \log Z_t$

and $g_t = \log \mathcal{G}_t$, where $\mathcal{G}_t$ is the Home CPI–PPI ratio of the environment under consideration. In addition, $g_t^* \equiv \log(P_t^*/P_{F,t}^*)$, $s_t \equiv \log S_t$, and $s_t^* \equiv \log(P_{H,t}^*/P_{F,t}^*)$.

As in Appendix D, the Home planner chooses the Home allocation, namely c_t, g_t, y_t, n_t, the Home producer reset price and, under IPT, also s_t and the Home importer reset price, while taking the Foreign plan (c_t^* and, under IPT, s_t^* and the Foreign importer reset price) as given. Because the Foreign CPI relation enters the Home problem (Sec. 4.2), g_t^* is also a Home coordinate. Complete international risk sharing, Eq. (C.11), is substituted before differentiation and is exact in logs:

$$z_t = c_t + g_t - c_t^* - g_t^*. \tag{F.37}$$

Under IPT, the definitions of the two LOOP gaps give the exact identities $\psi_t = z_t - s_t$ and $\psi_t^* = -z_t - s_t^*$. Let $\widetilde{p}_{F,t} \equiv \log(\widetilde{P}_{F,t}/P_{H,t})$ and $\widetilde{p}_{H,t}^* \equiv \log(\widetilde{P}_{H,t}^*/P_{F,t}^*)$ denote the logarithms of the *cum-tariff* relative prices of imports faced by Home and Foreign households, respectively. Then

$$\begin{aligned} &\text{PPT:} \qquad \widetilde{p}_{F,t} = z_t + \log(1+\tau_{M,t}), \qquad \widetilde{p}_{H,t}^* = -z_t + \log(1+\tau_{M,t}^*), \\ &\text{IPT:} \qquad \widetilde{p}_{F,t} = s_t + \log\left(1+\tau_{M,t}e^{\psi_t}\right) + O(3), \qquad \widetilde{p}_{H,t}^* = s_t^* + \log\left(1+\tau_{M,t}^* e^{\psi_t^*}\right) + O(3) \end{aligned} \tag{F.38}$$

The IPT line follows from the additive-duty purchaser price in Eq. (2), $\widetilde{P}_{F,t}^{IPT}(\iota)/P_{F,t} = P_{F,t}(\iota)/P_{F,t} + \tau_{M,t}\Psi_t$, and its Foreign mirror. Because the *ex-tariff* index and the *cum-tariff* index are CES aggregates with the same elasticity ε_M, cross-sectional importer-price dispersion cancels from $\log(\widetilde{P}_{F,t}^{IPT}/P_{F,t})$ up to third order. In particular, the second-order expansion $\widetilde{p}_{H,t}^* = s_t^* + \tau_{M,t}^* + \tau_{M,t}^*\psi_t^* - (\tau_{M,t}^*)^2/2 + O(3)$ contains the interaction $\tau_{M,t}^*\psi_t^*$, which is retained below.

F.3.2 Second-Order Approximation of Utility

Since $N^{1+\varphi} = \mathcal{M}^{-1}$ in the zero-tariff steady state, the second-order expansion of the period utility in Eq. (1), before the linear terms are eliminated, is

$$u_{H,t} - u_H = c_t - \mathcal{M}^{-1}y_t - \frac{(1+\varphi)\mathcal{M}^{-1}}{2}y_t^2 - \mathcal{M}^{-1}\log\Delta_{H,t} + O(3). \tag{F.39}$$

Here $\log\Delta_{H,t}$ starts at second order around the zero-inflation steady state. Equation (F.39) alone is *not* a valid second-order policy loss around a distorted steady state. The Benigno–Woodford construction first cancels $c_t - \mathcal{M}^{-1}y_t$ using the first-order equilibrium restrictions and the multipliers in Appendix F.3.5. The remaining producer- and importer-Calvo dispersion terms contribute the inflation weights below.

F.3.3 Second-Order Approximation of Price Dispersion: Calvo Blocks

Applying Eq. (F.6) of Appendix F.1.3 to each Calvo block gives its zero-dispersion term and its dispersion term. For Home producers, Eq. (B.12) is equivalent to the reset condition with

$$\mathcal{F}_H(u, X_t) = e^{a_{H,t}}e^{-\varepsilon u}\left(e^u - e^{mc_{H,t}}\right), \qquad a_{H,t} \equiv y_t - c_t - g_t, \qquad mc_{H,t} = c_t + \varphi n_t + g_t,$$

where $a_{H,t} = \log\{Y_tP_{H,t}/(C_tP_t)\}$ comes from the household stochastic discount factor and $mc_{H,t} \equiv \log(\mathcal{M}MC_t^r)$ from Eq. (B.14). Since $\mathcal{F}_u = 1$ and $\mathcal{F}_{uu} = 1 - 2\varepsilon$ at the steady state, defining $\Theta_H^{AS} \equiv -\Theta_{R,H}/(1-\theta)$ gives

$$\mathcal{C}_{H,t} = \Theta_H^{AS}\left[mc_{H,t} + a_{H,t}mc_{H,t} + \frac{1}{2}mc_{H,t}^2 + \frac{\varepsilon}{2}V_{H,t}\right] + O(3). \tag{F.40}$$

For Home importers, Eq. (16) with $\tau = 1/\varepsilon_M$ is equivalent to the reset condition with

$$\mathcal{F}_M(u, X_t) = e^{a_{M,t}}e^u\left[\frac{e^u + \tau_{M,t}e^{\psi_t}}{1+\tau_{M,t}e^{\psi_t}}\right]^{-\varepsilon_M}\left[1 - \varepsilon_M\frac{e^u - (1-\varepsilon_M^{-1})e^{\psi_t}}{e^u + \tau_{M,t}e^{\psi_t}}\right],$$

where $a_{M,t} \equiv c_{F,t} + s_t - c_t - g_t = \log\{C_{F,t}P_{F,t}/(C_tP_t)\}$ and $c_{F,t} = c_t - \eta(\widetilde{p}_{F,t} - g_t)$ by Eq. (6). At the steady state $\mathcal{F}_u = -(\varepsilon_M - 1)$ and $\mathcal{F}_{uu} = (\varepsilon_M - 1)(2\varepsilon_M - 1)$, so defining $\Theta_M^{AS} \equiv (\varepsilon_M - 1)\Theta_{R,M}/(1 - \theta_M)$ gives

$$\mathcal{C}_{M,t} = \Theta_M^{AS}\left[e^{a_{M,t}}\frac{e^{\psi_t} - 1 + \tau_{M,t}e^{\psi_t}/(\varepsilon_M - 1)}{1 + \tau_{M,t}e^{\psi_t}} + \frac{\varepsilon_M}{2}V_{M,t}\right] + O(3). \tag{F.41}$$

The linear part of Eq. (F.41) is $\Theta_M^{AS}\{\psi_t + \tau_{M,t}/(\varepsilon_M - 1)\}$, the driving term of Eq. (18). Finally, $\widetilde{\Delta}_{H,t}^*$ in Eq. (F.42) is the quantity factor of the Foreign importer block; by the same argument, $\log\widetilde{\Delta}_{H,t}^* = \varepsilon_M V_{H,t}^*/2 + O(3)$ and $\sum_s \beta^s V_{H,s}^* = \kappa_M^{-1}\sum_s \beta^s(\pi_{H,s}^*)^2 + \text{t.i.p.}$

F.3.4 Second-Order Approximation of the Equilibrium Conditions

The Home CPI, the Foreign CPI, and Home goods-market clearing are approximated to second order by attaching their stationary multipliers, together with the Calvo blocks above. Because the steady-state multipliers are constant from the timeless perspective, the Home Lagrangian of Appendix D can be written, up to terms independent of policy, as $\mathrm{E}_0\sum_t \beta^t \mathcal{J}_{H,t}$ with

$$\begin{aligned}\mathcal{J}_{H,t} \;=\;& c_t - \frac{L}{1+\varphi}e^{(1+\varphi)n_t} + \Theta_g\left[g_t - \frac{1}{1-\eta}\log\left\{\omega_H + \omega_F e^{(1-\eta)\widetilde{p}_{F,t}}\right\}\right] \\ &+\Theta_{g^*}\left[g_t^* - \frac{1}{1-\eta}\log\left\{\omega_F^* + \omega_H^* e^{(1-\eta)\widetilde{p}_{H,t}^*}\right\}\right] \\ &+\Theta_Y\left[y_t - \log\left\{\omega_H e^{c_t+\eta g_t} + \omega_F e^{c_t^* - \eta(\widetilde{p}_{H,t}^* - g_t^*)}\widetilde{\Delta}_{H,t}^*\right\}\right] + \mathcal{C}_{H,t} + \mathcal{C}_{M,t}.\end{aligned} \tag{F.42}$$

Here $n_t = y_t + \log\Delta_{H,t}$ by Eq. (B.11); the first bracket represents the Home CPI restriction, Eq. (C.3) or Eq. (C.9); the bracket with Θ_{g^*} represents the Foreign CPI restriction, which constrains the Home problem under the national constraint sets in Sec. 4.2; the bracket with Θ_Y represents the Home goods-market-clearing condition, Eq. (C.8) or Eq. (C.16), after substituting the bilateral demands in Eqs. (C.4) and (C.10); $\widetilde{\Delta}_{H,t}^* \equiv (1-\nu)^{-1}\int_\nu^1(\widetilde{P}_{H,t}^*(\iota)/\widetilde{P}_{H,t}^*)^{-\varepsilon_M}d\iota$ is the physical-quantity factor in Eq. (C.16) ($\widetilde{\Delta}_{H,t}^* = 1$ under PPT); and $\mathcal{C}_{H,t}$ and $\mathcal{C}_{M,t}$ are the Calvo contributions of Home producers and Home importers in Eqs. (F.40) and (F.41) ($\mathcal{C}_{M,t} = 0$ under PPT). The multiplier on the Euler equation is zero at a steady state with $i > 0$. Expressing the CPI and goods-market restrictions in levels rather than logs, or retaining z_t as a separate variable with a multiplier on Eq. (F.37), yields the same reduced Hessian on the first-order equilibrium set.

The second-order terms of these restrictions are evaluated on the first-order equilibrium set. Under IPT, the first-order identities are

$$\begin{aligned} g_t &= \omega_F(z_t - \psi_t + \tau_{M,t}), \\ g_t^* &= \omega_H^*(-z_t - \psi_t^* + \tau_{M,t}^*), \\ c_t &= y_t + \omega_F(z_t - g_t + g_t^*) - \eta\{\omega_H g_t + \omega_F(g_t^* + z_t + \psi_t^* - \tau_{M,t}^*)\}, \\ c_t^* &= c_t - z_t + g_t - g_t^*, \\ mc_{H,t}^{(1)} &= c_t + \varphi y_t + g_t. \end{aligned} \tag{F.43}$$

The first two identities follow from a first-order Taylor expansion of the Home and Foreign CPI indices in Appendix C.2. The third and fourth identities follow by log-linearizing the international risk-sharing and goods-market-clearing conditions in Appendix C.2. The fifth follows from the Home household labor-supply condition and the definition of producer real marginal cost. Thus every expression on the right-hand side of Eq. (F.43) is linear in the four real coordinates and two tariffs. Under PPT, the same identities hold with $\psi_t = \psi_t^* = 0$.

F.3.5 Eliminating the Linear Terms

The steady-state multipliers are determined by the first-order stationarity conditions of the Home national Lagrangian $\mathcal{J}_{H,t}$ with respect to the Home choice variables. Equivalently, n_t and $mc_{H,t}$ can be kept as separate variables with multipliers Θ_n on $n_t = y_t + \log \Delta_{H,t}$ and $\Theta_{mc,H}$ on $mc_{H,t} = c_t + \varphi n_t + g_t$; the conditions with respect to n_t and $mc_{H,t}$ are then $-\mathcal{M}^{-1} + \Theta_n - \varphi\Theta_{mc,H} = 0$ and $\Theta_{mc,H} + \Theta_H^{AS} = 0$. Under IPT, $\Theta_{mc,M}$ is attached in the same way to the LOOP-gap identity $\psi_t = z_t - s_t$, so that $\Theta_{mc,M} + \Theta_M^{AS} = 0$. The condition with respect to g_t^* determines Θ_{g^*}. The resulting linear systems are Eq. (F.44) for PPT and Eq. (F.46) for IPT.

PPT. Under PPT, $\psi_t = \psi_t^* = 0$, so $s_t = z_t$ and $s_t^* = -z_t$. To first order, Eq. (F.38) gives $\widetilde{p}_{F,t} = z_t + \tau_{M,t}$ and $\widetilde{p}_{H,t}^* = \tau_{M,t}^* - z_t$. The Home choice variables are c_t, g_t, g_t^*, y_t, n_t, and $mc_{H,t}$. Attaching Θ_g^{PPT}, $\Theta_{g^*}^{PPT}$, Θ_Y^{PPT}, Θ_n^{PPT}, $\Theta_{mc,H}^{PPT}$, and $\Theta_H^{AS,PPT}$ to the Home CPI, Foreign CPI, Home goods-market, labor, producer-marginal-cost, and producer aggregate-supply restrictions, the absence of first-order terms in c_t, g_t, g_t^*, y_t, n_t, and $mc_{H,t}$ requires

$$\begin{aligned}
0 &= 1 - \omega_F\Theta_g^{PPT} + \omega_H^*\Theta_{g^*}^{PPT} - (\omega_H + \eta\omega_F)\Theta_Y^{PPT} - \Theta_{mc,H}^{PPT},\\
0 &= \omega_H\Theta_g^{PPT} + \omega_H^*\Theta_{g^*}^{PPT} - \eta\Theta_Y^{PPT} - \Theta_{mc,H}^{PPT},\\
0 &= \omega_F\Theta_g^{PPT} + \omega_F^*\Theta_{g^*}^{PPT},\\
0 &= \Theta_Y^{PPT} - \Theta_n^{PPT},\\
0 &= -\mathcal{M}^{-1} + \Theta_n^{PPT} - \varphi\Theta_{mc,H}^{PPT},\\
0 &= \Theta_{mc,H}^{PPT} + \Theta_H^{AS,PPT}.
\end{aligned} \tag{F.44}$$

Under PPT, c_t and g_t move z_t one-for-one, and g_t^* moves it one-for-one in the opposite direction, through Eq. (F.37). Therefore c_t enters the Home CPI restriction with weight ω_F, the Foreign CPI restriction with weight ω_H^*, and Home exports with weight $\eta\omega_F$, whereas g_t enters the Home CPI restriction with net weight $1 - \omega_F = \omega_H$, the Foreign CPI restriction with weight ω_H^*, and goods-market clearing with weight $\eta(\omega_H + \omega_F) = \eta$. The ratio g_t^* enters the Home CPI restriction with weight ω_F and the Foreign CPI restriction with net weight $1 - \omega_H^* = \omega_F^*$. No separate relative-price multiplier arises, because z_t has been eliminated by complete risk sharing, exactly as under IPT below. Solving Eq. (F.44) gives

$$\begin{aligned}
\Theta_Y^{PPT} = \Theta_n^{PPT} &= \frac{\varphi(1-\upsilon) + \omega_F^*\mathcal{M}^{-1}}{\omega_F^*(1+\eta\varphi) - (\eta-1)\varphi(1-\upsilon)\omega_H},\\
\Theta_{mc,H}^{PPT} &= \frac{\Theta_Y^{PPT} - \mathcal{M}^{-1}}{\varphi}, \qquad \Theta_g^{PPT} = 1 + (\eta-1)\omega_H\Theta_Y^{PPT},\\
\Theta_{g^*}^{PPT} &= -\frac{\omega_F}{\omega_F^*}\Theta_g^{PPT}, \qquad \Theta_H^{AS,PPT} = -\Theta_{mc,H}^{PPT}.
\end{aligned} \tag{F.45}$$

Under the baseline parameterization,

$$\begin{aligned}
&\Theta_Y^{PPT} = \Theta_n^{PPT} = 0.661773, \qquad \Theta_{mc,H}^{PPT} = -0.034122,\\
&\Theta_g^{PPT} = 1.258422, \qquad \Theta_{g^*}^{PPT} = -0.299885, \qquad \Theta_H^{AS,PPT} = 0.034122.
\end{aligned}$$

IPT. Under IPT, s_t is also a Home choice variable, and ψ_t is kept as a separate variable with the multiplier $\Theta_{mc,M}$ on $\psi_t = c_t + g_t - c_t^* - g_t^* - s_t$, which combines $\psi_t = z_t - s_t$ with Eq. (F.37). To first order, Eq. (F.38) gives $\widetilde{p}_{F,t} = s_t + \tau_{M,t}$, and $\widetilde{p}_{H,t}^*$ does not depend on the Home choice variables. The absence of first-order terms in c_t, g_t, g_t^*, y_t, n_t, $mc_{H,t}$, ψ_t, and s_t requires

$$\begin{aligned}
0 &= 1 - \omega_H\Theta_Y - \Theta_{mc,H} - \Theta_{mc,M},\\
0 &= \Theta_g - \eta\omega_H\Theta_Y - \Theta_{mc,H} - \Theta_{mc,M},
\end{aligned}$$

$$
\begin{aligned}
0 &= \Theta_{g^*} - \eta\omega_F\Theta_Y + \Theta_{mc,M},\\
0 &= \Theta_Y - \Theta_n,\\
0 &= -\mathcal{M}^{-1} + \Theta_n - \varphi\Theta_{mc,H},\\
0 &= \Theta_{mc,H} + \Theta_H^{AS},\\
0 &= \Theta_{mc,M} + \Theta_M^{AS},\\
0 &= -\omega_F\Theta_g + \Theta_{mc,M}.
\end{aligned}
\tag{F.46}
$$

Its solution is

$$
\begin{aligned}
\Theta_Y = \Theta_n &= \frac{1 + \mathcal{M}^{-1}/(\omega_H\varphi)}{\omega_H + \eta\omega_F + 1/(\omega_H\varphi)},\\
\Theta_{mc,H} &= \frac{\Theta_Y - \mathcal{M}^{-1}}{\varphi},\\
\Theta_g &= \eta\Theta_Y + \frac{\Theta_{mc,H}}{\omega_H},\\
\Theta_{mc,M} &= \omega_F\Theta_g,\\
\Theta_H^{AS} &= -\Theta_{mc,H}, \qquad \Theta_M^{AS} = -\Theta_{mc,M},\\
\Theta_{g^*} &= \omega_F(\eta\Theta_Y - \Theta_g) = -\frac{\omega_F}{\omega_H}\Theta_{mc,H}.
\end{aligned}
\tag{F.47}
$$

All right-hand sides are functions of the structural parameters only.

