Tariff-Inclusive Prices: Optimal Monetary Policy under Dominant-Currency Pricing

Eiji Okano*
Nagoya City University

Yang Zhou
Nagoya City University

Masataka Eguchi
Nagoya City University

August 2026

Abstract

This paper studies optimal monetary policy in the country that imposes an import tariff and asks why the sign of the optimal nominal interest-rate response can differ across pricing environments. We distinguish a pre-tariff-price convention (PreTP), under which the sticky destination price excludes the tariff, from a post-tariff-price convention (PostTP), under which the sticky destination price includes it. The two empirical dimensions motivating our focal specification are dominant-currency pricing, documented by Gopinath et al. [21], and tariff-incidence patterns with adjustment in purchaser-facing prices, margins, and pre-tariff prices, documented by Cavallo et al. [10]. We combine these findings in DCP–PostTP; the latter evidence motivates, rather than structurally estimates, our PostTP convention. The analysis proceeds in stages. Importer LCP (ILCP) in a small open economy isolates the PreTP–PostTP mechanism in its simplest form; full LCP then moves price setting to exporters and shares the tariffed Foreign-to-Home block with DCP; DCP finally adds the dominant-currency asymmetry. The optimal rate crosses zero between PreTP and PostTP in all three regimes, so the sequence also provides cross-pricing-regime robustness. In the small open economy, ILCP changes the impact response from −75.95 to +57.68 basis points. In an equal-size two-country Nash–Ramsey economy, the response changes from −31.92 to +43.09 basis points under ILCP, from +49.49 to −4.98 under full LCP, and from +57.59 to −4.29 under Home-currency DCP. These zero crossings survive changes in country size, tariff size down to 5 percentage points, trade openness, and the removal of markup-correcting subsidies; DCP becomes sensitive only at very high substitutability. Cooperation can remove the zero crossings, showing that the international policy game is a distinct margin. In the substantially richer Bergin–Corsetti [6] private economy, holding the national Nash–Ramsey policy game fixed restores the sign reversal under both full LCP and DCP. Thus tariff location can determine the sign of the optimal monetary stance across distinct pricing architectures and richer model environments.

Keywords: import tariffs; optimal monetary policy; tariff pass-through; local-currency pricing; dominant currency pricing; Nash–Ramsey policy.
JEL Classification: E31, E52, F13, F41, F42.

*Corresponding author. Graduate School of Economics, Nagoya City University, 1, Aza, Yamanobata, Mizuho-cho, Mizuho-ku, Nagoya-shi Aichi, 467-8501, Japan. Tel.: +81-52-872-5721; Fax: +81-52-872-1531; E-mail: eiji_okano@econ.nagoya-cu.ac.jp

1 Introduction

What should monetary policy do in the country that imposes an import tariff: raise the policy rate or lower it? In the standard open-economy New Keynesian mechanism, an import tariff shifts expenditure away from foreign goods and toward Home goods, raising the Home output gap and creating a motive for monetary tightening. Bergin and Corsetti [6], for example, obtain tightening by the tariff-imposing country in their principal producer-currency-pricing (PCP) and exporter local-currency-pricing (LCP) cases. The question is whether this policy prescription is robust once international pricing behavior is taken seriously. Other recent analyses likewise show that the monetary transmission of tariffs depends on the surrounding international-price and exchange-rate environment [25, 28, 7]. The relevant issue is therefore not a mechanical reaction to the first-round consumer price index (CPI) effect of a tariff, but how nominal rigidities shape the welfare-relevant stabilization problem in the tariff-imposing economy.

A natural place to ask whether this tightening prescription is robust is international pricing behavior. Bergin and Corsetti [6] already make pricing behavior central by comparing producer-currency pricing (PCP), local-currency pricing (LCP), and dominant-currency pricing (DCP). That dimension is empirically important. Gopinath, Itskhoki, and Rigobon [20] show that exchange-rate pass-through depends strongly on the currency of invoicing, and Gopinath et al. [21] document the dominant role of a small set of currencies, especially the U.S. dollar, in international price adjustment and trade transmission. Dominant-currency pricing is therefore a natural pricing environment for asking how the central bank of a tariff-imposing country should respond.

The tariff evidence points, however, to a second dimension of pricing behavior that is distinct from the currency of invoicing: *where the tariff enters relative to the sticky destination price.* Amiti, Redding, and Weinstein [1] find very high pass-through of U.S. tariffs into domestic import prices. Using border and retail micro data, Cavallo, Gopinath, Neiman, and Tang [10] likewise find high tariff pass-through at the border but substantially more muted adjustment at the retail level, with part of the tariff absorbed through lower margins; they also document adjustment in pre-tariff export prices in response to retaliatory tariffs. We take this evidence to mean that tariff incidence need not be implemented only by mechanically adding the tariff on top of a sticky tariff-exclusive price: adjustment can instead occur in margins or net-of-tariff prices while the purchaser-facing price adjusts more slowly. This empirical pattern motivates our post-tariff-price (PostTP) convention. We do not claim that Cavallo et al. structurally estimate PostTP. Rather, their evidence motivates the location-of-the-tariff dimension that PostTP captures. Combined with Gopinath et al.'s evidence for dominant-currency pricing, it makes DCP–PostTP the empirically motivated focal configuration of our two-country analysis: DCP determines the currency in which the international price is sticky, while PostTP determines whether the tariff lies inside that sticky destination-price contract.

This observation matters because DCP and PostTP capture two distinct empirically motivated dimensions of international pricing. In Bergin and Corsetti [6], including their dominant-currency specification, the tariff is imposed on top of a sticky tariff-exclusive exporter price. We call this the pre-tariff-price convention, or PreTP. We contrast it with the PostTP convention, under which the tariff-inclusive destination price is the sticky object. Under PostTP the tariff enters the acquisition cost or net revenue relevant for the price setter before the sticky purchaser-facing price can fully adjust. Our DCP–PreTP case therefore holds the dominant-currency architecture fixed and changes only the location of the tariff relative to the sticky price; DCP–PostTP is the focal case that combines the two empirical dimensions.

The two timing conventions are deliberately constructed so that this difference is not a disguised wealth effect. With lump-sum rebate of tariff revenue and representative-household ownership of firms, PreTP and PostTP have the same consolidated household-side tariff wedge. What differs is the price-setting constraint. Under PreTP the tariff lies outside the sticky-price setter's reset problem; under PostTP it enters the relevant marginal cost or net-revenue margin. The corresponding first-order conditions and Phillips curves therefore differ even though the household-side incidence is the same. This is not a secondary modeling detail: because it changes the private-sector price-setting constraint faced by the Ramsey authority, the PreTP/PostTP distinction can determine the sign of the optimal interest-rate response itself. Our question is therefore whether this additional dimension of pricing behavior makes the tariff-imposing country optimally raise or lower its policy rate.

It can. We organize the analysis deliberately from the simplest device for understanding the mechanism to the empirically focal pricing configuration. We begin with importer LCP in a small open economy because it lets us study LCP and the PreTP–PostTP distinction without Foreign monetary-policy feedback or dominant-currency asymmetry. ILCP is used here as a transparent analytical device, not as an approximation to DCP. Under PCP the tariff raises the Home policy rate by 18.56 basis points. Under ILCP, however, the same 20-percentage-point tariff innovation produces a 75.95-basis-point cut under PreTP and a 57.68-basis-point increase under PostTP. The sign reversal comes from the importer price-setting constraint: PreTP makes the tariff-exclusive import price sticky, whereas PostTP makes the cum-tariff import price sticky.

We then move to an equal-size two-country economy, $\nu = 0.5$, with markup-correcting subsidies and national Nash–Ramsey policy under commitment in the sense of Faia and Monacelli [17]. First, retaining ILCP verifies that the simple SOE mechanism survives endogenous Foreign quantities and strategic monetary interaction: the Home response changes from -31.92 basis points under PreTP to $+43.09$ under PostTP. We next move to full LCP. This is the natural bridge from ILCP to DCP because the Home import price is now set by the Foreign exporter and, for the unilateral Home tariff, the tariffed Foreign-to-Home price-setting block is the same one that appears under DCP. The Home response again crosses zero, from $+49.49$ to -4.98 basis points. Finally, under Home-currency DCP—the pricing environment motivated by the dominant-currency evidence—the response changes from $+57.59$ to -4.29 basis points. Thus ILCP and full LCP do double duty: they make the mechanism progressively easier to understand on the way to DCP, and their own zero crossings are cross-pricing-regime robustness checks showing that the result is neither importer-specific nor peculiar to dominant-currency asymmetry. The same qualitative sign pattern survives changes in country size. Section 7.1 treats the steady-state-distortion exercise separately, defines the Distorted specification as one with neither producer nor importer markup-correcting subsidies, and re-solves ILCP under that definition.

The policy implication is especially transparent in a U.S.-like calibration. We set $\nu = 0.27$ to match the U.S. share of world current-dollar GDP, and with $\upsilon = 0.30$ this implies a Home import expenditure weight of $\upsilon(1-\nu) = 0.219$. The latter is close to the 22.3 percent average ratio of U.S. imports of goods and services to personal consumption expenditures (PCE) over 2000–2025; importantly, that match is an implication of ν and υ, not a separate calibration target [23, 27]. Under the empirically motivated DCP–PostTP configuration, the tariff-imposing country optimally lowers its policy rate on impact, and this conclusion is essentially unchanged by whether the steady-state markup distortion is removed: the impact response is -6.34 basis points with the markup-correcting producer subsidies and -6.30 basis points without them. Under DCP–PreTP, by contrast, the same U.S.-like calibration calls for monetary tightening, with impact responses of $+90.59$ and $+89.15$ basis points, respectively. The PreTP result is therefore qualitatively consistent with the tightening obtained by Bergin and Corsetti [6]; our results do not contradict their policy conclusion under that pricing convention. Holding DCP fixed, what changes the prescription is the location of the tariff relative to the sticky price. Under PostTP the tariff-inclusive Home-currency destination price is sticky and the Foreign exporter absorbs the tariff through its net-of-tariff revenue margin. Thus the focal DCP–PostTP case combines the dominant-currency regularity documented by Gopinath et al. with the tariff-incidence pattern motivating PostTP, and that combination is sufficient to turn the optimal impact response from tightening to easing.

The central contribution is stronger than showing that an additional pricing assumption changes the quantitative transmission of tariffs. We identify an overlooked dimension of the nominal pricing contract that can determine the sign of optimal monetary policy: whether the tariff is imposed outside a sticky tariff-exclusive price or is included inside the sticky purchaser-facing price. This distinction is easy to miss because it is usually embedded in the price-setting convention rather than treated as a separate economic dimension. Yet, holding fixed the invoicing-currency regime and the consolidated household-side tariff wedge, moving the tariff from one side of the sticky price to the other changes the private-sector implementability constraint faced by the Ramsey authority and can reverse the policy prescription from tightening to easing, or vice versa. The mechanism is simple once the distinction is made explicit, but its consequence is first order: policy conclusions obtained under a given pricing currency can depend qualitatively on where the tariff enters the nominal pricing problem. The welfare criterion determines which output, inflation, and relative-price deviations matter, while the PreTP–PostTP price-setting constraints determine which combinations of those objects are feasible in the short run.

The sign reversal also survives a broad set of within-model robustness checks. It remains present when the tariff innovation is reduced from 20 to 5 percentage points, when trade openness is varied, and when the markup-correcting subsidies are removed. Across the elasticity exercises, ILCP and full LCP retain a zero crossing throughout the reported range. DCP retains it through $\eta = 2.9$ in the equal-size economy and through the determinate $\eta = 2.4$ U.S.-size case, but becomes sensitive at still higher substitutability. A separate robustness question is whether the sign reversal is merely a feature of our parsimonious private economy or instead survives substantial model enrichment. We address that question while controlling separately for the international policy game. Motivated by the fact that Bergin and Corsetti [6] solve their quantitative model cooperatively, we first re-solve the equal-size Baseline at the Benchmark Parameterization under cooperative Ramsey policy. The zero crossings disappear, showing that policy coordination is itself an important margin. This observation also means that the absence of a sign reversal in the original cooperative Bergin–Corsetti environment cannot by itself be attributed to its richer trade and production structure. We therefore perform the sharper comparison: we hold the Bergin–Corsetti private economy fixed and solve it under the national Nash–Ramsey policy game under commitment used in our two-country analysis, following Faia and Monacelli [17]. The sign reversal returns under both full LCP and DCP, with the tariff-imposing country's impact response changing from $+576.61$ to -20.57 basis points and from $+222.67$ to -9.00 basis points, respectively. This is our strongest external-model robustness result: even after moving from the parsimonious Calvo model to the substantially richer Bergin–Corsetti economy, the PreTP–PostTP sign reversal survives when the policy game is held fixed. The cooperative comparison then identifies international coordination as a separate determinant of whether the timing-induced change in stance crosses zero. Trade structure clearly matters for the magnitude and transmission of tariff shocks, but our comparison points to a more fundamental determinant of the sign of optimal monetary policy: how the tariff enters the marginal-cost or net-revenue margin in the relevant sticky-price-setting problem. In this sense, our results redirect attention from the richness of the trade block itself to the location of the tariff within the nominal pricing problem.

The rest of the paper is organized as follows. Section 2 relates the analysis to the tariff–monetary-policy and international-pricing literatures. Section 3 uses small-open-economy ILCP to isolate the LCP and PreTP–PostTP mechanism in its simplest form. Section 4 introduces the two-country national Nash–Ramsey model, retains ILCP as a first check, then moves to exporter-set full LCP as the bridge to DCP before presenting the dominant-currency block. Section 5 presents the equal-size Baseline and shows that the timing-induced zero crossing occurs under ILCP, full LCP, and DCP, providing cross-pricing-regime robustness while progressively approaching the focal DCP environment. Section 6 repeats the analysis for a U.S.-like country size, $\nu = 0.27$, while retaining the Baseline subsidy configuration. Section 7 provides further robustness checks over steady-state distortions, tariff size, the trade elasticity, country size, and trade openness before turning to the Bianchi–Coulibaly external-model check, the role of international policy coordination, and the richer Bergin–Corsetti [6] model. Section 8 concludes.

2 Related Literature

This paper belongs first to the relatively small recent literature on optimal monetary policy in the country that imposes a tariff. Bergin and Corsetti [6] study cooperative Ramsey stabilization under PCP and extend the analysis to exporter local-currency and dominant-currency pricing. For the tariff-imposing country, their principal PCP and exporter-LCP results imply monetary tightening. Their LCP specification is exporter LCP: exporters set destination-specific sticky prices in the buyer's currency and the tariff is imposed on top of that exporter-set price. In the terminology of the present paper, this structure is closest to full-LCP PreTP. Monacelli [25] derives optimal policy for import and export tariffs in a small open economy, while Werning, Lorenzoni, and Guerrieri [28] study tariffs on imported intermediate inputs as cost-push shocks. The later Bergin–Corsetti analysis [7] emphasizes exchange-rate management in the cooperative stabilization of unilateral tariffs, and Auray, Devereux, and Eyquem [2] study monetary-rule design when tariffs are endogenous strategic choices. These contributions show that the monetary stabilization of tariffs depends on the real and nominal structure of the open economy. Our focus within this literature is deliberately narrower: we ask whether international pricing behavior itself can determine the sign of the optimal policy response.

Bianchi and Coulibaly [8] provide an important comparison precisely because their scope is different. International pricing behavior is not the object of their analysis. Their reference specification is a small-open-economy Ramsey model with Rotemberg rigidity in Home producer prices and lump-sum rebates of tariff revenue. The tariff creates a fiscal externality because private import decisions respond to the tariff-inclusive price whereas the tariff payment is a domestic transfer from the social point of view. Their optimal policy is therefore expansionary relative to the look-through allocation in the sense that the Ramsey authority raises employment and imports and tolerates more Home-producer inflation. This allocation-based characterization should not be read mechanically as a statement that the impact nominal policy rate must fall. Section 7.5 extends their environment with importer local-currency pricing and applies the same PreTP–PostTP distinction as in Section 3. The exercise is useful because it asks whether tariff timing can still determine the sign of the optimal nominal-rate response when the real welfare mechanism is the Bianchi–Coulibaly fiscal externality rather than the pricing mechanism in our parsimonious Baseline specification.

A second strand shows directly that tariff transmission depends on international pricing behavior. Hwang and Turnovsky [22] compare PCP and LCP in a two-country tariff model and show that real effects can differ sharply across pricing regimes. More recently, Auray, Devereux, and Eyquem [3] compare PCP, LCP, and DCP and show that invoicing currency has first-order effects on the response to a unilateral tariff. Bergin and Corsetti [6] are especially close to our question because they analyze optimal monetary policy under alternative pricing currencies, including dominant-currency pricing. Their analysis establishes that the currency in which the sticky international price is set matters for the monetary-policy prescription. Our contribution is to ask whether the position of the tariff relative to that sticky price matters separately.

The empirical pricing literature motivates both dimensions that define our focal DCP–PostTP specification. Gopinath, Itskhoki, and Rigobon [20] document that exchange-rate pass-through depends strongly on the currency of invoicing, and Gopinath et al. [21] show that dominant currencies, especially the U.S. dollar, govern a large share of international price adjustment and transmission. This evidence motivates DCP. Separately, Amiti, Redding, and Weinstein [1] document very high pass-through of U.S. tariffs into domestic import prices, while Cavallo, Gopinath, Neiman, and Tang [10] find high pass-through at the border but much more muted retail-price adjustment, with part of the tariff absorbed through lower margins, together with adjustment in pre-tariff export prices under retaliatory tariffs. These findings motivate PostTP by showing that tariff incidence can be absorbed in margins or net-of-tariff prices while purchaser-facing prices adjust more slowly. They do not constitute a structural estimate of our PostTP specification, and we do not claim otherwise. The point is that the empirical literature supports the two relevant pricing dimensions separately; our DCP–PostTP model combines them. We reach that focal case in steps. ILCP is the simplest environment in which LCP and the PreTP–PostTP distinction can be understood, including in a small open economy. Full LCP then moves price setting from the importer to exporters and, for the tariffed Foreign-to-Home transaction, has the same reset-price block as DCP. These intermediate regimes therefore have a dual role: they clarify the mechanism on the way to DCP and, because the interest-rate response also crosses zero in each of them, provide cross-pricing-regime robustness for the central timing result.

An earlier border-tax literature already separates tax-inclusive from tax-exclusive nominal rigidity. Buiter [9] makes this distinction explicitly. Barbiero, Farhi, Gopinath, and Itskhoki [4] provide an especially close pricing precedent, but only for part of our architecture. In their benchmark LCP specification, destination-currency consumer prices are preset inclusive of the border adjustment, so the border tax is absorbed through the exporter's net receipt during a price spell. In our terminology, this is closely analogous to full-LCP PostTP. Their DCP specification is different: both import and export border prices are sticky in dollars, and on the U.S. import side the border adjustment is applied to the preset dollar border price, so the purchaser-facing import price responds immediately to the tax. In our terminology, that tariffed Foreign-to-Home block is analogous to PreTP, not PostTP. Our DCP–PostTP specification instead makes the tariff-inclusive dominant-currency destination price itself sticky. Barbiero et al. also consider an alternative LCP formulation in which the border tax is levied on top of the initially preset import price, reinforcing the point that the location of the tax relative to the sticky price and the currency of price setting are distinct dimensions. Their setting concerns a border-adjustment tax reform under rule-based monetary policy rather than a unilateral import tariff under Ramsey policy, and does not conduct our systematic PreTP–PostTP comparison across ILCP, full LCP, and DCP.

This distinction is orthogonal to the invoicing-currency distinction. A model can feature DCP and still be PreTP if the tariff is simply added on top of a sticky dominant-currency exporter price, as in the Bergin–Corsetti convention. Conversely, a PostTP model can be formulated under importer LCP, full LCP, or DCP. The pricing architecture therefore has two separate dimensions: the currency and identity of the sticky price setter, and the location of the tariff relative to that sticky price. Our results show that the second dimension can overturn the policy implication associated with the first.

The broader open-economy monetary literature provides the welfare logic for this comparison. Devereux and Engel [15] show that the optimal degree of exchange-rate flexibility depends fundamentally on whether prices are set in producer or local currency. Corsetti, Dedola, and Leduc [12, 13] emphasize that incomplete pass-through introduces additional stabilization margins, including LOOP and international relative-price misalignments, that are absent from the standard PCP case. Corsetti, Dedola, and Leduc [14] provide a particularly sharp example in which the optimal monetary stance changes sign with the degree of exchange-rate pass-through. Fujiwara and Wang [18] solve cooperative and noncooperative Ramsey problems under LCP and use quadratic national welfare representations to make the output, inflation, and LOOP stabilization margins transparent. Their work is an important methodological precedent for the welfare decomposition used below. In the present paper, the nonlinear national Nash–Ramsey system determines the policy allocation, while the Benigno–Woodford criterion is used to interpret the welfare margins behind that allocation.

To our knowledge, existing work has not directly compared Ramsey-optimal monetary policy for the tariff-imposing country under PreTP and PostTP across importer LCP, full LCP, and DCP. The PreTP/PostTP comparison therefore asks whether the sign of the monetary-policy prescription is robust to an empirically motivated difference in where nominal adjustment occurs along the pricing chain. It is not. The timing convention changes the relevant Phillips curve and can reverse the sign of optimal monetary policy, with the direction of the reversal depending on whether pricing is importer LCP, full LCP, or DCP.

3 Small open economy: importer LCP and the tariff-timing mechanism

We begin with the small open economy because it isolates the tariff-timing mechanism from foreign monetary feedback and strategic interaction between national planners. Importer local-currency pricing (ILCP) is the simplest device for understanding both LCP and the distinction between a sticky tariff-exclusive and a sticky tariff-inclusive purchaser price, and it can be analyzed without building the full two-country exporter-pricing structure. This is a pedagogical choice: ILCP is not intended to approximate DCP. Its result will also serve as the first leg of the cross-pricing-regime robustness comparison once we move to full LCP and DCP. PCP is retained only as a control in the figure.

3.1 Environment

The small-open-economy environment follows Monacelli [25] in its CES consumption structure, complete international risk sharing, lump-sum rebate of tariff revenue, and Calvo Home-price block, but it is not identical to his model. There are two differences that matter for the Ramsey problem. First, Monacelli characterizes optimal policy from a steady state with the market-power distortion; in the terminology defined in Section 4.5.2, that is a Distorted specification. Our Baseline specification instead uses constant markup-correcting subsidies to remove the producer and, under ILCP, importer monopoly-markup distortions. Second, we retain variety-level production and the associated Calvo price dispersion in the aggregate labor requirement and in the Ramsey problem.[1] We also add importer local-currency pricing in order to isolate the tariff-timing mechanism.

The Home household has period utility

$$U_t \equiv \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi}. \tag{1}$$

Aggregate consumption is the CES index

$$C_t = \left[(1-\upsilon)^{1/\eta} C_{H,t}^{(\eta-1)/\eta} + \upsilon^{1/\eta} C_{F,t}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}, \tag{2}$$

where $C_{H,t}$ and $C_{F,t}$ are Home consumption of Home and Foreign goods, respectively, and $\upsilon \in (0,1)$ is the CES weight on Foreign goods; at the normalized zero-tariff steady state it is also the import-expenditure share and hence a measure of openness. If $P_{H,t}$ is the Home producer-price index and $P_{F,t}$ is the tariff-exclusive Home-currency price of Foreign goods, the corresponding consumption-price index is

$$P_t = \left[(1-\upsilon) P_{H,t}^{1-\eta} + \upsilon \widetilde{P}_{F,t}^{1-\eta}\right]^{1/(1-\eta)}. \tag{3}$$

Here $\widetilde{P}_{F,t} \equiv (1+\tau_{M,t})P_{F,t}$ is the purchaser-facing, tariff-inclusive import price.

In nominal terms, the representative household's budget constraint can be written

$$P_{H,t}C_{H,t} + \widetilde{P}_{F,t}C_{F,t} + \sum_{h^{t+1}} \xi_{t,t+1}(h^{t+1}|h^t)B_{t+1}(h^{t+1}) = B_t + W_t N_t + PR_t + T_t, \tag{4}$$

where B_{t+1} is the portfolio of state-contingent nominal claims, $\xi_{t,t+1}$ is the corresponding state price, PR_t denotes nominal profits, and T_t denotes lump-sum transfers, including rebated tariff revenue. The household-side accounting in Eq. (4) is common to PreTP and PostTP. In either convention the Home household purchases the Foreign good at $\widetilde{P}_{F,t}$, and tariff revenue is returned through the lump-sum transfer T_t. The timing convention therefore does not create a different household budget constraint or a different household-side tariff wedge. What changes is which nominal price is sticky and, consequently, where an unanticipated change in the realized tariff is absorbed. Any resulting change in firm profits enters household income through PR_t; this is an endogenous change within the same budget constraint rather than a change in the budget constraint itself.

[1] Monacelli [25] writes the variety-level production technology in the main text as $Y_t(j) = F(N_t(j))$. However, the first-order conditions reported in his Appendix C appear to use the aggregate relation $Y_t = N_t$ in the Ramsey problem. With Calvo price dispersion, this suggests that price dispersion is effectively ignored, or equivalently set to one, in that step. We retain the price-dispersion term explicitly when solving the Ramsey problem.

The intratemporal optimality condition between consumption and labor is

$$\frac{W_t}{P_t} = C_t N_t^{\varphi}. \tag{5}$$

The within-period expenditure allocation also implies $C_{H,t} = (1-\upsilon)(P_{H,t}/P_t)^{-\eta} C_t$ and $C_{F,t} = \upsilon(\widetilde{P}_{F,t}/P_t)^{-\eta} C_t$.

The inflation rate of the purchaser-facing import price satisfies

$$\widetilde{\pi}_{F,t} = \pi_{F,t} + \Delta \tau_{M,t}, \tag{6}$$

where $\Delta x_t \equiv x_t - x_{t-1}$ denotes the first-difference operator.