F.3.6 Inflation and Output Weights

Given the multipliers, the policy-dependent second-order Home welfare contribution is $\frac{1}{2}x_t'\nabla^2\mathcal{J}_H x_t$ evaluated on the first-order equilibrium set, where x_t collects the log deviations in Appendix F.3.1 and the two tariffs. Terms linear in the Foreign variables taken as given, (c_t^*, s_t^*), are independent of the Home planner's choices and are included in s.o.t.i.p.. The Foreign-plan variables are expressed as log deviations and enter the remaining second-order terms through the first-order mapping in Eq. (F.43). The coefficients on $V_{H,t}$, $V_{M,t}$ and $V_{H,t}^*$ give the three sticky-inflation weights:

$$
\Lambda_{\pi,H} = \frac{\varepsilon}{\kappa_H}(\Theta_n - \Theta_H^{AS}), \qquad \Lambda_{\pi,F} = -\frac{\varepsilon_M}{\kappa_M}\Theta_M^{AS}, \qquad \Lambda_{\pi_H^*} = \frac{\varepsilon_M}{\kappa_M}\omega_F\Theta_Y,
\tag{F.48}
$$

where the last two are present only under IPT. On the first-order equilibrium set $n_t = y_t$ and $mc_{H,t} = (1+\varphi)y_t + (c_t + g_t - y_t)$, while Eq. (F.43) shows that $c_t + g_t - y_t$, the importer terms, the CPI term, and the goods-market term do not depend on y_t. Hence the producer term reduces to $\Theta_H^{AS}[(1+\varphi)^2 y_t^2 - (c_t + g_t - y_t)^2]/2$, output separates from all other coordinates, $y_t^{w,\cdot} = 0$, and in both environments

$$
\Lambda_y = (1+\varphi)\mathcal{M}^{-1} + (1+\varphi)^2\Theta_{mc,H} = (1+\varphi)(\Theta_n - \Theta_H^{AS}) = \frac{(1+\varphi)\kappa_H}{\varepsilon}\Lambda_{\pi,H}.
\tag{F.49}
$$

Write the real part as $\frac{1}{2}v_t'Qv_t - v_t'B\boldsymbol{\tau}_t$ + s.o.t.i.p., where $v_t = (y_t, z_t)'$ under PPT and $v_t = (y_t, \psi_t, z_t, \psi_t^*)'$ under IPT, and $\boldsymbol{\tau}_t = (\tau_{M,t}, \tau_{M,t}^*)'$. The weights and welfare-relevant variables are obtained by completing the square in one coordinate of v_t at a time, in the order listed. Start from $\Lambda_k^{(0)} = Q_{kk}$, $C_{kj}^{(0)} = -Q_{kj}$, and $h_{k,t}^{(0)} = \sum_m B_{km}\tau_{m,t}$. At step k, the welfare-relevant value of $v_{k,t}$ is

$$
v_{k,t}^w = \frac{h_{k,t}^{(k-1)} + \sum_{j>k} C_{kj}^{(k-1)} v_{j,t}}{\Lambda_k^{(k-1)}},
\tag{F.50}
$$

its weight is $\Lambda_k^{(k-1)}$, and, for $j,l > k$, the coefficients on the remaining coordinates are updated according to

$$\begin{aligned}
\Lambda_j^{(k)} &= \Lambda_j^{(k-1)} - \frac{[C_{kj}^{(k-1)}]^2}{\Lambda_k^{(k-1)}}, \\
C_{jl}^{(k)} &= C_{jl}^{(k-1)} + \frac{C_{kj}^{(k-1)} C_{kl}^{(k-1)}}{\Lambda_k^{(k-1)}}, \\
h_{j,t}^{(k)} &= h_{j,t}^{(k-1)} + \frac{h_{k,t}^{(k-1)} C_{kj}^{(k-1)}}{\Lambda_k^{(k-1)}}.
\end{aligned} \tag{F.51}$$

The tariff persistence does not enter any weight. The calculation was performed independently using exact rational arithmetic and the nonlinear restrictions in Appendices C and D. Its restricted Hessian coincides exactly with the explicit polynomials in Eqs. (F.52) and (F.58), and Eq. (F.6) was checked on exact Calvo cross-sections, with approximation errors shrinking at the cubic rate.

F.3.7 Welfare Cost Function under PPT

Specializing Eq. (F.42) to PPT, the policy-dependent real second-order Home welfare contribution is

$$\begin{aligned}
\mathcal{W}_{H,t}^{(2),real,PPT} &= -\frac{(1+\varphi)\mathcal{M}^{-1}}{2} y_t^2 - \frac{\omega_H \omega_F (1-\eta)\Theta_g^{PPT}}{2} (z_t + \tau_{M,t})^2 \\
&\quad - \frac{\omega_H \omega_F \Theta_Y^{PPT}}{2} \left[(c_t + \eta g_t) - \{ c_t^* - \eta(\tau_{M,t}^* - z_t - g_t^*) \} \right]^2 \\
&\quad + \Theta_{mc,H}^{PPT} \left[mc_{H,t}^{(1)} (c_t + g_t - y_t) - \frac{1}{2} (mc_{H,t}^{(1)})^2 \right] \\
&\quad - \frac{\omega_H^* \omega_F^* (1-\eta)\Theta_{g^*}^{PPT}}{2} (\tau_{M,t}^* - z_t)^2 + \text{s.o.t.i.p.},
\end{aligned} \tag{F.52}$$

with $g_t = \omega_F(z_t + \tau_{M,t})$, $g_t^* = \omega_H^*(\tau_{M,t}^* - z_t)$, c_t and c_t^* given by Eq. (F.43) at $\psi_t = \psi_t^* = 0$, and $mc_{H,t}^{(1)} = c_t + \varphi y_t + g_t$. The five terms represent, respectively, the direct utility expansion, the second-order Home CPI contribution, the Home goods-market-clearing contribution reflecting Home and Foreign demand for Home goods, the producer Calvo contribution in Eq. (F.40), and the second-order Foreign CPI contribution. Using $\omega_H - \omega_H^* = 1 - \upsilon$ and $\omega_H + \omega_F^* = 2 - \upsilon$, the first-order equilibrium set implies

$$\begin{aligned}
c_t + g_t - y_t &= -(\eta-1)(2-\upsilon)\omega_F z_t - (\eta-1)\omega_H \omega_F \tau_{M,t} \\
&\quad + (\eta\omega_F^* + \omega_H^*)\omega_F \tau_{M,t}^*, \\
(c_t + \eta g_t) - \{ c_t^* - \eta(\tau_{M,t}^* - z_t - g_t^*) \} &= -(\eta-1)(1-\upsilon) z_t + (\eta-1)\omega_F \tau_{M,t} \\
&\quad + (\eta\omega_F^* + \omega_H^*)\tau_{M,t}^*, \\
mc_{H,t}^{(1)} &= (1+\varphi) y_t + (c_t + g_t - y_t).
\end{aligned} \tag{F.53}$$

Substituting Eq. (F.53) into Eq. (F.52) and writing $\mathcal{L}_{H,t}^{PPT,real} = \frac{1}{2} v_t' Q^{PPT} v_t - v_t' B^{PPT} (\tau_{M,t}, \tau_{M,t}^*)'$ with $v_t = (y_t, z_t)'$ gives

$$\begin{aligned}
Q_{11}^{PPT} &= (1+\varphi)\mathcal{M}^{-1} + (1+\varphi)^2 \Theta_{mc,H}^{PPT}, \qquad Q_{12}^{PPT} = 0, \qquad B_{11}^{PPT} = B_{12}^{PPT} = 0, \\
Q_{22}^{PPT} &= (\eta-1)\omega_H \omega_F \left[(\eta-1)(1-\upsilon)^2 \Theta_Y^{PPT} - \Theta_g^{PPT} \right] - (\eta-1)^2 (2-\upsilon)^2 \omega_F^2 \Theta_{mc,H}^{PPT} \\
&\quad + (1-\eta)\omega_H^* \omega_F^* \Theta_{g^*}^{PPT}, \\
B_{21}^{PPT} &= (\eta-1)\omega_H \omega_F \left[\Theta_g^{PPT} + (\eta-1)\omega_F \left\{ (1-\upsilon)\Theta_Y^{PPT} + (2-\upsilon)\Theta_{mc,H}^{PPT} \right\} \right],
\end{aligned}$$

$$
\begin{aligned}
B_{22}^{PPT} &= (\eta-1)(\eta\omega_F^*+\omega_H^*)\omega_F\left[(1-\upsilon)\omega_H\Theta_Y^{PPT}-(2-\upsilon)\omega_F\Theta_{mc,H}^{PPT}\right] \\
&\quad +(1-\eta)\omega_H^*\omega_F^*\Theta_{g^*}^{PPT}. \qquad \text{(F.54)}
\end{aligned}
$$

Since $Q_{12}^{PPT}=0$, the output–relative-price cross term vanishes. Completing squares yields the welfare-relevant variables entering Eq. (28):

$$
y_t^{w,PPT}=0, \qquad \text{(F.55)}
$$

$$
z_t^{w,PPT}=\frac{B_{21}^{PPT}\tau_{M,t}+B_{22}^{PPT}\tau_{M,t}^*}{\Lambda_z^{PPT}}, \qquad \text{(F.56)}
$$

and the corresponding PPT weights are

$$
\begin{aligned}
&\Lambda_y^{PPT}=Q_{11}^{PPT}=(1+\varphi)(\Theta_n^{PPT}-\Theta_H^{AS,PPT}), \qquad \Lambda_z^{PPT}=Q_{22}^{PPT}, \\
&\Lambda_{\pi,H}^{PPT}=\frac{\varepsilon}{\kappa_H}(\Theta_n^{PPT}-\Theta_H^{AS,PPT}). \qquad \text{(F.57)}
\end{aligned}
$$

Inserting the structural parameters from Table 1 and $\nu=0.27$ gives

$$
Q^{PPT}=\begin{pmatrix}2.008482 & 0\\ 0 & -0.081410\end{pmatrix}, \qquad B^{PPT}=\begin{pmatrix}0 & 0\\ 0.111414 & 0.071012\end{pmatrix},
$$

$$
\begin{aligned}
(\Lambda_y^{PPT},\Lambda_z^{PPT},\Lambda_{\pi,H}^{PPT}) &= (2.008482,\ -0.081410,\ 45.866779), \\
z_t^{w,PPT} &= -1.368559\,\tau_{M,t}-0.872271\,\tau_{M,t}^*.
\end{aligned}
$$

These numerical values agree, up to rounding, with the PPT coefficients reported in Table 3. Equation (F.56) defines the (logarithmic) welfare-relevant *ex-tariff* relative border price of imports. Since $\Lambda_z^{PPT}<0$, the associated completed square is not a stabilization objective; it reflects the relative-price externality discussed in Sec. 4.4.

F.3.8 Welfare Cost Function under IPT

Specializing Eq. (F.42) to IPT, the policy-dependent real second-order *welfare contribution* (the negative of the real welfare loss, before completing squares) is

$$
\begin{aligned}
\mathcal{W}_{H,t}^{(2),real} &= -\frac{(1+\varphi)\mathcal{M}^{-1}}{2}y_t^2-\frac{\omega_H\omega_F(1-\eta)\Theta_g}{2}(z_t-\psi_t+\tau_{M,t})^2 \\
&\quad -\frac{\omega_H\omega_F\Theta_Y}{2}\left[(c_t+\eta g_t)-\{c_t^*-\eta(-z_t-\psi_t^*+\tau_{M,t}^*-g_t^*)\}\right]^2 \\
&\quad +\Theta_{mc,H}\left[mc_{H,t}^{(1)}(c_t+g_t-y_t)-\frac{1}{2}(mc_{H,t}^{(1)})^2\right] \\
&\quad -\frac{\omega_F\Theta_g}{\varepsilon_M-1}\Bigg[\{(\varepsilon_M-1)\psi_t+\tau_{M,t}\}\{-\eta(z_t-\psi_t+\tau_{M,t}-g_t)+(z_t-\psi_t)-g_t\} \\
&\qquad +\frac{\varepsilon_M-1}{2}\psi_t^2+\tau_{M,t}\psi_t-\frac{\varepsilon_M+1}{2}\tau_{M,t}^2\Bigg] \\
&\quad +\eta\omega_F\Theta_Y\tau_{M,t}^*\psi_t^*-\omega_H^*\Theta_{g^*}\tau_{M,t}^*\psi_t^* \\
&\quad -\frac{\omega_H^*\omega_F^*(1-\eta)\Theta_{g^*}}{2}(-z_t-\psi_t^*+\tau_{M,t}^*)^2+\text{s.o.t.i.p.} \qquad \text{(F.58)}
\end{aligned}
$$

Each line has a direct structural origin in Eq. (F.42): the first term is the expansion of utility; the second is the second-order term of the Home CPI aggregator; the third is the second-order

term of Home goods-market clearing; the fourth is the producer Calvo block in Eq. (F.40); and the fifth is the importer Calvo block in Eq. (F.41), combined with the second-order part $\tau_{M,t}\psi_t$ of $\widetilde{p}_{F,t}$ in Eq. (F.38) as it enters the CPI restriction. The terms on the penultimate line arise from the second-order interaction $\tau^*_{M,t}\psi^*_t$ in the Foreign purchaser price $\widetilde{p}^*_{H,t}$ in Eq. (F.38). This interaction enters Home exports with the weight $-\eta\omega_F$ and is priced by Θ_Y; it also enters the Foreign CPI restriction with the weight ω^*_H and is priced by Θ_{g^*}. The final term is the second-order term of the Foreign CPI restriction. The interaction between the Foreign import tariff and the Foreign LOOP gap enters the tariff-loading matrix through B_{42}. It affects the loading of $\psi_t^{*,w,IPT}$ on $\tau^*_{M,t}$, while leaving the Home welfare-relevant LOOP-gap loading unchanged. In particular, the importer term retains the additive-duty price structure of Eq. (2); it is not obtained by multiplying ex-tariff sticky import prices by $1+\tau_{M,t}$.

For direct coefficient matching in the second-order polynomial, the three mixed real-price and allocation expressions in Eq. (F.58) reduce, using the first-order identities above, to

$$\begin{aligned}
c_t+g_t-y_t &= -(\eta-1)(2-\upsilon)\omega_F z_t+(\eta-1)\omega_H\omega_F(\psi_t-\tau_{M,t})\\
&\quad -\omega_F(\eta\omega^*_F+\omega^*_H)(\psi^*_t-\tau^*_{M,t}),\\
(c_t+\eta g_t)-\{c^*_t-\eta(-z_t-\psi^*_t+\tau^*_{M,t}-g^*_t)\} &= -(\eta-1)(1-\upsilon)z_t-(\eta-1)\omega_F(\psi_t-\tau_{M,t})\\
&\quad -(\eta\omega^*_F+\omega^*_H)(\psi^*_t-\tau^*_{M,t}),\\
-\eta(z_t-\psi_t+\tau_{M,t}-g_t)+(z_t-\psi_t)-g_t &= -(\eta-1)\omega_H(z_t-\psi_t)-[\eta-(\eta-1)\omega_F]\tau_{M,t}. \qquad \text{(F.59)}
\end{aligned}$$

Inserting Eq. (F.59) into Eq. (F.58) and collecting the coefficients on $(y_t,\psi_t,z_t,\psi^*_t)$ and the two tariffs gives Q and B in Eq. (F.60).

Set $v_t=(y_t,\psi_t,z_t,\psi^*_t)'$ and $\boldsymbol{\tau}_t=(\tau_{M,t},\tau^*_{M,t})'$. The real loss obtained from Eq. (F.58) is

$$\mathcal{L}^{real}_{H,t}=\frac{1}{2}v_t'Qv_t-v_t'B\boldsymbol{\tau}_t+\text{s.o.t.i.p.},\quad Q_{ij}=-\frac{\partial^2\mathcal{W}^{(2),real}_{H,t}}{\partial v_{i,t}\partial v_{j,t}},\quad B_{ij}=\frac{\partial^2\mathcal{W}^{(2),real}_{H,t}}{\partial v_{i,t}\partial\boldsymbol{\tau}_{j,t}}. \qquad \text{(F.60)}$$

With the multipliers in Eq. (F.47), $Q_{11}=(1+\varphi)\mathcal{M}^{-1}+(1+\varphi)^2\Theta_{mc,H}$, $Q_{1j}=0$ for $j>1$, $B_{11}=B_{12}=0$, and the remaining entries are

$$\begin{aligned}
Q_{22} &= \omega_F\Theta_g\{1+(\eta-1)\omega_H\}+(\eta-1)^2\omega_H\omega_F^2(\omega_F\Theta_Y-\omega_H\Theta_{mc,H}),\\
Q_{23} &= (\eta-1)^2\omega_H\omega_F^2\{(1-\upsilon)\Theta_Y+(2-\upsilon)\Theta_{mc,H}\},\\
Q_{24} &= (\eta-1)(\eta\omega^*_F+\omega^*_H)\omega_H\omega_F^2(\Theta_Y+\Theta_{mc,H}),\\
Q_{33} &= (\eta-1)\{(\eta-1)(1-\upsilon)^2\omega_H\omega_F\Theta_Y-\omega_H\omega_F\Theta_g-(\eta-1)(2-\upsilon)^2\omega_F^2\Theta_{mc,H}-\omega^*_H\omega^*_F\Theta_{g^*}\},\\
Q_{34} &= (\eta-1)[(\eta\omega^*_F+\omega^*_H)\omega_F\{(1-\upsilon)\omega_H\Theta_Y-(2-\upsilon)\omega_F\Theta_{mc,H}\}-\omega^*_H\omega^*_F\Theta_{g^*}],\\
Q_{44} &= (\eta\omega^*_F+\omega^*_H)^2\omega_F(\omega_H\Theta_Y-\omega_F\Theta_{mc,H})-(\eta-1)\omega^*_H\omega^*_F\Theta_{g^*},\\
B_{21} &= \omega_F\Theta_g\left\{1-\frac{1+(\eta-1)\omega_H}{\varepsilon_M-1}\right\}+(\eta-1)^2\omega_H\omega_F^2(\omega_F\Theta_Y-\omega_H\Theta_{mc,H}),\\
B_{31} &= (\eta-1)\omega_H\omega_F\left[\frac{\varepsilon_M}{\varepsilon_M-1}\Theta_g+(\eta-1)\omega_F\{(1-\upsilon)\Theta_Y+(2-\upsilon)\Theta_{mc,H}\}\right],\\
B_{42} &= Q_{44}-\omega^*_H\Theta_{g^*}+\eta\omega_F\Theta_Y,\qquad B_{22}=B_{41}=Q_{24},\qquad B_{32}=Q_{34}. \qquad \text{(F.61)–(F.62)}
\end{aligned}$$

The equalities in the last line hold because, apart from the importer block and the terms in $\tau^*_{M,t}\psi^*_t$, the tariffs enter Eq. (F.58) only through $\tau_{M,t}-\psi_t$ and $\tau^*_{M,t}-\psi^*_t$; the terms in $\tau^*_{M,t}\psi^*_t$ make B_{42} differ from Q_{44}.

Since $Q_{1j}=0$ for $j>1$, sequential Schur completion in the order y,ψ,z,ψ^* yields

$$\begin{aligned}
&\Lambda^{IPT}_y=Q_{11},\qquad \Lambda^{IPT}_\psi=Q_{22},\\
&\Lambda^{IPT}_z=Q_{33}-\frac{Q_{23}^2}{Q_{22}},\\
&\Lambda^{IPT}_{\psi^*}=Q_{44}-\frac{Q_{24}^2}{Q_{22}}-\frac{(Q_{34}-Q_{23}Q_{24}/Q_{22})^2}{Q_{33}-Q_{23}^2/Q_{22}},\\
&\Gamma_{\text{own}}=\frac{B_{21}}{Q_{22}},\qquad \Gamma_{\text{cross}}=\frac{B_{22}}{Q_{22}}=\frac{Q_{24}}{Q_{22}},\qquad \Gamma_z=-\frac{Q_{23}}{Q_{22}}.
\end{aligned} \qquad \text{(F.63)}$$

Substituting Eq. (F.62) into $\Gamma_{\text{own}} = B_{21}/Q_{22}$ separates the own-tariff loading into three parts:

$$\Gamma_{\text{own}} = \frac{1}{\Lambda_{\psi}^{IPT}} \Bigg[\underbrace{\omega_F \Theta_g \left\{ 1 - \frac{1 + (\eta - 1)\omega_H}{\varepsilon_M - 1} \right\}}_{\text{CPI and importer pricing}} + \underbrace{(\eta - 1)^2 \omega_H \omega_F^3 \Theta_Y}_{\text{Home-goods allocation}} - \underbrace{(\eta - 1)^2 \omega_H^2 \omega_F^2 \Theta_{mc,H}}_{\text{producer pricing}} \Bigg]. \tag{F.64}$$

The first part combines the CES price response of the Home CPI with the importer restriction weighted by the *ex-tariff* reset price, including the additive-duty term $1/(\varepsilon_M - 1)$. The second reflects Home goods-market clearing and the allocation of Home goods across countries. The third reflects Home producer marginal cost and the producer Calvo restriction.

Sequential completion of squares in the order y, ψ, z, ψ^* yields $y_t^{w,IPT} = 0$, because $Q_{1j} = 0$ for $j > 1$ and $B_{11} = B_{12} = 0$, and the Home welfare-relevant LOOP gap in Eq. (30), with Γ_{own}, Γ_{cross}, and Γ_z given in Eq. (F.63). Because $B_{22} = Q_{24}$, the Foreign import tariff and the Foreign LOOP gap enter it through $\tau_{M,t}^* - \psi_t^*$. The starred coefficients in Eq. (F.71) follow from the same derivation with Home and Foreign labels interchanged. The remaining two references are

$$\begin{aligned} z_t^{w,IPT} &= \frac{1}{\Lambda_z^{IPT}} \left[\left(B_{31} - \frac{Q_{23} B_{21}}{Q_{22}} \right) \tau_{M,t} + \left(B_{32} - \frac{Q_{23} B_{22}}{Q_{22}} \right) \tau_{M,t}^* - \left(Q_{34} - \frac{Q_{23} Q_{24}}{Q_{22}} \right) \psi_t^* \right], \\ \psi_t^{*,w,IPT} &= \frac{1}{\Lambda_{\psi^*}^{IPT}} \sum_{j=1}^{2} \left[B_{4j} - \frac{Q_{24} B_{2j}}{Q_{22}} - \frac{(Q_{34} - Q_{23} Q_{24}/Q_{22})\,(B_{3j} - Q_{23} B_{2j}/Q_{22})}{\Lambda_z^{IPT}} \right] \boldsymbol{\tau}_{j,t}. \end{aligned} \tag{F.65}$$

The inflation weights are given by Eq. (F.48). In this representation, $y_t^{w,IPT}$ is the (logarithmic) welfare-relevant output, $\psi_t^{w,IPT}$ is the (logarithmic) Home welfare-relevant LOOP gap, and $z_t^{w,IPT}$ is the (logarithmic) welfare-relevant *ex-tariff* relative border price of imports. The variable $\psi_t^{*,w,IPT}$ is the welfare-relevant value associated with the Foreign LOOP gap ψ_t^* in the Home welfare cost function, distinct from the Foreign planner's own welfare-relevant LOOP gap $\psi_t^{w*,IPT}$. In particular, Eq. (30) makes explicit that the Home welfare-relevant LOOP gap depends not only on the Home tariff but also on the endogenous international relative price z_t and the Foreign LOOP gap ψ_t^*. These welfare-relevant variables are obtained by completed squares and are not additional private-equilibrium variables.

Inserting the structural parameters from Table 1 and $\nu = 0.27$ produces the matrices

$$Q = \begin{pmatrix} 2.860011 & 0 & 0 & 0 \\ 0 & 0.406343 & 0.006318 & 0.024430 \\ 0 & 0.006318 & -0.097218 & 0.071398 \\ 0 & 0.024430 & 0.071398 & 0.303265 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 0 \\ 0.147982 & 0.024430 \\ 0.160665 & 0.071398 \\ 0.024430 & 0.581868 \end{pmatrix}. \tag{F.66}$$

These intermediate matrices permit direct verification of the welfare coefficients reported in Table 3 through sequential completion of squares. The structural pivots are

$$(\Lambda_y^{IPT}, \Lambda_{\psi}^{IPT}, \Lambda_z^{IPT}, \Lambda_{\psi^*}^{IPT}) = (2.860011,\ 0.406343,\ -0.097316,\ 0.353623), \tag{F.67}$$

and the three Calvo-inflation coefficients, computed from Eq. (F.48), are

$$(\Lambda_{\pi,H}^{IPT}, \Lambda_{\pi,F}^{IPT}, \Lambda_{\pi_H^*}^{IPT}) = (65.312754,\ 21.282917,\ 13.518748). \tag{F.68}$$

The Home welfare-relevant LOOP-gap coefficients are $(\Gamma_{\text{own}}, \Gamma_{\text{cross}}, \Gamma_z) = (0.364180, 0.060123, -0.015548)$. Interchanging the country labels, as described at the beginning of Appendix F.3, gives $(\Gamma_{\text{own}}^*, \Gamma_{\text{cross}}^*, \Gamma_z^*) = (0.328075, 0.024116, 0.006695)$, with Γ_z^* expressed in the Home z_t coordinate. These values agree, up to rounding, with the coefficients reported in Sec. 4.4. Equation (F.65) gives the remaining two welfare-relevant references,

$$z_t^{w,IPT} = -1.627314\,\tau_{M,t} - 0.729770\,(\tau_{M,t}^* - \psi_t^*), \qquad \psi_t^{*,w,IPT} = 0.370741\,\tau_{M,t} + 1.787856\,\tau_{M,t}^*.$$

In line with Eq. (F.49), the multipliers $\Theta_n = \Theta_Y = 0.844719$ and $\Theta_H^{AS} = -0.049035$ give $\Lambda_y^{IPT} = (1+\varphi)(\Theta_n - \Theta_H^{AS}) = 2.860011$ and $\Lambda_{\pi,H}^{IPT} = (\varepsilon/\kappa_H)(\Theta_n - \Theta_H^{AS}) = 65.312754$.

Since $\Lambda_z^{IPT} < 0$, the associated completed square is not a stabilization objective; it reflects the relative-price externality discussed in Sec. 4.4, and the quadratic form is not positive definite over unrestricted $(y_t, \psi_t, z_t, \psi_t^*)$.

F.3.9 Decomposition of the z_t Coefficient

The coefficient Γ_z in Eq. (30) is $-Q_{23}/Q_{22}$, as shown in Eq. (F.63). In Q_{23} in Eq. (F.61), the contributions from the Home CPI and the Home importer-pricing terms cancel, and two terms remain. The first term in braces comes from Home goods-market clearing and captures the effect of home bias on the international allocation of Home goods: under home bias, an increase in z_t shifts the allocation of Home goods toward Foreign households, and a lower Home LOOP gap counteracts this shift by increasing Home households' consumption of Home goods relative to Foreign households, for given Home output. The second comes from Home producer pricing. From the steady-state multiplier equations in Eq. (F.47), this latter component satisfies

$$\Theta_{mc,H} = \frac{(1-\mathcal{M}^{-1}) - \mathcal{M}^{-1}(\eta-1)\omega_F}{\varphi(\omega_H + \eta\omega_F) + 1/\omega_H}. \tag{F.69}$$

The two terms in the numerator reflect the competing incentives to expand Home output by reducing the distortion from the producer markup and to restrict Home output to improve the terms of trade. Under the baseline parameterization, the two terms in braces in Eq. (F.61) contribute 0.005537 and 0.000781, respectively, so $Q_{23} = 0.006318 > 0$ and $\Gamma_z = -0.015548$. Both components therefore contribute to the negative coefficient: holding the Home and Foreign tariffs and the Foreign LOOP gap fixed, an increase in z_t lowers the Home welfare-relevant LOOP gap, whereas a decline in z_t raises it.

For the symmetric case without home bias, $\upsilon = 1$ and $\nu = 0.5$, the definitions of the expenditure weights give $\omega_H = \omega_F = \omega_H^* = 1/2$. The first term in Eq. (F.61) then vanishes, and

$$Q_{23} = \frac{(\eta-1)^2}{8}\Theta_{mc,H}, \qquad \Gamma_z = 0 \iff \Theta_{mc,H} = 0 \iff \mathcal{M}^{-1} = \frac{2}{1+\eta}, \quad (\eta > 1,\ Q_{22} > 0). \tag{F.70}$$

Thus, in this symmetric case, the z_t coefficient vanishes when the two producer-side incentives offset each other. This condition is not the same as eliminating the producer markup distortion ($\mathcal{M} = 1$).

F.3.10 Second-Order Condition

The negative Schur pivots Λ_z^{PPT} and Λ_z^{IPT} show that the real quadratic form is not positive definite on the unrestricted coordinate space v_t. The relevant second-order condition, however, concerns only the directions in which the Home planner can move while taking the Foreign plan as given, namely $dc_t^* = 0$ and, under IPT, $ds_t^* = 0$, which implies $d\psi_t^* = -dz_t$. Under PPT, $dc_t^* = 0$ implies $dy_t = 1.105\, dz_t$ at the baseline, and the restricted coefficient is $\Lambda_y^{PPT}(1.105)^2 + \Lambda_z^{PPT} = 2.372 > 0$. Under IPT, $dc_t^* = 0$ implies $dy_t = 0.867\, dz_t - 0.086\, d\psi_t$, and the restricted 2×2 matrix in (z_t, ψ_t) has eigenvalues 0.398 and 2.240. Since the inflation weights are positive, the quadratic welfare-cost approximation is positive definite when restricted to the first-order feasible directions of the Home national problem. The same holds in the High Substitution Case ($\eta = 3$): the restricted PPT coefficient is 2.959, and the restricted IPT eigenvalues are 0.873 and 2.924.

F.3.11 Welfare Weights in the Baseline and the High Substitution Case

The following tables present the coefficient values for the two-country IPT baseline and its High Substitution Case.

Table F.1: Home welfare weights across IPT calibrations

Variable	Baseline	High Substitution
$y_t - y_t^w$	2.860	2.221
$\psi_t - \psi_t^w$	0.406	1.206
$z_t - z_t^w$	−0.097	−0.480
$\psi_t^* - \psi_t^{*,w}$	0.354	1.436
$\pi_{H,t}$	65.313	50.711
$\pi_{F,t}$	21.283	33.686
$\pi_{H,t}^*$	13.519	11.320

Note: The IPT superscripts on the welfare-relevant variables are omitted from the row labels.

Table F.2: Loadings of the Home welfare-relevant LOOP gap across IPT calibrations

Term	Baseline	High Substitution
$\tau_{M,t}$ (Γ_{own})	0.364	0.053
$\tau_{M,t}^* - \psi_t^*$ (Γ_{cross})	0.060	0.122
z_t (Γ_z)	−0.016	−0.059

Note: Each entry is the coefficient on the corresponding term in the Home welfare-relevant LOOP gap; the structural formulas are given in Appendix F.3.8.

High Substitution Case. The High Substitution Case applies the same welfare-cost derivation at $\eta = 3$; no separate derivation is required. The resulting Home welfare weights are reported in Table F.1.

Baseline Reciprocal Tariff Case. The baseline reciprocal-tariff experiment uses the same two-country IPT welfare-cost function and structural parameters as the Baseline Case of the unilateral tariff. Under reciprocal tariffs, both $\tau_{M,t}$ and $\tau_{M,t}^*$ are nonzero, while the welfare weights remain unchanged. Hence no separate welfare-cost derivation is required.

F.4 Foreign Tariffs, the FAC, and the Foreign Welfare-Relevant LOOP Gap

Under reciprocal tariffs, the Foreign LOOP gap falls on impact, so both the Foreign tariff increase and this decline raise the Home welfare-relevant LOOP gap through the FAC. Under the unilateral tariff, by contrast, the Foreign LOOP gap rises on impact, and the FAC contribution is negative (Sec. 4.5).

An increase in the Foreign import tariff raises the *cum-tariff* relative price of Home goods faced by Foreign households, inducing inefficient underconsumption of imports in Foreign. Under IPT, the same import-allocation mechanism gives the Foreign Ramsey planner an incentive to move the actual Foreign LOOP gap toward the Foreign welfare-relevant LOOP gap to mitigate this distortion. The Foreign welfare-relevant LOOP gap is given by the Home–Foreign mirror of Eq. (30):

$$\psi_t^{w*,IPT} = \Gamma_{\text{own}}^* \tau_{M,t}^* + \Gamma_{\text{cross}}^* (\tau_{M,t} - \psi_t) + \Gamma_z^* z_t, \tag{F.71}$$

where the starred Γ coefficients are the exact Home–Foreign mirrors of the corresponding Home loadings in Eq. (30). As stated at the beginning of Appendix F.3, they follow from Eqs. (F.47), (F.62), and (F.63) after interchanging the country labels: ν is replaced by $1 - \nu$, so that $(\omega_H, \omega_F, \omega_H^*, \omega_F^*)$ is replaced by $(\omega_F^*, \omega_H^*, \omega_F, \omega_H)$, and z_t by $-z_t$. Denoting the resulting entries

by Q^*_{ij} and B^*_{ij},

$$\Gamma^*_{\text{own}} = \frac{B^*_{21}}{Q^*_{22}}, \qquad \Gamma^*_{\text{cross}} = \frac{Q^*_{24}}{Q^*_{22}}, \qquad \Gamma^*_z = \frac{Q^*_{23}}{Q^*_{22}}, \tag{F.72}$$

where Γ^*_z has the sign opposite to that in Eq. (F.63) because it is expressed in the Home coordinate z_t. Under the baseline parameterization, $\Gamma^*_{\text{own}} = 0.328$, $\Gamma^*_{\text{cross}} = 0.024$, and $\Gamma^*_z = 0.007$. Thus the positive Foreign-tariff coefficient reflects the same welfare mechanism as the positive Home-tariff coefficient in Eq. (30). The Home import tariff and the Home LOOP gap enter Eq. (F.71) because they affect Home households' consumption of Foreign goods relative to Home goods. A higher Home tariff reduces this relative consumption, resulting in inefficient overconsumption of Home goods in Home and underconsumption of Home goods in Foreign. The Foreign welfare-relevant LOOP gap therefore rises to mitigate this distortion by encouraging Foreign households to consume more Home goods relative to Foreign goods. Conversely, a higher Home LOOP gap increases Home households' relative consumption of Foreign goods, reducing the need for this adjustment and lowering the Foreign welfare-relevant LOOP gap.

Even in the two-country economy, deviations from the LOOP under the IPT environment can be interpreted as currency misalignment, as discussed in Sec. 3.4. For the Home economy, define the welfare-relevant nominal exchange rate in the two-country economy as $e^w_t \equiv p_{F,t} - p^*_{F,t} + \psi^{w,IPT}_t$. It follows that the Home welfare-relevant LOOP-gap deviation satisfies $\psi_t - \psi^{w,IPT}_t = e_t - e^w_t$. Analogously, define the Foreign welfare-relevant nominal exchange rate by

$$e^{w*}_t \equiv p_{H,t} - p^*_{H,t} - \psi^{w*,IPT}_t. \tag{F.73}$$

The log-linearized definition of the Foreign LOOP gap, $\psi^*_t = p_{H,t} - e_t - p^*_{H,t}$, then implies

$$\psi^*_t - \psi^{w*,IPT}_t = -(e_t - e^{w*}_t). \tag{F.74}$$

Thus the same bilateral nominal exchange rate e_t is compared with two country-specific welfare-relevant nominal exchange rates. Moving e_t toward e^w_t reduces the Home welfare-relevant LOOP-gap deviation, whereas moving it toward e^{w*}_t reduces the Foreign welfare-relevant LOOP-gap deviation. In general, e^w_t and e^{w*}_t differ because the Home and Foreign planners face different national welfare trade-offs.

G Foreign Responses in the Baseline

Figures G.1 and G.2 report the Foreign responses to the unilateral Home tariff in Sec. 4.5 and to the reciprocal tariffs in Sec. 4.6, respectively.

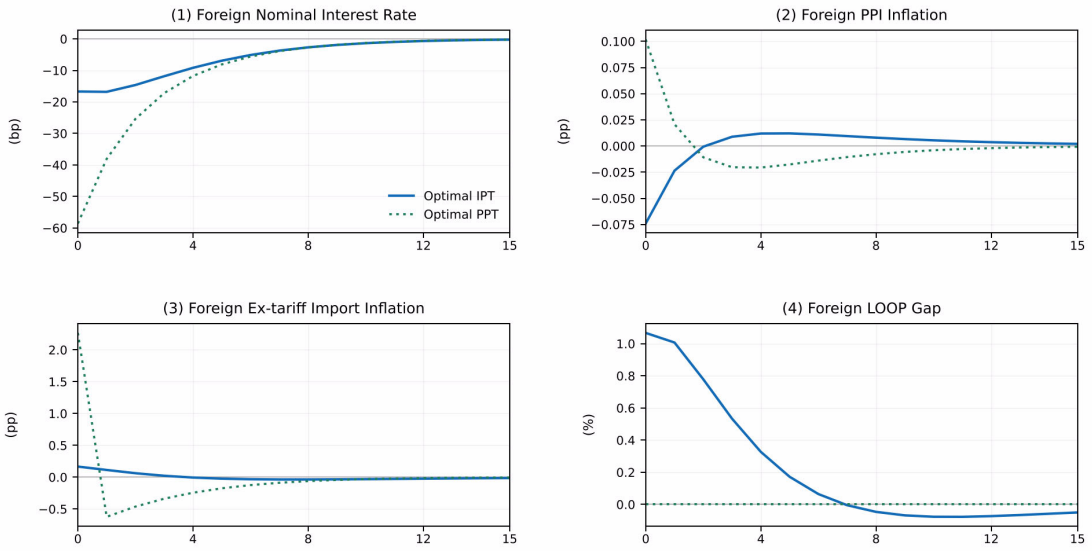


Figure G.1: Two-country unilateral tariff: Foreign optimal PPT and IPT responses to a 20-percentage-point increase in the Home import tariff. Panels 1–4 report Foreign nominal interest rate, Foreign PPI inflation, Foreign *ex-tariff* import inflation, and Foreign LOOP gap, respectively. The interest-rate deviation is in basis points, and the other series are in percent or percentage points.

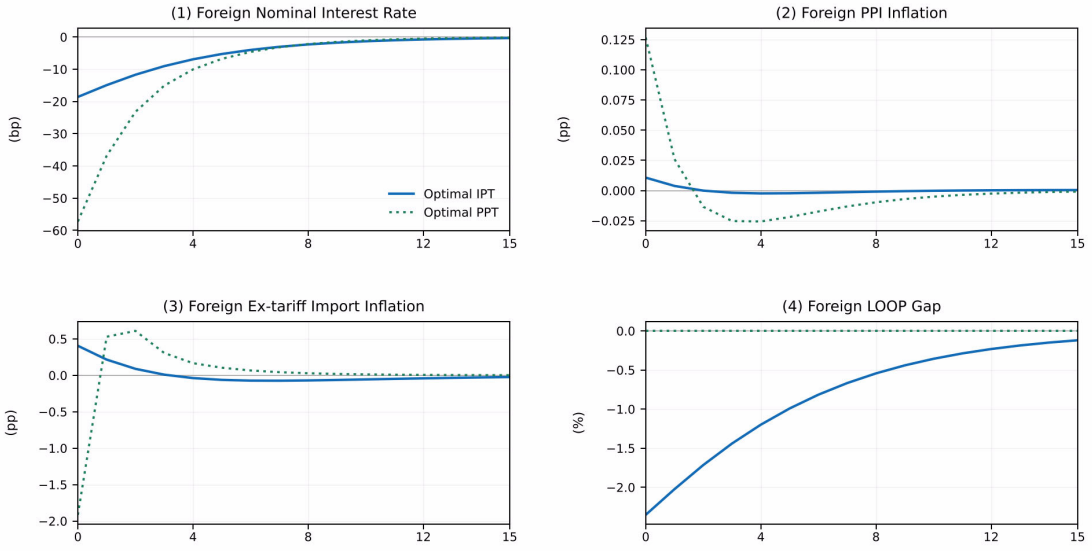


Figure G.2: Two-country reciprocal tariffs: Foreign optimal PPT and IPT responses to simultaneous 20-percentage-point increases in the Home and Foreign import tariffs. Panels 1–4 report Foreign nominal interest rate, Foreign PPI inflation, Foreign *ex-tariff* import inflation, and Foreign LOOP gap, respectively. The interest-rate deviation is in basis points, and the other series are in percent or percentage points.

H Analytical Benchmarks and Price-Rigidity Sensitivity: Proofs and Details

H.1 Proof of Proposition 1

Let $x_t \equiv (y_t, \psi_t)'$. We normalize $p_F^* = 0$, so that $e_t = \psi_t + p_{F,t}$, and set the pre-shock values $p_{F,-1} = e_{-1} = \psi_{-1} = 0$.

Step 1: Differentiating the Nonlinear Ramsey Conditions with $\theta = 0$. With $\theta = 0$, every producer resets its price each period, so $\Delta_{H,t} = 1$ and the producer pricing block reduces to $MC_t^r = \mathcal{M}^{-1}$. By Eqs. (B.14) and (9), $MC_t^r = Z_t Y_t^{\varphi}$, so the block is $Z_t Y_t^{\varphi} = \mathcal{M}^{-1}$, whose first-order approximation is $z_t + \varphi y_t = 0$. Combining it with the first-order relation $z_t = p_* y_t + k_*(\psi_t - \tau_{M,t})$ of Appendix F.1 gives

$$y_t = g(\tau_{M,t} - \psi_t), \qquad z_t = -\varphi g(\tau_{M,t} - \psi_t), \tag{H.1}$$

where g is defined in Proposition 1. The following lemma expresses the linearization of the original nonlinear Ramsey first-order conditions in terms of the Hessian of the Ramsey Lagrangian.