Complete international risk sharing under log utility is

$$\Psi_t S_t = \frac{C_t}{C_t^*} \mathcal{G}(S_t, \tau_{M,t}), \tag{7}$$

where $S_t \equiv P_{F,t}/P_{H,t}$ is the Home ex-tariff relative price of imports and $\Psi_t \equiv \mathcal{E}_t P_{F,t}^*/P_{F,t}$ is the Home LOOP gap. Here $\mathcal{E}_t$ denotes the nominal exchange rate, expressed as units of Home currency per unit of Foreign currency, and $P_{F,t}^*$ denotes the Foreign-currency price of Foreign goods at the border. If the law of one price holds at the border, $P_{F,t} = \mathcal{E}_t P_{F,t}^*$ and hence $\Psi_t = 1$. Importer LCP allows $\Psi_t \neq 1$ away from the flexible-price allocation. Here $\mathcal{G}(S_t, \tau_{M,t}) \equiv P_t/P_{H,t}$ denotes the Home CPI–PPI ratio. Using Eq. (3), this ratio can be written as

$$\mathcal{G}(S_t, \tau_{M,t}) = \Big[(1-\upsilon) + \upsilon(1+\tau_{M,t})^{1-\eta} S_t^{1-\eta}\Big]^{1/(1-\eta)}.$$

Home goods-market clearing is

$$Y_t = (1-\upsilon)\mathcal{G}(S_t, \tau_{M,t})^{\eta} C_t + \upsilon \left(\frac{\Psi_t S_t}{1+\tau_{X,t}}\right)^{\eta} C_t^*. \tag{8}$$

The experiment sets $\tau_{X,t} = 0$ and holds C_t^* fixed. The household Euler equation is

$$\frac{1}{1+i_t} = \beta \frac{C_t \mathcal{G}(S_t, \tau_{M,t})}{C_{t+1}\mathcal{G}(S_{t+1}, \tau_{M,t+1})\Pi_{H,t+1}}. \tag{9}$$

We assume that an analogous Euler equation holds for the Foreign household. Combining the Home and Foreign Euler equations with complete international risk sharing yields the uncovered interest parity (UIP) condition

$$1 + i_t = (1+i_t^*)\frac{\mathrm{E}_t(\mathcal{E}_{t+1})}{\mathcal{E}_t},$$

where i_t^* denotes the Foreign nominal interest rate. Equation (9) is an implementation constraint, not by itself an explanation of the sign of optimal monetary policy; the sign is determined jointly with the timing-specific price-setting constraints below.

Each Home variety is produced according to the linear technology

$$Y_t(j) = A_t N_t(j).$$

CES demand for variety j is $Y_t(j) = (P_{H,t}(j)/P_{H,t})^{-\varepsilon} Y_t$. Aggregating labor across producers therefore gives

$$N_t = \frac{Y_t Z_{H,t}}{A_t}.$$

where $Z_{H,t} \equiv \int_0^1 (P_{H,t}(j)/P_{H,t})^{-\varepsilon} dj$ denotes Home producer-price dispersion and A_t denotes productivity. We retain this price-dispersion term explicitly when solving the Ramsey problem. We assume $A_t = 1$ throughout the paper.

The zero-tariff, zero-inflation deterministic steady state is normalized to

$$\bar{S} = \bar{\Psi} = \bar{C} = \bar{Y} = \bar{N} = 1. \tag{10}$$

3.2 PCP

Before introducing ILCP, it is useful to state producer-currency pricing (PCP). Under PCP a Home producer sets its price in Home currency and there is no separate sticky import-price block. If a Home producer resets its price at t and does not reset again before $t+k$, the resetting producer solves

$$\max_{\bar{P}_{H,t}} \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \left[\bar{P}_{H,t} - (1-\tau) P_{H,t+k} MC^r_{t+k}\right] Y_{t+k|t}, \tag{11}$$

where $Y_{t+k|t} \equiv (\bar{P}_{H,t}/P_{H,t+k})^{-\varepsilon} Y_{t+k}$ is demand in period $t+k$ for a variety whose price was reset at t, and $\Lambda_{t,t+k} \equiv \beta^k (C_t/C_{t+k})(P_t/P_{t+k})$. Here MC_t is nominal marginal cost and $MC^r_t \equiv MC_t/P_{H,t}$ is real marginal cost, with deterministic steady-state value MC^r. Since $MC_{t+k} = W_{t+k}/A_{t+k}$, the household intratemporal condition in Eq. (5) and $P_{t+k}/P_{H,t+k} = \mathcal{G}(S_{t+k}, \tau_{M,t+k})$ imply

$$MC^r_{t+k} = \frac{C_{t+k} N^{\varphi}_{t+k} \mathcal{G}(S_{t+k}, \tau_{M,t+k})}{A_{t+k}} = C_{t+k} N^{\varphi}_{t+k} \mathcal{G}(S_{t+k}, \tau_{M,t+k}),$$

where the second equality uses $A_{t+k} = 1$. The first-order condition can be written with the producer's chosen reset price on the left-hand side:

$$\bar{P}_{H,t} = (1-\tau)\mathcal{M}_P \frac{\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} Y_{t+k|t} P_{H,t+k} MC^r_{t+k}}{\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} Y_{t+k|t}}. \tag{12}$$

where $\mathcal{M}_P \equiv \varepsilon/(\varepsilon-1)$ is the desired producer markup. Define $\widetilde{p}_{H,t} \equiv \bar{P}_{H,t}/P_{H,t}$, $\mathcal{K}_{H,t} \equiv \widetilde{p}^{\varepsilon}_{H,t} \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} Y_{t+k|t}$, and $\mathcal{Z}_{H,t} \equiv (\widetilde{p}^{\varepsilon}_{H,t}/P_{H,t}) \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} Y_{t+k|t} P_{H,t+k} MC^r_{t+k}$. Using these definitions, Eq. (12) can be written compactly as

$$\widetilde{p}_{H,t} = (1-\tau)\mathcal{M}_P \frac{\mathcal{Z}_{H,t}}{\mathcal{K}_{H,t}}.$$

Linearization of the producer reset-price condition and the Calvo price law gives

$$\pi_{H,t} = \beta \mathrm{E}_t \pi_{H,t+1} + \kappa mc^r_t. \tag{13}$$

Here $\pi_{H,t} \equiv \log \Pi_{H,t} = \log(P_{H,t}/P_{H,t-1})$, $mc^r_t \equiv \log(MC^r_t/MC^r)$, and $\kappa \equiv (1-\theta)(1-\beta\theta)/\theta$. We impose the same Calvo parameter θ on the importer price-setting block below, so κ is the common Phillips-curve slope for the producer and importer blocks. PCP therefore contains only the Home producer-price rigidity.

3.3 ILCP price setting: PreTP and PostTP

Relative to PCP, ILCP introduces a Home importer that sets a separate local-currency destination price for Foreign goods. The PreTP formulation follows the low-pass-through importer-LCP setup of Monacelli [24]; the Home producer-price block remains the PCP block above. The PreTP–PostTP distinction is entirely about which import price is the object of the importer's Calvo reset problem. Under PreTP, the importer optimizes the ex-tariff reset price $\overline{P}_{F,t}$. Under PostTP, the importer optimizes the cum-tariff reset price $\overline{\widetilde{P}}_{F,t}$.

To distinguish the timing convention from the product-variety index, let $\mathcal{P} \in \{\mathrm{PreTP}, \mathrm{PostTP}\}$ and denote the demand faced by an importer whose price was reset at t and remains unchanged through $t+k$ by $C^{\mathcal{P}}_{F,t+k|t}$. Specifically, $C^{\mathrm{PreTP}}_{F,t+k|t} \equiv (\overline{P}_{F,t}/P_{F,t+k})^{-\varepsilon_M} C_{F,t+k}$, whereas $C^{\mathrm{PostTP}}_{F,t+k|t} \equiv (\overline{\widetilde{P}}_{F,t}/\widetilde{P}_{F,t+k})^{-\varepsilon_M} C_{F,t+k}$.

The resetting importer solves

$$\mathrm{PreTP}: \quad \max_{\overline{P}_{F,t}} \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \left[\overline{P}_{F,t} - (1-\tau_I)\mathcal{E}_{t+k} P^*_{F,t+k}\right] C^{\mathrm{PreTP}}_{F,t+k|t}, \tag{14}$$

$$\text{PostTP}: \quad \max_{\overline{\overline{P}}_{F,t}} \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \left[\overline{\overline{P}}_{F,t} - (1-\tau_I)(1+\tau_{M,t+k})\mathcal{E}_{t+k} P^*_{F,t+k}\right] C^{\text{PostTP}}_{F,t+k|t}, \tag{15}$$

Equations (14) and (15) make the distinction explicit. In Eq. (14), PreTP means that the importer optimizes the ex-tariff price $\overline{P}_{F,t}$. In Eq. (15), PostTP means that the importer optimizes the cum-tariff price $\overline{\overline{P}}_{F,t}$. Thus PreTP makes the ex-tariff import price sticky, whereas PostTP makes the cum-tariff import price sticky.

The first-order conditions can be written directly in terms of the prices chosen by the resetting importer:

$$\text{PreTP}: \quad \overline{P}_{F,t} = (1-\tau_I)\mathcal{M}_I \frac{\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} C^{\text{PreTP}}_{F,t+k|t} \mathcal{E}_{t+k} P^*_{F,t+k}}{\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} C^{\text{PreTP}}_{F,t+k|t}}, \tag{16}$$

$$\text{PostTP}: \quad \overline{\overline{P}}_{F,t} = (1-\tau_I)\mathcal{M}_I \frac{\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} C^{\text{PostTP}}_{F,t+k|t} (1+\tau_{M,t+k}) \mathcal{E}_{t+k} P^*_{F,t+k}}{\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} C^{\text{PostTP}}_{F,t+k|t}}. \tag{17}$$

Here, the desired monopolistic-competition markups are $\mathcal{M}_P \equiv \varepsilon/(\varepsilon-1)$ for Home producers and $\mathcal{M}_I \equiv \varepsilon_M/(\varepsilon_M - 1)$ for importers. Under PreTP, define $\widetilde{p}^{\text{PreTP}}_{F,t} \equiv \bar{P}_{F,t}/P_{F,t}$, $\mathcal{K}^{\text{PreTP}}_{F,t} \equiv (\widetilde{p}^{\text{PreTP}}_{F,t})^{\varepsilon_M} \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} C^{\text{PreTP}}_{F,t+k|t}$, and $\mathcal{Z}^{\text{PreTP}}_{F,t} \equiv ((\widetilde{p}^{\text{PreTP}}_{F,t})^{\varepsilon_M}/P_{F,t}) \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} C^{\text{PreTP}}_{F,t+k|t} \mathcal{E}_{t+k} P^*_{F,t+k}$. Equation (16) can then be written compactly as

$$\widetilde{p}^{\text{PreTP}}_{F,t} = (1-\tau_I)\mathcal{M}_I \frac{\mathcal{Z}^{\text{PreTP}}_{F,t}}{\mathcal{K}^{\text{PreTP}}_{F,t}}.$$

Under PostTP, define $\widetilde{p}^{\text{PostTP}}_{F,t} \equiv \overline{\overline{P}}_{F,t}/\widetilde{P}_{F,t}$, $\mathcal{K}^{\text{PostTP}}_{F,t} \equiv (\widetilde{p}^{\text{PostTP}}_{F,t})^{\varepsilon_M} \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} C^{\text{PostTP}}_{F,t+k|t}$, and $\mathcal{Z}^{\text{PostTP}}_{F,t} \equiv ((\widetilde{p}^{\text{PostTP}}_{F,t})^{\varepsilon_M}/\widetilde{P}_{F,t}) \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} C^{\text{PostTP}}_{F,t+k|t} (1+\tau_{M,t+k}) \mathcal{E}_{t+k} P^*_{F,t+k}$. Equation (17) can then be written compactly as

$$\widetilde{p}^{\text{PostTP}}_{F,t} = (1-\tau_I)\mathcal{M}_I \frac{\mathcal{Z}^{\text{PostTP}}_{F,t}}{\mathcal{K}^{\text{PostTP}}_{F,t}}.$$

The constant markup-correcting subsidies are distinct from the time-varying import tariff. In the Baseline specification we set $\tau = 1/\varepsilon$ and $\tau_I = 1/\varepsilon_M$. Consequently $(1-\tau)\mathcal{M}_P = 1$ and $(1-\tau_I)\mathcal{M}_I = 1$, so both deterministic monopoly-markup distortions are removed.

Written this way, the two first-order conditions have the same discounted-demand structure but differ in the object whose expected path is averaged into the reset price. In PreTP, the reset price is tariff exclusive and the statutory tariff never enters the importer's marginal acquisition cost. In PostTP, the reset cum-tariff price reflects the entire expected future path of tariff-inclusive acquisition costs over the stochastic duration of the price spell. Each future tariff rate is weighted jointly by Calvo survival, the stochastic discount factor, and future demand, rather than by a simple arithmetic average or by the tariff rate at any particular date. Hence the tariff compresses the non-reset importer's markup under PostTP but not under PreTP.

The accounting behind this PostTP margin compression is important. If an importer resets at t and cannot reset through $t+k$, the purchaser-facing price of that variety remains at the preset level $\overline{\overline{P}}_{F,t}$ even though the tariff actually levied in period $t+k$ is $\tau_{M,t+k}$. The Home household therefore continues to pay the preset cum-tariff price for that variety. At the same time, Eq. (15) shows that the importer's landed acquisition cost is proportional to $(1+\tau_{M,t+k})\mathcal{E}_{t+k}P^*_{F,t+k}$. Thus a higher realized tariff during the Calvo spell is not mechanically added to the household's price; with the cum-tariff price fixed, it is absorbed by a lower importer margin and hence lower importer profits, with the converse for a lower tariff. Tariff revenue is still rebated through T_t. Accordingly, PostTP changes the importer's price-setting constraint and profit income, but not the form of the household budget constraint in Eq. (4).

The Calvo price-index laws are

$$P_{F,t}^{1-\varepsilon_M} = \theta P_{F,t-1}^{1-\varepsilon_M} + (1-\theta)\overline{P}_{F,t}^{1-\varepsilon_M}, \qquad \text{PreTP}, \tag{18}$$

$$\widetilde{P}_{F,t}^{1-\varepsilon_M} = \theta \widetilde{P}_{F,t-1}^{1-\varepsilon_M} + (1-\theta)\overline{\widetilde{P}}_{F,t}^{1-\varepsilon_M}, \qquad \text{PostTP}. \tag{19}$$

The Calvo formulas are mechanically identical, but they describe distributions of different nominal objects: Eq. (18) evolves the tariff-exclusive price index $P_{F,t}$, whereas Eq. (19) evolves the purchaser-facing cum-tariff index $\widetilde{P}_{F,t}$. Thus the same Calvo parameter does not imply the same short-run pass-through of the tariff. Linearizing the reset-price condition and the corresponding Calvo law gives

$$\pi_{F,t} = \beta \mathrm{E}_t \pi_{F,t+1} + \kappa \psi_t, \qquad \text{PreTP}, \tag{20}$$

$$\widetilde{\pi}_{F,t} = \beta \mathrm{E}_t \widetilde{\pi}_{F,t+1} + \kappa \psi_t, \qquad \text{PostTP}, \tag{21}$$

Here $\psi_t \equiv \log(\Psi_t/\bar{\Psi})$ denotes the log deviation of the LOOP gap from its deterministic steady state. Equations (20) and (21) have the same common slope κ defined in Section 3.2 and the same LOOP-gap real term, but the inflation variable on the left-hand side differs. PreTP stabilizes tariff-exclusive import-price inflation $\pi_{F,t}$; PostTP stabilizes cum-tariff import-price inflation $\widetilde{\pi}_{F,t}$. Rewriting PostTP in the same tariff-exclusive coordinate therefore introduces the current and expected future change in the tariff directly, as shown next. Using the definition of $\widetilde{P}_{F,t}$, the PostTP equation in the same tariff-exclusive inflation coordinate becomes

$$\pi_{F,t} = \beta \mathrm{E}_t \pi_{F,t+1} + \kappa \psi_t - \Delta\tau_{M,t} + \beta \mathrm{E}_t \Delta\tau_{M,t+1}. \tag{22}$$

This is the central theoretical distinction. PreTP and PostTP face the same flexible-price tariff wedge, but they impose different nominal constraints on the Ramsey allocation.

3.4 Ramsey policy and welfare interpretation

Following Monacelli [25], we state the nonlinear Ramsey problems under PCP and ILCP explicitly. Under PCP, the SOE Ramsey authority solves

$$\begin{gathered} \max_{\{\mathcal{X}_t^{PCP}\}_{t\geq 0}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \\ \text{subject to (68)–(76) with } \Psi_t = 1, \qquad i_t \geq 0. \end{gathered} \tag{23a}$$

Under ILCP, the authority instead solves

$$\begin{gathered} \max_{\{\mathcal{X}_t^{ILCP,\mathcal{P}}\}_{t\geq 0}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \\ \text{subject to (68)–(76)}, \quad i_t \geq 0, \quad \begin{cases} \text{(77)–(82)}, & \mathcal{P} = \text{PreTP}, \\ \text{(83)–(88)}, & \mathcal{P} = \text{PostTP}. \end{cases} \end{gathered} \tag{23b}$$

Let $\mathcal{P} \in \{\text{PreTP}, \text{PostTP}\}$. The period-$t$ choice sets are

$$\begin{aligned} \mathcal{X}_t^{PCP} &\equiv \{C_t, S_t, N_t, Y_t, i_t, \Pi_{H,t}, \widetilde{p}_{H,t}, \mathcal{K}_{H,t}, \mathcal{Z}_{H,t}, Z_{H,t}\}, \\ \mathcal{X}_t^{ILCP,\mathcal{P}} &\equiv \mathcal{X}_t^{PCP} \cup \{\Psi_t, \Pi_{F,t}^{\mathcal{P}}, \widetilde{p}_{F,t}^{\mathcal{P}}, \mathcal{K}_{F,t}^{\mathcal{P}}, \mathcal{Z}_{F,t}^{\mathcal{P}}, Z_{F,t}\}. \end{aligned}$$

Relative to PCP, ILCP adds the LOOP gap Ψ_t and the sticky import-price block to the planner's choice set. In particular, import-price inflation $\Pi_{F,t}^{\mathcal{P}}$ becomes an additional Ramsey choice variable. The underlying sticky nominal object, however, depends on tariff timing: under PreTP, the tariff-exclusive import price $P_{F,t}$ is sticky and $\Pi_{F,t}^{\text{PreTP}} \equiv P_{F,t}/P_{F,t-1}$ is the corresponding import-price inflation rate, whereas under PostTP, the tariff-inclusive purchaser price $\widetilde{P}_{F,t}$ is sticky and

$\Pi_{F,t}^{\text{PostTP}} \equiv \widetilde{P}_{F,t}/\widetilde{P}_{F,t-1}$ is the corresponding purchaser-price inflation rate. Accordingly, the planner chooses tariff-exclusive import-price inflation under PreTP and tariff-inclusive purchaser-price inflation under PostTP. The remaining variables $\widetilde{p}_{F,t}^{\mathcal{P}}$, $\mathcal{K}_{F,t}^{\mathcal{P}}$, $\mathcal{Z}_{F,t}^{\mathcal{P}}$, and $Z_{F,t}$ implement the corresponding nonlinear Calvo block.

Here $Z_{H,t}$ and $Z_{F,t}$ denote producer- and importer-price dispersion, respectively. The recursive representations of the $\mathcal{K}/\mathcal{Z}$ variables, together with the full nonlinear implementability system and FONCs, are given in Appendix D.1.

To make the social planner's stabilization objective transparent, it is useful first to display the PCP welfare cost function. Under the Baseline, the welfare-relevant output object is common to PCP and ILCP, so we use y_t^w and $\hat{y}_t^w \equiv y_t - y_t^w$ without a pricing-regime superscript throughout this section.[2] Under PCP there is no sticky importer-price block, and the second-order period loss is

$$\mathcal{L}_t^{SOE,PCP} = \frac{1}{2}\left[\Lambda_y(\hat{y}_t^w)^2 + \Lambda_{\pi,H}\pi_{H,t}^2\right].$$

Thus, under PCP the planner trades off stabilization of the welfare-relevant output gap against stabilization of Home PPI inflation. For interpretation, the corresponding second-order welfare criterion under ILCP can be written

$$\mathcal{L}_t^{SOE,\mathcal{P}} = \frac{1}{2}\Big[\Lambda_y(\hat{y}_t^w)^2 + \Lambda_\psi(\hat{\psi}_t^{w,\mathcal{P}})^2 + \Lambda_{\pi,H}\pi_{H,t}^2 + \Lambda_{\pi,F}(\pi_{F,t}^{\mathcal{P}})^2\Big], \qquad \mathcal{P} \in \{\text{PreTP}, \text{PostTP}\}, \quad (24)$$

Relative to $\mathcal{L}_t^{SOE,PCP}$, Eq. (24) changes the set of welfare-relevant target variables: ILCP retains the welfare-relevant output gap and Home PPI inflation, and adds the welfare-relevant LOOP-gap deviation and sticky import-price inflation. In particular, the import-price target is $\pi_{F,t}$ under PreTP but $\widetilde{\pi}_{F,t}$ under PostTP. Thus the PreTP–PostTP distinction operates through which sticky import-price inflation rate the planner stabilizes, rather than through a different weight on the common PCP margins.

Here $y_t \equiv \log\frac{Y_t}{Y}$. Let the Γ coefficients below denote the symbolic cross coefficients implied by the Benigno–Woodford second-order welfare representation.[3]

$$y_t^w \equiv \frac{\Gamma_{y\psi}}{\Lambda_y}\psi_t + \frac{\Gamma_{yM}}{\Lambda_y}\tau_{M,t} + \frac{\Gamma_{yX}}{\Lambda_y}\tau_{X,t}, \quad (25)$$

The dependence of y_t^w on ψ_t, $\tau_{M,t}$, and $\tau_{X,t}$ can be related directly to the demand components of Eq. (8). From the definitions of S_t and Ψ_t,

$$\Psi_t S_t = \frac{\mathcal{E}_t P_{F,t}^*}{P_{H,t}}.$$

Log-linearizing the Foreign-demand component of Eq. (8), $EX_t \equiv \upsilon\left(\frac{\Psi_t S_t}{1+\tau_{X,t}}\right)^\eta C_t^*$, around the zero-tariff deterministic steady state gives

$$ex_t = \eta(\psi_t + s_t - \tau_{X,t}) + c_t^*.$$

Here $s_t \equiv \log(S_t/\bar{S})$ denotes the log deviation of the ex-tariff relative price of imports from its deterministic steady state. With $ex_t \equiv \log\frac{EX_t}{EX}$ and $c_t^* \equiv \log\frac{C_t^*}{C^*}$, $\partial ex_t/\partial\psi_t = \eta > 0$. Thus an increase in Ψ_t lowers the Home producer price relative to the Foreign-good border price and shifts Foreign expenditure toward Home goods, raising Home output. The welfare reduction correspondingly

[2] This notational simplification is specific to the Baseline with markup-correcting subsidies. When the producer and importer subsidies are removed in Section 7.1, the corresponding PCP and ILCP second-order welfare representations are regime specific, so their welfare-relevant output objects need not coincide.

[3] Following Monacelli [25], in log-linear and second-order expressions $\tau_{M,t}$ and $\tau_{X,t}$ denote the log deviations of the corresponding gross tariff wedges. Around the zero-tariff steady state, these coincide to first order with the tariff rates.

has $\Gamma_{y\psi} > 0$ under the Benchmark Parameterization. Likewise, log-linearizing the Home-demand component of Eq. (8), $C_{H,t} \equiv (1-\upsilon)\mathcal{G}(S_t, \tau_{M,t})^{\eta} C_t$, around the same steady state gives

$$c_{H,t} = c_t + \eta\upsilon(s_t + \tau_{M,t}).$$

With $c_{H,t} \equiv \log\frac{C_{H,t}}{C_H}$ and $c_t \equiv \log\frac{C_t}{C}$, $\partial c_{H,t}/\partial \tau_{M,t} = \eta\upsilon > 0$. Here EX, C_H, C, and C^* denote their deterministic steady-state values. A higher import tariff raises the purchaser-facing relative price of Foreign goods and shifts Home expenditure toward Home goods; this is reflected in $\Gamma_{yM} > 0$.

By contrast, $\partial ex_t/\partial \tau_{X,t} = -\eta < 0$, so the direct expenditure-switching effect of a higher export tariff is to reduce Foreign demand for Home goods and Home output. Nevertheless, evaluation of the symbolic Benigno–Woodford coefficient under the Benchmark Parameterization gives $\Gamma_{yX} > 0$. This is not a contradiction: Γ_{yX} is a coefficient of the welfare center, not the derivative of goods demand. It is obtained only after the second-order utility, CPI, goods-market, and producer price-setting restrictions are combined; those welfare terms overturn the direct negative demand effect of the export tariff. The distinction is also visible in the footnote below, where the export-tariff coefficient in flexible-price natural output is negative. There is no PreTP/PostTP superscript on y_t^w because these output-side coefficients are common to the two timing conventions.

The welfare-relevant LOOP-gap deviation is $\hat{\psi}_t^{w,\mathcal{P}} \equiv \psi_t - \psi_t^{w,\mathcal{P}}$, where $\psi_t^{w,\mathcal{P}}$ is the welfare-relevant LOOP gap, which is given by

$$\psi_t^{w,\mathcal{P}} \equiv \frac{\Gamma_{\psi M}^{\mathcal{P}} + \Gamma_{y\psi}\Gamma_{yM}/\Lambda_y}{\Lambda_\psi}\tau_{M,t} + \frac{\Gamma_{\psi X} + \Gamma_{y\psi}\Gamma_{yX}/\Lambda_y}{\Lambda_\psi}\tau_{X,t}. \tag{26}$$

The direction in which the two tariffs shift the welfare-relevant LOOP gap $\psi_t^{w,\mathcal{P}}$ can be traced directly to Eqs. (7) and (8). Log-linearizing the risk-sharing condition in Eq. (7) around the zero-tariff deterministic steady state gives

$$\psi_t + s_t = c_t - c_t^* + \upsilon(s_t + \tau_{M,t}).$$

Thus, for given $c_t - c_t^*$ and s_t, a higher import tariff raises the CPI–PPI ratio $\mathcal{G}(S_t, \tau_{M,t})$ and therefore raises the LOOP gap ψ_t. The export-tariff effect follows from the Foreign-demand component of Eq. (8) derived above,

$$ex_t = \eta(\psi_t + s_t - \tau_{X,t}) + c_t^*.$$

A higher export tariff lowers Foreign demand for Home goods at a given ψ_t; offsetting this expenditure-switching effect requires a higher, rather than a lower, LOOP gap. Equivalently, for given ex_t, s_t, and c_t^*, the equation implies $d\psi_t/d\tau_{X,t} = 1$. Hence both tariffs raise the welfare-relevant LOOP gap $\psi_t^{w,\mathcal{P}}$ under the Benchmark Parameterization, although through different demand margins: the import tariff through the Home CPI–PPI ratio and the export tariff through Foreign expenditure switching. The superscript $\mathcal{P}$ is needed only for the import-tariff component: PreTP and PostTP change how the import tariff enters the importer price-setting restriction and hence $\Gamma_{\psi M}^{\mathcal{P}}$, whereas the export-tariff component is common to the two timing conventions. Appendix F gives the general square-completion construction.

Although the Baseline subsidies eliminate the producer and importer monopoly-markup distortions and the deterministic steady state has zero tariffs, welfare-relevant output y_t^w does not in general coincide with logarithmic flexible-price natural output y_t^n after a tariff innovation.[4]

[4]Let $y_t^n \equiv \log(Y_t^n/\bar{Y})$. Under flexible producer and importer prices in the Baseline, $mc_t^{r,n} = 0$ and $\psi_t^n \equiv \log(\Psi_t^n/\bar{\Psi}) = 0$. The first-order versions of Eqs. (7) and (8) then imply

$$y_t^n = \frac{\upsilon(1-\upsilon)(\eta-1)\tau_{M,t} - \eta\upsilon\tau_{X,t}}{1 + \varphi\{1 + (\eta-1)\upsilon(2-\upsilon)\}}.$$

By contrast, evaluating Eq. (25) at the flexible-import-price allocation gives

$$y_t^w|_{\psi_t = \psi_t^n} = \frac{\Gamma_{yM}}{\Lambda_y}\tau_{M,t} + \frac{\Gamma_{yX}}{\Lambda_y}\tau_{X,t}.$$

Section 7.1 relaxes the markup-correction assumption by removing the producer and importer subsidies and checks whether the main sign results survive.