Lemma 2. *Write the nonlinear timeless-perspective Ramsey problem under perfect foresight as the maximization of $\sum_{t\geq 0} \beta^t U(X_t)$ subject to $\Phi(\Xi_t) = 0$ for $t \geq 0$, where $\Xi_t \equiv (X_{t-1}', X_t', X_{t+1}', \tau_{M,t})'$, with Lagrangian $\sum_{t\geq 0} \beta^t [U(X_t) + \lambda_t' \Phi(\Xi_t)]$ and lagged multipliers fixed at their steady-state values. Let (X, λ) be the zero-tariff steady state of the Ramsey first-order conditions, and let $\mathcal{H}$ denote the Hessian of the period-t term $U(X_t) + \lambda' \Phi(\Xi_t)$ with respect to the* stacked *vector Ξ_t at that steady state. $\mathcal{H}$ therefore contains the intertemporal second derivatives $\partial^2(\lambda'\Phi)/\partial X_{t-1}\partial X_t$, $\partial^2(\lambda'\Phi)/\partial X_t \partial X_{t+1}$, and $\partial^2(\lambda'\Phi)/\partial X_{t-1}\partial X_{t+1}$. Then the first-order approximation of the nonlinear Ramsey first-order conditions and constraints, in $(dX_t, d\lambda_t)$, is identical to the system of first-order conditions and constraints of*

$$\max \sum_{t\geq 0} \beta^t \, \frac{1}{2} \, d\Xi_t' \, \mathcal{H} \, d\Xi_t \quad \textit{subject to} \quad \Phi_\Xi \, d\Xi_t = 0, \qquad t \geq 0,$$

with $d\lambda_t$ as the multipliers and $d\lambda_{-1} = 0$.

Proof. X_t enters the period-$(t-1)$, period-t, and period-$(t+1)$ terms of the Lagrangian. Its nonlinear first-order condition, which sets the derivative of the Lagrangian with respect to X_t to zero, collects the derivatives of these three terms. Differentiating this condition at (X, λ) with respect to $(dX_s)_{s\geq -1}$, $(d\lambda_s)_{s\geq -1}$, and $(d\tau_{M,s})_{s\geq 0}$ gives the second derivatives of the three period terms, evaluated at λ, multiplying $dX_{t-2}, \ldots, dX_{t+2}$ and the tariffs, plus the transposed constraint Jacobians multiplying $d\lambda_{t-1}, d\lambda_t, d\lambda_{t+1}$. Because each period term is a function of Ξ_t alone, these second derivatives are exactly the blocks of $\mathcal{H}$, including its intertemporal blocks. The result is the stationarity condition of the displayed quadratic problem with multipliers $d\lambda_t$. Differentiating $\Phi(\Xi_t) = 0$ gives the displayed linear constraints. The timeless-perspective initial condition fixes the lagged multipliers at λ, so $d\lambda_{-1} = 0$. □

The coefficients Θ in Eqs. (F.14) and (F.15) are determined by the steady-state first-order conditions of the original Ramsey Lagrangian. In the normalization used in Appendix F.1, they are the steady-state multipliers λ on the corresponding constraints, including the reset-price-weighted aggregate-supply restrictions. Rescaling a constraint rescales its multiplier inversely and leaves $\lambda'\Phi$, and hence $\mathcal{H}$, unchanged. The matrix Q in Eq. (F.25) is the negative of the policy–policy Hessian block of $\mathcal{H}$, whereas B is the policy–tariff cross-derivative block, after the private-equilibrium identities are used to eliminate C_t and z_t. Eliminating variables through equality constraints that hold exactly does not change the reduced Hessian of the Lagrangian. With $\theta = 0$, intertemporal second derivatives arise in the importer block: in the Calvo price-index and dispersion laws and in

the importer reset-price condition, which involves future variables. After the private-equilibrium restrictions are eliminated and the Calvo aggregation in Appendix F.1 is applied, the dynamic second-order terms that remain are those in import-price inflation, $\frac{1}{2}\Lambda_{\pi,F}\pi_{F,t}^2$ in Eq. (F.30), with $\pi_{F,t} = p_{F,t} - p_{F,t-1}$. No producer-inflation term appears, because with $\theta = 0$ there is no producer price dispersion. By Lemma 2, the differentiated nonlinear Ramsey first-order conditions therefore form a linear system whose coefficient matrices are those of the quadratic form

$$\mathcal{L}_t = \frac{1}{2}x_t'Qx_t - x_t'B\tau_{M,t} + \frac{1}{2}\Lambda_{\pi,F}\pi_{F,t}^2,$$

together with the differentiated private-equilibrium conditions, with the lagged multipliers at zero deviation. $\mathcal{L}_t$ is used here only as a compact notation for the coefficients of the differentiated original first-order conditions. It is not an objective that is assumed, and it is not the objective of the policy computations in the paper, which solve the nonlinear Ramsey first-order conditions. The differentiated private-equilibrium conditions are Eq. (H.1), the importer Phillips curve (18), the relative-price identity $s_t = s_{t-1} + \pi_{F,t} - \pi_{H,t}$ with $s_t = z_t - \psi_t$, and the Euler equation. Because $\pi_{H,t}$ carries no welfare weight and is otherwise unrestricted, the relative-price identity only determines $\pi_{H,t}$; the Euler equation only determines i_t (Lemma 1). Neither restricts $(\psi_t, \pi_{F,t})$. The differentiated system therefore reduces to the conditions for $\{\psi_t, \pi_{F,t}\}_{t\geq 0}$ associated with Eq. (18) alone.

Step 2: Reduction of the Coefficients. Substituting $x_t = (g(\tau_{M,t} - \psi_t), \psi_t)'$ into $\mathcal{L}_t$ gives

$$\begin{aligned}\frac{1}{2}x_t'Qx_t - x_t'B\tau_{M,t} &= \frac{1}{2}\left(Q_{11}g^2 - 2Q_{12}g + Q_{22}\right)\psi_t^2 - \left[B_2 - (B_1 + Q_{12})g + Q_{11}g^2\right]\psi_t\tau_{M,t} + \text{t.i.p.} \\ &= \frac{1}{2}\tilde{\Lambda}_\psi\left(\psi_t - \tilde{\Gamma}_{\text{own}}\tau_{M,t}\right)^2 + \text{t.i.p.},\end{aligned}$$

where t.i.p. collects terms independent of policy. Since $\tilde{\Lambda}_\psi = v'Qv$ with $v = (-g, 1)'$ and Q is positive definite, $\tilde{\Lambda}_\psi > 0$; also $\Lambda_{\pi,F} > 0$, so $\vartheta > 0$.

Step 3: The Differentiated First-Order Conditions. Let μ_t denote the deviation of the original Ramsey multiplier on the importer pricing block, expressed in the normalization of Eq. (18) written as $\pi_{F,t} - \beta\pi_{F,t+1} - \kappa_M[\psi_t + \tau_{M,t}/(\varepsilon_M - 1)] = 0$. By Steps 1 and 2, the differentiated first-order conditions for ψ_t and $\pi_{F,t}$, $t \geq 0$, are

$$\tilde{\Lambda}_\psi\left(\psi_t - \tilde{\Gamma}_{\text{own}}\tau_{M,t}\right) - \kappa_M\mu_t = 0, \qquad \Lambda_{\pi,F}\pi_{F,t} + \mu_t - \mu_{t-1} = 0,$$

where, from the timeless perspective, the lagged multiplier μ_{-1} is at its steady-state value, which is zero in deviation form. Define $\zeta_t \equiv \psi_t - \tilde{\Gamma}_{\text{own}}\tau_{M,t}$, so that $\zeta_{-1} = \kappa_M\mu_{-1}/\tilde{\Lambda}_\psi = 0$. Eliminating μ_t gives $\Lambda_{\pi,F}\pi_{F,t} = -(\tilde{\Lambda}_\psi/\kappa_M)(\zeta_t - \zeta_{t-1})$. Summing from date 0 and using $p_{F,-1} = 0$ yields

$$\psi_t - \tilde{\Gamma}_{\text{own}}\tau_{M,t} = -\vartheta p_{F,t}, \qquad t \geq 0. \tag{H.2}$$

Step 4: The Import Price Path. Substituting Eq. (H.2) into Eq. (18) with $\pi_{F,t} = p_{F,t} - p_{F,t-1}$ gives

$$\beta p_{F,t+1} - (1 + \beta + \kappa_M\vartheta)p_{F,t} + p_{F,t-1} = -\kappa_M\left(\tilde{\Gamma}_{\text{own}} + \frac{1}{\varepsilon_M - 1}\right)\tau_{M,t}. \tag{H.3}$$

Let $f(\ell) \equiv \beta\ell^2 - (1 + \beta + \kappa_M\vartheta)\ell + 1 = (1 - \ell)(1 - \beta\ell) - \kappa_M\vartheta\ell$. Since $f(0) = 1 > 0$ and $f(1) = -\kappa_M\vartheta < 0$, f has one root $\delta \in (0, 1)$, and its other root is $1/(\beta\delta) > 1$. Imposing boundedness eliminates the explosive root and yields the unique solution

$$p_{F,t} = \delta p_{F,t-1} + \delta\kappa_M\left(\tilde{\Gamma}_{\text{own}} + \frac{1}{\varepsilon_M - 1}\right)\sum_{k\geq 0}(\beta\delta)^k\tau_{M,t+k}.$$

With no further innovations after the date-0 shock, the AR(1) tariff process implies $\tau_{M,t+k} = \rho_\tau^k \tau_{M,t}$, so $p_{F,t} = \delta p_{F,t-1} + A\tau_{M,t}$, where $A \equiv \delta\kappa_M[\tilde{\Gamma}_{\text{own}} + 1/(\varepsilon_M - 1)]/(1 - \beta\delta\rho_\tau) > 0$. Hence $p_{F,0} = A\tau_{M,0}$, $p_{F,1} = (\delta + \rho_\tau)A\tau_{M,0}$, and $\pi_{F,1} = (\delta + \rho_\tau - 1)A\tau_{M,0}$.

Step 5: The Impact Interest-Rate Response. From $e_t = \psi_t + p_{F,t}$ and Eq. (H.2), $e_t = \tilde{\Gamma}_{\text{own}}\tau_{M,t} + (1 - \vartheta)p_{F,t}$. Lemma 1 then gives

$$\hat{\imath}_0 = e_1 - e_0 = -\tilde{\Gamma}_{\text{own}}(1 - \rho_\tau)\tau_{M,0} + (1 - \vartheta)(\delta + \rho_\tau - 1)A\tau_{M,0}.$$

Because $f(\delta) = 0$ is equivalent to $\kappa_M\vartheta\delta = (1 - \delta)(1 - \beta\delta)$,

$$(1 - \vartheta)A = \frac{1 - \vartheta}{\vartheta} \frac{(1 - \delta)(1 - \beta\delta)}{1 - \beta\delta\rho_\tau} \left(\tilde{\Gamma}_{\text{own}} + \frac{1}{\varepsilon_M - 1}\right),$$

which yields Eq. (A.2).

Step 6: The Sign. Since $\tau_{M,0} = \epsilon_{\tau,0} > 0$, $\rho_\tau < 1$, and $\tilde{\Gamma}_{\text{own}} > 0$, the first term of Eq. (A.2) is strictly negative. In the second term, $(1 - \delta)(1 - \beta\delta)/(1 - \beta\delta\rho_\tau) > 0$ and $\tilde{\Gamma}_{\text{own}} + 1/(\varepsilon_M - 1) > 0$, while $(1 - \vartheta)(\delta + \rho_\tau - 1)/\vartheta \leq 0$ under the stated condition. Hence $\hat{\imath}_0 < 0$. Finally, f is convex with $f(0) > 0$, $f(\delta) = 0$, and $f(1) < 0$, so for $\ell \in [0, 1]$, $f(\ell) \geq 0$ if and only if $\ell \leq \delta$. Applying this to $\ell = 1 - \rho_\tau$, $\delta + \rho_\tau \geq 1$ if and only if $f(1 - \rho_\tau) \geq 0$, that is, $\rho_\tau\{1 - \beta(1 - \rho_\tau)\} \geq \kappa_M\vartheta(1 - \rho_\tau)$, which is the stated bound. □

Remarks. The proposition concerns the notional interest rate. In the flexible-producer-price benchmark, a 20-percentage-point tariff shock implies a notional rate decline exceeding 101.01 basis points, the distance from the steady-state nominal interest rate to the ZLB. Within the first-order approximation, the sign does not depend on the size of $\epsilon_{\tau,0}$; the finite-shock responses in Sec. 3.5 are computed from the nonlinear Ramsey first-order conditions.

Additional interpretation and numerical checks for Proposition 1. The proof differentiates the nonlinear Ramsey first-order conditions directly; no quadratic welfare criterion is minimized. The coefficients Q, B, and $\Lambda_{\pi,F}$ appear in Proposition 1 because the coefficient matrix of the differentiated first-order conditions is the Hessian of the nonlinear Ramsey Lagrangian evaluated at the steady-state multipliers (Lemma 2), which is the object computed in Appendix F.1.[1]

The first term captures the tariff-induced change in the welfare-relevant LOOP gap and its subsequent decay. When $\tilde{\Gamma}_{\text{own}} > 0$, the tariff raises the welfare-relevant LOOP gap; because it subsequently declines as the tariff shock dissipates, the Home currency is expected to appreciate after impact, and Lemma 1 translates this expected appreciation into a rate cut. The second term captures the interaction between the HIAC-induced incentive to narrow the welfare-relevant LOOP-gap deviation and the competing objective of stabilizing import-price inflation: under commitment, the planner sets $\psi_t - \tilde{\Gamma}_{\text{own}}\tau_{M,t} = -\vartheta p_{F,t}$ (Eq. (H.2)). When $\vartheta \geq 1$, the planner weighs import-price stability more heavily than closing the LOOP-gap deviation; when $\delta + \rho_\tau \geq 1$, the import price level is still rising at date 1. Together, the two conditions make the second term reinforce the cut. Because $\kappa_M\Lambda_{\pi,F} = -\varepsilon_M C^{1-\eta}\Theta_M^{AS}$ by Eq. (F.30), ϑ does not depend on θ_M; importer stickiness enters only through κ_M in the persistence condition, which is easier to satisfy when import prices are stickier. Under PPT with $\theta = 0$, by contrast, the producer block, risk sharing, and goods-market clearing pin down y_t and z_t for any monetary policy, so there is no welfare motive for any interest-rate response. The IPT rate cut in Proposition 1 is therefore generated by the importer-pricing mechanism alone: it arises without any producer-price rigidity.

[1] As a check, solving the differentiated nonlinear Ramsey first-order conditions (Lemma 2) at the baseline calibration ($\theta = 0.8$) gives $\partial \log(1 + i_0)/\partial\tau_{M,0} = -0.025903$ for $\eta = 1.5$ and -0.058840 for $\eta = 3$. With $\theta = 10^{-8}$, the same calculation gives -0.160890974 for $\eta = 1.5$ and -0.087397486 for $\eta = 3$, against -0.160890996 and -0.087397488 from Eq. (A.2).

At the baseline values of the remaining parameters, $\tilde{\Gamma}_{\text{own}} = 0.414$, $\vartheta = 2.79$, $\delta = 0.687$, and $\kappa_M \vartheta = 0.145$, well below the bound 1.640; for $\eta = 3$, the corresponding values are 0.250, 1.54, 0.757, and 0.080. Both conditions of Proposition 1 therefore hold. In this benchmark, a one-percentage-point tariff increase lowers the notional net nominal rate on impact by 16.25 basis points for $\eta = 1.5$ and by 8.83 basis points for $\eta = 3$.[2]

H.2 Proof of Proposition 2 and Corollary 1

Step 1: Importer Pricing with $\theta_M = 0$. With $\theta_M = 0$, only the $k = 0$ term of the importer reset-price condition (16) remains, and every importer resets its price each period. With $\tau = 1/\varepsilon_M$, the condition gives, exactly,

$$P_{F,t}(\iota) = \overline{P}_{F,t} = \left(1 + \frac{\tau_{M,t}}{\varepsilon_M - 1}\right) \mathcal{E}_t P_F^* \qquad \text{for all } \iota,$$

as in Eq. (17). Hence there is no importer price dispersion, and by Eq. (2) every purchaser price equals $P_{F,t} + \tau_{M,t}\mathcal{E}_t P_F^* = (1 + \tilde{\tau}_{M,t})\mathcal{E}_t P_F^*$, with $\tilde{\tau}_{M,t}$ defined in Eq. (A.3). Equation (4) then gives $\widetilde{P}_{F,t}^{IPT} = (1 + \tilde{\tau}_{M,t})\mathcal{E}_t P_F^*$.

Step 2: Equivalence of the Two Ramsey Problems. Under PPT, $\widetilde{P}_{F,t}^{PPT} = (1 + \tau_{M,t})\mathcal{E}_t P_F^*$ and the LOOP holds, so $S_t = Z_t$. By Step 1, the IPT household-facing import price is the PPT one with $\tau_{M,t}$ replaced by $\tilde{\tau}_{M,t}$; hence $P_t^{IPT}/P_{H,t} = \mathcal{G}^{PPT}(\tilde{\tau}_{M,t}, Z_t)$. Substituting this into the remaining IPT constraints listed in Sec. 3.2, namely import demand, the intratemporal and Euler conditions, risk sharing (9), the producer block (B.11)–(B.15), goods-market clearing (19), and the ZLB conditions (20), yields exactly the PPT constraints evaluated at $\tilde{\tau}_{M,t}$, with S_t replaced by Z_t. The importer variables Ψ_t, $\Pi_{F,t}$, $\overline{P}_{F,t}$, $P_{F,t}$, and $\widetilde{P}_{F,t}^{IPT}$ are determined by the exchange-rate and tariff paths through Step 1 and introduce no additional independent constraints, so eliminating them leaves the feasible set of the remaining variables unchanged. Tariff revenue, the importer subsidy, and importer dividends enter only the lump-sum transfers, which are not Ramsey constraints under complete markets. The objective (1) is the same in both environments. After eliminating the importer-specific variables, the IPT Ramsey problem under $\{\tau_{M,t}\}$ is equivalent to the PPT Ramsey problem under $\{\tilde{\tau}_{M,t}\}$, including the ZLB constraints and timeless-perspective initial conditions. Consequently, the two problems yield identical optimal paths for the common allocation and monetary-policy variables. This proves Proposition 2. The impact response in this benchmark is characterized as follows.

Corollary 1. *Under the assumptions of Proposition 2, let the tariff follow $\tau_{M,t} = \rho_\tau \tau_{M,t-1} + \epsilon_{\tau,t}$ with $\rho_\tau \in [0, 1)$, hit at date 0 by an unanticipated innovation $\epsilon_{\tau,0} > 0$ from the zero-tariff steady state, with perfect foresight thereafter, and consider the first-order approximation of the nonlinear timeless-perspective Ramsey first-order conditions with a non-binding ZLB. Let p_*, k_*, and g be as in Proposition 1, and define*

$$\Omega \equiv \frac{B^{SOE,PPT}}{Q^{SOE,PPT}}, \qquad \chi \equiv \frac{\kappa_H(p_* + \varphi)\Lambda_{\pi,H}^{SOE,PPT}}{Q^{SOE,PPT}},$$

with $Q^{SOE,PPT}$ and $B^{SOE,PPT}$ as in Eq. (F.25) and $\Lambda_{\pi,H}^{SOE,PPT}$ as in Eq. (F.30). Let $\delta_H \in (0, 1)$ be the root of $\beta\delta^2 - [1 + \beta + \kappa_H(p_* + \varphi)\chi]\delta + 1 = 0$ inside the unit circle. Then

$$\frac{\hat{\imath}_0^{IPT}}{\tau_{M,0}} = \frac{\varepsilon_M}{\varepsilon_M - 1}\left[(k_* - p_*\Omega)(1 - \rho_\tau) + \frac{1 - p_*\chi}{\chi}(\Omega - g)\frac{(1 - \delta_H)(1 - \beta\delta_H)(\delta_H + \rho_\tau - 1)}{1 - \beta\delta_H\rho_\tau}\right]. \quad \text{(H.4)}$$

The bracket is $\hat{\imath}_0^{PPT}/\tilde{\tau}_{M,0}$. The optimal (notional) nominal interest rate under IPT with $\theta_M = 0$ therefore rises on impact if and only if the bracket is positive. A sufficient condition is $p_*\Omega < k_*$ together with $(1 - p_*\chi)(\Omega - g)(\delta_H + \rho_\tau - 1) \geq 0$.

[2] To first order, the change in the net rate is $di_0 = \beta^{-1} d\hat{\imath}_0$, so these values equal β^{-1} times Eq. (A.2) evaluated at $\tau_{M,0} = 0.01$.

Step 3: First-Order PPT Response. By Proposition 2, $\hat{\imath}_0^{IPT}(\{\tau_{M,t}\}) = \hat{\imath}_0^{PPT}(\{\tilde{\tau}_{M,t}\})$, so it suffices to derive $\hat{\imath}_0^{PPT}/\tilde{\tau}_{M,0}$ and multiply by $\varepsilon_M/(\varepsilon_M-1)$. Applying Lemma 2 to the PPT problem, the differentiated nonlinear Ramsey first-order conditions have the coefficients of the quadratic form $\frac{1}{2}Q^{SOE,PPT}y_t^2 - B^{SOE,PPT}y_t\tilde{\tau}_{M,t} + \frac{1}{2}\Lambda_{\pi,H}^{SOE,PPT}\pi_{H,t}^2$, together with the differentiated private-equilibrium conditions. Under PPT, $\psi_t = 0$, so $z_t = p_*y_t - k_*\tilde{\tau}_{M,t}$ and real marginal cost is $mc_t = z_t + \varphi y_t = (p_*+\varphi)(y_t - g\tilde{\tau}_{M,t})$. Writing the producer Phillips curve (B.16) as $\pi_{H,t} - \beta\pi_{H,t+1} - \kappa_H(p_*+\varphi)(y_t - g\tilde{\tau}_{M,t}) = 0$ and letting μ_t denote the deviation of the original Ramsey multiplier on the producer pricing block in this normalization, the differentiated first-order conditions for y_t and $\pi_{H,t}$, $t \geq 0$, are

$$Q^{SOE,PPT}\,(y_t - \Omega\tilde{\tau}_{M,t}) - \kappa_H(p_*+\varphi)\mu_t = 0, \qquad \Lambda_{\pi,H}^{SOE,PPT}\pi_{H,t} + \mu_t - \mu_{t-1} = 0,$$

with $\mu_{-1} = 0$. The relative-price and Euler conditions only determine i_t. Eliminating μ_t and summing from date 0 with $p_{H,-1} = 0$ gives

$$y_t - \Omega\tilde{\tau}_{M,t} = -\chi p_{H,t}, \qquad t \geq 0. \tag{H.5}$$

Step 4: The Producer Price Path. Substituting Eq. (H.5) into the Phillips curve with $\pi_{H,t} = p_{H,t} - p_{H,t-1}$ gives

$$\beta p_{H,t+1} - [1 + \beta + \kappa_H(p_*+\varphi)\chi]p_{H,t} + p_{H,t-1} = -\kappa_H(p_*+\varphi)(\Omega - g)\tilde{\tau}_{M,t}.$$

The characteristic polynomial $f_H(\ell) = (1-\ell)(1-\beta\ell) - \kappa_H(p_*+\varphi)\chi\ell$ satisfies $f_H(0) = 1 > 0$ and $f_H(1) < 0$, because $\chi > 0$. It therefore has one root $\delta_H \in (0,1)$, and the other root is $1/(\beta\delta_H) > 1$. As in the proof of Proposition 1, imposing boundedness eliminates the explosive root and yields $p_{H,t} = \delta_H p_{H,t-1} + A_H\tilde{\tau}_{M,t}$, where $A_H \equiv \delta_H\kappa_H(p_*+\varphi)(\Omega-g)/(1-\beta\delta_H\rho_\tau)$. Hence $\pi_{H,1} = (\delta_H + \rho_\tau - 1)A_H\tilde{\tau}_{M,0}$.

Step 5: The Impact Interest-Rate Response. Lemma 1 holds under PPT as well, because Eq. (B.10) with $S_t = Z_t$ gives $C_tP_t^{PPT} = \mathcal{E}_tP_F^*$. With $p_F^* = 0$, $e_t = z_t + p_{H,t}$. Using $z_t = p_*y_t - k_*\tilde{\tau}_{M,t}$ and Eq. (H.5), $e_t = (p_*\Omega - k_*)\tilde{\tau}_{M,t} + (1 - p_*\chi)p_{H,t}$. Therefore

$$\hat{\imath}_0^{PPT} = e_1 - e_0 = (k_* - p_*\Omega)(1-\rho_\tau)\tilde{\tau}_{M,0} + (1-p_*\chi)(\delta_H + \rho_\tau - 1)A_H\tilde{\tau}_{M,0}.$$

Since $f_H(\delta_H) = 0$ is equivalent to $\kappa_H(p_*+\varphi)\chi\delta_H = (1-\delta_H)(1-\beta\delta_H)$,

$$(1-p_*\chi)A_H = \frac{1-p_*\chi}{\chi}\,(\Omega - g)\,\frac{(1-\delta_H)(1-\beta\delta_H)}{1-\beta\delta_H\rho_\tau}.$$

Multiplying by $\tilde{\tau}_{M,0}/\tau_{M,0} = \varepsilon_M/(\varepsilon_M - 1)$ yields Eq. (H.4). Under the sufficient condition, the first term is strictly positive and the second is non-negative, so $\hat{\imath}_0^{IPT} > 0$. □

Multiplying the PPT values of the bracket, 0.010506 for $\eta = 1.5$ and 0.004713 for $\eta = 3$, by $\varepsilon_M/(\varepsilon_M - 1)$ gives $\hat{\imath}_0^{IPT}/\tau_{M,0} = 0.014258$ and 0.006397, respectively.

Further interpretation of Proposition 2. With the fixed importer subsidy, the opportunistic-pricing incentive magnifies the effective tariff by the factor $\varepsilon_M/(\varepsilon_M - 1)$. If the importer subsidy were instead indexed to the tariff, as in Eq. (A.4), the *ex-tariff* import price would equal the border price, and the IPT economy with $\theta_M = 0$ would coincide with PPT for the same tariff path. In either case, the sign of the IPT response with flexible import prices is the sign of a PPT response, which can be characterized in closed form to first order.

Evaluation of Corollary 1. Equation (H.4) mirrors Eq. (A.2). Its first term is the decay of the tariff-induced change in the welfare-relevant *ex-tariff* relative border price, $z_t^w = (p_*\Omega - k_*)\tilde{\tau}_{M,t}$. When $p_*\Omega < k_*$, the tariff calls for a welfare-relevant real appreciation; as it decays, the

Home currency is expected to depreciate, and Lemma 1 translates this into a rate increase. In Proposition 1, by contrast, the tariff raises the welfare-relevant LOOP gap and calls for a welfare-relevant depreciation whose decay implies a rate cut. The second term reflects the anchoring of the producer price level under commitment, $y_t - \Omega\tilde{\tau}_{M,t} = -\chi p_{H,t}$ (Eq. (H.5)).

At the baseline values of the remaining parameters, $p_*\Omega - k_* = -0.039$, $\Omega = 0.060$, $g = 0.029$, $\chi = 3.71$, and $\delta_H = 0.477$ for $\eta = 1.5$; for $\eta = 3$, the corresponding values are -0.033, 0.397, 0.085, 3.61, and 0.500. The bracket in Eq. (H.4) is positive for both elasticities, although the simple sufficient condition does not hold because $1 - p_*\chi < 0$, while $\Omega - g > 0$ and $\delta_H + \rho_\tau - 1 > 0$, for both elasticities. To first order, a 20-percentage-point tariff with flexible import prices therefore raises the notional net rate on impact by 28.8 basis points for $\eta = 1.5$ and by 12.9 basis points for $\eta = 3$. The exact nonlinear values implied by Proposition 2, obtained by solving PPT with the effective tariff $\tilde{\tau}_{M,0} = 0.2714$, are +21.49 and +1.90 basis points, respectively. The sign is not universal: for $\eta = 3$ and $\rho_\tau = 0.9$, the bracket is negative, because PPT itself calls for a cut there. For $\eta = 1.5$, the same solution implies that relative import consumption falls by 29.03 percent on impact, compared with 24.19 percent under IPT with $\theta_M = 0.8$.

H.3 Price-Rigidity Sensitivity in the Small Open Economy

The main purpose of the parameter map is to connect the two analytical limits and examine their implications in the fully nonlinear Ramsey model for a finite tariff shock. Proposition 1 establishes conditions for a cut in the limit of flexible producer prices with sticky import prices; Proposition 2 establishes exact equivalence to PPT with a magnified tariff at the flexible-import-price boundary. At our calibration, the latter boundary calls for a rate increase. Figures A.1 and A.2 connect these two limits. They report the fully nonlinear Ramsey responses to a 20-percentage-point tariff with $\rho_\tau = 0.7$ on a grid of producer and importer Calvo parameters, $\theta \in \{0.1, \ldots, 0.8\}$ and $\theta_M \in \{0, 0.1, \ldots, 0.8\}$, for $\eta = 1.5$ and $\eta = 3$. The ZLB is not imposed, so the figures report the notional rate. For $\theta_M > 0$, each node solves the nonlinear IPT Ramsey problem. For $\theta_M = 0$, each node solves the PPT Ramsey problem with the effective tariff $\tilde{\tau}_{M,0} = (3.8/2.8) \times 0.20 = 0.2714$, which is exactly equivalent by Proposition 2. All 144 nodes converge, with maximum residuals below 2×10^{-12}. At 10 of the 144 nodes (4 for $\eta = 1.5$ and 6 for $\eta = 3$, all with $\theta \leq 0.2$), the optimal notional rate becomes negative at some date; at all other nodes the ZLB would not bind, so the results coincide with those under the ZLB.

Three features stand out. First, the two edges behave as the propositions predict. At $\theta = 0.1$ and the baseline importer stickiness $\theta_M = 0.8$, the optimal rate is cut by 61.7 basis points for $\eta = 1.5$ and by 150.5 basis points for $\eta = 3$, in line with Proposition 1. Along $\theta_M = 0$, the rate rises for every θ, by between 21.5 and 34.7 basis points for $\eta = 1.5$ and between 1.9 and 60.2 basis points for $\eta = 3$, in line with Proposition 2 and Corollary 1. Convergence toward the $\theta_M = 0$ edge is slow when producer prices are sticky: for $\eta = 3$ and $\theta \geq 0.6$, the response at $\theta_M = 0.1$ is still a cut, so the sign change occurs between $\theta_M = 0.1$ and 0, and the interpolated contour there should not be read literally.

Second, the initial response is a cut over most of the interior, including the baseline: it is negative at 44 of the 48 nodes with $\theta_M \geq 0.3$ for $\eta = 1.5$ and at 43 of 48 for $\eta = 3$. Along the baseline importer stickiness, however, the response is hump-shaped in θ (Fig. A.2). Starting from the baseline $\theta = 0.8$, lower producer-price stickiness first raises the response and turns the cut into an increase at intermediate θ, peaking at 27.7 basis points ($\eta = 1.5$, $\theta = 0.3$) and 22.2 basis points ($\eta = 3$, $\theta = 0.4$); the response then falls again, and the cut returns as θ approaches the limit in Proposition 1. The interior hump is a secondary numerical finding that reflects the interaction of producer-price adjustment with the importer-pricing block, not producer-price rigidity in isolation. Along $\theta_M = 0$, where Proposition 2 applies, the initial response is monotonic in θ: it falls from +34.7 to +21.5 basis points as θ rises from 0.1 to 0.8 for $\eta = 1.5$, and from +60.2 to +1.9 for $\eta = 3$. By contrast, with $\theta_M = 0.8$ the response peaks at intermediate θ, as noted above. The difference between these two rows is largest at $\theta = 0.1$, closest to zero at $\theta = 0.3$ for $\eta = 1.5$ and at $\theta = 0.5$ for $\eta = 3$, and larger again at $\theta = 0.8$.

One economic interpretation follows from the structure of the inflation weights in Eq. (F.30). Each weight is inversely proportional to the slope of the corresponding Phillips curve, so that $\kappa_H \Lambda_{\pi,H}^{SOE,IPT}$ and $\kappa_M \Lambda_{\pi,F}^{SOE,IPT}$ depend on neither θ nor θ_M. For a given path of the relevant Phillips-curve driving term—real marginal cost for producer inflation (Eq. (B.16)) and $\psi_t + \tau_{M,t}/(\varepsilon_M - 1)$ for import-price inflation (Eq. (18))—the inflation contribution to the welfare cost *rises* with flexibility of the corresponding price: the larger inflation response more than offsets the smaller weight. As prices become fully flexible, however, price setting itself drives that term to zero—$mc_t^r = 0$ for producers in the limit of Proposition 1 and $\psi_t = -\tau_{M,t}/(\varepsilon_M - 1)$ for importers at the boundary of Proposition 2—and the corresponding stabilization motive vanishes. When prices are very sticky, inflation responds little. The welfare importance of stabilizing each inflation rate is therefore greatest at intermediate rigidity. This structure is the same as that of the wage-inflation term in Galí and Monacelli [4], where the volatility of wage inflation rises with wage flexibility while its welfare weight falls, so that welfare losses are hump-shaped in wage rigidity under a currency peg or an exchange-rate-oriented rule.

The HIAC gives the Ramsey planner an incentive to ease monetary policy by moving the actual LOOP gap toward the welfare-relevant LOOP gap. This incentive is balanced against the objectives of stabilizing import-price inflation and Home PPI inflation. The relative importance of these objectives varies with producer- and importer-price stickiness. Accordingly, when import prices are sticky, the initial response is hump-shaped in θ: at intermediate θ the PPI-stabilization motive offsets most of the easing, and at $\theta_M = 0.8$ enough of it to produce an increase. Conversely, for given θ the cut is deepest at intermediate importer stickiness—at θ_M between 0.4 and 0.7 for every θ on the grid and both elasticities—and the response rises again toward $\theta_M = 0.8$. Along $\theta_M = 0$, there is no hump: producer-price stickiness still affects the optimal response through the producer-pricing block, but its effect is monotonic in θ on the grid.

The parallel with Galí and Monacelli [4] has limits. In their model, the exchange-rate orientation of policy is imposed by the monetary regime, and their non-monotonicity persists under complete import-price pass-through (their Fig. 12). In our model, the HIAC gives the Ramsey planner an incentive to adjust the LOOP gap through monetary policy when import prices are sticky. At $\theta_M = 0$, however, the LOOP gap is determined by the tariff independently of monetary policy. Their object is welfare under given monetary regimes; ours is the optimal interest-rate path. The argument above interprets the fully nonlinear Ramsey solutions; it is not a decomposition of the Ramsey first-order conditions, and the weights in Eq. (F.30) are not substituted for the original utility objective. The first-order solutions also display a hump along $\theta_M = 0.8$ but remain negative throughout that row; the positive peaks at intermediate θ in Fig. A.2 arise in the fully nonlinear 20-percentage-point experiment.

Third, the cumulative responses in the bottom row distinguish two kinds of rate increase. At $\theta_M = 0.8$, the cumulative 20-quarter response is negative for every θ, between -471 and -29 basis-point-quarters for $\eta = 1.5$ and between -513 and -105 for $\eta = 3$. At intermediate θ, the initial increase is therefore followed by rate cuts large enough to make the cumulative response negative. At $\theta_M = 0$, by contrast, both the initial and the cumulative responses are positive (between 108 and 130 basis-point-quarters for $\eta = 1.5$ and between 136 and 232 for $\eta = 3$). At $\theta_M = 0$, Eq. (17) pins down the actual LOOP gap as a function of the tariff, independently of monetary policy. The welfare-relevant LOOP-gap deviation therefore can no longer be adjusted through monetary policy. The Ramsey allocation is equivalent to that under PPT with the adjusted tariff specified in Proposition 2, and monetary policy tightens. The zero contour of the cumulative response rises with θ over most of the grid: cumulative easing prevails when import prices are very sticky or producer prices are nearly flexible, and cumulative tightening prevails when import prices are relatively flexible. The relative stickiness of the two prices alone does not determine the sign, however; for example, the cumulative response is $+38.9$ basis-point-quarters at $(\theta, \theta_M) = (0.4, 0.5)$ for $\eta = 1.5$ and -44.6 at $(0.8, 0.6)$ for $\eta = 3$. At the baseline, the initial cut is followed by later increases; the cumulative response is -33 basis-point-quarters for $\eta = 1.5$ and -107 for $\eta = 3$.

The rate cut under IPT is therefore not unconditional. It is robust when import prices are sticky and producer prices are either sticky, as in the baseline, or nearly flexible, as in Proposition 1;

it disappears when import prices are nearly flexible, as in Proposition 2; and at intermediate producer-price stickiness, where PPI stabilization matters most, the PPI-inflation stabilization objective can outweigh the HIAC-induced incentive for monetary easing, producing an initial rate increase followed by rate cuts.

I Robustness Checks

I.1 High Substitution Case

I.1.1 Small Open Economy

Figure I.1 presents the SOE experiment with the substitution elasticity raised to $\eta = 3$, holding the tariff shock and every other parameter fixed. *IPT continues to call for an initial interest-rate cut*, and the cut is larger than under the baseline elasticity (Panel 1). Equation (23) continues to characterize the welfare-relevant LOOP gap. Although the own-tariff coefficient $\Gamma_{\text{own}}^{SOE}$ declines from 0.421 to 0.293, the Home welfare-relevant LOOP gap still rises with the tariff, preserving the incentive for monetary easing through the HIAC. The magnitude of the optimal rate cut reflects this incentive together with the inflation-stabilization objectives.

By comparison, under PPT a higher elasticity of substitution strengthens expenditure switching toward Home goods following an import-tariff increase, intensifying upward pressure on Home PPI inflation (Panels 2 and 6). However, monetary tightening exerts appreciation pressure on the Home currency, lowering the *ex-tariff* relative price of imports and shifting expenditure back toward Foreign goods. With a higher elasticity of substitution, this countervailing expenditure-switching effect is stronger. Consequently, the stronger expenditure-switching response to nominal appreciation allows the Ramsey planner to limit the increase in the nominal interest rate, despite greater upward pressure on Home PPI inflation. The optimal policy remains a rate increase, and the policy rate rises further in subsequent quarters. The impact responses are -47.08 basis points under IPT and $+3.31$ basis points under PPT. Thus the SOE produces an optimal IPT rate cut under both elasticities, while the policy-sign contrast with PPT also persists.

I.1.2 Two-Country Economy: Unilateral Tariff

Figures I.2 and I.3 report the unilateral tariff experiment when $\eta = 3$. The key IPT finding persists: Home initially cuts its nominal interest rate, by 51.35 basis points (Panel 1). The welfare-relevant LOOP gap remains positive on impact (Panel 10), although the own-tariff loading Γ_{own} in Eq. (30) declines from 0.364 to 0.053. At the same time, the FAC coefficient Γ_{cross} rises, and the welfare weight on the Home welfare-relevant LOOP-gap deviation increases. However, the Foreign LOOP gap rises under the unilateral tariff increase (Panel 4 of Fig. I.3). The higher FAC coefficient therefore amplifies its negative contribution to the Home welfare-relevant LOOP gap. The Home welfare-relevant LOOP gap is smaller on impact than in the baseline, even though the initial Home rate cut is larger.

The comparative PPT result is different from the baseline. Under PPT, the stronger direct substitution away from Foreign goods raises demand for Home goods and puts upward pressure on Home PPI inflation. However, it also produces a much larger contraction in Foreign demand, inducing a sharp reduction in the Foreign policy rate. Foreign reaches the ZLB on impact (Panel 1, Fig. I.3), and the Home currency appreciates strongly (Panel 11, Fig. I.2). Under PPT, Home-currency appreciation immediately lowers the *cum-tariff* relative price of imports faced by Home households, inducing expenditure switching toward Foreign goods. This restrains Home demand and exerts downward pressure on Home PPI inflation. The Home planner therefore cuts its rate even under PPT, by 44.27 basis points on impact. Home PPI inflation nevertheless remains positive: the exchange-rate-induced pressure partially offsets, rather than fully eliminates, the direct effect of the Home tariff. A binding Foreign ZLB accompanies this policy interaction, but the sign of the Home PPT rate cut is not attributable to the ZLB constraint alone.