Following Monacelli [25], we set $\beta = 0.99$, $\upsilon = 0.30$, $\eta = 1.5$, $\varepsilon = 3.8$, and $\theta = 0.8$. We set the importer elasticity to $\varepsilon_M = 3.8$ and, as imposed above, use the same Calvo parameter θ for the importer sector. The inverse Frisch elasticity is $\varphi = 2.2$. Tariff persistence is 0.7, and the tariff innovation raises the Home import tariff by 20 percentage points on impact. Thus the basic parameterization is borrowed from Monacelli's November 2025 calibration; the importer-pricing block and the PreTP–PostTP comparison are the additions studied here.

The sticky import inflation variables are $\pi_{F,t}^{\text{PreTP}} \equiv \pi_{F,t}$ and $\pi_{F,t}^{\text{PostTP}} \equiv \widetilde{\pi}_{F,t}$; $\pi_{H,t}$ was defined immediately after Eq. (13).

Table 1: SOE Baseline: PCP and ILCP welfare coefficients

Coefficients		Λ_y	Λ_ψ	$\Lambda_{\pi,H}$	$\Lambda_{\pi,F}$
Target variable		y_t^w	$\hat{\psi}_t^w$	$\pi_{H,t}$	$\pi_{F,t}$
Pricing	PCP	1.6236	—	38.5689	—
Pricing	ILCP	1.6236	0.5054	38.5689	27.1051

Notes: $\Lambda_y \equiv -2C_{yy}$, $\Lambda_\psi \equiv \bar{\Lambda}_\psi - \Gamma_{y\psi}^2/\Lambda_y$, $\Lambda_{\pi,H} \equiv (\varepsilon/\kappa)(\Theta_n - \Theta_H^{AS})$, and $\Lambda_{\pi,F} \equiv -(\varepsilon_M/\kappa)\Theta_M^{AS}$. Under the Baseline, the output coefficient Λ_y and the Home-PPI coefficient $\Lambda_{\pi,H}$ are common to PCP and ILCP, so no pricing-regime superscripts are used. $\bar{\Lambda}_\psi$ is the pre-completion ILCP LOOP-gap quadratic coefficient, and Θ_n, Θ_H^{AS}, and Θ_M^{AS} are the corresponding Benigno–Woodford undetermined coefficients. The common Phillips-curve slope is κ because producer and importer prices use the same Calvo parameter.

Table 1 shows that the basic stabilization priorities are similar under PCP and ILCP. The Home-PPI inflation weight, $\Lambda_{\pi,H} = 38.5689$, is much larger than the output weight, $\Lambda_y = 1.6236$; under ILCP the import-price-inflation weight, $\Lambda_{\pi,F} = 27.1051$, is likewise much larger than the LOOP-gap weight, $\Lambda_\psi = 0.5054$. Hence, under both PCP and ILCP, the welfare criterion places the dominant quantitative weight on stabilizing inflation rates associated with sticky prices. The crucial distinction between PreTP and PostTP is which sticky import-price inflation rate enters the ILCP criterion: tariff-exclusive import-price inflation $\pi_{F,t}$ under PreTP and cum-tariff import-price inflation $\widetilde{\pi}_{F,t}$ under PostTP. The welfare-relevant LOOP gap can also differ through $\Gamma_{\psi M}^{\mathcal{P}}$, while the output-side coefficients are common to the two timing conventions.

Analytical benchmark: tariff location and the monetary stance. The pricing equations imply a directional monetary-policy force before solving the full Ramsey system. Because $\Lambda_{\pi,F}$ is large relative to Λ_ψ, consider the limiting benchmark in which the planner stabilizes the relevant sticky import-price inflation closely. Under PreTP this means $\pi_{F,t} \simeq 0$. Under PostTP it means instead $\widetilde{\pi}_{F,t} \simeq 0$. Using Eq. (6) gives

$$\pi_{F,t} \simeq -\Delta\tau_{M,t}. \tag{27}$$

Since $\pi_{F,t} = \Delta p_{F,t}$, where $p_{F,t} \equiv \log(P_{F,t}/P_F)$, a positive tariff innovation therefore requires a lower tariff-exclusive import price under PostTP than under PreTP, starting from the same predetermined lagged price. From the LOOP-gap identity $\Psi_t = \mathcal{E}_t P_{F,t}^*/P_{F,t}$, for a given LOOP gap this lower $P_{F,t}$ requires a lower Home-currency ex-tariff border price $\mathcal{E}_t P_{F,t}^*$. Since $P_{F,t}^*$ is exogenous in the small open economy, this means a lower $\mathcal{E}_t$, that is, an appreciation relative to the PreTP path. With the Foreign interest rate exogenous, UIP then maps this relative appreciation force into a higher Home nominal interest rate, conditional on the expected future exchange rate. PreTP lacks the requirement that the tariff be absorbed by the ex-tariff price and therefore carries the opposite relative force, toward depreciation and monetary easing.

The same result is visible directly from the two Phillips curves. Rewriting PostTP in the tariff-exclusive coordinate as in Eq. (22), the timing-specific term relative to PreTP is

$$-\Delta\tau_{M,t} + \beta \mathrm{E}_t \Delta\tau_{M,t+1}. \tag{28}$$

Thus both objects are zero at the zero-tariff deterministic steady state, but their tariff-response coefficients need not coincide. The markup-correcting subsidies remove the monopoly-markup distortions; they do not make the time-varying tariff allocation identical to the welfare center of the second-order Ramsey criterion.

If, to first order, the tariff follows $\mathrm{E}_t\tau_{M,t+1} = \rho\tau_{M,t}$, $0 \leq \rho < 1$, then on impact from the zero-tariff steady state Eq. (28) equals

$$-\left[1 + \beta(1-\rho)\right]\tau_{M,0} < 0. \tag{29}$$

Hence PostTP shifts the ex-tariff import-price adjustment downward and, through the border-price identity and UIP, shifts the monetary stance toward appreciation and tightening relative to PreTP. This benchmark signs the *timing-induced shift* in the monetary-policy force; it does not by itself assert that the complete Ramsey interest-rate response must have opposite signs, because output, Home PPI inflation, and the LOOP-gap target remain part of the welfare tradeoff. The quantitative question is whether this timing-induced shift is large enough to cross zero. The SOE Ramsey solution below shows that it is.

3.5 SOE Baseline results

The SOE Baseline result is the first quantitative result of the paper. Figure 1 reports the impulse responses to the 20-percentage-point Home import-tariff shock in the small open economy. It compares ILCP under PreTP and PostTP, with PCP included as a control. To highlight the central sign result, Table 2 summarizes the impact responses of the Home nominal interest rate, in basis points from its deterministic steady state. Table 2 is a compact sign comparison, not

Table 2: SOE Baseline: Home impact nominal-interest-rate responses

Pricing regime	PreTP	PostTP
PCP control	+18.56	
Importer LCP	−75.95	+57.68

a separate robustness exercise: PCP has no timing pair, while ILCP changes from easing under PreTP to tightening under PostTP. Thus importer LCP generates a timing-induced zero crossing already in the small open economy: the optimal policy switches from a 75.95-basis-point cut under PreTP to a 57.68-basis-point increase under PostTP. Because Foreign consumption and Foreign monetary policy are exogenous in the SOE, this result does not rely on a two-country strategic monetary-policy interaction.

Figure 1 shows why the two interest-rate responses differ. Under ILCP PreTP, ex-tariff import-price inflation is stabilized closely (Panel (e)). The tariff increase reduces consumption of Foreign goods (Panel (n)), weakening import demand and putting downward pressure on ex-tariff import-price inflation. Offsetting this pressure requires a higher domestic-currency ex-tariff import price at the border, $\mathcal{E}_t P^*_{F,t}$. A nominal depreciation raises this border price and, through UIP, is implemented by a lower Home nominal interest rate (Panel (a)). Under ILCP PostTP, by contrast, the sticky object is tariff-inclusive import-price inflation rather than ex-tariff import-price inflation. The tariff increase directly raises the tariff-inclusive acquisition cost of imports. Stabilizing the sticky tariff-inclusive import price therefore requires a lower domestic-currency ex-tariff import price at the border, $\mathcal{E}_t P^*_{F,t}$. This requires a nominal appreciation, which, through UIP, is implemented by a higher Home nominal interest rate (Panel (a)). Thus the change in tariff timing reverses the required exchange-rate adjustment and, consequently, the direction of the optimal nominal-interest-rate response. Note that under PostTP the LOOP gap nevertheless rises substantially (Panel (h)), while tariff-inclusive import-price inflation remains relatively stable (Panel (f)). These responses are consistent with Eq. (21), since the relatively small Phillips-curve slope κ limits the effect of the LOOP gap on tariff-inclusive import-price inflation.

Thus, under ILCP, the direction of the optimal nominal-interest-rate response depends on whether the sticky retail import price is tariff exclusive or tariff inclusive. Under PreTP, the planner lowers the Home nominal interest rate to induce depreciation and raise the domestic-currency ex-tariff border price, thereby offsetting the downward pressure on the sticky ex-tariff import price. Under PostTP, the planner instead raises the Home nominal interest rate to induce appreciation and lower the domestic-currency ex-tariff border price, thereby offsetting the direct upward pressure of the tariff on the sticky tariff-inclusive import price. The sign reversal in the optimal nominal-interest-rate response therefore follows from the opposite exchange-rate adjustments required to stabilize the relevant sticky import price under PreTP and PostTP, rather than from a mechanical response to CPI inflation.

Finally, Home PPI inflation is no longer stabilized as closely under ILCP as under PCP, under either PreTP or PostTP (Panel (b)). Under PCP, Home PPI inflation is the dominant stabilization target because its welfare weight is much larger than the output-gap weight. Under ILCP, the planner must additionally stabilize the LOOP-gap distortion and sticky import-price inflation, creating a trade-off with PPI stabilization. Consequently, PPI inflation need not remain close to zero under ILCP.[5]

4 Two-country model and pricing regimes

4.1 Environment

A mass ν of households resides in Home and a mass $1-\nu$ in Foreign. The Home and Foreign consumption indexes are

$$C_t = \left[\omega_H^{1/\eta} C_{H,t}^{(\eta-1)/\eta} + \omega_F^{1/\eta} C_{F,t}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}, \tag{30}$$

$$C_t^* = \left[(\omega_F^*)^{1/\eta} (C_{F,t}^*)^{(\eta-1)/\eta} + (\omega_H^*)^{1/\eta} (C_{H,t}^*)^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}, \tag{31}$$

where $C_{H,t}$ and $C_{F,t}$ are Home household purchases of Home and Foreign goods, and $C_{F,t}^*$ and $C_{H,t}^*$ are the corresponding Foreign-household purchases. The expenditure weights are $\omega_F = \upsilon(1-\nu)$, $\omega_H = 1-\omega_F$, $\omega_H^* = \upsilon\nu$, and $\omega_F^* = 1-\omega_H^*$. The associated Home CPI is

$$P_t = \left(\omega_H P_{H,t}^{1-\eta} + \omega_F \widetilde{P}_{F,t}^{1-\eta}\right)^{1/(1-\eta)}, \tag{32}$$

with an exact starred mirror for Foreign. Thus the two-country model preserves the same CES demand system and household-side tariff wedge as the SOE; what changes is that Foreign consumption, prices, and monetary policy are now endogenous.

Financial markets are complete and tariff revenue is rebated lump sum. The Home and Foreign authorities play a national Nash–Ramsey game under commitment in the allocation-game sense of Faia and Monacelli [17]: each authority chooses its own complete contingent allocation while taking the other country's complete contingent allocation as given. The Baseline specification uses the same markup-correcting subsidies as the SOE. We set $\tau = \tau^* = 1/\varepsilon$ and, where importer sectors exist, $\tau_I = \tau_I^* = 1/\varepsilon_M$, so $(1-\tau)\mathcal{M}_P = (1-\tau^*)\mathcal{M}_P = 1$ and $(1-\tau_I)\mathcal{M}_I = (1-\tau_I^*)\mathcal{M}_I = 1$. Section 7.1 separately removes all of these subsidies.

Home real marginal cost is the SOE object MC_t^r. For Foreign, let MC_t^* denote nominal marginal cost and define $MC_t^{r,*} \equiv MC_t^*/P_{F,t}^*$, with deterministic steady-state value $MC^{r,*}$. Thus the star alone distinguishes Foreign from Home marginal cost.

4.2 PCP and importer LCP

The PCP and importer-LCP pricing blocks are direct two-country counterparts of the SOE blocks in Sections 3.2 and 3.3, so we do not repeat the same reset-price problems and first-order conditions.

[5] CPI inflation is also particularly well stabilized under ILCP PostTP (Panel (d)), although CPI inflation is not itself a direct welfare target in Eq. (24). The higher nominal interest rate appreciates the currency and restrains tariff-inclusive import-price inflation by lowering the importer's border acquisition cost. At the same time, tighter monetary policy reduces consumption and puts downward pressure on Home PPI inflation. The small remaining increase in tariff-inclusive import-price inflation is therefore offset by the decline in PPI inflation, leaving CPI inflation close to zero.

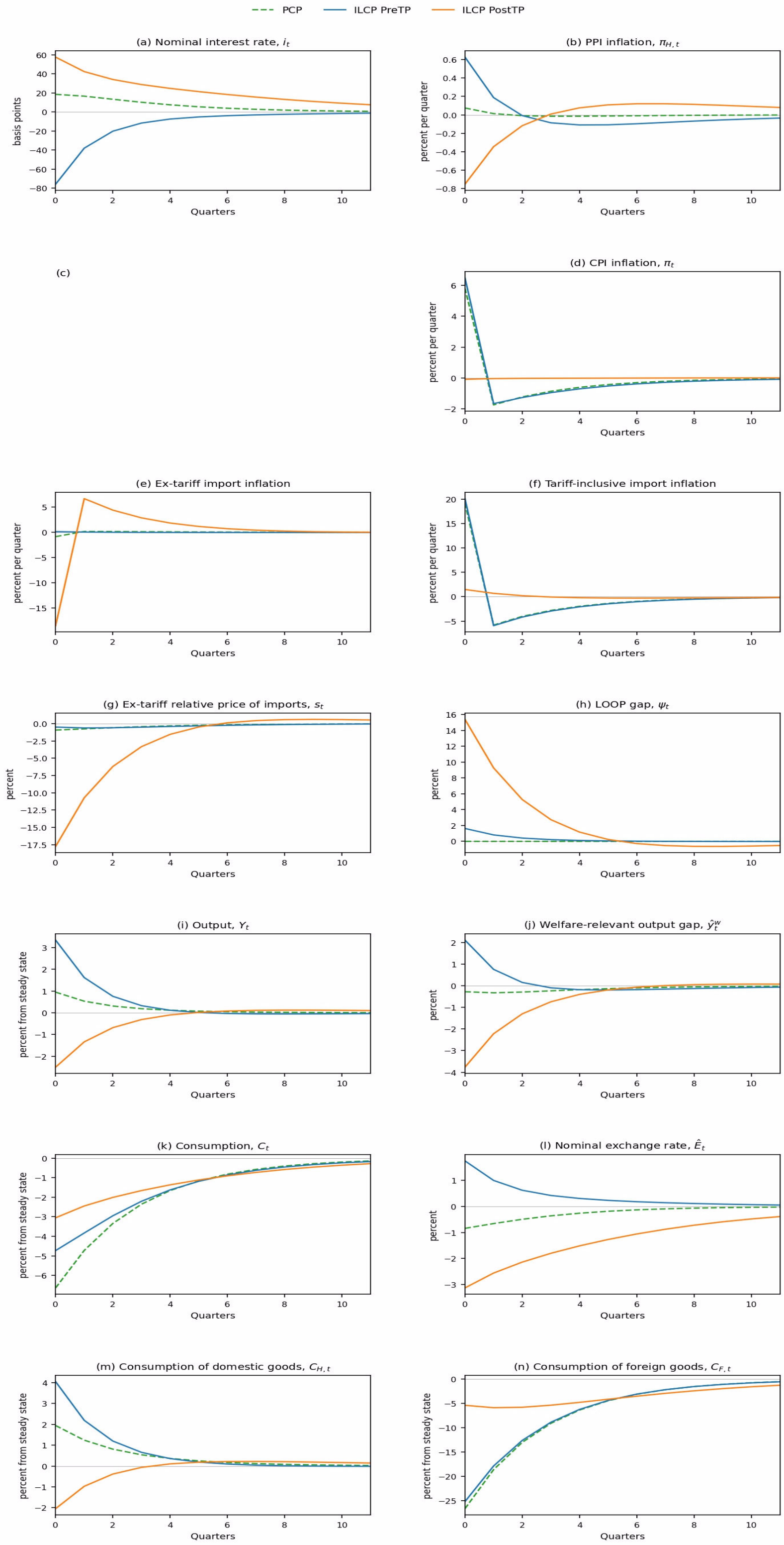

Figure 1: Small-open-economy Baseline: importer LCP under PreTP and PostTP, with PCP as a control. The nominal interest rate is shown in basis points from steady state; other panels are in percent, with inflation rates in percent per quarter.

Under PCP, the Home producer block is exactly Eqs. (11)–(13), while Foreign has the starred mirror block. PCP therefore still has no PreTP–PostTP pair. Relative to the SOE, the difference is that Foreign consumption, prices, and monetary policy are endogenous.

Under importer LCP, the Home importer block is exactly Eqs. (14)–(22): PreTP is the Monacelli [24] tariff-exclusive importer-LCP formulation, whereas PostTP makes the cum-tariff purchaser price sticky. The two-country model additionally contains the exact starred mirror importer block for Foreign, with tariff $\tau^*_{M,t}$ and importer subsidy τ^*_I. Hence the pricing equations themselves introduce no new mechanism relative to the SOE; the two-country extension endogenizes Foreign quantities and policy and adds the Foreign mirror price-setting block.

4.3 Full LCP

Full LCP is the intermediate pricing architecture between the simple ILCP mechanism and DCP. The Home import price is now set by the Foreign exporter rather than by a Home importer, so under PostTP the tariff is absorbed through the exporter's net-of-tariff revenue rather than through an importer margin. More importantly, for the unilateral Home tariff, the Foreign-to-Home price-setting block is exactly the one used under DCP. Full LCP is therefore structurally much closer to DCP than ILCP, while still avoiding the dominant-currency asymmetry in the opposite Home-to-Foreign block. A PreTP–PostTP sign reversal here is consequently both a bridge to the DCP analysis and a robustness check that the ILCP result does not depend on importer price setting.

4.3.1 Home-to-Home and Home-to-Foreign pricing

Under full LCP, producers set destination-specific Calvo prices in the buyer's currency. For Home producers, the domestic-market pricing block sets the Home-currency price $P_{H,t}$ and faces only Home demand for Home goods, $C_{H,t}$. If the domestic price is reset at t and remains unchanged through $t+k$, the relevant demand is

$$C_{H,t+k|t} \equiv \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} C_{H,t+k}.$$

The resetting Home domestic producer therefore solves

$$\max_{\bar{P}_{H,t}} \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \left[\bar{P}_{H,t} - (1-\tau) P_{H,t+k} MC^r_{t+k}\right] C_{H,t+k|t}. \tag{33}$$

The Home export-market pricing block is separate. A Home exporter selling in Foreign sets the Foreign-currency destination price $P^*_{H,t}$ and faces only Foreign demand for Home goods, $C^*_{H,t}$. In the unilateral Home-tariff experiment, $\tau^*_{M,t} = 0$, so this Home-to-Foreign block is identical under PreTP and PostTP. Defining

$$C^*_{H,t+k|t} \equiv \left(\frac{\bar{P}^*_{H,t}}{P^*_{H,t+k}}\right)^{-\varepsilon} C^*_{H,t+k},$$

the resetting Home exporter solves

$$\max_{\bar{P}^*_{H,t}} \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \left[\mathcal{E}_{t+k}\bar{P}^*_{H,t} - (1-\tau) P_{H,t+k} MC^r_{t+k}\right] C^*_{H,t+k|t}. \tag{34}$$

Thus the Home domestic and export blocks face distinct destination-specific demands and set distinct sticky prices, even though both use the same Home production marginal cost.

The corresponding first-order conditions are

$$\bar{P}_{H,t} = (1-\tau)\mathcal{M}_P \frac{\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} C_{H,t+k|t} P_{H,t+k} MC^r_{t+k}}{\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} C_{H,t+k|t}}, \tag{35}$$

$$\bar{P}^*_{H,t} = (1-\tau)\mathcal{M}_P \frac{\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} C^*_{H,t+k|t} P_{H,t+k} MC^r_{t+k}}{\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} C^*_{H,t+k|t} \mathcal{E}_{t+k}}. \tag{36}$$

The associated Calvo price-index laws are

$$P^{1-\varepsilon}_{H,t} = \theta P^{1-\varepsilon}_{H,t-1} + (1-\theta)\bar{P}^{1-\varepsilon}_{H,t}, \tag{37}$$

$$(P^*_{H,t})^{1-\varepsilon} = \theta (P^*_{H,t-1})^{1-\varepsilon} + (1-\theta)(\bar{P}^*_{H,t})^{1-\varepsilon}. \tag{38}$$

For the Home-to-Foreign block, define the Foreign-destination LOOP gap by

$$\Psi^*_t \equiv \frac{P_{H,t}}{\mathcal{E}_t P^*_{H,t}}, \qquad \psi^*_t \equiv \log(\Psi^*_t/\bar{\Psi}^*),$$

and let $\pi^*_{H,t} \equiv \log(P^*_{H,t}/P^*_{H,t-1})$. Linearizing Eqs. (35)–(38) gives the two Home-producer Phillips curves

$$\pi_{H,t} = \beta \mathrm{E}_t \pi_{H,t+1} + \kappa mc^r_t, \tag{39}$$

$$\pi^*_{H,t} = \beta \mathrm{E}_t \pi^*_{H,t+1} + \kappa \left(\psi^*_t + mc^r_t\right). \tag{40}$$

The domestic Phillips curve is driven only by Home real marginal cost, whereas the export-price Phillips curve also contains the Foreign-destination LOOP gap because the Home export price is sticky in Foreign currency.

4.3.2 Foreign-to-Home pricing

The remaining export-market block is the Foreign exporter selling in Home. This exporter sets the Home-currency destination price $P_{F,t}$ and faces Home demand for Foreign goods, $C_{F,t}$. Unlike the Home-to-Foreign block, it is directly affected by the Home import tariff $\tau_{M,t}$ and is therefore the block in which the PreTP–PostTP distinction matters in the unilateral experiment. We consequently spell out this block in detail below. The PreTP exporter-pricing formulation follows the local-currency-pricing environments of Corsetti, Dedola, and Leduc [11] and Okano [26].

For $\mathcal{P} \in \{\text{PreTP}, \text{PostTP}\}$, let $C^{\mathcal{P}}_{F,t+k|t}$ denote Home demand for a Foreign export variety whose destination price was reset at t and remains unchanged through $t+k$. Here $C^{\text{PreTP}}_{F,t+k|t} \equiv (\bar{P}_{F,t}/P_{F,t+k})^{-\varepsilon} C_{F,t+k}$ and $C^{\text{PostTP}}_{F,t+k|t} \equiv (\bar{\widetilde{P}}_{F,t}/\widetilde{P}_{F,t+k})^{-\varepsilon} C_{F,t+k}$.

The resetting Foreign exporter solves

$$\text{PreTP}: \quad \max_{\bar{P}_{F,t}} \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda^*_{t,t+k} \left[\frac{\bar{P}_{F,t}}{\mathcal{E}_{t+k}} - (1-\tau^*) P^*_{F,t+k} MC^{r,*}_{t+k} \right] C^{\text{PreTP}}_{F,t+k|t}, \tag{41}$$

$$\text{PostTP}: \quad \max_{\bar{\widetilde{P}}_{F,t}} \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda^*_{t,t+k} \left[\frac{\bar{\widetilde{P}}_{F,t}}{(1+\tau_{M,t+k})\mathcal{E}_{t+k}} - (1-\tau^*) P^*_{F,t+k} MC^{r,*}_{t+k} \right] C^{\text{PostTP}}_{F,t+k|t}. \tag{42}$$

The Foreign household intratemporal condition is $W^*_t/P^*_t = C^*_t (N^*_t)^{\varphi}$. It implies that the Foreign real marginal cost in Eqs. (41) and (42) is

$$MC^{r,*}_{t+k} = \frac{C^*_{t+k}(N^*_{t+k})^{\varphi}}{A^*_{t+k}} \mathcal{G}^*(S_{t+k}, \tau^*_{M,t+k}) = C^*_{t+k}(N^*_{t+k})^{\varphi} \mathcal{G}^*(S_{t+k}, \tau^*_{M,t+k}),$$

where $\mathcal{G}^*(S_t, \tau^*_{M,t}) \equiv P^*_t/P^*_{F,t}$ denotes the CPI–PPI ratio in country F, and the second equality uses the Foreign mirror normalization $A^*_{t+k} = 1$.

The revenue term in Eq. (42) has a direct interpretation. Under PostTP, the sticky object is the purchaser-facing cum-tariff price $\bar{\widetilde{P}}_{F,t}$. If that price remains in effect at $t+k$, the Foreign exporter receives $\bar{\widetilde{P}}_{F,t}/(1+\tau_{M,t+k})$ in Home currency per unit sold, or $\bar{\widetilde{P}}_{F,t}/[(1+\tau_{M,t+k})\mathcal{E}_{t+k}]$ in Foreign currency, as shown directly in Eq. (42). Thus a higher realized tariff during a Calvo price spell lowers the non-resetting exporter's net-of-tariff receipt and markup, while leaving its production cost unchanged. By contrast, under PreTP the ex-tariff destination price itself is sticky, so the purchaser-facing cum-tariff price moves with the realized tariff.

The Home household budget constraint is unchanged across these timing conventions. The household always pays the purchaser-facing import price, while tariff revenue is rebated lump sum. What changes under FLCP PostTP is Foreign exporter profit: any discrepancy between the tariff anticipated when $\bar{\widetilde{P}}_{F,t}$ was chosen and the tariff subsequently realized appears in the exporter's net receipt and hence in Foreign household income.

The first-order conditions can again be written with the exporter's chosen destination price on the left-hand side:

$$\text{PreTP}: \quad \bar{P}_{F,t} = (1-\tau^*)\mathcal{M}_P \frac{\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda^*_{t,t+k} C^{\mathrm{PreTP}}_{F,t+k|t} P^*_{F,t+k} MC^{r,*}_{t+k}}{\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda^*_{t,t+k} C^{\mathrm{PreTP}}_{F,t+k|t} / \mathcal{E}_{t+k}}, \tag{43}$$

$$\text{PostTP}: \quad \bar{\widetilde{P}}_{F,t} = (1-\tau^*)\mathcal{M}_P \frac{\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda^*_{t,t+k} C^{\mathrm{PostTP}}_{F,t+k|t} P^*_{F,t+k} MC^{r,*}_{t+k}}{\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda^*_{t,t+k} C^{\mathrm{PostTP}}_{F,t+k|t} / [(1+\tau_{M,t+k})\mathcal{E}_{t+k}]}. \tag{44}$$

Using the normalized reset-price ratios $\widetilde{p}^{\mathrm{PreTP}}_{F,t}$ and $\widetilde{p}^{\mathrm{PostTP}}_{F,t}$ defined in the SOE importer-LCP block, define the Foreign exporter's Calvo auxiliaries under PreTP as

$$\begin{aligned}
\mathcal{K}^{*,\mathrm{PreTP}}_{F,t} &\equiv (\widetilde{p}^{\mathrm{PreTP}}_{F,t})^{\varepsilon} \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda^*_{t,t+k} \frac{C^{\mathrm{PreTP}}_{F,t+k|t}}{\mathcal{E}_{t+k}}, \\
\mathcal{Z}^{*,\mathrm{PreTP}}_{F,t} &\equiv \frac{(\widetilde{p}^{\mathrm{PreTP}}_{F,t})^{\varepsilon}}{P_{F,t}} \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda^*_{t,t+k} C^{\mathrm{PreTP}}_{F,t+k|t} P^*_{F,t+k} MC^{r,*}_{t+k}.
\end{aligned}$$

Then Eq. (43) can be written compactly as

$$\widetilde{p}^{\mathrm{PreTP}}_{F,t} = (1-\tau^*)\mathcal{M}_P \frac{\mathcal{Z}^{*,\mathrm{PreTP}}_{F,t}}{\mathcal{K}^{*,\mathrm{PreTP}}_{F,t}}.$$

Under PostTP, define

$$\begin{aligned}
\mathcal{K}^{*,\mathrm{PostTP}}_{F,t} &\equiv (\widetilde{p}^{\mathrm{PostTP}}_{F,t})^{\varepsilon} \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda^*_{t,t+k} \frac{C^{\mathrm{PostTP}}_{F,t+k|t}}{(1+\tau_{M,t+k})\mathcal{E}_{t+k}}, \\
\mathcal{Z}^{*,\mathrm{PostTP}}_{F,t} &\equiv \frac{(\widetilde{p}^{\mathrm{PostTP}}_{F,t})^{\varepsilon}}{\widetilde{P}_{F,t}} \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda^*_{t,t+k} C^{\mathrm{PostTP}}_{F,t+k|t} P^*_{F,t+k} MC^{r,*}_{t+k}.
\end{aligned}$$

Then Eq. (44) can be written compactly as

$$\widetilde{p}^{\mathrm{PostTP}}_{F,t} = (1-\tau^*)\mathcal{M}_P \frac{\mathcal{Z}^{*,\mathrm{PostTP}}_{F,t}}{\mathcal{K}^{*,\mathrm{PostTP}}_{F,t}}.$$

In the PostTP condition, the expected tariff path enters the denominator because the chosen object is the purchaser-facing cum-tariff price while the exporter receives that price net of the tariff. Thus the exporter also chooses one reset price against the entire expected tariff path over the stochastic price spell, but the weighting is through expected net revenue rather than through tariff-inclusive marginal acquisition cost as under ILCP.

The Calvo laws are

$$P^{1-\varepsilon}_{F,t} = \theta P^{1-\varepsilon}_{F,t-1} + (1-\theta)\bar{P}^{1-\varepsilon}_{F,t}, \qquad \text{PreTP}, \tag{45}$$

$$\widetilde{P}^{1-\varepsilon}_{F,t} = \theta \widetilde{P}^{1-\varepsilon}_{F,t-1} + (1-\theta)\bar{\widetilde{P}}^{1-\varepsilon}_{F,t}, \qquad \text{PostTP}. \tag{46}$$

The Home-import Phillips curves are

$$\pi_{F,t} = \beta \mathrm{E}_t \pi_{F,t+1} + \kappa \left(\psi_t + mc_t^{r,*}\right), \qquad \text{PreTP,} \tag{47}$$

$$\widetilde{\pi}_{F,t} = \beta \mathrm{E}_t \widetilde{\pi}_{F,t+1} + \kappa \left(\psi_t + mc_t^{r,*}\right), \qquad \text{PostTP.} \tag{48}$$

Equations (47) and (48) have the same slope and the same real forcing term, where $mc_t^{r,*} \equiv \log(MC_t^{r,*}/MC^{r,*})$. Their difference is the sticky inflation object: PreTP governs tariff-exclusive destination-price inflation $\pi_{F,t}$, whereas PostTP governs cum-tariff destination-price inflation $\widetilde{\pi}_{F,t}$. Hence, if the PostTP equation is rewritten in tariff-exclusive inflation, current and expected future tariff growth enter explicitly, exactly as in the SOE importer-LCP case. Thus the tariffed Home-market price is set by the Foreign exporter, and under PostTP the non-resetting exporter absorbs tariff movements through its net-of-tariff revenue.

4.3.3 Foreign-to-Foreign pricing

The Foreign domestic-market pricing block is the exact starred mirror of the Home-to-Home block in Section 4.3. A Foreign domestic producer sets the Foreign-currency price $P_{F,t}^*$ and faces only Foreign demand for Foreign goods, $C_{F,t}^*$. Its reset-price problem, first-order condition, Calvo price-index law, and Phillips curve are obtained by applying the starred mapping to the corresponding Home domestic-market equations above. Because this block is not directly affected by the unilateral Home import tariff $\tau_{M,t}$, we do not repeat those equations here.

4.4 Dominant-currency pricing

Having isolated the timing mechanism under ILCP and then moved to exporter-set destination prices under full LCP, we now turn to the empirically focal pricing environment. Dominant-currency pricing (DCP) means that international export prices in both directions are quoted and remain sticky in a common currency. We take Home currency to be the dominant currency. The currency-of-pricing architecture follows Bergin and Corsetti [6]: Home firms use producer-currency pricing, so the same sticky Home-currency producer price governs both Home-to-Home and Home-to-Foreign sales, whereas Foreign exporters selling in Home use local-currency pricing in Home currency. Foreign domestic sales are priced in Foreign currency. Thus the Home side is a single PCP block, Foreign-to-Home prices are sticky in Home currency, and Foreign-to-Foreign prices are sticky in Foreign currency. Our PreTP–PostTP distinction is a separate dimension: it concerns whether an import tariff lies outside or inside the sticky destination-price contract.

4.4.1 Home-to-Home and Home-to-Foreign pricing

Following Bergin and Corsetti [6], both Home-to-Home and Home-to-Foreign sales are governed by producer-currency pricing. Hence the Home side of DCP is simply the PCP block already stated in Sections 3.2 and 4.2. There is one Home-currency producer price, $P_{H,t}$, and one associated inflation rate, $\pi_{H,t}$; no separate Home-to-Foreign price or inflation notation is required.

In the Baseline unilateral Home-tariff experiment, Foreign does not levy an import tariff, $\tau_{M,t}^* = 0$, so there is no PreTP–PostTP distinction on the Home side. Relative to full LCP, the only change on the Home side is therefore that the two destination-specific pricing blocks are replaced by this single PCP block.

4.4.2 Foreign-to-Home pricing

The Foreign-to-Home block is directly hit by the Home tariff. Under both full LCP and DCP, the Foreign exporter sets its Home-market destination price in Home currency, so the price setter and the currency of denomination are identical in the two regimes. The reset-price problems are therefore exactly the full-LCP problems,

$$\text{PreTP}: \quad \max_{\bar{P}_{F,t}} \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda^*_{t,t+k} \left[\frac{\bar{P}_{F,t}}{\mathcal{E}_{t+k}} - (1-\tau^*) P^*_{F,t+k} MC^{r,*}_{t+k} \right] C^{\text{PreTP}}_{F,t+k|t},$$

$$\text{PostTP}: \quad \max_{\overline{\overline{P}}_{F,t}} \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda^*_{t,t+k} \left[\frac{\overline{\overline{P}}_{F,t}}{(1+\tau_{M,t+k})\mathcal{E}_{t+k}} - (1-\tau^*) P^*_{F,t+k} MC^{r,*}_{t+k} \right] C^{\text{PostTP}}_{F,t+k|t}.$$

These are exactly Eqs. (41) and (42). Hence the corresponding first-order conditions, compact $\mathcal{K}/\mathcal{Z}$ representations, Calvo laws, Phillips curves, and the PostTP net-receipt interpretation are also exactly those in the full-LCP Foreign-to-Home block in Section 4.3. In the Baseline unilateral Home-tariff experiment, this is the only DCP pricing block in which the PreTP–PostTP distinction is active.

4.4.3 Foreign-to-Foreign pricing

The Foreign-to-Foreign block is the standard Foreign domestic producer block. A Foreign domestic producer sets the Foreign-currency price $P^*_{F,t}$ and faces only Foreign demand for Foreign goods, $C^*_{F,t}$. This block is identical under full LCP and DCP and is the exact starred mirror of the Home-to-Home block. Because it is not directly affected by the unilateral Home import tariff, its reset-price problem, first-order condition, Calvo law, and Phillips curve are not repeated here.

Thus, for the unilateral Home tariff, full LCP and DCP differ on the Home side because full LCP uses separate destination-specific Home-to-Home and Home-to-Foreign prices, whereas DCP follows Bergin and Corsetti [6] and uses one common Home-currency PCP price for both markets. By contrast, the tariffed Foreign-to-Home block is identical across the two regimes, and it is this common block that generates the PreTP–PostTP timing distinction in the Baseline.

4.5 Ramsey policy and welfare interpretation

4.5.1 Ramsey problem and choice set

For each pricing regime $R \in \{PCP, ILCP, FLCP, DCP\}$ and, where relevant, timing convention $\mathcal{P} \in \{\text{PreTP}, \text{PostTP}\}$, we adopt the same national Nash–Ramsey equilibrium concept as Faia and Monacelli [17]: under commitment and in the allocation-game sense, each national authority chooses its own complete contingent allocation while taking the other country's complete contingent allocation as given. Accordingly, the Home national Ramsey planner maximizes the welfare of the Home representative household over its complete contingent choice allocation. In the same form as the SOE problem in Section 3.4, the Home problem is

$$\begin{aligned} &\max_{\{\mathcal{X}^{R,\mathcal{P}}_{H,t}\}_{t\geq 0}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \\ &\text{subject to the regime-specific nonlinear implementability restrictions} \\ &\qquad \text{in Appendix D.2}, \qquad i_t \geq 0. \end{aligned} \tag{49}$$

The Foreign planner solves the exact starred mirror problem, taking the complete contingent Home allocation as given. Its policy rate i^*_t is likewise one element of the Foreign complete contingent choice allocation and is subject to $i^*_t \geq 0$. Appendix D.2 gives the complete nonlinear constraint systems, national Lagrangians, and first-order conditions for both planners; they are not repeated here.

As in Section 3.4, the period-t choice variables can be written directly as sets rather than as a separate table. The Home choice sets are

$$\mathcal{X}^{PCP}_{H,t} \equiv \{C_t, S_t, N_t, Y_t, i_t, \Pi_{H,t}, \widetilde{p}_{H,t}, \mathcal{K}_{H,t}, \mathcal{Z}_{H,t}, Z_{H,t}\},$$

$$\mathcal{X}^{ILCP,\mathcal{P}}_{H,t} \equiv \mathcal{X}^{PCP}_{H,t} \cup \{\Psi_t, \Pi^{\mathcal{P}}_{F,t}, \widetilde{p}^{\mathcal{P}}_{F,t}, \mathcal{K}^{\mathcal{P}}_{F,t}, \mathcal{Z}^{\mathcal{P}}_{F,t}, Z_{F,t}\},$$

$$
\begin{aligned}
\mathcal{X}_{H,t}^{FLCP,\mathcal{P}} \equiv \{ & C_t, S_t, N_t, C_{H,t}, C_{H,t}^*, S_t^{*,\mathcal{P}}, i_t, \Pi_{H,t}, \Pi_{H,t}^{*,\mathcal{P}}, \widetilde{p}_{H,t}, \widetilde{p}_{H,t}^{*,\mathcal{P}}, \\
& \mathcal{K}_{H,t}, \mathcal{Z}_{H,t}, \mathcal{K}_{H,t}^{*,\mathcal{P}}, \mathcal{Z}_{H,t}^{*,\mathcal{P}}, Z_{H,t}, Z_{H,t}^* \} = \mathcal{X}_{H,t}^{DCP,\mathcal{P}}.
\end{aligned}
$$

For computational bookkeeping, the DCP implementation retains the same-dimensional Home and Foreign destination-price variables as full LCP. Under the Baseline DCP PCP restriction, however, any destination-tagged Home-to-Foreign bookkeeping variables are merely computational copies of the common Home producer-price block; they do not represent an independent Home reset-price choice. The Foreign choice sets are the corresponding Home–Foreign mirrors.

The common PreTP–PostTP structure is most transparent in these choice sets. In every timing-sensitive destination-price block, PreTP places the tariff-exclusive sticky-price inflation rate in the relevant planner's choice set, whereas PostTP replaces it with the corresponding tariff-inclusive inflation rate. Under ILCP, the Home import price is set by a Home importer, so the timing-specific object $\Pi_{F,t}^{\mathcal{P}}$ belongs to the Home planner's choice set: it is $\Pi_{F,t}$ under PreTP and $\widetilde{\Pi}_{F,t}$ under PostTP. The associated real pricing margin is the importer LOOP gap Ψ_t (or ψ_t in log-linear form).

Under full LCP, by contrast, the tariffed Home-market price is set by the Foreign exporter. Hence the same timing distinction—tariff-exclusive destination-price inflation under PreTP versus tariff-inclusive destination-price inflation under PostTP—appears in the Foreign planner's choice set rather than the Home planner's. The real forcing term of the corresponding Phillips curve contains Foreign producer marginal cost, together with the LOOP component, as in Eqs. (47)–(48). Thus the agent absorbing the PostTP tariff movement also changes: it is the Home importer under ILCP but the Foreign exporter under full LCP.

Under DCP, the Foreign-to-Home block directly hit by the Home tariff is exactly the full-LCP block, whereas Home-to-Home and Home-to-Foreign sales are governed by the PCP block in Section 4.4.1. In the Baseline unilateral Home-tariff experiment, $\tau_{M,t}^* = 0$, so the Home side has no PreTP–PostTP distinction and there is no separate Home-to-Foreign price-inflation object. The DCP timing difference therefore comes entirely from whether the tariffed Foreign-to-Home block uses tariff-exclusive inflation $\Pi_{F,t}$ or tariff-inclusive inflation $\widetilde{\Pi}_{F,t}$. PCP has no separate sticky destination-price block and therefore no PreTP–PostTP pair.

4.5.2 Welfare cost function

To interpret the nonlinear Nash–Ramsey allocation, we use the second-order Home national-welfare representation. Let $\mathcal{P} \in \{\text{PreTP}, \text{PostTP}\}$. The four period welfare cost functions are

$$
\mathcal{L}_{H,t}^{PCP} = \frac{1}{2}\left[\Lambda_y^{PCP}(\hat{y}_t^w)^2 + \Lambda_s^{PCP}(\hat{s}_t^w)^2 + \Lambda_{\pi,H}^{PCP}\pi_{H,t}^2\right], \tag{50}
$$

$$
\begin{aligned}
\mathcal{L}_{H,t}^{ILCP,\mathcal{P}} = \frac{1}{2}\Big[& \Lambda_y^{ILCP}(\hat{y}_t^w)^2 + \Lambda_\psi^{ILCP}(\hat{\psi}_t^w)^2 + \Lambda_s^{ILCP}(\hat{s}_t^w)^2 + \Lambda_{\psi^*}^{ILCP}(\hat{\psi}_t^{*,w})^2 \\
& + \Lambda_{\pi,H}^{ILCP}\pi_{H,t}^2 + \Lambda_{\pi,F}^{ILCP}(\pi_{F,t}^{\mathcal{P}})^2 + \Lambda_{\pi,H^*}^{ILCP}(\pi_{H,t}^{*,\mathcal{P}})^2\Big],
\end{aligned} \tag{51}
$$

$$
\begin{aligned}
\mathcal{L}_{H,t}^{FLCP,\mathcal{P}} = \frac{1}{2}\Big[& \Lambda_y^{FLCP}(\hat{y}_t^w)^2 + \Lambda_{\psi^*}^{FLCP}(\hat{\psi}_t^{*,w})^2 + \Lambda_s^{FLCP}(\hat{s}_t^w)^2 + \Lambda_\psi^{FLCP}(\hat{\psi}_t^w)^2 \\
& + \Lambda_{\pi,H}^{FLCP}\pi_{H,t}^2 + \Lambda_{\pi,H^*}^{FLCP}(\pi_{H,t}^{*,\mathcal{P}})^2\Big],
\end{aligned} \tag{52}
$$

$$
\begin{aligned}
\mathcal{L}_{H,t}^{DCP,\mathcal{P}} = \frac{1}{2}\Big[& \Lambda_y^{DCP}(\hat{y}_t^w)^2 + \Lambda_{\psi^*}^{DCP}(\hat{\psi}_t^{*,w})^2 + \Lambda_s^{DCP}(\hat{s}_t^w)^2 + \Lambda_\psi^{DCP}(\hat{\psi}_t^w)^2 \\
& + \Lambda_{\pi,H}^{DCP}(\hat{\pi}_{H,t}^{w,DCP,\mathcal{P}})^2 + \Lambda_\zeta^{DCP}(\zeta_t^{\mathcal{P}})^2\Big].
\end{aligned} \tag{53}
$$

The hatted real variables in Eqs. (50)–(53) are deviations of Home output, the ex-tariff relative import price, the Home LOOP gap, and the Foreign-destination LOOP gap from their welfare-relevant centers. These centers are generated by sequential square completion of the exact second-order national-welfare polynomial after the first-order equilibrium restrictions have been substituted.

Their symbolic construction, completion order, and the numerical equal-size Baseline centers are reported in Appendix F, rather than repeated here.

The nominal objects have a direct pricing interpretation, but not every nominal term in Home welfare is a Home policy choice. $\pi_{H,t}$ is Home PPI inflation. Under ILCP, $\pi_{F,t}^{\mathcal{P}}$ is inflation in the sticky Home import price: tariff-exclusive import-price inflation under PreTP and tariff-inclusive purchaser-price inflation under PostTP. This importer-price inflation belongs to the Home contingent allocation and is therefore a direct stabilization margin for the Home planner. By contrast, $\pi_{H,t}^{*,\mathcal{P}}$ is inflation in the sticky Foreign-market import price of Home goods, set by the Foreign importer. It belongs to the Foreign contingent allocation and is taken as given in the Home best response; it is therefore not an independently controlled Home stabilization target.

Nevertheless, $\pi_{H,t}^{*,\mathcal{P}}$ enters Home national welfare because Foreign importer price dispersion affects the resources required to supply Home goods abroad. Under ILCP the Home resource restriction is

$$Y_t = C_{H,t} + \frac{1-\nu}{\nu} C_{H,t}^{*} Z_{H,t}^{*}, \tag{54}$$

where $Z_{H,t}^{*}$ is the dispersion of the Foreign import prices of Home goods; equivalently, this is the first restriction in Eq. (107). A rise in $Z_{H,t}^{*}$ raises the Home output required to deliver a given $C_{H,t}^{*}$ and, through $N_t = Y_t Z_{H,t}$, raises Home labor input and labor disutility. Under Calvo pricing, the second-order variation in $Z_{H,t}^{*}$ is generated by variation in the corresponding sticky-price inflation rate, so after substituting the equilibrium restrictions into the second-order Home welfare expansion this resource cost appears as the positive term $\Lambda_{\pi,H^*}^{ILCP}(\pi_{H,t}^{*,\mathcal{P}})^2$. Hence the $\pi_{H,t}^{*,\mathcal{P}}$ term is a welfare cost transmitted from Foreign pricing into Home production, not a price that the Home planner directly chooses. In the Baseline unilateral Home-tariff experiment, $\tau_{M,t}^{*} = 0$, so this Foreign-importer block itself has no PreTP–PostTP distinction.

DCP requires one additional definition because Home domestic-price inflation and Home-currency Foreign-destination inflation are linked. Define

$$\zeta_t^{\text{PreTP}} \equiv \Delta\psi_t^{*}, \qquad \zeta_t^{\text{PostTP}} \equiv \Delta\psi_t^{*} - \Delta\tau_{M,t}^{*}.$$

The welfare-relevant Home-PPI center is

$$\pi_{H,t}^{w,DCP,\mathcal{P}} \equiv -\frac{(\eta-1)\omega_F\Theta_n}{1-(\eta-1)\omega_F\Theta_n}\zeta_t^{\mathcal{P}}.$$

The hatted DCP PPI term in Eq. (53) is the deviation of Home PPI inflation from this center. In the unilateral Home-tariff experiment, $\tau_{M,t}^{*} = 0$, so $\zeta_t^{\text{PreTP}} = \zeta_t^{\text{PostTP}} = \Delta\psi_t^{*}$. Appendix F gives the square-completion derivation and defines Θ_n and all remaining symbolic coefficients.

Parameterization. We define the *Benchmark Parameterization* as $\beta = 0.99$, $\upsilon = 0.30$, $\varphi = 2.2$, $\eta = 1.5$, $\varepsilon = \varepsilon_M = 3.8$, $\theta = 0.8$, tariff persistence 0.7, and, in the two-country model, equal country size $\nu = 0.50$. Throughout the paper, the term *Benchmark* refers only to this parameter set; for the SOE the same parameterization applies without the country-size parameter. Unless otherwise stated, the quantitative exercises use a 20-percentage-point unilateral Home import-tariff innovation, but the shock size is not part of the Benchmark Parameterization. We use *Baseline* for the specification with markup-correcting subsidies, $\tau = \tau^* = 1/\varepsilon$ and, where importer sectors exist, $\tau_I = \tau_I^* = 1/\varepsilon_M$, so the deterministic monopoly-markup distortions are removed. We use *Distorted* for the no-subsidy specification. Under the equal-size Benchmark Parameterization, $\omega_H = \omega_F^* = 0.85$ and $\omega_F = \omega_H^* = 0.15$, so in the zero-tariff steady state each household assigns 85 percent of its CES expenditure weight to its own country's goods and 15 percent to imports.

At this Benchmark Parameterization, Table 3 reports the coefficients in a format parallel to Table 1.

Table 3 makes the nominal terms in Home welfare transparent. The largest positive coefficient attached to a Home choice is the coefficient on Home PPI inflation, which lies between 54.4 and

Table 3: Equal-size Baseline: Home welfare coefficients

Coefficients		Λ_y	Λ_s	Λ_ψ	Λ_{ψ^*}	$\Lambda_{\pi,H}$	$\Lambda_{\pi,H}^{DCP}$	$\Lambda_{\pi,F}$	Λ_{π,H^*}	Λ_ζ
Target variable		y_t^w	$\hat{s}_t^w$	$\hat{\psi}_t^w$	$\hat{\psi}_t^{w,*}$	$\pi_{H,t}$	$\hat{\pi}_{H,t}$	$\pi_{F,t}$	$\pi_{H,t}^*$	ζ_t
Pricing	PCP	2.3818	-0.0704	—	—	54.3921	—	—	—	—
Pricing	ILCP	2.9831	-0.0962	0.2328	0.2974	68.1246	—	15.4031	10.4508	—
Pricing	FLCP	2.9718	-0.0660	0.0292	-0.3307	73.0769	—	—	-5.2120	—
Pricing	DCP	2.9718	-0.0660	0.0292	-0.3307	—	67.8649	—	—	-5.6123

Notes: The first header row reports coefficient symbols and the second reports the corresponding target variables. Regime superscripts are suppressed where unambiguous. The DCP target $\hat{\pi}_{H,t}$ abbreviates $\hat{\pi}_{H,t}^{w,DCP,\mathcal{P}}$; $\pi_{F,t}$, $\pi_{H,t}^*$, and ζ_t are timing-specific where applicable. Negative entries are signed national-welfare coefficients generated by strategic international relative-price margins; they are not standalone stabilization penalties.

73.1, compared with an output coefficient between 2.38 and 2.98. Under ILCP, the coefficient on the Home import-price inflation $\pi_{F,t}^{\mathcal{P}}$ is 15.40; this is a direct Home stabilization margin. The coefficient on $\pi_{H,t}^{*,\mathcal{P}}$ is 10.45, but this term has a different interpretation: the Foreign importer sets that price and the Home planner takes it as given. Its presence in Home welfare reflects the Home resource and labor cost of Foreign import-price dispersion described in Eq. (54). Thus a positive coefficient in the Home welfare representation need not identify an independently controlled Home stabilization target. PreTP and PostTP have the same coefficient values within a pricing regime; tariff timing changes the relevant sticky-price constraints and feasible paths, rather than the underlying welfare coefficients. Under DCP, the economically dominant nominal term is the deviation of Home PPI inflation from its welfare-relevant center, whose coefficient is 67.86. The remaining $\zeta_t^{\mathcal{P}}$ term is a signed residual generated by square completion of the DCP nominal block and is not a separate inflation-stabilization objective.

Some coefficients in Table 3 are negative because this is a national Nash–Ramsey criterion, not a cooperative world-welfare loss composed only of domestic distortion squares. Each national planner takes the other country's contingent allocation as given and therefore has a strategic incentive to manipulate international relative prices, LOOP gaps, and destination-specific export markups in favor of its own residents. After the equilibrium restrictions are substituted, those terms make the national-welfare quadratic form signed: the negative Λ_s terms and, under full LCP/DCP, the negative Λ_{ψ^*} term reflect strategic international relative-price margins rather than a desire to create instability. The negative full-LCP coefficient on Foreign-destination inflation has the same origin: inflation in the Home export price changes the destination-specific markup and the terms at which Home sells abroad, so its national-welfare contribution is not a pure price-dispersion cost. Under DCP, the common-currency price identity combines this signed export-destination term with the positive Home-PPI dispersion term; square completion therefore leaves a negative coefficient on $(\zeta_t^{\mathcal{P}})^2$. These negative coefficients must be interpreted jointly with the positive inflation/output terms and the equilibrium constraints. They do not imply that the unconstrained planner values price dispersion or arbitrarily large relative-price volatility.

5 Equal-size two-country Baseline

5.1 Cross-regime comparison of the impact policy response

Before examining the regime-specific IRFs, Table 4 isolates the paper's central quantitative comparison: the impact response of the Home policy rate to the same tariff shock under each pricing regime and timing convention. The ordering of the rows is deliberate. ILCP provides the simplest mechanism, full LCP replaces importer pricing with exporter-set destination prices and forms the bridge to DCP, and DCP is the empirically focal environment. The fact that the policy response crosses zero between PreTP and PostTP in all three regimes is itself a cross-pricing-regime robustness result; Section 7 subsequently asks whether that result also survives changes in parameters, distortions, model structure, and the policy game.