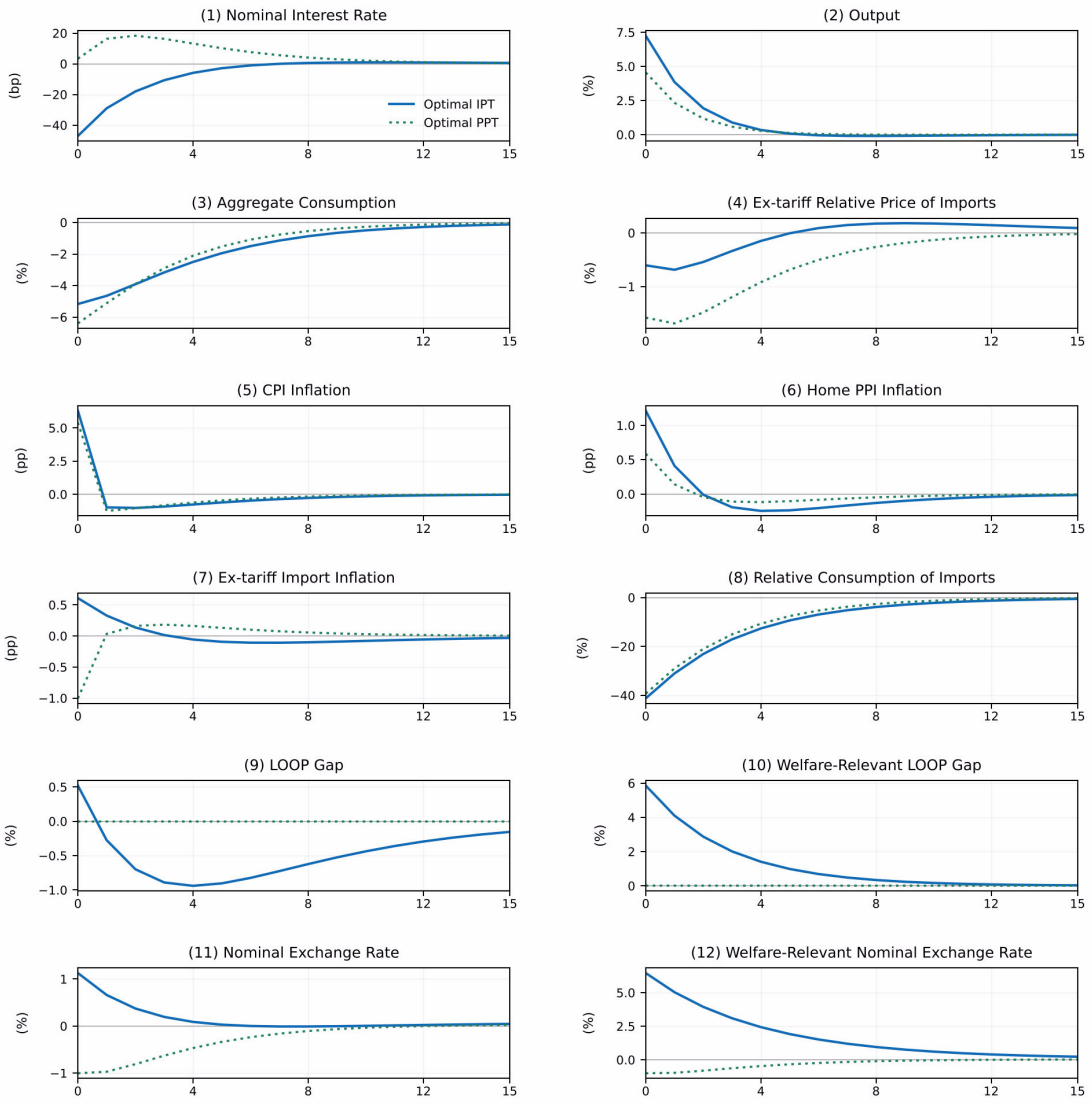


Figure I.1: Small open economy: responses under the High Substitution Case ($\eta = 3$), holding other parameters and the unilateral Home tariff shock fixed. The panels, units, and PPT/IPT line styles are as in Fig. 2.

Under IPT, both countries cut their nominal interest rates in response to the Home tariff increase; the Home rate cut is larger on impact, whereas the Foreign rate cut is more persistent (Panel 1, Figs. I.2 and I.3). The nominal exchange rate appreciates, and this appreciation raises the Foreign LOOP gap Ψ_t^* (Panel 4, Fig. I.3). With the Foreign tariff at zero, this increase makes the FAC contribution to the Home welfare-relevant LOOP gap negative, partially offsetting the Home tariff's upward effect (Fig. 1). The Home IPT rate cut exceeds the PPT cut on impact by 7.08 basis points. Thus *the IPT rate cut is robust to higher substitution elasticity*, even though the PPT rate increase is not. In both environments Home subsequently raises the policy rate temporarily above its steady-state level, while Home PPI inflation turns negative after its initial positive response (Panels 1 and 6). Thus the planner permits an initial increase in producer-price inflation and subsequently generates disinflation to restrain the cumulative change in the Home producer-price level.

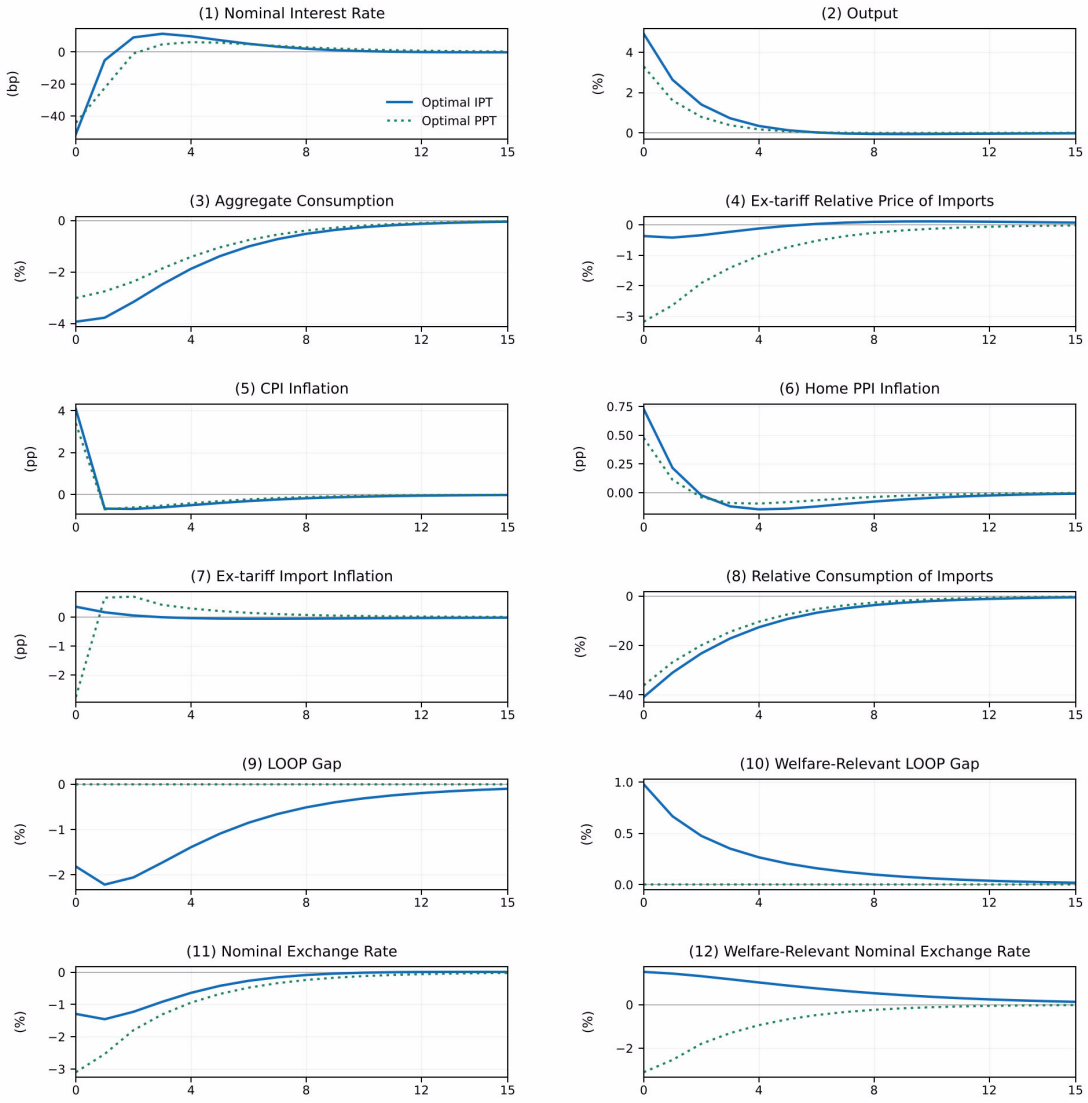


Figure I.2: Two-country unilateral tariffs, High Substitution Case ($\eta = 3$): Home PPT and IPT responses. Panel definitions and units match Fig. 3.

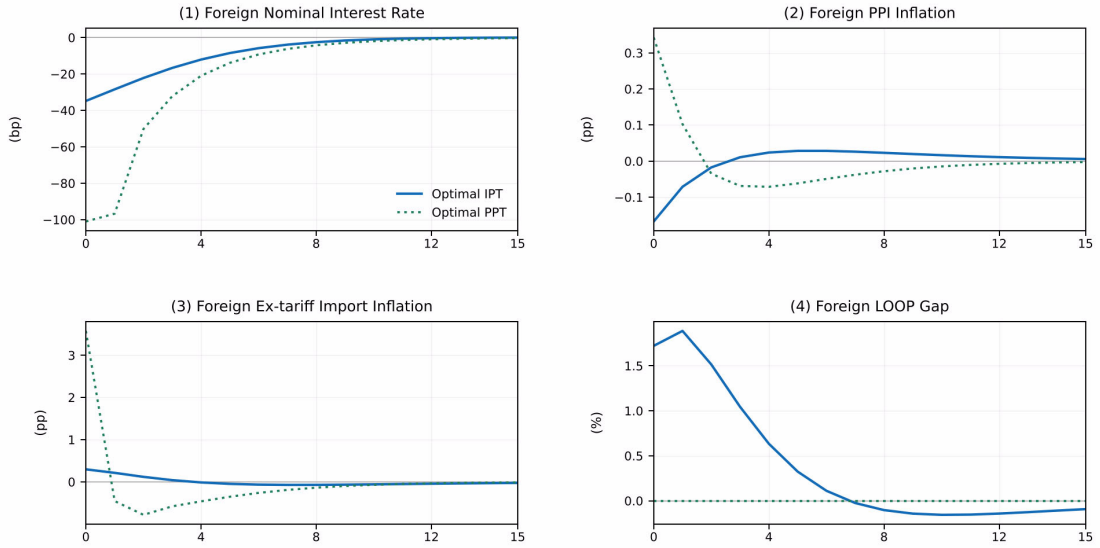


Figure I.3: Two-country unilateral tariffs, High Substitution Case ($\eta = 3$): Foreign PPT and IPT responses. Panel definitions and units match Fig. G.1.

I.1.3 Two-Country Economy: Reciprocal Tariffs

We retain the reciprocal tariff shocks of Sec. 4.6 and raise only the Home–Foreign substitution elasticity from the baseline value $\eta = 1.5$ to $\eta = 3$, the High Substitution Case considered by Monacelli [6]. The experiment examines how stronger expenditure switching affects optimal monetary policy, holding all other structural parameters and the tariff paths unchanged. Figures I.4 and I.5 compare optimal PPT and IPT responses at $\eta = 3$; the corresponding baseline paths are reported in Figs. 4 and G.2.

The IPT policy response remains a substantial rate cut. The Home nominal interest rate reaches the ZLB for two quarters, whereas it remains positive in the baseline; the Foreign IPT nominal rate remains positive (Panel 1 of Figs. I.4 and I.5). The Home rate cut reflects both the Home welfare-relevant LOOP gap and the external-demand effects of reciprocal tariffs.

For comparison, under PPT a higher substitution elasticity strengthens expenditure switching in both countries. The Home tariff shifts Home expenditure from Foreign goods toward Home goods, while the Foreign tariff reduces Foreign demand for Home goods. The latter effect creates an incentive for Home monetary easing: under PPT, Home-currency depreciation lowers the *cum-tariff* relative price of Home goods faced by Foreign households, supporting their demand for Home goods. This incentive operates against the inflationary pressure arising from the Home tariff's direct expenditure-switching effect. In the High Substitution Case, the optimal response is nevertheless a sufficiently large rate cut for the Home nominal interest rate to reach the ZLB for four quarters, compared with two quarters in the baseline (Panel 1 of Fig. I.4). Foreign also cuts its nominal rate to the ZLB for two quarters, whereas its rate remains positive in the baseline (Panel 1 of Fig. I.5). The Home currency depreciates on impact under PPT (Panel 11).

Under IPT, the Home welfare cost function remains the two-country criterion in Eq. (29), with its coefficients and welfare-relevant reference values re-evaluated at $\eta = 3$. Tables F.1 and F.2 in Appendix F.3 report the corresponding welfare weights and the loadings of the Home welfare-relevant LOOP gap, respectively. A key change is the stronger *Foreign Allocation Channel* (FAC). In Eq. (30), the coefficient on $\tau^*_{M,t} - \psi^*_t$ rises from 0.060 in the baseline to 0.122 at $\eta = 3$. An increase in $\tau^*_{M,t} - \psi^*_t$ raises the *cum-tariff* relative price of Home goods faced by Foreign households, reducing their consumption of Home goods relative to Foreign goods. Greater substitutability amplifies this reduction in relative consumption and the resulting inefficient shift of Home-good

consumption toward Home households. In this reciprocal-tariff experiment, the Foreign LOOP gap falls on impact under both elasticities (Panel 4 of Figs. G.2 and I.5); together with the positive Foreign tariff, its negative sign makes the FAC contribution positive. The increase in the FAC loading strengthens this contribution without requiring a substantially larger initial fall in the Foreign LOOP gap.

At the same time, the own-tariff loading Γ_{own} declines from 0.364 to 0.053. Consequently, the *total* initial increase in the Home welfare-relevant LOOP gap is smaller at $\eta = 3$ than under the baseline, even though the Foreign-side contribution is larger (Panel 10 of Fig. I.4). The welfare weight on the Home welfare-relevant LOOP-gap deviation, Λ_ψ, nevertheless rises from 0.406 to 1.206 (Table F.1). A higher elasticity makes relative demands more sensitive to prices, increasing the welfare consequences of deviations from the welfare-relevant allocation. The weight on ex-tariff import-price inflation also rises, limiting the extent to which monetary accommodation can be used to adjust the actual LOOP gap.

Consistent with these competing considerations, the initial actual Home LOOP gap increases more under IPT at $\eta = 3$ than in the baseline, while the Home currency depreciates (Panels 9 and 11 of Fig. I.4). As explained above, the higher elasticity amplifies the external-demand effect of the Foreign tariff, so that the Home ZLB binds under IPT at $\eta = 3$ but not in the baseline. Thus, at $\eta = 3$, reciprocal tariffs generate a Home ZLB episode in both pricing environments and a Foreign ZLB episode only under PPT. The constrained Home policy rates alone cannot rank the desired unconstrained interest-rate reductions. We therefore also solve the *same* nonlinear Nash–Ramsey systems with the Home and Foreign ZLB constraints removed, leaving all structural parameters, tariff paths, and timeless-perspective initial conditions unchanged. Table I.1 reports the minimum quarterly notional interest-rate *levels*, alongside the constrained ZLB durations. Under PPT the notional Home rate reaches −192.18 basis points, compared with −62.06 basis points under IPT. For Foreign the minima are −66.29 and 52.68 basis points, respectively. Hence PPT requires more accommodation in both economies and produces a Foreign ZLB episode, even though the HIAC provides an additional welfare-based incentive for Home monetary easing under IPT.

Table I.1: Reciprocal tariffs at $\eta = 3$: notional minima and ZLB durations

	PPT		IPT	
Economy	Min. notional rate (bp)	ZLB quarters	Min. notional rate (bp)	ZLB quarters
Home	−192.18	4	−62.06	2
Foreign	−66.29	2	52.68	0

Note: Minimum notional rates are quarterly nominal interest-rate *levels*, not deviations from steady state; the steady-state nominal rate is 101.01 bp. Notional solutions omit both national ZLB constraints. ZLB durations refer to the corresponding constrained Nash–Ramsey allocations. The tariff increases by 20 percentage points in each country and follows the baseline persistence $\rho_\tau = 0.7$.

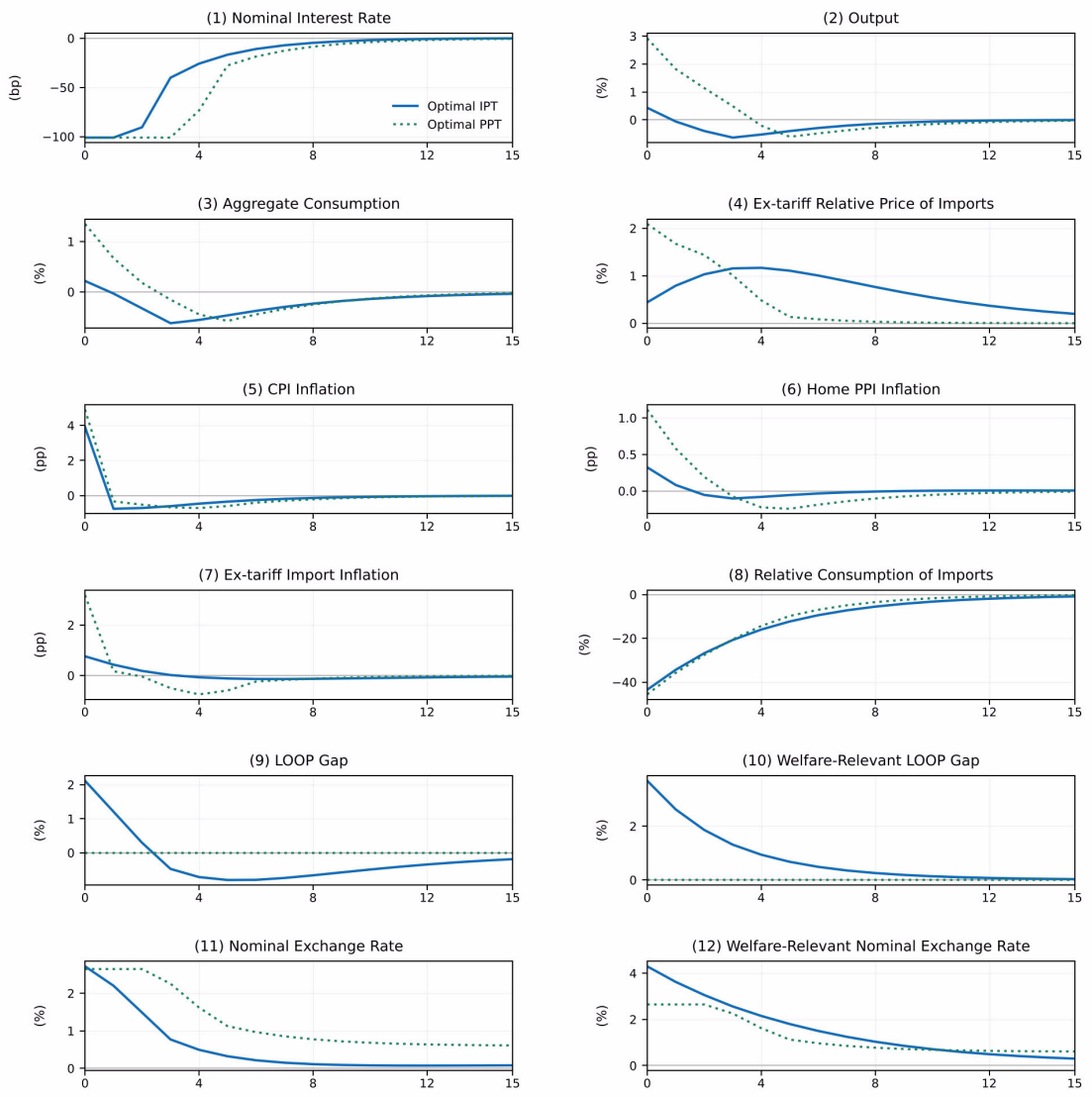


Figure I.4: Two-country reciprocal tariffs, High Substitution Case ($\eta = 3$): Home PPT and IPT responses. Panel definitions and units match Fig. 4.