Table 4: Equal-size Baseline at the Benchmark Parameterization: Home impact nominal-interest-rate responses

Pricing	PreTP	PostTP
PCP control	5.48	
Importer LCP	-31.92	43.09
Full LCP	49.49	-4.98
DCP	57.59	-4.29

Notes: Basis-point deviations from the deterministic steady-state quarterly nominal rate. PCP has no PreTP/PostTP pair. Its entry is the unconstrained linear Nash–Ramsey solution.[6]

5.2 Importer LCP

Figures 2 and 3 show the first leg of the staged comparison. ILCP keeps the price-setting mechanism transparent: the Home rate falls by 31.92 basis points under PreTP and rises by 43.09 basis points under PostTP. This reproduces the SOE zero crossing after Foreign quantities and monetary policy are made endogenous.

To an increase in the import taiff in country H, the tariff inclusive inflation increases through an increase in the cum-tariff price of imports under the Pre TP (Panel (f), Fig. 2). This increase in the cum-tariff price of imports decreases the consumption of foreign goods (Panel (n), 2). While the consumption of domestic goods increases through expenditure switching effect, the consumption in country H decreases (Panels (k) and (m), Fig. 2). This decrease in the consumption applies pressure to decrease the PPI inflation, as shown in the NKPC Eq.(13). For now, as shown in Tab. 3, stabilizing the PPI inflation is the most important target variable. In addition, stabilizing the ex-tariff import inflation is another importa target to stabilize. Then, the nominal interest rate is lowered (Panel (a), 2). A decrease in the nominal interest rate bolsters the PPI inflation through pressure to increase the marginal cost resulting from pressure to increase the consumption. Consequently, the PPI inflation is stabilized although it slightly increases (Panlel (b), 2). Transmission to stabilize the ex-tariff import inflation has already been refered in Section 3.5. Although the nominal exchange rate appreciates in equilibrium because the Foreign policy rate falls even more sharply, the Home rate cut exerts depreciating pressure and thereby mitigates the decline in the Home-currency border price of imports. Therefore, presusre to decrease the ex-tariff import inflation is mitigated and is stabilized (Panel (e)).

Under the Post TP, an increase in the cum-tariff import inflation is less than that under the Pre TP because the cum-tariff import inflation is more sticky. Therefore, a decrease in the consumption is less than that under the Pre TP because a decrease in the consumption of foreign goods is less under than that under the Pre TP (Panels (k) and (n), 2). Pressure to decrease the PPI inflation is less under the Post TP because pressure to decrease the marginal cost resulting from a decrease in the consumption is less and the social planner focuses on stabilizing another important target, namely, the cum-tariff import inflation. Transmission to stabilize the cum-tariff import inflation has already been refered in Section 3.5. An increase in the nominal interest rate suppresses cum-tariff import inflation through appreciation in the nominal exchange rate (Panel (l)).

To an increase in the import tariff in country H, under the Pre TP, the nominal interest is lowered while it is hiked under the Post TP. The opposite policy signs are therefore tied to the different inflation objects in Eqs. (20) and (21).

The Foreign rate falls by 93.33 basis points under PreTP but rises by 11.31 basis points under PostTP. These spillovers matter quantitatively, but the Home sign reversal remains the same one isolated in the SOE.

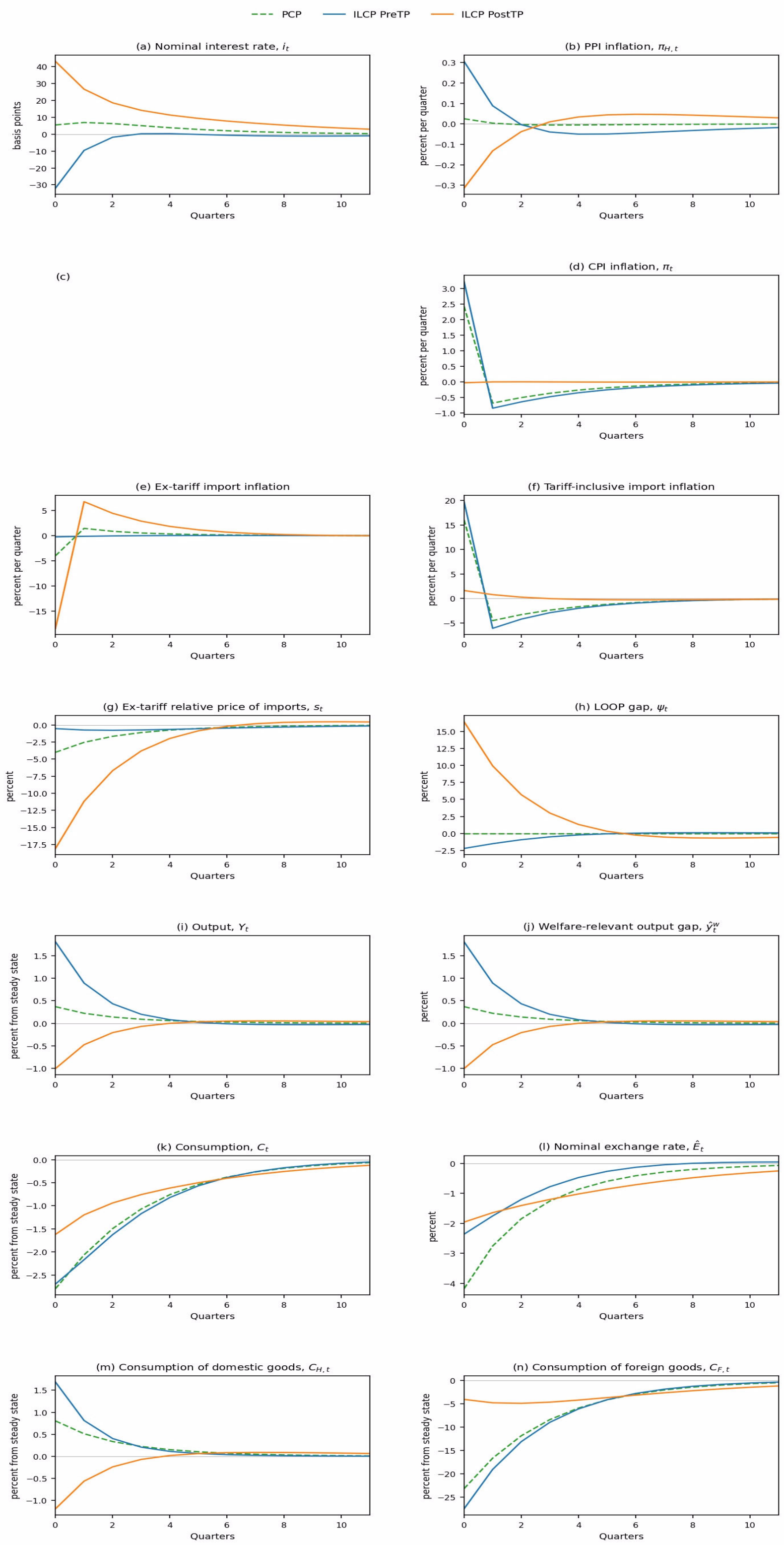

Figure 2: Equal-size Baseline two-country importer LCP: Home country, PreTP versus PostTP, with PCP as a control.

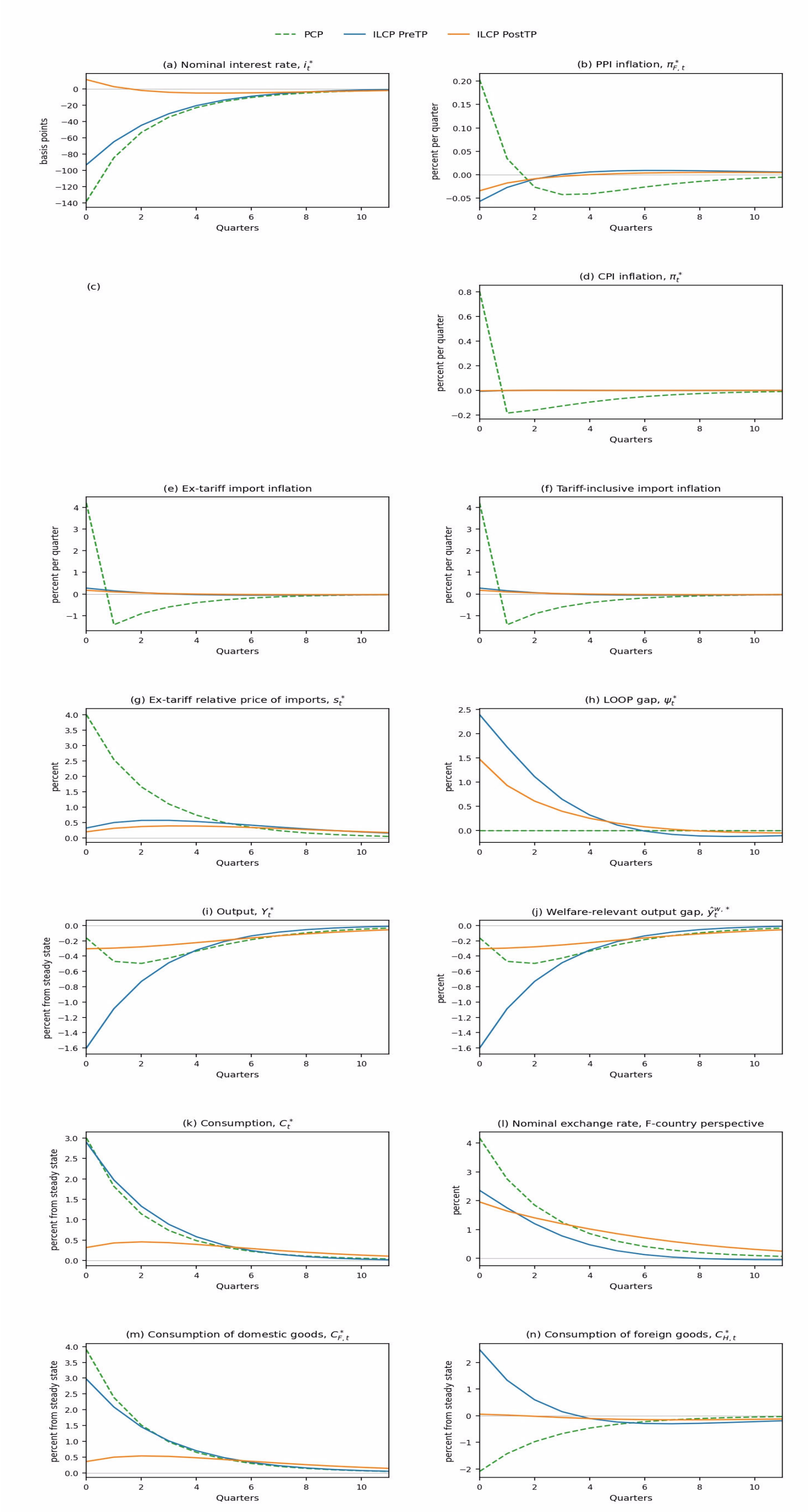

Figure 3: Equal-size Baseline two-country importer LCP: Foreign country, PreTP versus PostTP, with PCP as a control.

5.3 Full LCP

Figures 4 and 5 provide the next step toward DCP. After replacing the Home importer with Foreign exporter destination pricing, the Home policy response reverses direction relative to ILCP: the Home rate rises by 49.49 basis points under PreTP but falls by 4.98 basis points under PostTP. The Foreign rate rises by 36.69 and 9.90 basis points, respectively.

Under PreTP, the tariff-inclusive import price jumps because the Home tariff is imposed on top of the sticky tariff-exclusive destination price set by the Foreign exporter (Panel (f), Fig. 4). Consumption of Foreign goods therefore falls sharply, while consumption of Home goods rises only slightly through expenditure switching, so aggregate Home consumption falls substantially (Panels (k), (m), and (n), Fig. 4). The fall in consumption puts downward pressure on Home real marginal cost and hence on Home PPI inflation through Eq. (39); PPI inflation is indeed slightly negative on impact (Panel (b)). At the same time, the tariff-exclusive Home import price is set by the Foreign exporter, whose PreTP revenue is $P_{F,t}/\mathcal{E}_t$ in Foreign currency, as shown in Eq. (41). The Home rate rises by more than the Foreign rate, exerting appreciation pressure on the Home currency (Panel (l)). A lower $\mathcal{E}_t$ lowers the Home-currency destination price consistent with a given Foreign-currency revenue margin, helping the ex-tariff import price and the ex-tariff relative price of imports fall (Panels (e) and (g)). This partially offsets the tariff-induced increase in the purchaser-facing import price and the associated contraction in import demand. The resulting small PPI deflation is therefore part of the Ramsey tradeoff rather than the object that mechanically determines the sign of the policy response. Indeed, because Home PPI inflation is negative and carries the large positive welfare weight $\Lambda_{\pi,H}^{FLCP} = 73.08$, the PPI term by itself favors a smaller rate increase. The PreTP tightening therefore reflects the remaining signed relative-price and LOOP terms together with the Foreign exporter's price-setting constraint, rather than a desire to reduce Home PPI inflation.

Under PostTP, the sticky object is instead the tariff-inclusive Home-currency destination price. The tariff enters the Foreign exporter's net-revenue margin in Eq. (42), so the exporter absorbs most of the tariff through a reduction in its ex-tariff price. Accordingly, ex-tariff import inflation and the ex-tariff relative price fall sharply on impact (Panels (e) and (g), Fig. 4), while tariff-inclusive import inflation rises only modestly (Panel (f)). Consumption of Foreign goods and aggregate Home consumption therefore decline much less than under PreTP (Panels (k) and (n)), and the downward pressure on Home marginal cost and PPI inflation is correspondingly weaker (Panel (b)). Because the Foreign exporter's PostTP price-setting block itself produces most of the required decline in the ex-tariff import price, the strong appreciation associated with PreTP tightening is no longer required. The Home rate instead falls by 4.98 basis points. Although the nominal exchange rate still appreciates slightly in equilibrium, the Home rate cut relative to the Foreign rate exerts depreciating pressure and makes the appreciation much smaller than under PreTP (Panel (l)); this supports Home consumption and PPI inflation.

Thus, under full LCP, the zero crossing does not arise because the Home planner directly stabilizes the Foreign exporter's price inflation. Rather, tariff timing changes the Foreign exporter's sticky price and net-revenue constraint, and thereby changes the exchange-rate adjustment needed to limit the purchaser-price and demand effects of the tariff. Under PreTP this calls for Home tightening and stronger appreciation; under PostTP the exporter absorbs the tariff in its net-of-tariff margin, so the remaining Home welfare tradeoff implies mild easing.

5.4 Dominant-currency pricing

Figures 6 and 7 present the focal DCP case. The Home side is now the single PCP block described in Section 4.4.1, while the tariffed Foreign-to-Home block is exactly the same exporter-pricing block as under full LCP. The Home rate rises by 57.59 basis points under PreTP but falls by 4.29 basis points under PostTP; the Foreign rate rises by 39.41 and 9.50 basis points, respectively.

Under PreTP, the tariff-inclusive import price again jumps because the tariff is added on top of the sticky tariff-exclusive Foreign-to-Home price (Panel (f), Fig. 6). Consumption of Foreign goods falls sharply, the increase in consumption of Home goods is small, and aggregate Home consumption

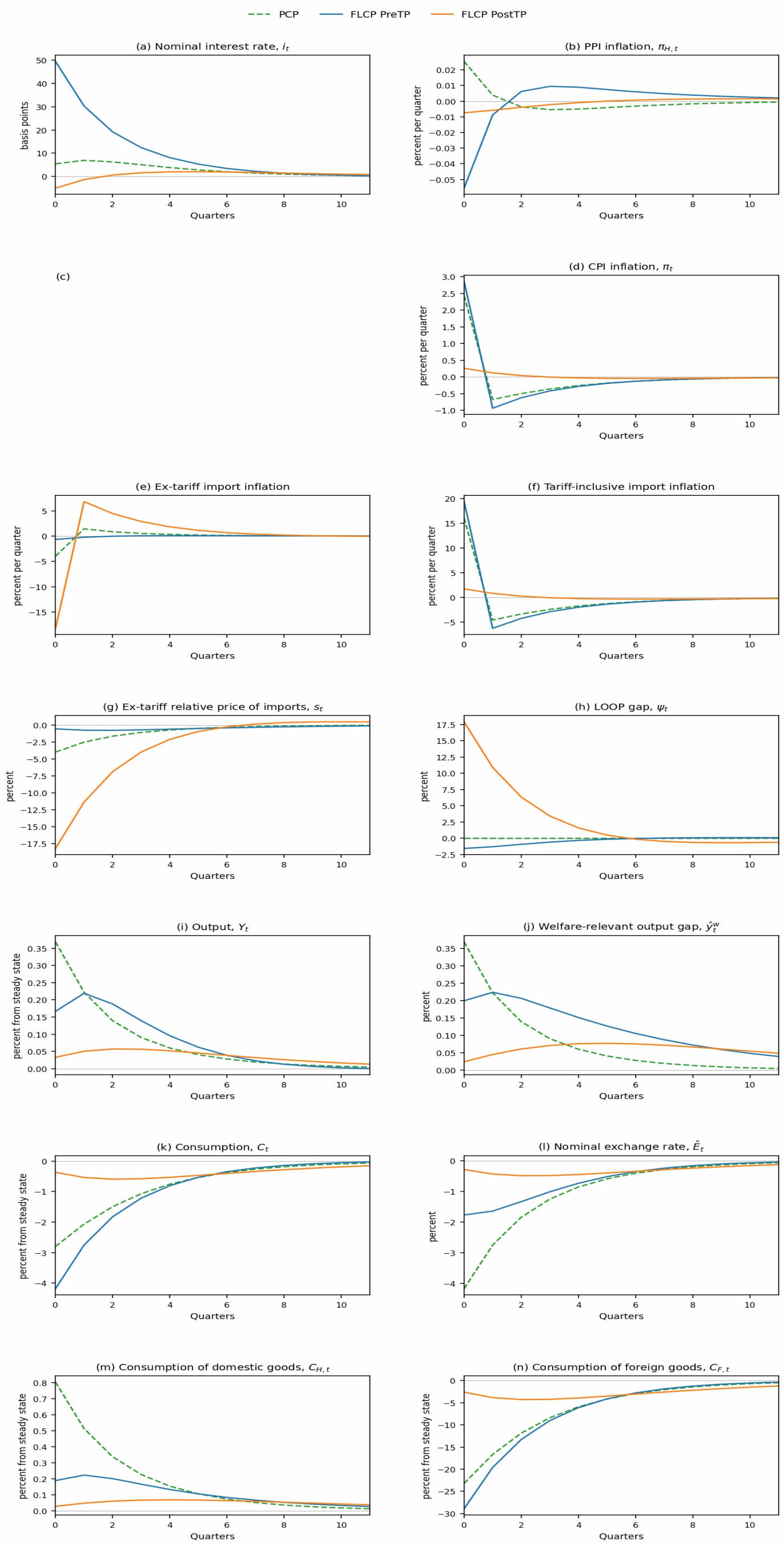

Figure 4: Equal-size Baseline two-country full LCP: Home country, PreTP versus PostTP, with PCP as a control.

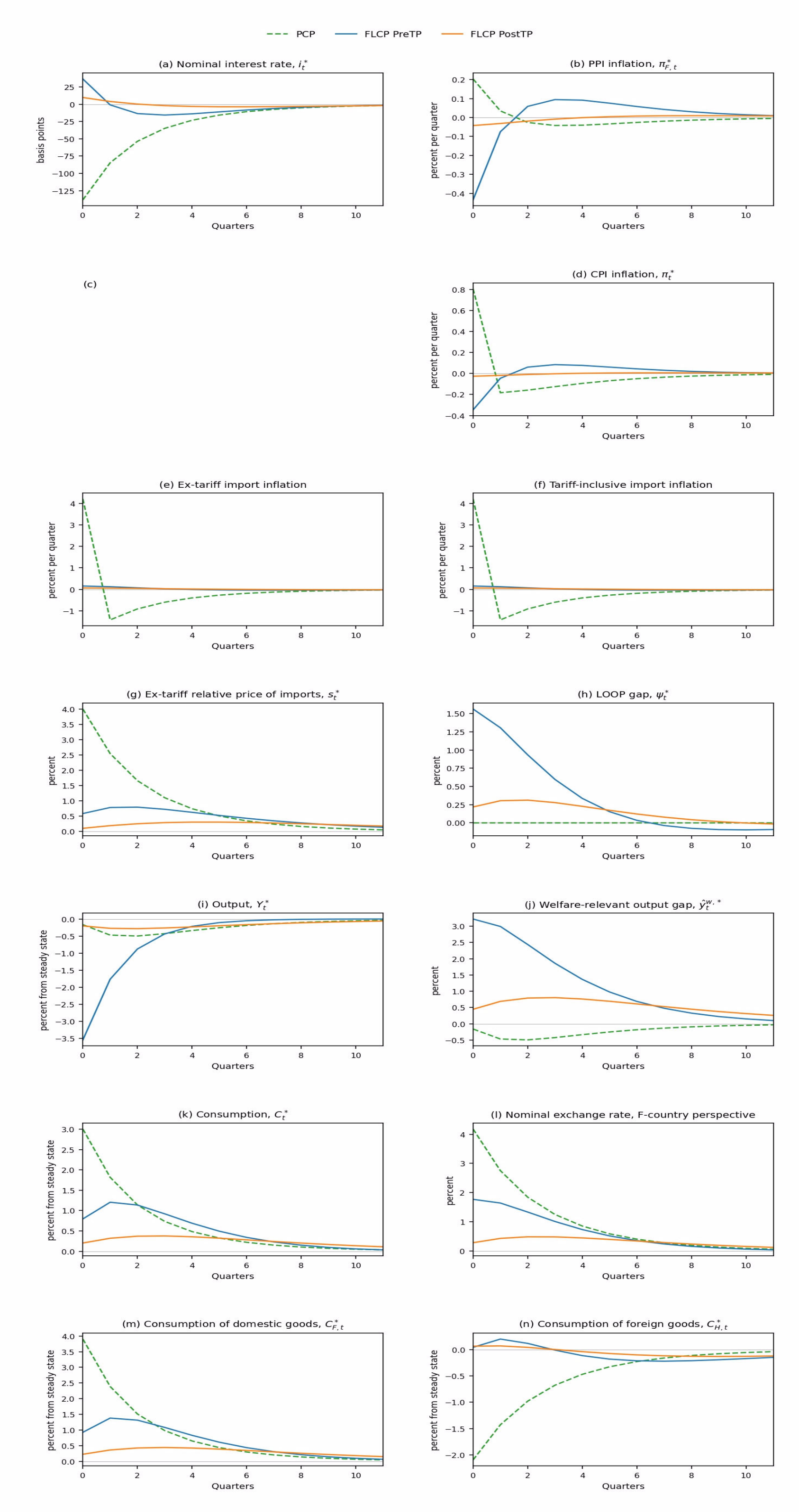

Figure 5: Equal-size Baseline two-country full LCP: Foreign country, PreTP versus PostTP, with PCP as a control.

declines substantially (Panels (k), (m), and (n), Fig. 6). This fall in consumption puts downward pressure on Home marginal cost and on the common Home PPI inflation governed by the PCP Phillips curve in Eq. (13). The welfare-relevant PPI gap therefore falls on impact (Panel (c)). Since this gap enters the Home loss with the large positive weight $\Lambda^{DCP}_{\pi,H} = 67.86$, the PPI term by itself favors a smaller rate increase. Nevertheless, the Home rate is raised because the signed real relative-price and LOOP margins, together with the Foreign-to-Home pricing constraint, determine the Ramsey tradeoff jointly. Because the tariffed Foreign-to-Home block is identical to full LCP, the same exchange-rate mechanism operates: the Home rate rises by more than the Foreign rate, exerting appreciation pressure (Panel (l)), and the appreciation helps lower the Home-currency ex-tariff destination price set by the Foreign exporter. Ex-tariff import inflation and the ex-tariff relative price consequently fall (Panels (e) and (g)), partially offsetting the large tariff-induced increase in the purchaser-facing import price.

Under PostTP, the tariff-inclusive destination price is the sticky object and the Foreign exporter absorbs the tariff through its net-of-tariff revenue margin. The ex-tariff import price therefore falls sharply, whereas tariff-inclusive import inflation rises only modestly (Panels (e)–(g), Fig. 6). Consumption of Foreign goods and aggregate Home consumption decline much less than under PreTP (Panels (k) and (n)), so the downward pressure on Home marginal cost and PPI inflation is also much smaller. The welfare-relevant PPI gap remains close to zero (Panel (c)). Since the PostTP exporter-pricing block itself absorbs most of the tariff through the ex-tariff price, strong appreciation through Home monetary tightening is no longer needed. The Home rate instead falls by 4.29 basis points. Although the nominal exchange rate still appreciates slightly in equilibrium, the Home rate cut relative to the Foreign rate exerts depreciating pressure and substantially limits the appreciation relative to PreTP (Panel (l)), supporting Home consumption and the PPI margin.

The DCP comparison is especially sharp because the Home welfare criterion in Eq. (53) has the same coefficients under PreTP and PostTP, and the Home PCP block is also common to the two cases. The completed Home PPI square carries weight 67.86 under both timings. Hence the change from strong tightening to mild easing is not generated by a change in welfare weights or by a different Home pricing block. It comes from the only timing-sensitive element in the Baseline DCP economy: whether the tariff lies outside the sticky Foreign-to-Home price under PreTP or inside that sticky purchaser-facing price under PostTP. This is the tariff-location mechanism derived in Section 4.4.

6 U.S.-like country size: $\nu = 0.27$ Baseline

This section changes country size only. We set $\nu = 0.27$, close to the U.S. share of world current-dollar GDP, while retaining the Baseline subsidy configuration and every other structural parameter apart from country size. Hence $\tau = \tau^* = 1/\varepsilon$ and, under ILCP, $\tau_I = \tau_I^* = 1/\varepsilon_M$. There is no reason to combine the country-size exercise with an arbitrary hybrid distortion in which producer subsidies are removed but importer subsidies are retained; doing so would confound the country-size comparison. The fully no-subsidy experiment is therefore left to Section 7.1, where it is defined consistently for every relevant sector.

The openness parameter remains $\upsilon = 0.30$, following Monacelli [25]. Together with $\nu = 0.27$, it implies a Home import expenditure weight $\omega_F = \upsilon(1-\nu) = 0.219$. This 21.9 percent share is an implication of country size and openness, not an additional calibration target; it is close to the 22.3 percent average ratio of U.S. imports of goods and services to personal consumption expenditures over 2000–2025 [27]. Thus the exercise asks whether the sign results survive a U.S.-like country size while keeping the deterministic steady state free of monopoly-markup distortions.

6.1 Welfare cost functions and numerical weights

The Home welfare cost functions have exactly the same form as Eqs. (50)–(53). The welfare-relevant centers, the hatted variables, and $\zeta_t^{\mathcal{P}}$ are defined as in Section 4.5.2; only the coefficients are re-evaluated at $\nu = 0.27$. Table 5 reports these coefficients in the same format as Table 3.