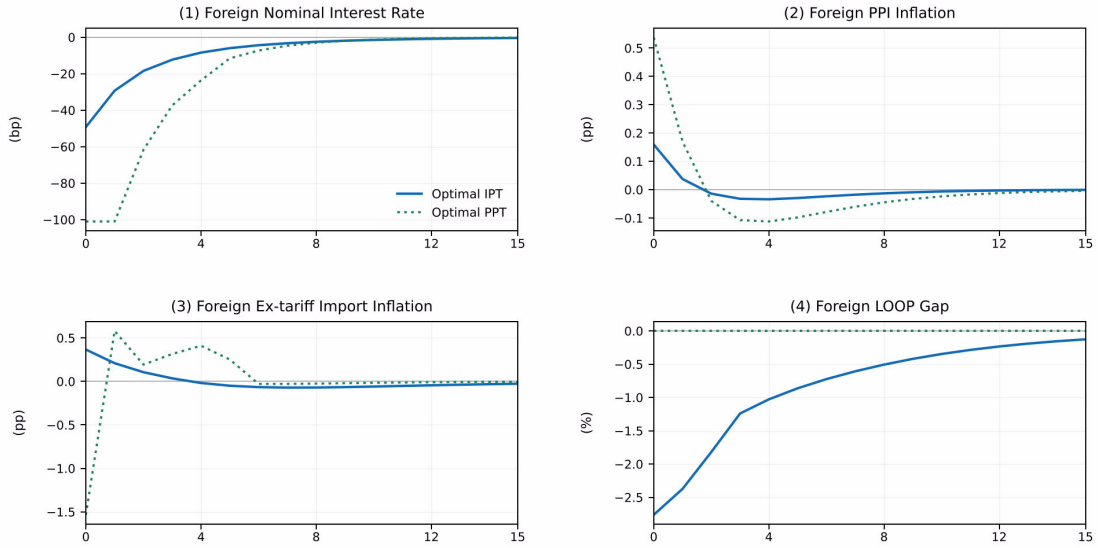


Figure I.5: Two-country reciprocal tariffs, High Substitution Case ($\eta = 3$): Foreign PPT and IPT responses. Panel definitions and units match Fig. G.2.

I.2 Equal Country Sizes

The baseline sets the Home economic-size share to $\nu = 0.27$ (Sec. 4.3). This subsection repeats the unilateral and reciprocal tariff experiments with equal country sizes, $\nu = 0.5$, holding every other parameter, the tariff paths, and the two noncooperative timeless-perspective Ramsey problems unchanged. With $\upsilon = 0.3$, Home's expenditure weight on Foreign goods, $\omega_F = \upsilon(1-\nu)$, falls from 0.219 to 0.150. Table I.2 reports the impact responses, the minimum notional rates, and the ZLB durations, and Fig. I.6 overlays the Home responses to the unilateral tariff on the baseline responses.

Under the unilateral Home tariff, the Home IPT planner still cuts the nominal interest rate on impact, by 4.26 basis points, compared with 17.23 basis points in the baseline (Panel 1 of Fig. I.6). The smaller cut is consistent with the welfare weights at $\nu = 0.5$: the weight on the Home welfare-relevant LOOP-gap deviation, Λ_{ψ}^{IPT}, falls from 0.406 to 0.293, and the own-tariff loading Γ_{own} in Eq. (30) falls from 0.364 to 0.345, so the HIAC provides a weaker incentive for monetary easing. The impact Home response under PPT is close to zero: $+0.03$ basis points in the constrained solution and -0.24 basis points in the notional solution. The Home PPT rate rises from the following quarter (Panel 1 of Fig. I.6). Under PPT, Foreign reaches the ZLB on impact. Thus, the IPT rate cut remains robust to equal country sizes, whereas the PPT impact rate increase becomes quantitatively negligible in the constrained solution and turns into a small cut in the notional solution. As in the baseline (Sec. 4.5), the cumulative Home IPT–PPT interest-rate difference is negative at every horizon within the first 20 quarters.

Under reciprocal tariffs with equal country sizes, the Home and Foreign nominal interest-rate paths coincide in each pricing environment. The stronger Home easing in the baseline reciprocal experiment (Sec. 4.6) therefore reflects the asymmetric country sizes. The need for monetary accommodation remains stronger under PPT than under IPT: the impact rate cut is 100.94 basis points under PPT and 43.15 basis points under IPT. Under PPT, the minimum notional rate is 0.07 basis points, so the ZLB does not bind; the Home ZLB episode under PPT in the baseline reciprocal experiment therefore depends on the country-size calibration.

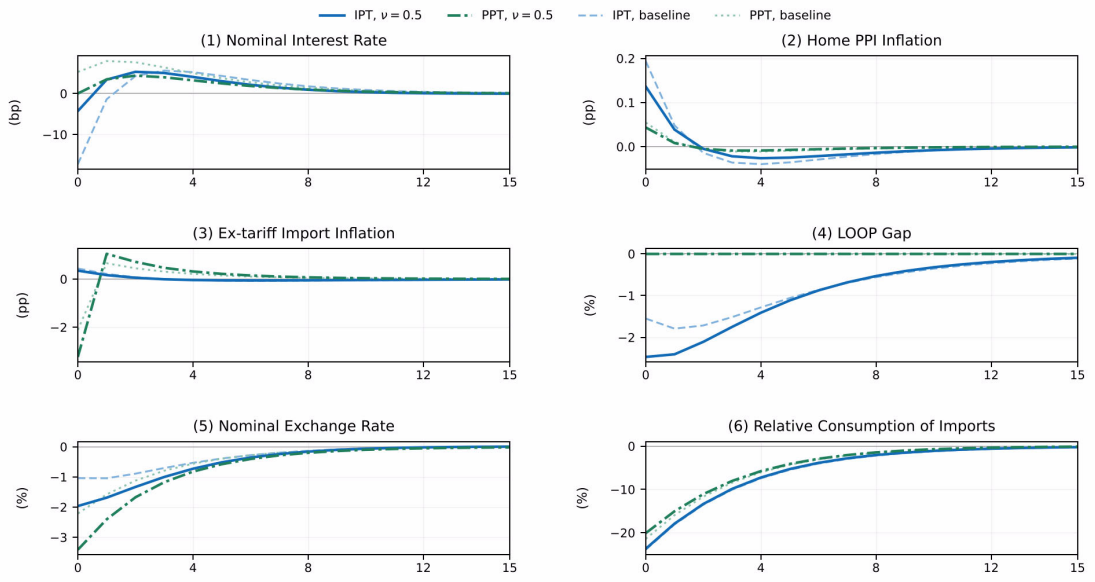


Figure I.6: Equal country sizes: Home responses to a 20-percentage-point unilateral Home import tariff with $\nu = 0.5$, overlaid on the baseline responses ($\nu = 0.27$). Panel 1 is in basis points, Panel 6 is in percent deviation from steady state, and other variables are in percent or percentage points.

Table I.2: Two-country economy with equal country sizes: impact responses, notional minima, and ZLB durations

			Impact response (bp)		Min. notional rate (bp)		ZLB quarters
Tariff	Pricing	ν	Home	Foreign	Home	Foreign	Home/Foreign
Unilateral	PPT	0.27	5.27	−58.58	101.01	42.43	0/0
		0.5	0.03	−101.01	100.77	−3.14	0/1
	IPT	0.27	−17.23	−16.70	83.78	84.20	0/0
		0.5	−4.26	−32.22	96.75	68.61	0/0
Reciprocal	PPT	0.27	−101.01	−57.23	−36.12	42.27	2/0
		0.5	−100.94	−100.94	0.07	0.07	0/0
	IPT	0.27	−75.98	−18.58	25.03	82.43	0/0
		0.5	−43.15	−43.15	57.86	57.86	0/0

Note: Impact responses are deviations of the constrained optimal nominal interest rates from their steady state. Minimum notional rates are quarterly nominal interest-rate *levels* from the solutions without the ZLB; the steady-state level is 101.01 bp. ZLB durations refer to the constrained Nash–Ramsey allocations. The import tariff rises by 20 percentage points in Home only (unilateral) or in each country (reciprocal), with baseline persistence $\rho_\tau = 0.7$. All other parameters are at their baseline values.

I.3 Tariff Persistence

This subsection raises the tariff persistence from $\rho_\tau = 0.7$ to $\rho_\tau = 0.9$, holding all other parameters at their baseline values. Figures I.7 and I.8 overlay the Home responses on the baseline responses. In the SOE, the initial IPT rate cut is almost unchanged (-32.98 basis points), and the IPT rate returns to its steady-state level more slowly, whereas the PPT rate increase shrinks to $+0.84$ basis points (Panel 1 of Fig. I.7). In the two-country unilateral tariff experiment, the impact Home rate response changes from -17.23 to -23.68 basis points under IPT and from $+5.27$ to -4.41 basis points under PPT, and the IPT rate cut is also deeper in the following quarter (Panel 1 of Fig. I.8). Thus, the IPT rate cut on impact is robust to tariff persistence in both economies.

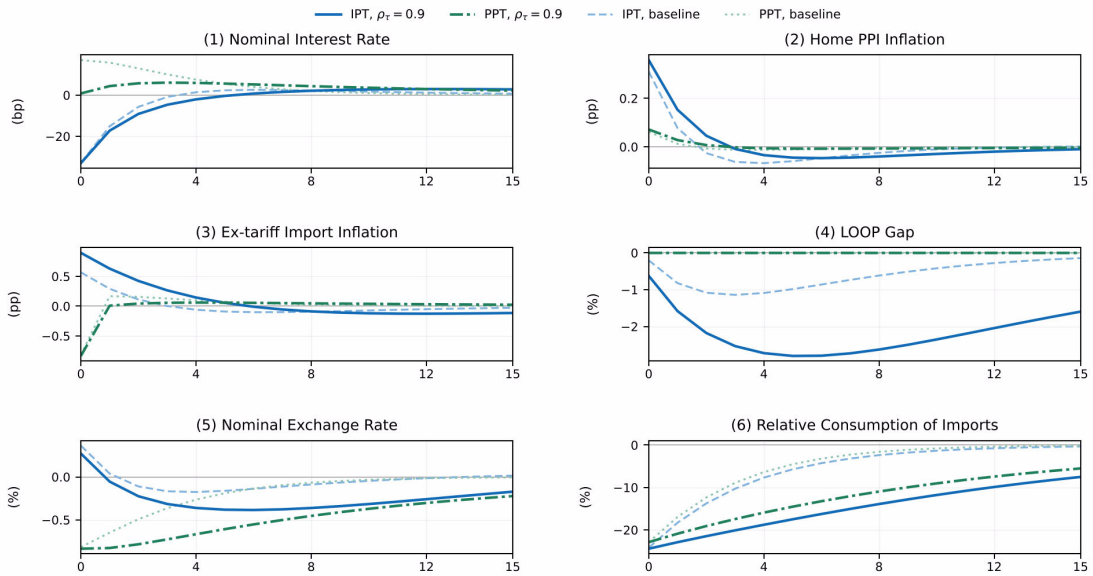


Figure I.7: Tariff persistence, small open economy: responses to a 20-percentage-point Home import tariff with $\rho_\tau = 0.9$, overlaid on the baseline responses ($\rho_\tau = 0.7$). Panel 1 is in basis points, Panel 6 is in percent deviation from steady state, and other variables are in percent or percentage points.

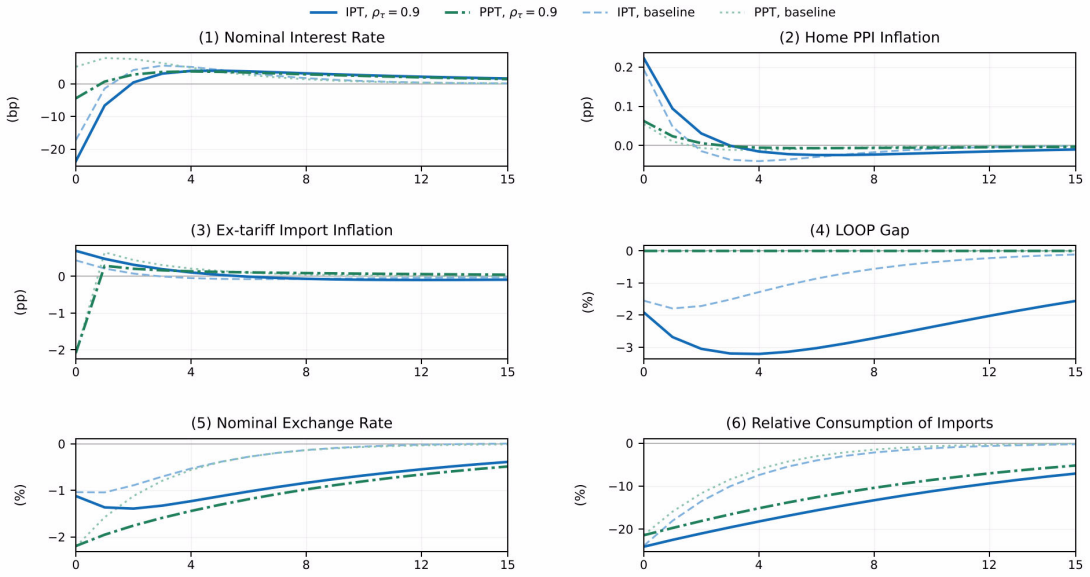


Figure I.8: Tariff persistence, two-country economy: Home responses to a 20-percentage-point unilateral Home import tariff with $\rho_\tau = 0.9$, overlaid on the baseline responses ($\rho_\tau = 0.7$). Panel 1 is in basis points, Panel 6 is in percent deviation from steady state, and other variables are in percent or percentage points.

I.4 Lower Stickiness Case

This subsection lowers the producer and importer Calvo parameters from $\theta = \theta_M = 0.8$ to $\theta = \theta_M = 0.75$, the benchmark values in Monacelli [5], holding all other parameters at their baseline values. Figures I.9 and I.10 overlay the Home responses on the baseline responses. In the SOE, the impact rate response is -32.04 basis points under IPT and $+17.76$ basis points under PPT, compared with -32.87 and $+17.03$ basis points in the baseline, and the IPT rate returns to its steady-state level one quarter earlier (Panel 1 of Fig. I.9). In the two-country unilateral tariff experiment, the impact Home rate response is -13.60 basis points under IPT and $+6.27$ basis points under PPT, compared with -17.23 and $+5.27$ basis points in the baseline, and the IPT rate cut is confined to the impact quarter, whereas in the baseline it lasts two quarters (Panel 1 of Fig. I.10).

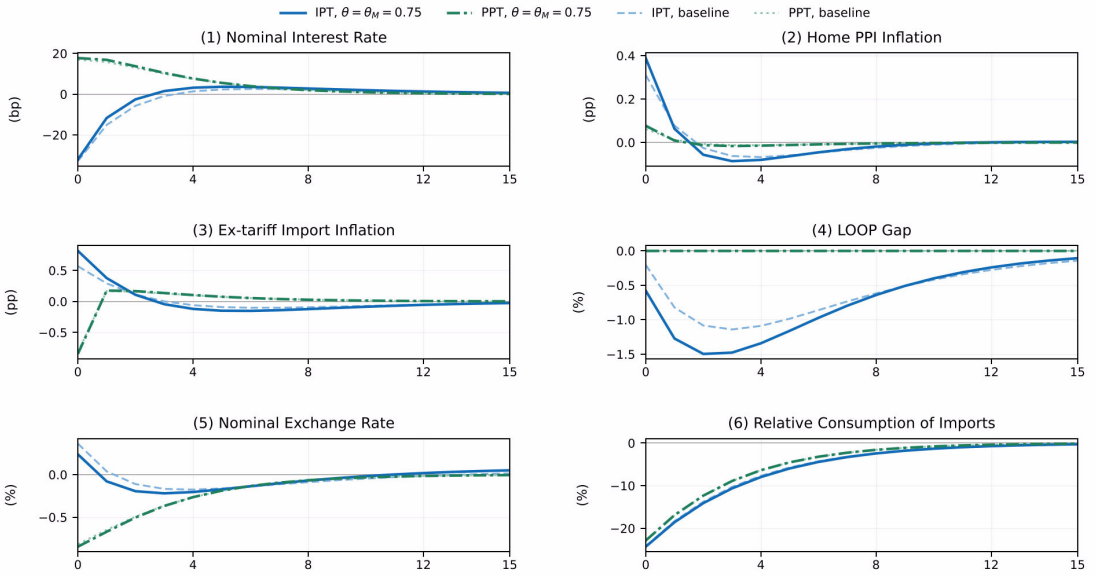


Figure I.9: Lower Stickiness Case, small open economy: responses to a 20-percentage-point Home import tariff with $\theta = \theta_M = 0.75$, overlaid on the baseline responses ($\theta = \theta_M = 0.8$). Panel 1 is in basis points, Panel 6 is in percent deviation from steady state, and other variables are in percent or percentage points.

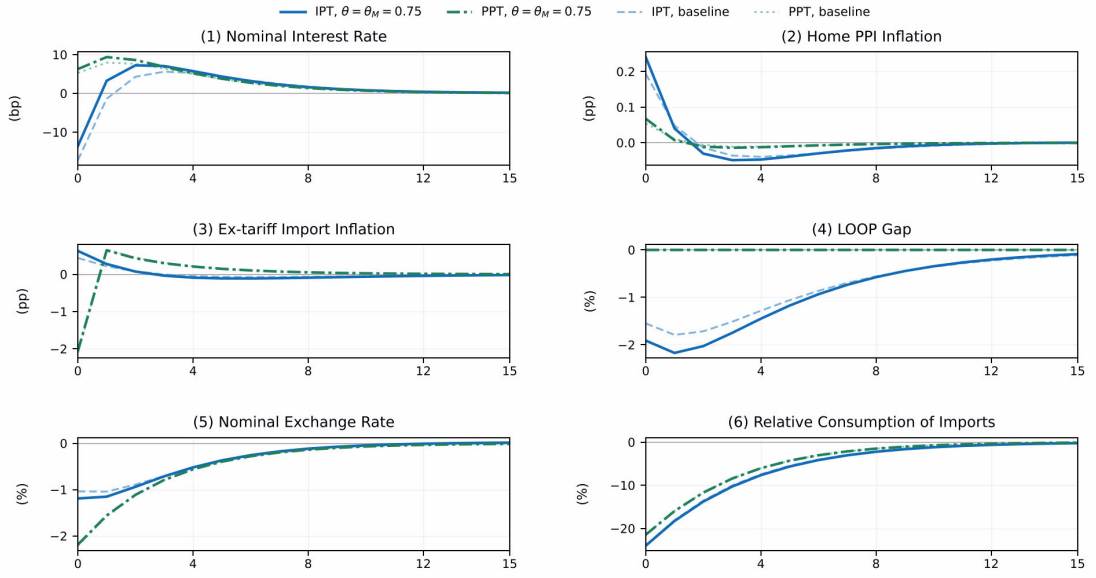


Figure I.10: Lower Stickiness Case, two-country economy: Home responses to a 20-percentage-point unilateral Home import tariff with $\theta = \theta_M = 0.75$, overlaid on the baseline responses ($\theta = \theta_M = 0.8$). Panel 1 is in basis points, Panel 6 is in percent deviation from steady state, and other variables are in percent or percentage points.

I.5 Tariff-Indexed Importer Subsidy

This subsection replaces the fixed importer subsidy $\tau = 1/\varepsilon_M$ with the tariff-indexed subsidy in Eq. (A.4), which removes the opportunistic-pricing effect, holding all other parameters at their baseline values. Figures I.11 and I.12 compare the IPT responses under the two subsidies in the SOE and in the two-country unilateral tariff experiment, respectively.