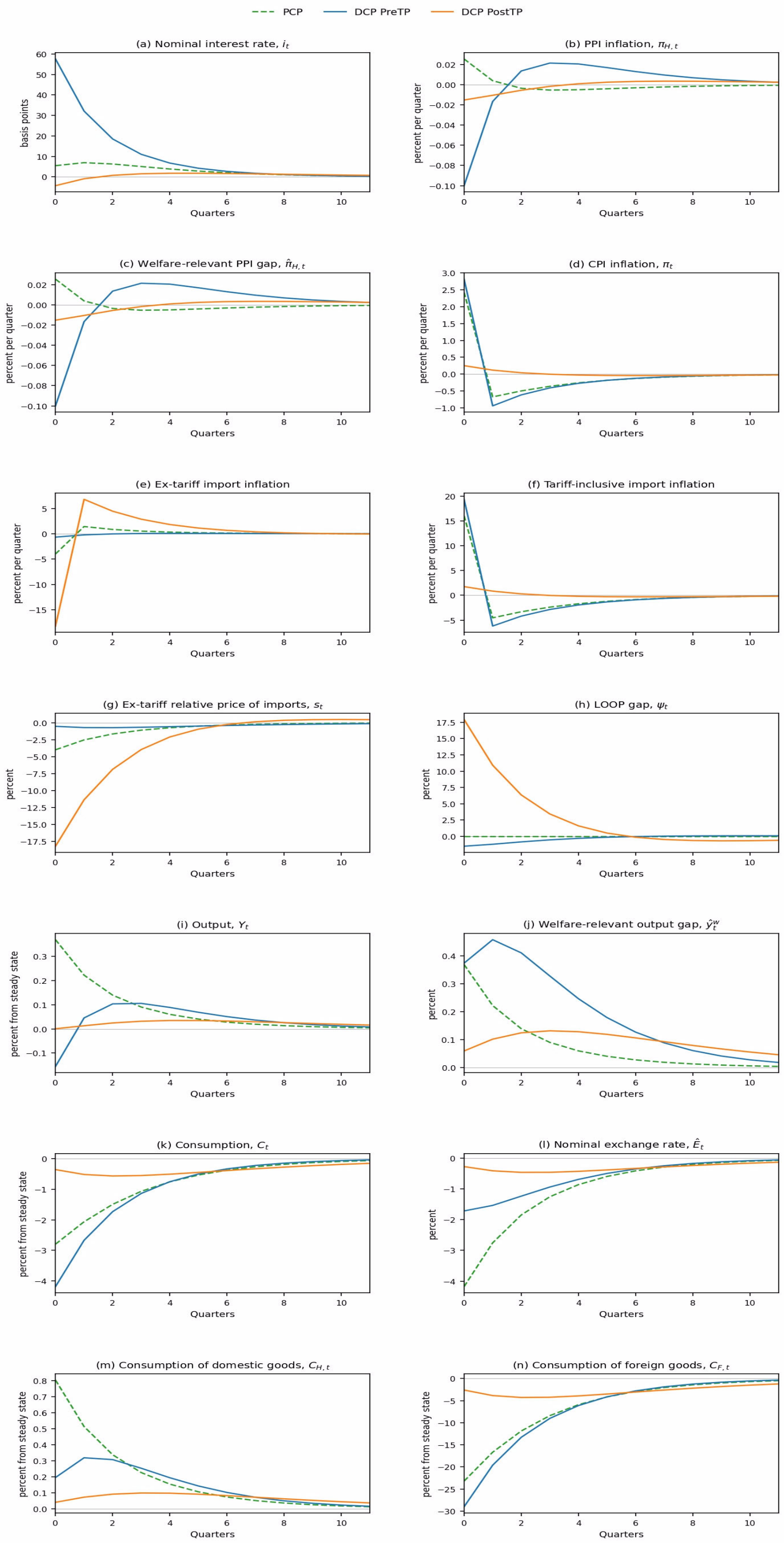

Figure 6: Equal-size Baseline two-country dominant-currency pricing: Home country, PreTP versus PostTP, with PCP as a control.

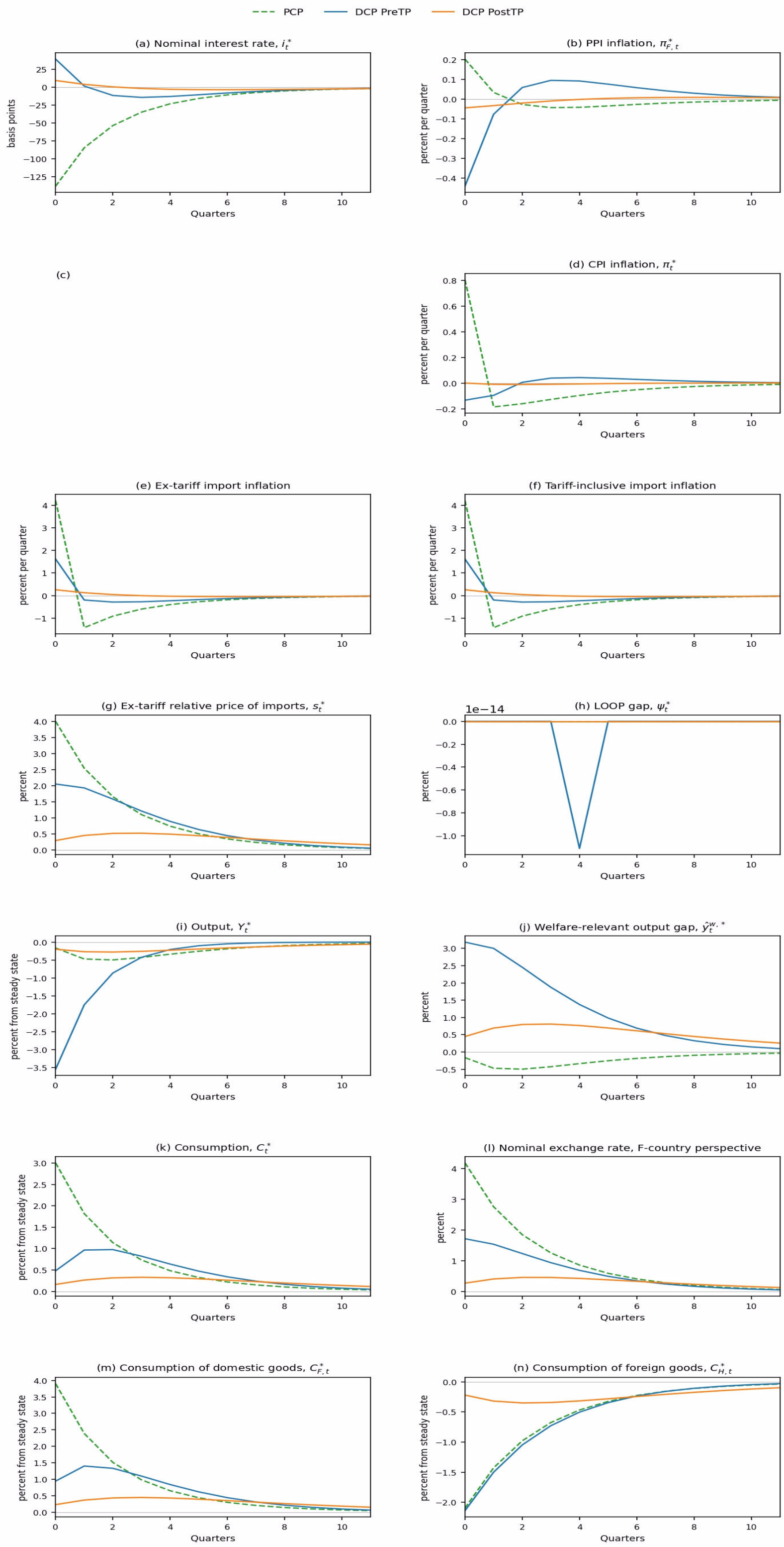

Figure 7: Equal-size Baseline two-country dominant-currency pricing: Foreign country, PreTP versus PostTP, with PCP as a control.

Table 5: $\nu = 0.27$ Baseline: Home welfare coefficients

Coefficients		Λ_y	Λ_s	Λ_ψ	Λ_{ψ^*}	$\Lambda_{\pi,H}$	$\Lambda_{\pi,H}^{DCP}$	$\Lambda_{\pi,F}$	Λ_{π,H^*}	Λ_ζ
Target variable		y_t^w	$\hat{s}_t^w$	$\hat{\psi}_t^w$	$\hat{\psi}_t^{w,*}$	$\pi_{H,t}$	$\hat{\pi}_{H,t}$	$\pi_{F,t}$	$\pi_{H,t}^*$	ζ_t
Pricing	PCP	2.0487	-0.0899	—	—	46.7859	—	—	—	—
Pricing	ILCP	2.8987	-0.1207	0.3331	0.4141	66.1960	—	21.8488	14.9678	—
Pricing	FLCP	2.8741	-0.0788	0.0507	-0.6420	73.0769	—	—	-7.4417	—
Pricing	DCP	2.8741	-0.0788	0.0507	-0.6420	—	65.6352	—	—	-8.2854

Notes: The first header row reports coefficient symbols and the second reports the corresponding target variables. Regime superscripts are suppressed where unambiguous. The DCP target $\hat{\pi}_{H,t}$ abbreviates $\hat{\pi}_{H,t}^{w,DCP,\mathcal{P}}$; $\pi_{F,t}$, $\pi_{H,t}^*$, and ζ_t are timing-specific where applicable. Negative entries are signed national-welfare coefficients generated by strategic international relative-price margins; they are not standalone stabilization penalties.

Table 5 has the same qualitative structure as Table 3. The largest positive coefficient is again attached to Home PPI inflation. Under ILCP, the additional positive nominal coefficients are attached to the sticky Home import-price inflation and to Foreign-market import-price inflation of Home goods, while full LCP and DCP retain the signed international relative-price terms described in Section 4.5.2. Changing country size therefore alters the numerical weights without changing the basic structure of the Home national-welfare criterion.

6.2 Home nominal-interest-rate responses

Table 6: $\nu = 0.27$ Baseline: Home impact nominal-interest-rate responses

Pricing	PreTP	PostTP
PCP control	10.46	
Importer LCP	-62.36	71.57
Full LCP	72.20	-7.31
DCP	90.59	-6.34

Notes: Basis-point deviations from the deterministic steady-state quarterly nominal rate. The Baseline subsidy configuration is retained; relative to Section 5, the substantive change is country size, $\nu = 0.27$.

The country-size change strengthens the impact responses but does not change their qualitative ordering. ILCP still moves from easing under PreTP to tightening under PostTP, whereas full LCP and DCP still move from tightening under PreTP to easing under PostTP. In particular, the focal DCP–PostTP case remains accommodative at the U.S.-like country size. Comparing Tables 4 and 6 therefore isolates country size cleanly: the PreTP–PostTP sign reversals survive without introducing any steady-state markup distortion. The common welfare message from Table 5 is that policy is primarily concerned with PPI inflation and, where relevant, sticky import/destination-price inflation; timing changes the feasible path of those nominal targets rather than replacing the objective itself.

7 Further robustness

The pricing-regime comparison in Sections 3–5 already provides an important robustness check: the optimal policy rate crosses zero between PreTP and PostTP under ILCP, full LCP, and DCP even though the identity of the price setter and the currency architecture change across those cases. ILCP and full LCP are therefore not merely preliminary models used to reach DCP; their results show that the tariff-timing sign reversal is robust across distinct sticky-price architectures. This section asks whether that cross-regime result survives further changes in steady-state distortions, tariff size, substitution elasticity, country size and openness, model structure, and the international policy game.

7.1 Distortions

We first ask whether the PreTP–PostTP sign reversals depend on the markup-correcting subsidies. Consistent with the terminology defined in Section 4.5.2, *Baseline* denotes $\tau = \tau^* = 1/\varepsilon$ and, wherever an importer sector is present, $\tau_I = \tau_I^* = 1/\varepsilon_M$, whereas *Distorted* denotes $\tau = \tau^* = \tau_I = \tau_I^* = 0$. For PCP, full LCP, and DCP there is no separate importer sector, so the latter change removes the producer markup correction; for ILCP it removes both the producer and importer markup corrections. Table 7 reports the Home impact nominal-interest-rate response first in the SOE and then in the equal-size two-country economy.

Table 7: Robustness to steady-state distortions: Home impact nominal-interest-rate responses

Panel A: Small open economy

Specification	ILCP PreTP	ILCP PostTP	PCP control
Baseline	-75.95	57.68	18.56
Distorted	-60.88	47.91	20.04

Panel B: Equal-size two-country economy, $\nu = 0.50$

Specification	ILCP		Full LCP		DCP		PCP
	PreTP	PostTP	PreTP	PostTP	PreTP	PostTP	
Baseline	-31.92	43.09	49.49	-4.98	57.59	-4.29	5.48
Distorted	-25.85	33.65	49.37	-4.98	57.07	-4.27	6.25

Notes: Entries are basis-point deviations of the Home nominal interest rate from steady state. Panel A isolates the effect of removing the producer and importer markup corrections without a Foreign monetary-policy best response. Panel B repeats the exercise in the equal-size two-country economy. PCP has no PreTP/PostTP distinction, and full LCP/DCP have no separate importer sector.

The timing-induced ILCP zero crossing therefore does not require either a Foreign monetary-policy best response or the markup-correcting subsidies. In the SOE, removing all markup corrections changes the ILCP impact response from -75.95 to -60.88 basis points under PreTP and from $+57.68$ to $+47.91$ under PostTP. In the equal-size two-country economy, the corresponding responses remain negative under PreTP and positive under PostTP, while full LCP and DCP continue to move from tightening under PreTP to easing under PostTP. Thus the steady-state monopoly distortions affect magnitudes but not the sign pattern.

We also vary trade openness in the equal-size Baseline economy. For $\upsilon = 0.15$, 0.30, and 0.45, the same qualitative ordering is preserved: ILCP implies easing under PreTP and tightening under PostTP, whereas full LCP and DCP imply tightening under PreTP and easing under PostTP. Appendix A reports the detailed responses.

7.2 Tariff Size

We next vary the size of the unilateral Home import-tariff innovation in both the small open economy and the equal-size two-country Baseline. In each economy the Benchmark Parameterization and the Baseline subsidy configuration are otherwise unchanged. Table 8 reports the Home impact nominal-interest-rate response for tariff increases of 20, 15, 10, and 5 percentage points.

The timing result is unchanged even when the tariff innovation is reduced to 5 percentage points. In the SOE, ILCP continues to imply easing under PreTP and tightening under PostTP. In the equal-size two-country economy, ILCP has the same ordering, whereas full LCP and DCP continue to imply tightening under PreTP and easing under PostTP. The response magnitudes decline with the tariff size, but the PreTP–PostTP sign reversals persist at every tariff size reported in Table 8. Hence the central timing result is not specific to the 20-percentage-point reference shock.

7.3 Elasticity of Substitution

We next vary the elasticity of substitution between Home and Foreign goods, η, while holding all other parameters and the Baseline subsidy configuration. Monacelli [25] uses $\eta = 0.5$, 1.5, and 3

Table 8: Tariff-size robustness: Home impact nominal-interest-rate responses

Panel A: Small open economy

Tariff increase	ILCP PreTP	ILCP PostTP	PCP control
20%	-75.95	57.68	18.56
15%	-56.96	43.26	13.92
10%	-37.97	28.84	9.28
5%	-18.99	14.42	4.64

Panel B: Equal-size two-country economy, $\nu = 0.50$

Tariff increase	ILCP		Full LCP		DCP		PCP
	PreTP	PostTP	PreTP	PostTP	PreTP	PostTP	
20%	-31.92	43.09	49.49	-4.98	57.59	-4.29	5.48
15%	-23.94	32.32	37.11	-3.73	43.19	-3.22	4.11
10%	-15.96	21.55	24.74	-2.49	28.80	-2.15	2.74
5%	-7.98	10.77	12.37	-1.24	14.40	-1.07	1.37

Notes: Entries are basis-point deviations of the Home nominal interest rate from steady state. Panel A uses the SOE Baseline of Section 3.5; Panel B uses the equal-size Baseline of Section 5. PCP has no PreTP/PostTP distinction. In Panel B the PCP column follows the same control convention as Table 4 and reports the unconstrained linear Nash–Ramsey response. The 20-percentage-point reference shock does not activate the ZLB in any of the ILCP, full-LCP, or DCP cases shown here, so the smaller tariff innovations remain in the same linear regime.

as low-, central-, and high-substitution cases, respectively. We retain those values and additionally report $\eta = 2$, the high-substitution calibration used by Galí and Monacelli [19]. For the two-country model we keep equal country sizes, $\nu = 0.50$. The tariff shock is the 20-percentage-point Home import-tariff innovation. The LCP and DCP entries are the ZLB-constrained piecewise-linear solutions. As in Section 5, the PCP two-country control reports the unconstrained linear Nash–Ramsey solution, since PCP has no PreTP–PostTP pair. An entry of -101.01 basis points means that the Home nominal rate reaches the ZLB on impact.

Table 9: Elasticity-of-substitution robustness: Home impact nominal-interest-rate responses

Panel A: Small open economy

η	ILCP PreTP	ILCP PostTP	PCP control
0.5	-101.01	53.53	-61.78
1.5 (Benchmark Parameterization)	-75.95	57.68	18.56
2.0	-92.96	52.66	16.93
3.0	-101.01	43.42	-27.49

Panel B: Equal-size two-country economy, $\nu = 0.50$

η	ILCP		Full LCP		DCP		PCP
	PreTP	PostTP	PreTP	PostTP	PreTP	PostTP	
0.5	-55.19	21.50	-47.96	5.01	-46.32	4.75	-30.24
1.5 (Benchmark Parameterization)	-31.92	43.09	49.49	-4.98	57.59	-4.29	5.48
2.0	-48.59	55.59	103.33	-9.24	143.97	-8.44	-5.60
3.0	-101.01	84.03	222.99	-17.09	1785.25	12.15	-64.55

Notes: Entries are basis-point deviations of the Home nominal interest rate from steady state. Panel A reports the SOE; Panel B reports the equal-size two-country economy. An entry of -101.01 means that the Home nominal rate reaches the ZLB on impact.

Panel A of Table 9 shows that the SOE sign reversal is robust throughout these calibrations. In particular, at $\eta = 2$ the ILCP impact response moves from -92.96 basis points under PreTP to $+52.66$ under PostTP. At both $\eta = 0.5$ and $\eta = 3$, the PreTP response reaches the ZLB on impact while the PostTP response remains positive. The PCP control is much more sensitive to η and changes sign across calibrations, but there is no timing comparison under PCP.

Panel B shows that the two-country sign reversal is especially clear at $\eta = 2$: the Home impact rate moves from -48.59 basis points under PreTP to $+55.59$ under PostTP in ILCP, from $+103.33$ to -9.24 in full LCP, and from $+143.97$ to -8.44 in DCP. Thus all three local-currency/destination-pricing regimes cross zero at $\eta = 2$.

The more extreme Monacelli high-substitution case, $\eta = 3$, provides an informative qualification rather than overturning the timing mechanism. ILCP still reverses from -101.01 to $+84.03$ basis points and full LCP from $+222.99$ to -17.09. DCP is the only case in which the response no longer crosses zero at $\eta = 3$: it moves from an exceptionally large tightening of $+1785.25$ basis points under PreTP to a much smaller tightening of $+12.15$ under PostTP. Hence PostTP still shifts the optimal stance sharply in the accommodative direction—by about 1773.10 basis points on impact—even though the shift stops just short of crossing zero. Moreover, this failure of the zero crossing occurs only at the very high end of the substitution range: at $\eta = 2.9$, DCP still reverses sign, from $+996.92$ basis points under PreTP to -4.78 under PostTP. The DCP evidence therefore points to a strong and persistent tariff-timing effect whose exact crossing of zero becomes sensitive only when Home and Foreign goods are made extremely substitutable. Importer LCP remains the strongest robustness case, with a PreTP–PostTP zero crossing in both the SOE and the two-country economy at every reported value of η.

7.4 US-size Country Robustness

We now collect the robustness results for the U.S.-size calibration, $\nu = 0.27$, rather than mixing them with the equal-size exercises. Table 10 reports three checks: Panel A revisits steady-state distortions, Panel B varies tariff size, and Panel C varies the elasticity of substitution η. Removing all markup-correcting subsidies changes the ILCP impact response from -62.36 to -47.21 basis points under PreTP and from $+71.57$ to $+55.14$ under PostTP. Full LCP and DCP are almost unchanged and continue to move from tightening under PreTP to easing under PostTP. Thus the timing-induced sign reversals remain intact at the U.S.-size calibration with or without the steady-state markup corrections.

Table 10: US-size country robustness: Home impact nominal-interest-rate responses

Panel A: Distortions, $\nu = 0.27$

Specification	ILCP		Full LCP		DCP		PCP
	PreTP	PostTP	PreTP	PostTP	PreTP	PostTP	
Baseline	-62.36	71.57	72.20	-7.31	90.59	-6.34	10.46
Distorted	-47.21	55.14	72.12	-7.32	89.15	-6.30	11.52

Panel B: Tariff size, $\nu = 0.27$

Tariff increase	ILCP		Full LCP		DCP		PCP
	PreTP	PostTP	PreTP	PostTP	PreTP	PostTP	
20%	-62.36	71.57	72.20	-7.31	90.59	-6.34	10.46
15%	-46.77	53.68	54.15	-5.49	67.95	-4.75	7.84
10%	-31.18	35.79	36.10	-3.66	45.30	-3.17	5.23
5%	-15.59	17.89	18.05	-1.83	22.65	-1.58	2.62

Panel C: Elasticity of substitution, $\nu = 0.27$

η	ILCP		Full LCP		DCP		PCP
	PreTP	PostTP	PreTP	PostTP	PreTP	PostTP	
0.5	-85.81	36.05	-70.28	7.60	-66.38	6.93	-45.07
1.5	-62.36	71.57	72.20	-7.31	90.59	-6.34	10.46
2.0	-91.34	91.12	148.24	-13.91	285.96	-11.95	0.99
3.0	-101.01	133.92	310.40	-26.18	*n.d.*	*n.d.*	-59.19

Notes: Entries are basis-point deviations of the Home nominal interest rate from steady state. Panel A compares the Baseline and Distorted specifications. Panel B holds the $\nu = 0.27$ Baseline fixed and varies the unilateral Home tariff innovation. Panel C holds the $\nu = 0.27$ Baseline and the 20-percentage-point tariff innovation fixed and varies η. An entry of -101.01 means that the Home nominal rate reaches the ZLB on impact. In Panel C, "n.d." denotes that the DCP linearized equilibrium does not satisfy the determinacy condition at $\eta = 3$, so no impact response is reported. At $\eta = 2.4$, however, the DCP solution is determinate and the Home impact response reverses from $+1023.42$ basis points under PreTP to -7.12 basis points under PostTP. Thus the missing DCP entry at $\eta = 3$ should be read as an extreme-calibration determinacy issue, not as evidence that the zero crossing has already disappeared. PCP has no PreTP/PostTP distinction and follows the control convention used in the main two-country tables.

Panel B shows that the U.S.-size result is not driven by the 20-percentage-point reference tariff. The ILCP response remains negative under PreTP and positive under PostTP as the tariff innovation is reduced from 20 to 5 percentage points. Full LCP and DCP retain the opposite ordering at every reported tariff size. Hence the PreTP–PostTP zero crossings survive even for a 5-percentage-point tariff innovation at $\nu = 0.27$.

Panel C repeats the elasticity exercise in Table 9 at the U.S.-size calibration. The ILCP zero crossing is particularly robust: the Home rate is below steady state under PreTP and above steady state under PostTP at every reported value of η, with the PreTP rate reaching the ZLB on impact at $\eta = 3$. Full LCP and DCP also display timing-induced zero crossings at $\eta = 0.5$, 1.5, and 2. At $\eta = 0.5$, their direction is reversed relative to the $\eta = 1.5$ case—full LCP moves from -70.28 basis points under PreTP to $+7.60$ under PostTP, and DCP from -66.38 to $+6.93$. At $\eta = 2$, the $\eta = 1.5$ ordering is restored: full LCP moves from $+148.24$ to -13.91 basis points and DCP from $+285.96$ to -11.95. The DCP zero crossing continues beyond $\eta = 2$: at $\eta = 2.4$, for which the linearized equilibrium remains determinate, the Home impact response moves from $+1023.42$ basis points under PreTP to -7.12 basis points under PostTP. At $\eta = 3$, full LCP continues to reverse, from $+310.40$ to -26.18 basis points, whereas the DCP linearized equilibrium no longer satisfies the determinacy condition at $\nu = 0.27$, so Panel C does not report a DCP impact response for that calibration. The $\eta = 2.4$ result therefore makes clear that the missing DCP entry at $\eta = 3$ is a very-high-substitutability determinacy issue rather than evidence that the PreTP–PostTP zero crossing has already disappeared. Thus the U.S.-size elasticity exercise reinforces the tariff-timing mechanism while also showing that the extreme DCP edge case becomes less well behaved when country sizes are asymmetric.

Trade-openness robustness gives the same message. Holding $\nu = 0.27$ fixed and varying υ over 0.15, 0.30, and 0.45 preserves the timing ordering in ILCP, full LCP, and DCP. At the highest openness value, $\upsilon = 0.45$, the ILCP PreTP response is sufficiently accommodative to reach the ZLB on impact, while ILCP PostTP continues to imply tightening; full LCP and DCP continue to imply tightening under PreTP and easing under PostTP. Appendix A reports the detailed responses.

7.5 Bianchi and Coulibaly (2025) Model

Bianchi and Coulibaly [8] provide a useful external small-open-economy robustness check because the force behind their optimal policy is deliberately different from the mechanism emphasized in Section 3. Their quantitative model is solved as a nonlinear Ramsey problem with Rotemberg rigidity in Home producer prices. Tariff revenue is rebated to households, so private agents perceive imports at the tariff-inclusive price while the tariff payment is a domestic transfer from the social point of view. The resulting fiscal externality gives the Ramsey authority a motive to support employment and imports relative to the look-through allocation. We add an importer local-currency-pricing block with Rotemberg adjustment costs, set the importer elasticity and adjustment-cost parameter equal to their Home-producer counterparts, and use a markup-correcting importer subsidy so that the added sector does not introduce a new steady-state monopoly distortion. Because the Bianchi–Coulibaly framework is a small open economy, ILCP is the only destination-pricing regime among our LCP cases that has a direct counterpart here; full LCP and DCP require an explicit two-country structure.

For the main cross-model comparison, we use Bianchi and Coulibaly's own endogenous-terms-of-trade extension to relax the one-for-one exchange-rate/PPI mapping. In their reference specification the border terms of trade are fixed and the normalization implies $P_{H,t} = \mathcal{E}_t$. Once the border terms of trade are endogenous, the law of one price instead allows $\mathcal{E}_t/P_{H,t}$ to move with the international relative price. We then add the same Rotemberg ILCP block while retaining the tariff-revenue fiscal externality that is central to the Bianchi–Coulibaly mechanism. In this broader extension, the qualitative ILCP result is the same as in Section 3: the optimal impact nominal rate falls under PreTP and rises under PostTP. This is a robustness comparison, not a pure identification exercise for the restriction $P_{H,t} = \mathcal{E}_t$: endogenizing the terms of trade also makes export demand finite-elasticity and introduces the associated terms-of-trade margin. The result therefore shows that the PreTP–PostTP zero crossing also arises in a Rotemberg SOE in which the tariff-revenue fiscal externality is a central force behind the Ramsey allocation, while the exact direction of the crossing remains model dependent.

The welfare interpretation is correspondingly close to the one used in Section 3. Up to the additional wealth/net-foreign-asset component generated by incomplete international asset markets, the second-order criterion can be organized in terms of a welfare-relevant output gap $\hat{y}_t^w$, a welfare-relevant import LOOP gap $\hat{\psi}_t^w$, Home PPI inflation $\pi_{H,t}$, and the inflation rate of the sticky local import price. PreTP and PostTP have the same quadratic stabilization margins; what changes is the sticky inflation object and the constraint linking that object to the LOOP gap. With an independent exchange-rate margin, PreTP allows depreciation to move the border price relative to the tariff-exclusive sticky local import price, while PostTP places the tariff inside the importer's sticky purchaser-price margin. The resulting Ramsey tradeoff therefore has the same qualitative orientation as in Section 3: easing under PreTP and tightening under PostTP.

The exact Bianchi–Coulibaly reference normalization is informative precisely because it removes that independent margin. With exogenous border terms of trade and their law-of-one-price normalization, $P_{H,t} = \mathcal{E}_t$, so exchange-rate adjustment is simultaneously Home-producer-price adjustment. Table 11 reports the nonlinear Ramsey results for this case.

Table 11: Bianchi–Coulibaly ILCP extension under $P_{H,t} = \mathcal{E}_t$: impact nominal-rate responses

Tariff process	PCP	ILCP PreTP	ILCP PostTP
		basis points	
Permanent +15 pp	5.02	9.45	-6.95
+15 pp, $\rho = 0.976$	1.60	4.45	-4.14
+20 pp, $\rho = 0.70$	0.23	0.49	-0.21

Notes: PCP is a control and has no separate sticky importer-price block. The ILCP columns add a Rotemberg-sticky Home importer sector with $\varepsilon_M = \varepsilon$ and $\phi_M = \phi_H$, together with a markup-correcting importer subsidy. The Ramsey allocation is computed from the nonlinear equilibrium restrictions. Appendix B reports the exact-normalization equations and welfare interpretation.

Under the permanent 15-percentage-point tariff increase used in the Bianchi–Coulibaly quantitative experiment, the ILCP impact response is +9.45 basis points under PreTP and −6.95 basis points under PostTP. Table 11 also shows that this opposite-orientation zero crossing is not specific to a permanent tariff: with a 15-percentage-point shock and persistence 0.976, the response changes from +4.45 to −4.14 basis points, while with a 20-percentage-point shock and persistence 0.70 it changes from +0.49 to −0.21 basis points. Thus the exact-normalization exercise provides a robust sign reversal across the reported tariff processes, even though its direction is opposite to the SOE Baseline result. In the broader endogenous-terms-of-trade extension, the one-for-one mapping is relaxed and the qualitative ILCP ordering returns to that of Section 3. Because that extension also introduces a finite-elasticity export-demand and terms-of-trade margin, we do not attribute the change in direction to the relaxation of $P_{H,t} = \mathcal{E}_t$ alone. The robust conclusion is the existence of the timing-induced zero crossing, not a universal claim that a particular timing convention must always imply easing or tightening. Which side of zero each convention occupies depends on the full nominal and international-relative-price implementation structure.

7.6 Policy-Game Robustness: Cooperative Ramsey

Before turning to the Bergin–Corsetti economy, we isolate the policy-game margin within our own model. This exercise is motivated directly by Bergin and Corsetti [6], who solve their quantitative model under cooperative Ramsey policy. Comparing their cooperative allocation directly with our equal-size Baseline under national Nash–Ramsey policy would therefore change both the economic environment and the policy game at the same time. We first ask whether the PreTP–PostTP sign reversals in our Baseline specification survive when the two monetary authorities cooperate.

We re-solve the equal-size Baseline at the Benchmark Parameterization with a single population-weighted world planner. For each pricing regime R and timing convention $\mathcal{P}$, the cooperative social planner solves

$$\max_{\{\mathcal{X}_t^{R,\mathcal{P}}, i_t, i_t^*\}_{t\geq 0}} \quad \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \nu \left[\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right] + (1-\nu) \left[\log C_t^* - \frac{(N_t^*)^{1+\varphi}}{1+\varphi} \right] \right\} \tag{55}$$
$$\text{subject to} \quad \text{the world private-equilibrium system for } (R, \mathcal{P}), \qquad i_t \geq 0, \quad i_t^* \geq 0.$$

where $\mathcal{X}_t^{R,\mathcal{P}}$ collects the Home and Foreign allocation and sticky-price variables for the relevant regime. The private-equilibrium equations, the 20-percentage-point unilateral Home tariff innovation, the pricing convention, and the two national ZLB constraints are unchanged; only the two national objectives are replaced by the single population-weighted world objective in Eq. (55). Appendix C gives implementation details and numerical checks. Table 12 compares the resulting impact responses with the national Nash–Ramsey Baseline.

Table 12: Policy-game robustness: impact nominal-interest-rate responses in the equal-size Baseline

Pricing regime	Timing	Nash Home (bp)	Cooperative Home (bp)	Cooperative Foreign (bp)
ILCP	PreTP	-31.92	-3.57	-87.38
ILCP	PostTP	43.09	-10.48	16.60
Full LCP	PreTP	49.49	-2.25	-93.51
Full LCP	PostTP	-4.98	-12.26	7.41
DCP	PreTP	57.59	-6.70	-101.01[a]
DCP	PostTP	-4.29	-8.91	6.34

Notes: Entries are basis-point deviations from steady state after a 20-percentage-point unilateral Home tariff innovation. The Nash Home column is the national Nash–Ramsey solution for the equal-size Baseline at the Benchmark Parameterization reported in the main text. The cooperative columns maximize population-weighted world welfare and jointly choose both nominal rates while retaining the same private-equilibrium constraints and pricing convention. [a]The Foreign nominal rate reaches its zero lower bound on impact; with $\beta = 0.99$, the deviation from the steady-state nominal rate to zero is -101.01 basis points.

Cooperation removes all three timing-induced zero crossings. Under ILCP, the Nash response changes from -31.92 basis points under PreTP to $+43.09$ under PostTP, whereas the cooperative Home rate is negative under both conventions, moving from -3.57 to -10.48 basis points. Under full LCP, the Nash reversal from $+49.49$ to -4.98 becomes cooperative easing of -2.25 and -12.26 basis points. Under DCP, the Nash reversal from $+57.59$ to -4.29 becomes cooperative easing of -6.70 and -8.91 basis points. Thus the sign reversal is not invariant to the international policy game.

The reason is that cooperation removes the national strategic component of the stabilization problem. Under Nash–Ramsey policy, each authority evaluates the effects of its own monetary plan on resident welfare while taking the other country's contingent allocation as given. Cross-border movements in relative prices and foreign demand therefore remain strategic national-welfare margins. Under cooperation, those spillovers are internalized jointly: a relative-price or expenditure-switching movement that raises one country's welfare partly at the expense of the other no longer provides an independent reason for one central bank to manipulate the international allocation. What remains in the Baseline LCP/DCP economies is primarily the joint stabilization of the tariff-induced contraction in activity together with price-dispersion and sticky-relative-price distortions. Those residual motives call for Home accommodation under both timing conventions.

PostTP does not become irrelevant under cooperation. In fact, the cooperative Home rate generally moves further toward accommodation: from -3.57 to -10.48 basis points under ILCP, from -2.25 to -12.26 under full LCP, and from -6.70 to -8.91 under DCP. The timing logic is the same as in the Nash experiments. When the tariff-inclusive purchaser-facing price is sticky, the tariff is initially absorbed to a greater extent in the price setter's acquisition-cost or net-revenue margin rather than appearing immediately in the purchaser price. The short-run inflation cost of supporting demand is therefore smaller, so the cooperative planner can accommodate the contraction more aggressively. Cooperation eliminates the strategic force that makes the Nash response cross zero, but it does not eliminate the underlying PreTP–PostTP pricing mechanism.

This result is essential for interpreting the external-model exercise that follows. Because cooperation alone removes the zero crossings in our Baseline economy, the absence of a reversal in Bergin and Corsetti's cooperative simulations cannot tell us whether their richer private economy destroys the sign mechanism. To answer that question, the policy game must be held fixed. Section 7.7 therefore asks whether the sign reversal survives when the private economy is replaced by Bergin and Corsetti's substantially richer structure but the national Nash–Ramsey policy game is restored.

7.7 Bergin and Corsetti (2023) Model

The preceding cooperative experiment motivates a controlled external model robustness test. Bergin and Corsetti [6] use cooperative Ramsey policy, whereas our two-country analysis uses national Nash–Ramsey policy under commitment. Section 7.6 shows that cooperation alone can eliminate the zero crossings, so a direct comparison would confound policy-game effects with model-structure effects. We therefore use their official replication files in two stages. First, we preserve their calibration, private-equilibrium structure, tariff process, and cooperative Ramsey objective and introduce only the PreTP–PostTP distinction. Second, holding the Bergin–Corsetti private economy fixed, we replace cooperation with the national Nash–Ramsey equilibrium concept under commitment of Faia and Monacelli [17], which is also used in our main two-country analysis. This second stage asks whether the sign reversal survives a large change in the private economy once the policy game is held constant.

Bergin and Corsetti's full model incorporates international production chains with imported intermediate inputs, iceberg trade costs, firm entry dynamics, incomplete asset markets, and Rotemberg price adjustment. The mapping to our terminology is direct. Their local-currency-pricing specification has exporters in both countries set sticky export prices in the currency of the buyer. Because the tariff is then imposed on the importer on top of the exporter-set sticky price, their LCP case corresponds to our full-LCP PreTP convention. For DCP, we use the unilateral-tariff experiment corresponding, after relabeling Home and Foreign, to column (2) of their Fig. 7: Home is the dominant-currency issuer and also the tariff-imposing country; Home exporters use PCP, while Foreign exporters set their Home-market export price in Home currency. The original Bergin–Corsetti tariff again lies outside the sticky exporter-set price and is therefore PreTP in our terminology.

To construct the PostTP counterfactual, we change only the location of the tariff relative to the sticky Foreign-to-Home export price. The tariff-inclusive purchaser-facing export price becomes the sticky Rotemberg object, while the Foreign exporter receives that price net of the Home tariff. Thus the tariff enters the exporter's net-revenue margin before the sticky destination price can adjust. All other private-equilibrium equations, parameters, and shocks are held fixed.

Table 13 first reports the cooperative results. The original PreTP simulations reproduce Bergin and Corsetti's monetary-policy pattern: the tariff-imposing country tightens by 57.57 basis points under full LCP and by 51.60 basis points under DCP. Moving the tariff inside the sticky destination price reduces the cooperative tightening to 28.97 and 16.77 basis points, respectively. As in the cooperative version of our Baseline, PostTP moves policy in the accommodative direction but does not generate a zero crossing. The level differs—the Bergin–Corsetti cooperative planner still tightens—because its richer private economy contains additional production, trade, entry, and asset-market margins absent from our parsimonious model. The comparison does not identify any one of these features in isolation.

The final row of Table 13 delivers the central external-model robustness result. We solve the same Bergin–Corsetti private economy under the Faia–Monacelli national Nash–Ramsey equilibrium concept used in our main two-country analysis. Each national authority maximizes resident welfare over its own contingent allocation while taking the other national allocation as given, and the two Ramsey plans jointly satisfy the common international equilibrium mappings. Under full LCP, the tariff-imposing country's impact response changes from a 576.61-basis-point increase under PreTP to a 20.57-basis-point cut under PostTP. Under DCP, it changes from a 222.67-basis-point increase to a 9.00-basis-point cut. Thus the PreTP–PostTP sign reversal survives under both pricing regimes even after replacing our parsimonious model with the substantially richer Bergin–Corsetti economy. In particular, an environment featuring international production chains, imported intermediate inputs, iceberg trade costs, firm entry dynamics, incomplete asset markets, and Rotemberg price adjustment still produces the same qualitative zero crossing when the policy game is held fixed.[7] The Nash magnitudes, especially under full-LCP PreTP, are much larger than under cooperation, so we interpret this exercise primarily as a qualitative external robustness check of the sign mechanism rather than as a calibrated prediction of the size of strategic monetary responses.

Table 13: Bergin–Corsetti (2023) model: tariff-imposing-country impact policy-rate responses

	Full LCP		DCP	
Policy game	PreTP	PostTP	PreTP	PostTP
Cooperative Ramsey	57.57	28.97	51.60	16.77
National Nash–Ramsey	576.61	-20.57	222.67	-9.00

Notes: Entries are basis-point deviations of the tariff-imposing country's nominal interest rate from steady state after Bergin and Corsetti's six-percentage-point tariff innovation. In the DCP columns the tariff-imposing country is also the dominant-currency issuer. The full-LCP PreTP case corresponds to Bergin and Corsetti's symmetric LCP specification, while the DCP PreTP case corresponds to their unilateral-tariff DCP experiment after relabeling Home and Foreign. PostTP changes only the location of the tariff relative to the sticky export price. The cooperative row retains Bergin and Corsetti's world Ramsey objective. The national Nash–Ramsey row holds the same private economy fixed and uses the policy-game concept of the main two-country analysis.

Table 13 makes the sign result explicit. Under cooperative Ramsey, both PreTP and PostTP remain on the tightening side of zero. Under national Nash–Ramsey, by contrast, the tariff-imposing-country response reverses sign in both pricing regimes: full LCP changes from $+576.61$ basis points under PreTP to -20.57 under PostTP, and DCP changes from $+222.67$ to -9.00. Thus the table reports a genuine PreTP–PostTP zero crossing, not merely a change in the magnitude of tightening.

The underlying price responses clarify why tariff timing remains important before the policy game is changed. In the cooperative simulations, under PreTP the Home purchaser-facing import price jumps by about 4.9 percent on impact and Home CPI inflation rises by about one percentage point per quarter. Under PostTP the tariff-inclusive import price itself is sticky: its impact response is only about 0.3 percent, while the corresponding net border price falls by about 5.7 percent. The tariff is therefore absorbed mainly in the exporter's net-revenue margin rather than in an immediate purchaser-price jump. This alteration of the private price-setting constraint is common to the cooperative and Nash exercises; the policy game determines how the two national monetary authorities exploit the resulting relative-price and demand margins.

The comparison also sharpens the coordination result in Table 12. In our parsimonious model, replacing national Nash–Ramsey policy with a population-weighted cooperative planner eliminates all three timing-induced zero crossings. The Bergin–Corsetti model displays the same policy-game dependence from the other side of the comparison: its cooperative solution keeps the tariff-imposing country's rate above steady state under both timings, whereas the same private economy under national Nash–Ramsey policy generates sign reversals under both full LCP and DCP. The key implication is therefore stronger than the statement that PostTP merely changes the quantitative stance in a richer model. The sign reversal itself is robust to substantial model enrichment conditional on the policy game. Conversely, the cooperative results show that matching the private

[7] We validate the external-model implementation in two steps. Using the same nonlinear private-equilibrium block and steady-state normalization as in the Nash exercise, an independently generated cooperative Ramsey system reproduces the PreTP impact responses of 57.5700 basis points under full LCP and 51.6030 basis points under DCP. In the four Nash–Ramsey simulations, Dynare's generalized-eigenvalue count gives 77 unstable roots for 77 forward-looking variables in every case.

economy without matching the policy game can conceal that robustness. The absence of a zero crossing in the original cooperative Bergin–Corsetti environment should therefore be read as evidence of policy-game dependence, not as evidence that its richer trade and production structure eliminates the tariff-timing mechanism. More broadly, the exercise shows that adding a rich trade block does not displace the basic New Keynesian pricing logic. International production chains, imported intermediates, trade costs, and entry reshape propagation and magnitudes, but the sign of the optimal rate response remains tightly linked to which margin the tariff enters in the sticky-price setter's first-order condition. What matters most for the sign is therefore not the richness of the trade structure per se, but the location of the tariff in the nominal pricing problem.

8 Conclusion

The empirical motivation for the central two-country case has two components: Gopinath et al. [21] establish the importance of dominant-currency pricing, while Cavallo et al. [10] show tariff-incidence patterns that motivate PostTP. Our focal DCP–PostTP specification combines these two empirical dimensions; we do not interpret the latter evidence as a structural estimate of PostTP. The analysis reaches that focal case in stages. ILCP first isolates LCP and the PreTP–PostTP mechanism in a small open economy; full LCP then replaces the importer with exporter-set destination prices and shares the tariffed Foreign-to-Home block with DCP; DCP finally adds the dominant-currency asymmetry. This sequence is not merely pedagogical. Because the optimal Home rate crosses zero between PreTP and PostTP under ILCP, full LCP, and DCP, the first two regimes also provide cross-pricing-regime robustness for the central timing result. The timing-induced zero crossings additionally survive tariff innovations as small as 5 percentage points, changes in country size and trade openness, and the removal of steady-state markup-correcting subsidies. Across the elasticity exercises, ILCP and full LCP retain the zero crossing throughout the reported range. DCP retains it through $\eta = 2.9$ in the equal-size economy and through the determinate $\eta = 2.4$ U.S.-size case; at still higher substitutability the exact zero crossing becomes sensitive, and the U.S.-size DCP linearization is indeterminate at $\eta = 3$.

The welfare decompositions show why the result is not a mechanical reaction to CPI inflation. Across pricing regimes, the principal nominal objectives remain stabilization of Home PPI inflation and, under LCP/DCP, the relevant sticky import or destination-price inflation and LOOP/relative-price distortions. Tariff timing changes the private price law that constrains those targets. International coordination is a separate margin: cooperation can remove the zero crossing even though the underlying PreTP–PostTP constraint remains different. The policy implication is therefore narrow but robust: optimal monetary policy cannot be inferred from the tariff rate or invoicing currency alone; it also depends on whether the tariff is outside or inside the nominal price contract that is actually sticky.

A Robustness to trade openness

We vary the foreign-goods weight υ while retaining the markup-correcting subsidies. Table 14 reports the Home impact nominal-interest-rate response for the equal-size calibration and for the U.S.-size calibration. Since the Home import expenditure weight is $\omega_F = \upsilon(1 - \nu)$, changing υ directly changes the degree of Home trade openness at either country size.

In both panels, lower trade openness attenuates the quantitative policy response and higher openness amplifies it. More importantly, the qualitative timing result is unchanged throughout the reported range. ILCP implies easing under PreTP and tightening under PostTP, whereas full LCP and DCP imply tightening under PreTP and easing under PostTP. The only lower-bound episode in this table is ILCP PreTP at $\nu = 0.27$ and $\upsilon = 0.45$, which strengthens rather than weakens the distinction between the two timing conventions.

Table 14: Trade-openness robustness: Home impact nominal-interest-rate responses

Panel A: Equal-size two-country economy, $\nu = 0.50$						
υ	ILCP		Full LCP		DCP	
	PreTP	PostTP	PreTP	PostTP	PreTP	PostTP
0.15	-13.18	17.81	24.53	-2.61	26.19	-2.42
0.30	-31.92	43.09	49.49	-4.98	57.59	-4.29
0.45	-60.24	76.12	74.69	-7.21	95.46	-5.72

Panel B: US-size two-country economy, $\nu = 0.27$						
υ	ILCP		Full LCP		DCP	
	PreTP	PostTP	PreTP	PostTP	PreTP	PostTP
0.15	-22.97	28.05	35.72	-3.83	39.53	-3.56
0.30	-62.36	71.57	72.20	-7.31	90.59	-6.34
0.45	-101.01	131.16	108.99	-10.50	157.82	-8.47

Notes: Entries are basis-point deviations of the Home nominal interest rate from steady state. All rows use the Baseline subsidy configuration. The $\upsilon = 0.30$ rows are the main-text calibrations. ILCP, full-LCP, and DCP entries use the ZLB-constrained piecewise solution. In Panel B, the ILCP PreTP response reaches the ZLB on impact at $\upsilon = 0.45$; the corresponding unconstrained linear response is -122.80 basis points.

B Bianchi–Coulibaly ILCP extension under $P_{H,t} = \mathcal{E}_t$

This appendix records the exact-normalization case discussed in Section 7.5. Bianchi and Coulibaly [8] assume pre-tariff law of one price, constant foreign-currency prices, and exogenous border terms of trade $p = P_{F,t}/P_{H,t}$, normalized to one in their reference specification. With the foreign-currency price of the Home good normalized to one, these assumptions imply

$$P_{H,t} = \mathcal{E}_t. \tag{56}$$

Their uncovered-interest-parity condition is

$$R_t = R^* \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}, \tag{57}$$

so Eq. (56) implies

$$\frac{R_t}{R^*} = \frac{P_{H,t+1}}{P_{H,t}} = 1 + \pi_{H,t+1}. \tag{58}$$

Hence the current policy rate is directly tied to next-period Home PPI inflation in this particular SOE closure. This restriction is stronger than the implementation structure in Section 3, where the nominal exchange rate can move relative to the Home producer price.

We extend the Bianchi–Coulibaly private economy by adding a continuum of Home importers that set local-currency prices subject to Rotemberg adjustment costs. In the log-linear and second-order expressions below, let τ_t denote the log gross tariff wedge and define the tariff-exclusive import LOOP ratio

$$\Psi_t \equiv \frac{\mathcal{E}_t P^*_{F,t}}{P_{F,t}}, \qquad \psi_t \equiv \log \Psi_t. \tag{59}$$

Under PreTP the sticky object is the tariff-exclusive local import price $P_{F,t}$; under PostTP it is the tariff-inclusive purchaser price $\widetilde{P}_{F,t} = (1+\tau_t^{\text{level}})P_{F,t} = e^{\tau_t} P_{F,t}$ in the log-gross-wedge notation used below. The first-order importer Phillips curves can therefore be written in the same sticky-object coordinates as in Section 3,

$$\pi_{F,t} = \beta \mathrm{E}_t \pi_{F,t+1} + \kappa_M \psi_t, \qquad \text{PreTP}, \tag{60}$$

$$\widetilde{\pi}_{F,t} = \beta \mathrm{E}_t \widetilde{\pi}_{F,t+1} + \kappa_M \psi_t, \qquad \text{PostTP}, \tag{61}$$

where $\kappa_M = \varepsilon_M/\phi_M$ under the Rotemberg normalization used here. The tariff is outside the importer's sticky-price margin under PreTP and inside it under PostTP.

For welfare interpretation of this exact-normalization case, let s_H and s_F denote the zero-tariff steady-state expenditure shares on Home and foreign goods, with $s_H + s_F = 1$. Expanding household welfare and the consolidated resource constraint to second order around the zero-tariff deterministic steady state of this extension gives the following reduced-form quadratic organization of the timing-relevant margins:

$$\mathcal{L}_t^{BC,\mathcal{P}} = \frac{1}{2}\left[\Lambda_y^{BC}(\hat{y}_t^{w,\mathcal{P}})^2 + \Lambda_\psi^{BC}(\hat{\psi}_t^{w,\mathcal{P}})^2 + \phi_H \pi_{H,t}^2 + s_F \phi_M (\pi_{F,t}^{\mathcal{P}})^2\right] + \mathcal{L}_{b,t}, \tag{62}$$

for $\mathcal{P} \in \{\text{PreTP}, \text{PostTP}\}$, up to policy-independent terms and terms of order higher than two. Here the hatted objects are defined from the real quadratic block itself, not by an unspecified target. Let $q_t \equiv (y_t, \psi_t)'$ and write that block before completion as $\frac{1}{2}q_t' H_q^{BC,\mathcal{P}} q_t + q_t' g_\tau^{BC,\mathcal{P}} \tau_t$. Then

$$\begin{pmatrix} y_t^{w,BC,\mathcal{P}} \\ \psi_t^{w,BC,\mathcal{P}} \end{pmatrix} \equiv -(H_q^{BC,\mathcal{P}})^{-1} g_\tau^{BC,\mathcal{P}} \tau_t, \qquad \hat{y}_t^{w,\mathcal{P}} \equiv y_t - y_t^{w,BC,\mathcal{P}}, \quad \hat{\psi}_t^{w,\mathcal{P}} \equiv \psi_t - \psi_t^{w,BC,\mathcal{P}}.$$

This is exactly the center generated by completing the quadratic form: substituting the vector above rewrites the real block as $\frac{1}{2}(q_t - q_t^w)' H_q^{BC,\mathcal{P}} (q_t - q_t^w)$ plus terms independent of q_t. Here $\pi_{F,t}^{\text{PreTP}} = \pi_{F,t}$ and $\pi_{F,t}^{\text{PostTP}} = \widetilde{\pi}_{F,t}$. The term $\mathcal{L}_{b,t}$ collects the time-invariant marginal-utility/net-foreign-asset component associated with incomplete international asset markets. Equation (62) is used only to organize the welfare margins of the exogenous-terms-of-trade, $P_{H,t} = \mathcal{E}_t$ case; it is not asserted to be the complete reduced form of the endogenous-terms-of-trade extension, which introduces an additional international-relative-price margin. The timing comparison within the exact-normalization case relies only on the fact that the positive quadratic weights on Home PPI inflation and sticky import-price inflation are common across PreTP and PostTP; tariff timing changes the implementability constraints and the inflation object, not the planner's underlying preference for one timing convention.

Under Eq. (56), the LOOP accumulation equations become especially restrictive. With $p = 1$,

$$\psi_t - \psi_{t-1} = \pi_{H,t} - \pi_{F,t}, \qquad \text{PreTP}, \tag{63}$$

$$\psi_t - \psi_{t-1} = \pi_{H,t} - \widetilde{\pi}_{F,t} + (\tau_t - \tau_{t-1}), \qquad \text{PostTP}. \tag{64}$$

The Bianchi–Coulibaly fiscal externality makes private imports inefficiently low because households do not internalize that a larger import base raises tariff revenue rebated to the private sector. In the present coordinates, raising ψ_t lowers the private tariff-inclusive relative import price $\tau_t - \psi_t$ and therefore moves import demand in the direction preferred by the planner. Under PreTP, Eq. (63) requires this relative-price correction to be implemented through Home PPI inflation relative to sticky import-price inflation. Because Eq. (58) ties the current nominal rate to future Home PPI inflation, the resulting nonlinear Ramsey allocation produces an impact rate increase. Under PostTP, by contrast, the tariff innovation $\tau_t - \tau_{t-1}$ itself enters Eq. (64); the required relative-price movement no longer has to be generated by the same positive Home-PPI differential, and the optimal transition features a lower future Home inflation path and an impact rate cut.

The nonlinear Ramsey results for this exact-normalization case are reported in Table 11 in Section 7.5. The permanent-shock row uses the Bianchi–Coulibaly quantitative experiment, a 15-percentage-point permanent increase in the import tariff. The additional rows show that the opposite orientation of the PreTP–PostTP reversal under Eq. (56) is not specific to permanence.