In the SOE, under the tariff-indexed subsidy, *ex-tariff* import inflation rises much less on impact (Panel 3 of Fig. I.11), and the nominal exchange rate depreciates more and for longer (Panel 5). Consequently, the actual LOOP gap rises on impact instead of falling (Panel 4), and the underconsumption of imports is smaller (Panel 6). The Ramsey planner still cuts the nominal interest rate, by 27.90 basis points on impact, compared with 32.87 basis points under the fixed subsidy. Under the fixed subsidy, the rate rises above its steady-state level from the fourth quarter after impact, whereas under the tariff-indexed subsidy it remains below that level over the first 20 quarters (Panel 1). The cumulative response over the first 20 quarters is therefore −93.40 basis-point-quarters, compared with −32.68 under the fixed subsidy.

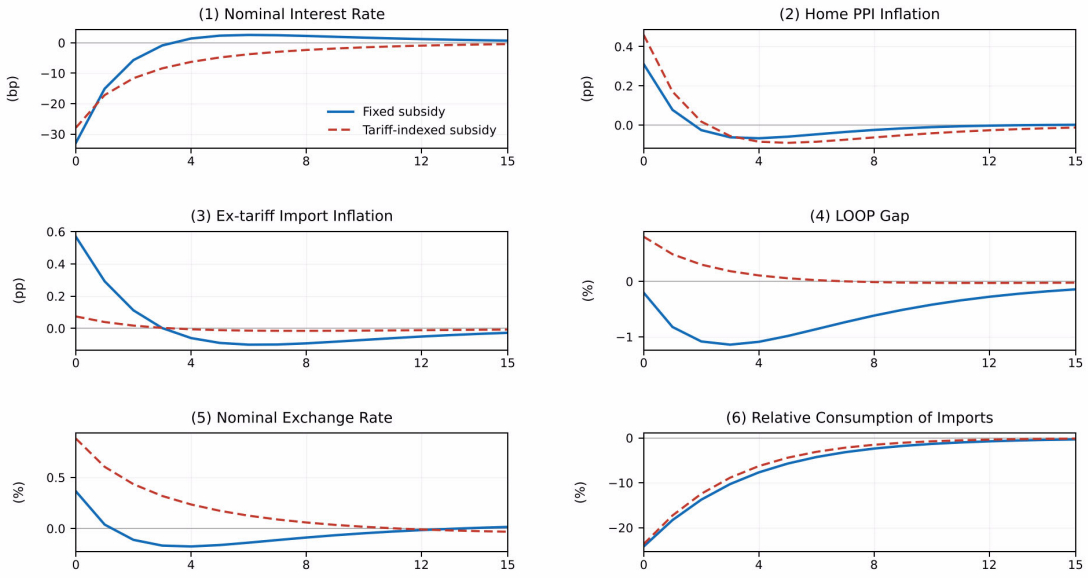


Figure I.11: Small open economy: optimal IPT responses to a 20-percentage-point Home import tariff under the fixed importer subsidy $\tau = 1/\varepsilon_M$ and under the tariff-indexed importer subsidy in Eq. (A.4). Panel 1 is in basis points, Panel 6 is in percent deviation from steady state, and other variables are in percent or percentage points.

In the two-country unilateral tariff experiment, the tariff-indexed subsidy leaves the initial Home rate cut almost unchanged (−17.44 basis points, compared with −17.23), but the Home rate does not rise above its steady-state level over the first 20 quarters, whereas under the fixed subsidy it rises above that level from the second quarter after impact (Panel 1 of Fig. I.12). The cumulative Home response over the first 20 quarters is −32.64 basis-point-quarters, compared with +12.50 under the fixed subsidy. The actual Home LOOP gap still falls on impact, but by 0.61 percent, about as much as the nominal appreciation of the Home currency, whereas under the fixed subsidy it falls by 1.55 percent, more than the 1.04 percent appreciation, as importers raise their *ex-tariff* import prices (Panels 3 to 5).

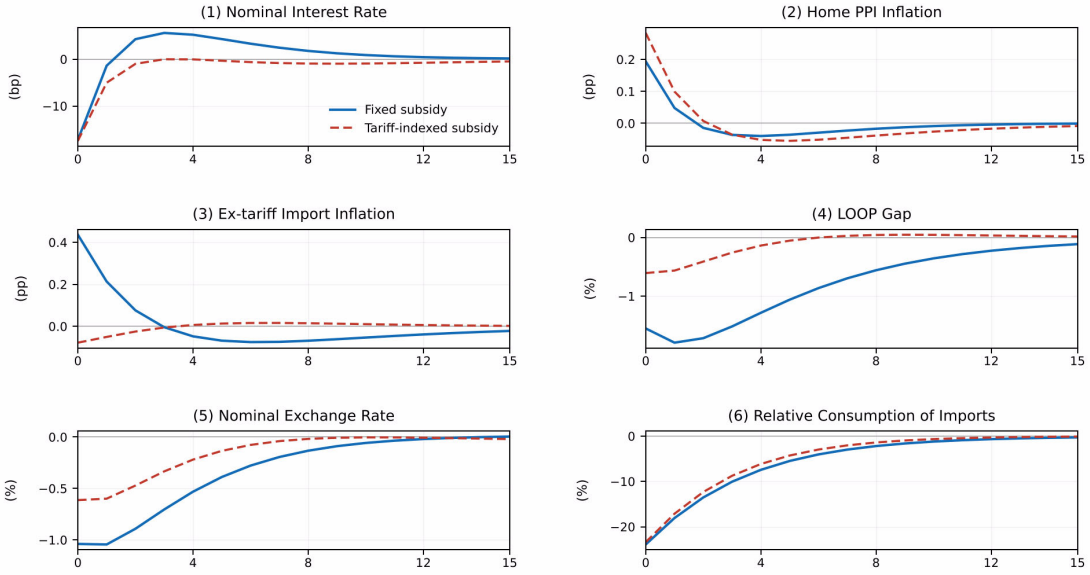


Figure I.12: Two-country unilateral tariff: optimal Home IPT responses to a 20-percentage-point Home import tariff under the fixed importer subsidy $\tau = 1/\varepsilon_M$ and under the tariff-indexed importer subsidy in Eq. (A.4). Panel 1 is in basis points, Panel 6 is in percent deviation from steady state, and other variables are in percent or percentage points.

I.6 Flexible Import Prices in the Two-Country Economy

This subsection describes the flexible-import-price benchmark used in Sec. 4.5. We solve the noncooperative Ramsey problem under IPT for the unilateral 20-percentage-point Home tariff with $\theta_M = 0$ in both countries, holding all other parameters at their baseline values. The private steady state does not depend on θ_M, but the steady-state Ramsey multipliers do and are recomputed for $\theta_M = 0$.

On impact, relative import consumption falls by 27.88 percent, compared with 23.91 percent under IPT with $\theta_M = 0.8$, and the Home planner raises the nominal interest rate by 22.37 basis points instead of cutting it by 17.23 basis points. The actual Home LOOP gap falls by 6.90 percent, as implied by Eq. (17), compared with 1.55 percent under IPT with $\theta_M = 0.8$.

At this allocation, the PPT private-equilibrium conditions hold under the scaled tariff in Eq. (A.3). Unlike in the SOE, however, the PPT Nash–Ramsey allocation under the scaled tariff differs from this allocation: relative import consumption falls by 27.28 percent on impact, and the Home nominal interest rate rises by 6.87 basis points. Under IPT, each planner takes the other country's *ex-tariff* import prices as given (Sec. 4.2), even when they are flexible.

J Welfare Cost of Following the Perfect Pass-Through Interest-Rate Path

This appendix measures the welfare cost to Home of setting the nominal interest rate under IPT along the path that is optimal under PPT. In both the SOE and the two-country economy, this counterfactual policy raises the Home rate on impact whereas optimal IPT policy cuts it (Secs. 3.5 and 4.5), and it lowers Home welfare relative to optimal IPT policy.

J.1 Counterfactual Allocation

Let i_t^{PPT} denote the optimal Home nominal interest rate under PPT, and $\Pi_{H,t}^{IPT}$ the optimal Home PPI inflation under IPT, both in response to the 20-percentage-point Home import tariff. In the two-country economy, both are taken from the noncooperative Ramsey equilibria. Imposing $i_t = i_t^{PPT}$ as an exogenous path on the IPT private equilibrium does not pin down a unique equilibrium: in the SOE, the resulting system has 144 eigenvalues outside the unit circle for 145 forward-looking variables. We therefore implement the PPT path through an interest-rate rule.

First, we estimate by least squares

$$i_t^{PPT} - i = \rho_i \left(i_{t-1}^{PPT} - i\right) + (1-\rho_i)\,\phi_\pi \left(\Pi_{H,t}^{IPT} - 1\right) + \varpi_t, \tag{J.1}$$

where $i \equiv \beta^{-1} - 1$ is the steady-state nominal interest rate and ϖ_t is the residual. The sample covers the 80 quarters (SOE) or 110 quarters (two-country economy) of the simulation horizon. In the SOE, the estimates are $\hat{\rho}_i = 0.906$ and $\hat{\phi}_\pi = 5.08$, with $R^2 = 0.970$. In the two-country economy, the unrestricted estimate of ρ_i exceeds one; we therefore fix ρ_i at its SOE estimate and obtain $\hat{\phi}_\pi = 2.95$.

Second, we add Eq. (J.1), with ϖ_t treated as an exogenous sequence, to the IPT private equilibrium in place of the Home Ramsey first-order conditions. Because i_t is endogenous, the rule with the fitted residuals does not reproduce i_t^{PPT}. We update ϖ_t until the simulated Home nominal interest rate equals i_t^{PPT} in every quarter, to within 10^{-3} basis points.[3] With ϖ_t given, the rule satisfies the Blanchard–Kahn conditions: the number of eigenvalues outside the unit circle equals the number of forward-looking variables (145 in the SOE and 434 in the two-country economy). In the two-country economy, the Foreign Ramsey planner keeps its noncooperative first-order conditions and re-optimizes in response to the new Home policy; the Foreign nominal interest rate remains above the ZLB.

Two checks support the construction. First, with the residuals implied by optimal IPT policy, Eq. (J.1) reproduces the optimal IPT allocation, and the welfare criterion of Appendix J.2 differs from its value under optimal IPT policy by less than 10^{-9}, five orders of magnitude below the welfare losses reported below. Second, replacing $\hat{\phi}_\pi$ with $\phi_\pi = 1.5$, 3, or 10 and repeating the residual update from zero changes the welfare loss reported below by less than one percent. The welfare loss is thus that of the equilibrium selected by Eq. (J.1) with the residuals that reproduce i_t^{PPT}.

J.2 Welfare Criterion

Let

$$\mathcal{V}_0 \equiv \sum_{t=0}^{T-1} \beta^t \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi}\right) \tag{J.2}$$

denote Home welfare over the simulation horizon, with $T = 80$ in the SOE and $T = 110$ in the two-country economy; the economy has returned to its steady state by T. The Ramsey problems are solved from the timeless perspective (Secs. 3.2 and 4.2): the lagged Lagrange multipliers are fixed at their steady-state values λ_k rather than reset to zero. The date-0 problem therefore contains, in addition to $\mathcal{V}_0$, the term

$$\mathcal{B}_0 \equiv \beta^{-1} \sum_k \lambda_k\, \Phi_{k,-1}, \tag{J.3}$$

where $\Phi_{k,-1}$ is constraint k of the Lagrangian in Appendix D dated $t = -1$, evaluated at the steady state for dates before 0 and at the allocation for date 0, and λ_k are the steady-state multipliers of

[3] The updates use Anderson acceleration with a memory of five iterations; the fixed point does not depend on the acceleration.

the optimal IPT problem (of the Home planner in the two-country economy). Timeless Ramsey policy maximizes

$$\mathcal{V}_0^{TP} \equiv \mathcal{V}_0 + \mathcal{B}_0, \tag{J.4}$$

not $\mathcal{V}_0$.

Following Benigno and Woodford [3], we rank policies by $\mathcal{V}_0^{TP}$. This criterion penalizes policies that do not honor the commitments that the planner is assumed to have made before date 0. Ranking by $\mathcal{V}_0$ alone can rank a suboptimal policy above the optimal policy, because a policy that departs from these commitments can raise $\mathcal{V}_0$.

For any variable X, let $\Delta X \equiv X^{IPT} - X^{CF}$ denote its value under optimal IPT policy minus its value in the counterfactual allocation. The welfare loss from the counterfactual policy is $\Delta\mathcal{V}_0^{TP} = \Delta\mathcal{V}_0 + \Delta\mathcal{B}_0$. We express it as the permanent percentage reduction in consumption that lowers $\mathcal{V}_0^{TP}$ by the same amount under log utility, $100\left[1 - \exp\left(-(1-\beta)\Delta\mathcal{V}_0^{TP}\right)\right]$. In the SOE, the counterfactual allocation satisfies every constraint of the optimal IPT problem from the same initial state, so $\Delta\mathcal{V}_0^{TP} \geq 0$ holds by construction. In the two-country economy, Foreign re-optimizes, so the counterfactual Home allocation need not be feasible for the Home planner, who takes the Foreign plan as given, and the sign of $\Delta\mathcal{V}_0^{TP}$ is not determined in advance.

J.3 Results

Table J.1: Welfare cost to Home of following the PPT interest-rate path under IPT

	$\Delta\mathcal{V}_0$	$\Delta\mathcal{B}_0$	$\Delta\mathcal{V}_0^{TP}$	Cons. equiv. (%)
SOE	8.996×10^{-4}	-6.858×10^{-4}	2.138×10^{-4}	2.14×10^{-4}
Two-country economy	-1.536×10^{-4}	4.262×10^{-4}	2.726×10^{-4}	2.73×10^{-4}

Note: $\Delta X \equiv X^{IPT} - X^{CF}$. $\mathcal{V}_0$, $\mathcal{B}_0$, and $\mathcal{V}_0^{TP}$ are defined in Eqs. (J.2)–(J.4). The last column reports the consumption equivalent, $100[1 - \exp(-(1-\beta)\Delta\mathcal{V}_0^{TP})]$.

Table J.1 reports the results. Following the PPT path lowers $\mathcal{V}_0^{TP}$ in both economies, by 2.14×10^{-4} percent of steady-state consumption in the SOE and by 2.73×10^{-4} percent in the two-country economy.

In both economies, $|\Delta\mathcal{B}_0|$ exceeds $\Delta\mathcal{V}_0^{TP}$, so $\Delta\mathcal{V}_0$ has the sign opposite to that of $\Delta\mathcal{B}_0$. In the SOE, $\Delta\mathcal{B}_0 < 0$ and $\Delta\mathcal{V}_0 > 0$, so both $\mathcal{V}_0$ and $\mathcal{V}_0^{TP}$ rank optimal IPT policy above the counterfactual. In the two-country economy, $\Delta\mathcal{V}_0 < 0$: the counterfactual policy raises $\mathcal{V}_0$ while lowering $\mathcal{V}_0^{TP}$, because it departs from the commitments in $\mathcal{B}_0$ ($\Delta\mathcal{B}_0 > 0$). Appendix K shows the underlying property at the two-country Nash equilibrium: small changes in a planner's policy-rate path change $\mathcal{V}_0$ to first order but $\mathcal{V}_0^{TP}$ only to second order.

K Unilateral-Deviation Test of the Two-Country Nash Equilibrium

This appendix checks, for the unilateral Home tariff under IPT, that each planner's Nash–Ramsey plan in Sec. 4.2 is a best response in the directions examined.

For each planner in turn, we fix the other planner's variables at their Nash paths and replace the planner's first-order conditions with an interest-rate rule of the form of Eq. (J.1), with ρ_i at its SOE estimate, $\phi_\pi = 3$, and an exogenous residual path; the Home rule responds to $\Pi_{H,t}$ and sets i_t, and the Foreign rule responds to $\Pi^*_{F,t}$ and sets i^*_t. With the residuals implied by the Nash allocation, the rule reproduces that allocation, and it satisfies the Blanchard–Kahn conditions. We then move the planner's policy-rate path away from its Nash path in nine directions: by γ basis points in period 0 only; by $\gamma\, 0.7^t$ basis points in period $t \geq 0$, with $\gamma = -5, -1, 1, 5$ in both cases;

and to the planner's rate path in the PPT Nash equilibrium. For each direction, the residuals are updated as in Appendix J.1 until the policy rate equals the target path to within 10^{-3} basis points. The deviation allocations satisfy the deviating planner's constraints, but not the other planner's, as in the definition of the game.

For any variable X, let ΔX now denote its value under the Nash plan minus its value under the deviation, where $\mathcal{V}_0^{TP}$ is the deviating planner's criterion in Eq. (J.4), computed with its own period utility and its own steady-state multipliers. A best response requires $\Delta\mathcal{V}_0^{TP} \geq 0$ for every deviation; $\Delta\mathcal{V}_0^{TP} > 0$ means that the deviation lowers the planner's criterion.

Table K.1: Unilateral-deviation test: $\Delta\mathcal{V}_0^{TP}$

Deviation of the planner's policy rate	Home planner	Foreign planner
Period 0 only, $\gamma = -5$	3.38×10^{-7}	4.22×10^{-7}
Period 0 only, $\gamma = -1$	1.36×10^{-8}	1.67×10^{-8}
Period 0 only, $\gamma = 1$	1.36×10^{-8}	1.67×10^{-8}
Period 0 only, $\gamma = 5$	3.37×10^{-7}	4.22×10^{-7}
$\gamma\, 0.7^t$, $\gamma = -5$	1.39×10^{-5}	1.61×10^{-5}
$\gamma\, 0.7^t$, $\gamma = -1$	5.49×10^{-7}	6.37×10^{-7}
$\gamma\, 0.7^t$, $\gamma = 1$	5.48×10^{-7}	6.37×10^{-7}
$\gamma\, 0.7^t$, $\gamma = 5$	1.38×10^{-5}	1.60×10^{-5}
PPT Nash rate path	1.41×10^{-5}	2.00×10^{-4}

Note: Unilateral Home tariff, IPT, baseline calibration. γ is in basis points. The other planner's variables are fixed at their IPT Nash paths. ΔX denotes the value of X under the Nash plan minus its value under the deviation.

Table K.1 reports the results. In all 18 cases, $\Delta\mathcal{V}_0^{TP} > 0$, and within each family $\Delta\mathcal{V}_0^{TP}$ is nearly symmetric in γ. Fitting $\Delta\mathcal{V}_0^{TP} = a\gamma + b\gamma^2$ within each family gives $b > 0$ and $|5a|/(25b) < 0.004$ in all four fits, so the first-order term is negligible and the second-order term is positive. The value for the PPT Nash rate path differs from that in Table J.1 because the Foreign variables are held at their Nash paths here, whereas Foreign re-optimizes in Appendix J.

The first-order change in $\mathcal{V}_0$ alone does not vanish. Across the four values of γ, $\Delta\mathcal{V}_0$ changes almost linearly in γ, by about 3×10^{-5} per basis point for period-0 changes and 2–3 $\times 10^{-4}$ per basis point for persistent changes, while $\Delta\mathcal{B}_0$ offsets this change to first order. The Nash plan is thus a stationary point of $\mathcal{V}_0^{TP}$, not of $\mathcal{V}_0$, as the timeless perspective implies.

The test covers the directions listed above and does not establish global optimality.

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[1] Benigno, Pierpaolo, and Michael Woodford (2003), "Optimal Monetary and Fiscal Policy: A Linear-Quadratic Approach," *NBER Working Paper* No. 9905.

[2] Benigno, Pierpaolo, and Michael Woodford (2005), "Inflation Stabilization and Welfare: The Case of a Distorted Steady State," *Journal of the European Economic Association*, 3(6), 1185–1236.

[3] Benigno, Pierpaolo, and Michael Woodford (2012), "Linear-Quadratic Approximation of Optimal Policy Problems," *Journal of Economic Theory*, 147(1), 1–42.

[4] Galí, Jordi, and Tommaso Monacelli (2016), "Understanding the Gains from Wage Flexibility: The Exchange Rate Connection," *American Economic Review*, 106(12), 3829–3868.

[5] Monacelli, Tommaso (2005), "Monetary Policy in a Low Pass-Through Environment," *Journal of Money, Credit and Banking*, 37(6), 1047–1066.

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