The opposite orientation relative to Section 3 is therefore consistent with the tighter nominal implementation structure imposed by Eq. (56). Bianchi and Coulibaly's own endogenous-terms-of-trade extension provides a broader robustness case in which that one-for-one mapping is relaxed. This extension is not a one-equation experiment: finite-elasticity export demand also introduces a terms-of-trade margin and changes the efficient tariff reference allocation. Let p_t denote the border terms of trade. After normalizing the foreign composite price, law of one price implies $P_{H,t} = \mathcal{E}_t / p_t$, so

$$\frac{R_t}{R^*} = \frac{p_{t+1}}{p_t}(1 + \pi_{H,t+1}), \tag{65}$$

and the PreTP LOOP law becomes

$$\psi_t - \psi_{t-1} = \log p_t - \log p_{t-1} + \pi_{H,t} - \pi_{F,t}, \tag{66}$$

with the corresponding $+(\tau_t - \tau_{t-1})$ term under PostTP when the sticky object is tariff inclusive. The international relative price can now absorb part of the LOOP correction without requiring the same movement in Home PPI, and the policy rate is no longer pinned down by future Home PPI inflation alone. In this broader extension the qualitative ILCP ordering returns to the Section 3 result: PreTP calls for easing and PostTP for tightening. Because the endogenous-terms-of-trade specification changes both the nominal mapping and the real international-price environment, this comparison does not identify the effect of Eq. (56) in isolation. It does establish the narrower robustness result needed here: the timing-induced zero crossing survives, while the direction of the crossing is model specific.

C Implementation details for cooperative Ramsey robustness

Section 7.6 reports the policy-game robustness results. This appendix records the cooperative planner problem and numerical checks. The main two-country analysis uses national Nash–Ramsey policy under commitment. For the cooperative exercise we instead use a single population-weighted world planner that maximizes

$$W_0^C = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \nu \left[\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right] + (1-\nu) \left[\log C_t^* - \frac{(N_t^*)^{1+\varphi}}{1+\varphi} \right] \right\}, \tag{67}$$

subject to the complete world private-equilibrium system for each pricing regime. Both national nominal rates are chosen jointly, with a separate ZLB constraint in each country. The calibration, the 20-percentage-point unilateral Home tariff innovation, and every PreTP/PostTP price-setting convention are unchanged. Thus the only policy experiment is to replace the two national objectives by Eq. (67); the private-sector pricing architecture is held fixed. We use this cooperative allocation as a diagnostic full-coordination comparison rather than as an alternative institutional reference for the paper.

The cooperative first-order conditions were generated directly from the nonlinear world Lagrangian and the unchanged private-equilibrium constraints. For ILCP PreTP, an independent implementation using the analytically expanded cooperative Ramsey first-order conditions gives the same impact responses, -3.5725 basis points at Home and -87.3801 basis points abroad, to numerical precision. Extending the solution horizon from 50 to 80 quarters likewise leaves the full-LCP PostTP impact responses unchanged to numerical precision. These checks are useful because the cooperative exercise changes the planner problem without changing the pricing equations that define PreTP and PostTP.

D Ramsey problems, Lagrangians, and first-order conditions

This appendix states the nonlinear Ramsey systems used for the SOE and two-country simulations. The purpose is to make explicit both the implementability restrictions and the variables with respect to which the planners differentiate. The tariff processes are exogenous and therefore carry no Ramsey multipliers. Price-dispersion states inherited from period $t-1$ are predetermined. Commitment is implemented from the timeless perspective, so inherited Ramsey multipliers are retained.

D.1 Small open economy

Using the CPI–PPI ratio $\mathcal{G}(S_t, \tau_{M,t})$ defined in Eq. (7), write

$$C_{F,t} \equiv \upsilon \left(\frac{(1+\tau_{M,t})S_t}{\mathcal{G}(S_t, \tau_{M,t})} \right)^{-\eta} C_t, \qquad MC_t^r \equiv N_t^{\varphi} C_t \mathcal{G}(S_t, \tau_{M,t}).$$

For the producer block, define the reset relative price $\widetilde{p}_{H,t} \equiv \bar{P}_{H,t}/P_{H,t}$ and the usual Calvo auxiliary variables $\mathcal{K}_{H,t}$ and $\mathcal{Z}_{H,t}$. As in the main text, $\mathcal{M}_P \equiv \varepsilon/(\varepsilon-1)$ is the desired producer markup; with the Baseline subsidy, the effective markup is $(1-\tau)\mathcal{M}_P = 1$. The common SOE implementability restrictions can be written

$$F_t^Y \equiv Y_t - (1-\upsilon)\mathcal{G}(S_t, \tau_{M,t})^{\eta} C_t - \upsilon \left(\frac{\Psi_t S_t}{1+\tau_{X,t}} \right)^{\eta} C_t^* = 0, \tag{68}$$

$$F_t^N \equiv N_t - Y_t Z_{H,t} = 0, \tag{69}$$

$$F_t^P \equiv 1 - \theta \Pi_{H,t}^{\varepsilon-1} - (1-\theta)\widetilde{p}_{H,t}^{1-\varepsilon} = 0, \tag{70}$$

$$F_t^K \equiv \mathcal{K}_{H,t} - Y_t - \beta\theta \frac{C_t \mathcal{G}(S_t, \tau_{M,t})}{C_{t+1}\mathcal{G}(S_{t+1}, \tau_{M,t+1})} \Pi_{H,t+1}^{\varepsilon-1} \mathcal{K}_{H,t+1} = 0, \tag{71}$$

$$F_t^Z \equiv \mathcal{Z}_{H,t} - Y_t MC_t^r - \beta\theta \frac{C_t \mathcal{G}(S_t, \tau_{M,t})}{C_{t+1}\mathcal{G}(S_{t+1}, \tau_{M,t+1})} \Pi_{H,t+1}^{\varepsilon} \mathcal{Z}_{H,t+1} = 0, \tag{72}$$

$$F_t^R \equiv \widetilde{p}_{H,t} - (1-\tau)\mathcal{M}_P \frac{\mathcal{Z}_{H,t}}{\mathcal{K}_{H,t}} = 0, \tag{73}$$

$$F_t^D \equiv Z_{H,t} - \theta \Pi_{H,t}^{\varepsilon} Z_{H,t-1} - (1-\theta)\widetilde{p}_{H,t}^{-\varepsilon} = 0, \tag{74}$$

$$F_t^{RS} \equiv \Psi_t S_t - \frac{C_t}{C_t^*}\mathcal{G}(S_t, \tau_{M,t}) = 0, \tag{75}$$

$$F_t^E \equiv \beta \frac{C_t \mathcal{G}(S_t, \tau_{M,t})}{C_{t+1}\mathcal{G}(S_{t+1}, \tau_{M,t+1})\Pi_{H,t+1}} - \frac{1}{1+i_t} = 0. \tag{76}$$

The first and eighth restrictions are Eqs. (8) and (7); the last is Eq. (9). The intervening equations write out the producer Calvo block and price dispersion that are used in the nonlinear simulations.

For the importer block, $\widetilde{p}_{F,t}^{\mathcal{P}}$ is the normalized importer reset price defined in Section 3.4, and $\mathcal{K}_{F,t}^{\mathcal{P}}$ and $\mathcal{Z}_{F,t}^{\mathcal{P}}$ are its denominator and numerator Calvo auxiliaries; $Z_{F,t}$ denotes importer-price dispersion. Here $\mathcal{M}_I \equiv \varepsilon_M/(\varepsilon_M - 1)$ is the desired importer markup, and the Baseline subsidy implies the effective markup $(1-\tau_I)\mathcal{M}_I = 1$. Under PreTP,

$$F_t^{PF,\mathrm{PreTP}} \equiv S_t - S_{t-1}\frac{\Pi_{F,t}}{\Pi_{H,t}} = 0, \tag{77}$$

$$F_t^{PM,\mathrm{PreTP}} \equiv 1 - \theta \Pi_{F,t}^{\varepsilon_M - 1} - (1-\theta)(\widetilde{p}_{F,t}^{\mathrm{PreTP}})^{1-\varepsilon_M} = 0, \tag{78}$$

$$F_t^{KM,\mathrm{PreTP}} \equiv \mathcal{K}_{F,t}^{\mathrm{PreTP}} - C_{F,t} - \beta\theta \frac{C_t \mathcal{G}(S_t, \tau_{M,t})}{C_{t+1}\mathcal{G}(S_{t+1}, \tau_{M,t+1})} \frac{S_{t+1}}{S_t} \Pi_{F,t+1}^{\varepsilon_M - 1} \mathcal{K}_{F,t+1}^{\mathrm{PreTP}} = 0, \tag{79}$$

$$F_t^{ZM,\mathrm{PreTP}} \equiv \mathcal{Z}_{F,t}^{\mathrm{PreTP}} - C_{F,t}\Psi_t - \beta\theta \frac{C_t \mathcal{G}(S_t, \tau_{M,t})}{C_{t+1}\mathcal{G}(S_{t+1}, \tau_{M,t+1})} \frac{S_{t+1}}{S_t} \Pi_{F,t+1}^{\varepsilon_M} \mathcal{Z}_{F,t+1}^{\mathrm{PreTP}} = 0, \tag{80}$$

$$F_t^{RM,\mathrm{PreTP}} \equiv \widetilde{p}_{F,t}^{\mathrm{PreTP}} - (1-\tau_I)\mathcal{M}_I \frac{\mathcal{Z}_{F,t}^{\mathrm{PreTP}}}{\mathcal{K}_{F,t}^{\mathrm{PreTP}}} = 0, \tag{81}$$

$$F_t^{DM,\mathrm{PreTP}} \equiv Z_{F,t} - \theta \Pi_{F,t}^{\varepsilon_M} Z_{F,t-1} - (1-\theta)(\widetilde{p}_{F,t}^{\mathrm{PreTP}})^{-\varepsilon_M} = 0. \tag{82}$$

Under PostTP the sticky object is $(1+\tau_{M,t})S_t$, and therefore

$$F_t^{PF,\mathrm{PostTP}} \equiv (1+\tau_{M,t})S_t - (1+\tau_{M,t-1})S_{t-1}\frac{\widetilde{\Pi}_{F,t}}{\Pi_{H,t}} = 0, \tag{83}$$

$$F_t^{PM,\mathrm{PostTP}} \equiv 1-\theta\widetilde{\Pi}_{F,t}^{\varepsilon_M-1}-(1-\theta)(\widetilde{p}_{F,t}^{\mathrm{PostTP}})^{1-\varepsilon_M}=0, \tag{84}$$

$$F_t^{KM,\mathrm{PostTP}} \equiv \mathcal{K}_{F,t}^{\mathrm{PostTP}}-C_{F,t}-\beta\theta\frac{C_t\mathcal{G}(S_t,\tau_{M,t})}{C_{t+1}\mathcal{G}(S_{t+1},\tau_{M,t+1})}\frac{(1+\tau_{M,t+1})S_{t+1}}{(1+\tau_{M,t})S_t}\widetilde{\Pi}_{F,t+1}^{\varepsilon_M-1}\mathcal{K}_{F,t+1}^{\mathrm{PostTP}}=0, \tag{85}$$

$$F_t^{ZM,\mathrm{PostTP}} \equiv \mathcal{Z}_{F,t}^{\mathrm{PostTP}}-C_{F,t}\Psi_t-\beta\theta\frac{C_t\mathcal{G}(S_t,\tau_{M,t})}{C_{t+1}\mathcal{G}(S_{t+1},\tau_{M,t+1})}\frac{(1+\tau_{M,t+1})S_{t+1}}{(1+\tau_{M,t})S_t}\widetilde{\Pi}_{F,t+1}^{\varepsilon_M}\mathcal{Z}_{F,t+1}^{\mathrm{PostTP}}=0, \tag{86}$$

$$F_t^{RM,\mathrm{PostTP}} \equiv \widetilde{p}_{F,t}^{\mathrm{PostTP}}-(1-\tau_I)\mathcal{M}_I\frac{\mathcal{Z}_{F,t}^{\mathrm{PostTP}}}{\mathcal{K}_{F,t}^{\mathrm{PostTP}}}=0, \tag{87}$$

$$F_t^{DM,\mathrm{PostTP}} \equiv Z_{F,t}-\theta\widetilde{\Pi}_{F,t}^{\varepsilon_M}Z_{F,t-1}-(1-\theta)(\widetilde{p}_{F,t}^{\mathrm{PostTP}})^{-\varepsilon_M}=0. \tag{88}$$

Thus the tariff enters the sticky-price implementability block only under PostTP. The acquisition-cost term in the $\mathcal{Z}_F$ recursion is Ψ_t under either notation because, under PostTP, both the tariff-inclusive acquisition cost and the sticky purchaser price contain the same factor $1+\tau_{M,t}$.

Let $\mathcal{P}\in\{\mathrm{PreTP},\mathrm{PostTP}\}$ and let

$$\mathcal{A}_{SOE}^{\mathcal{P}}=\{Y,N,P,K,Z,R,D,RS,E,PF_{\mathcal{P}},PM_{\mathcal{P}},KM_{\mathcal{P}},ZM_{\mathcal{P}},RM_{\mathcal{P}},DM_{\mathcal{P}}\}$$

denote the restrictions above, with the appropriate timing-specific importer block. The SOE Lagrangian is

$$\begin{aligned}\mathfrak{L}^{SOE,\mathcal{P}}=\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\Bigg[&\log C_t-\frac{N_t^{1+\varphi}}{1+\varphi}+\lambda_{Y,t}F_t^Y+\lambda_{N,t}F_t^N+\lambda_{P,t}F_t^P+\lambda_{K,t}F_t^K+\lambda_{Z,t}F_t^Z\\&+\lambda_{R,t}F_t^R+\lambda_{D,t}F_t^D+\lambda_{RS,t}F_t^{RS}+\lambda_{E,t}F_t^E+\lambda_{PF,t}F_t^{PF,\mathcal{P}}+\lambda_{PM,t}F_t^{PM,\mathcal{P}}\\&+\lambda_{KM,t}F_t^{KM,\mathcal{P}}+\lambda_{ZM,t}F_t^{ZM,\mathcal{P}}+\lambda_{RM,t}F_t^{RM,\mathcal{P}}+\lambda_{DM,t}F_t^{DM,\mathcal{P}}+\mu_{i,t}i_t\Bigg].\end{aligned} \tag{89}$$

For any SOE choice variable x_t, define the exact discounted Euler–Lagrange derivative of the constraint part by

$$\mathcal{D}_{x_t}^{SOE,\mathcal{P}}\equiv\sum_{a\in\mathcal{A}_{SOE}^{\mathcal{P}}}\left[\lambda_{a,t}\frac{\partial F_t^a}{\partial x_t}+\beta^{-1}\lambda_{a,t-1}\frac{\partial F_{t-1}^a}{\partial x_t}+\beta\lambda_{a,t+1}\frac{\partial F_{t+1}^a}{\partial x_t}\right]. \tag{90}$$

This expression is simply $\beta^{-t}\partial\mathfrak{L}^{SOE,\mathcal{P}}/\partial x_t$ after separating the direct utility derivative and the lower-bound term; it is not a linearization. The complete interior first-order conditions, apart from the policy-rate KKT condition, are

$$0=C_t^{-1}+\mathcal{D}_{C_t}^{SOE,\mathcal{P}},\qquad 0=-N_t^{\varphi}+\mathcal{D}_{N_t}^{SOE,\mathcal{P}}, \tag{91}$$

$$0=\mathcal{D}_{S_t}^{SOE,\mathcal{P}},\qquad 0=\mathcal{D}_{\Psi_t}^{SOE,\mathcal{P}},\qquad 0=\mathcal{D}_{Y_t}^{SOE,\mathcal{P}}, \tag{92}$$

$$0=\mathcal{D}_{\Pi_{H,t}}^{SOE,\mathcal{P}},\qquad 0=\mathcal{D}_{\widetilde{p}_{H,t}}^{SOE,\mathcal{P}},\qquad 0=\mathcal{D}_{\mathcal{K}_{H,t}}^{SOE,\mathcal{P}}, \tag{93}$$

$$0=\mathcal{D}_{\mathcal{Z}_{H,t}}^{SOE,\mathcal{P}},\qquad 0=\mathcal{D}_{Z_{H,t}}^{SOE,\mathcal{P}},\qquad 0=\mathcal{D}_{\Pi_{F,t}^{\mathcal{P}}}^{SOE,\mathcal{P}}, \tag{94}$$

$$0=\mathcal{D}_{\widetilde{p}_{F,t}^{\mathcal{P}}}^{SOE,\mathcal{P}},\qquad 0=\mathcal{D}_{\mathcal{K}_{F,t}^{\mathcal{P}}}^{SOE,\mathcal{P}},\qquad 0=\mathcal{D}_{\mathcal{Z}_{F,t}^{\mathcal{P}}}^{SOE,\mathcal{P}},\qquad 0=\mathcal{D}_{Z_{F,t}}^{SOE,\mathcal{P}}. \tag{95}$$

Here $\Pi_{F,t}^{\mathrm{PreTP}}=\Pi_{F,t}$ and $\Pi_{F,t}^{\mathrm{PostTP}}=\widetilde{\Pi}_{F,t}$. The policy-rate condition is

$$\frac{\lambda_{E,t}}{(1+i_t)^2}+\mu_{i,t}=0,\qquad i_t\geq 0,\qquad \mu_{i,t}\geq 0,\qquad \mu_{i,t}i_t=0. \tag{96}$$

Whenever the lower bound is slack, $\mu_{i,t}=0$ and hence $\lambda_{E,t}=0$.

D.2 Two-country national Nash–Ramsey problem

The Home and Foreign CPI–PPI ratios implied by the CES price indexes are

$$\mathcal{G}(S_t,\tau_{M,t}) = \left[\omega_H + \omega_F(1+\tau_{M,t})^{1-\eta}S_t^{1-\eta}\right]^{1/(1-\eta)},$$
$$\mathcal{G}^*(S_t^*,\tau_{M,t}^*) = \left[\omega_F^* + \omega_H^*(1+\tau_{M,t}^*)^{1-\eta}(S_t^*)^{1-\eta}\right]^{1/(1-\eta)}.$$

The demand and complete-markets mappings used before national differentiation are

$$C_{H,t} = \omega_H\mathcal{G}(S_t,\tau_{M,t})^{\eta}C_t, \qquad C_{F,t} = \omega_F\left(\frac{(1+\tau_{M,t})S_t}{\mathcal{G}(S_t,\tau_{M,t})}\right)^{-\eta}C_t, \tag{97}$$

$$C_{F,t}^* = \omega_F^*\mathcal{G}^*(S_t^*,\tau_{M,t}^*)^{\eta}C_t^*, \qquad C_{H,t}^* = \omega_H^*\left(\frac{(1+\tau_{M,t}^*)S_t^*}{\mathcal{G}^*(S_t^*,\tau_{M,t}^*)}\right)^{-\eta}C_t^*, \tag{98}$$

$$\Psi_t S_t = \frac{C_t\mathcal{G}(S_t,\tau_{M,t})}{C_t^*\mathcal{G}^*(S_t^*,\tau_{M,t}^*)}. \tag{99}$$

The Home Euler restriction is

$$F_{H,t}^E \equiv \beta\frac{C_t\mathcal{G}(S_t,\tau_{M,t})}{C_{t+1}\mathcal{G}(S_{t+1},\tau_{M,t+1})\Pi_{H,t+1}} - \frac{1}{1+i_t} = 0, \tag{100}$$

with the exact starred mirror for Foreign. Because the CPI–PPI ratio is substituted directly from the CES price index, it is not an additional Ramsey choice variable and there is no separate CPI auxiliary constraint. Under PCP, the law of one price imposes $\Psi_t = 1$ and Eq. (99) links S_t to the Home allocation, conditional on the Foreign contingent allocation. Under importer LCP, both S_t and Ψ_t are retained as Home own-plan variables and Eq. (99) is imposed as a complete-markets restriction. Thus neither planner treats the other country's contingent allocation as an own choice.

For compactness, a sticky price block b with elasticity ϵ_b, Calvo parameter θ_b, reset relative price $p_{b,t}$, gross inflation $\Pi_{b,t}$, demand $Q_{b,t}$, and real marginal-cost or net-revenue term $m_{b,t}$ is represented by the five residuals

$$F_{b,t}^P \equiv 1 - \theta_b\Pi_{b,t}^{\epsilon_b-1} - (1-\theta_b)p_{b,t}^{1-\epsilon_b} = 0, \tag{101}$$
$$F_{b,t}^K \equiv \mathcal{K}_{b,t} - Q_{b,t} - \beta\theta_b\Xi_{b,t,t+1}\Pi_{b,t+1}^{\epsilon_b-1}\mathcal{K}_{b,t+1} = 0, \tag{102}$$
$$F_{b,t}^Z \equiv \mathcal{Z}_{b,t} - Q_{b,t}m_{b,t} - \beta\theta_b\Xi_{b,t,t+1}\Pi_{b,t+1}^{\epsilon_b}\mathcal{Z}_{b,t+1} = 0, \tag{103}$$
$$F_{b,t}^R \equiv p_{b,t} - \mathcal{M}_b\frac{\mathcal{Z}_{b,t}}{\mathcal{K}_{b,t}} = 0, \tag{104}$$
$$F_{b,t}^D \equiv Z_{b,t} - \theta_b\Pi_{b,t}^{\epsilon_b}Z_{b,t-1} - (1-\theta_b)p_{b,t}^{-\epsilon_b} = 0. \tag{105}$$

Here $\mathcal{M}_b$ denotes the effective markup factor in block b—for example $(1-\tau)\mathcal{M}_P$ for a Home producer and $(1-\tau_I)\mathcal{M}_I$ for a Home importer—and $\Xi_{b,t,t+1}$ is the exact household-discount-factor and relative-price term appropriate to block b.

The Home resource/labor restrictions are regime specific. Under PCP,

$$F_{H,t}^{Y,PCP} \equiv Y_t - C_{H,t} - \frac{1-\nu}{\nu}C_{H,t}^* = 0, \qquad F_{H,t}^{N,PCP} \equiv N_t - Y_tZ_{H,t} = 0. \tag{106}$$

Under importer LCP,

$$F_{H,t}^{Y,ILCP} \equiv Y_t - C_{H,t} - \frac{1-\nu}{\nu}C_{H,t}^*Z_{H,t}^* = 0, \qquad F_{H,t}^{N,ILCP} \equiv N_t - Y_tZ_{H,t} = 0, \tag{107}$$

where $Z_{H,t}^*$ belongs to the Foreign contingent allocation and is held fixed in the Home best response. Under full LCP and DCP, Home production is allocated across the Home and Foreign destination blocks,

$$F_{H,t}^{N,LCP} \equiv N_t - C_{H,t}Z_{H,t} - \frac{1-\nu}{\nu}C_{H,t}^*Z_{H,t}^* = 0. \tag{108}$$

The Foreign restrictions are exact mirrors.

Let $\mathcal{B}_H^{R,\mathcal{P}}$ denote the Home-owned sticky-price blocks. They are

$$\mathcal{B}_H^{PCP} = \{H\}, \tag{109}$$
$$\mathcal{B}_H^{ILCP,\mathcal{P}} = \{H, M_H^{\mathcal{P}}\}, \tag{110}$$
$$\mathcal{B}_H^{FLCP,\mathcal{P}} = \{H, H^{*,\mathcal{P}}\}, \tag{111}$$
$$\mathcal{B}_H^{DCP,\mathcal{P}} = \{H, H^{*,\mathcal{P}}\}. \tag{112}$$

Here $M_H^{\mathcal{P}}$ is the Home importer's local-currency block. Under full LCP, the starred block $H^{*,\mathcal{P}}$ is the distinct Home-good price-setting block for the Foreign market and is quoted in Foreign currency. Under Baseline DCP, by contrast, Home-to-Home and Home-to-Foreign sales share the common Home-currency PCP price. The destination-tagged starred variables retained in the numerical implementation are therefore bookkeeping copies of the common Home producer-price block, not an independent Home reset-price choice. The tariffed Foreign-to-Home price-setting block is Foreign-owned under full LCP and DCP and is therefore held fixed when the Home national planner differentiates. Under PostTP the relevant cum-tariff price is used in that Foreign-owned block.

For reference, the Home own-plan choice sets (excluding i_t) are

$$\mathcal{X}_H^{PCP} = \{C_t, S_t, N_t, Y_t, \Pi_{H,t}, \widetilde{p}_{H,t}, \mathcal{K}_{H,t}, \mathcal{Z}_{H,t}, Z_{H,t}\}, \tag{113}$$
$$\mathcal{X}_H^{ILCP,\mathcal{P}} = \mathcal{X}_H^{PCP} \cup \{\Psi_t, \Pi_{F,t}^{\mathcal{P}}, \widetilde{p}_{F,t}^{\mathcal{P}}, \mathcal{K}_{F,t}^{\mathcal{P}}, \mathcal{Z}_{F,t}^{\mathcal{P}}, Z_{F,t}\}, \tag{114}$$
$$\mathcal{X}_H^{FLCP,\mathcal{P}} = \{C_t, S_t, N_t, C_{H,t}, C_{H,t}^*, S_t^{*,\mathcal{P}}, \Pi_{H,t}, \Pi_{H,t}^{*,\mathcal{P}}, \widetilde{p}_{H,t}, \widetilde{p}_{H,t}^{*,\mathcal{P}}, \mathcal{K}_{H,t}, \mathcal{Z}_{H,t}, \tag{115}$$
$$\mathcal{K}_{H,t}^{*,\mathcal{P}}, \mathcal{Z}_{H,t}^{*,\mathcal{P}}, Z_{H,t}, Z_{H,t}^*\}, \tag{116}$$
$$\mathcal{X}_H^{DCP,\mathcal{P}} = \{C_t, S_t, N_t, C_{H,t}, C_{H,t}^*, S_t^{*,\mathcal{P}}, \Pi_{H,t}, \Pi_{H,t}^{*,\mathcal{P}}, \widetilde{p}_{H,t}, \widetilde{p}_{H,t}^{*,\mathcal{P}}, \mathcal{K}_{H,t}, \mathcal{Z}_{H,t}, \tag{117}$$
$$\mathcal{K}_{H,t}^{*,\mathcal{P}}, \mathcal{Z}_{H,t}^{*,\mathcal{P}}, Z_{H,t}, Z_{H,t}^*\}. \tag{118}$$

Here $S_t^{*,\mathrm{PreTP}} = S_t^*$ and $S_t^{*,\mathrm{PostTP}} = (1+\tau_{M,t}^*)S_t^*$ when the Foreign import tariff is present; in the unilateral Home-tariff experiments $\tau_{M,t}^* = 0$. The Foreign choice sets are the exact Home–Foreign mirrors.

Let $\mathcal{J}_H^{R,\mathcal{P}}$ be the explicit set consisting of the Home Euler and complete-markets restrictions, the appropriate resource/labor restriction above, the relative-price law(s), and the five residuals in Eqs. (101)–(105) for every block in $\mathcal{B}_H^{R,\mathcal{P}}$. The Home national Lagrangian is

$$\boxed{\mathfrak{L}_H^{N,R,\mathcal{P}} = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} + \sum_{a \in \mathcal{J}_H^{R,\mathcal{P}}} \lambda_{H,a,t} F_{H,t}^{R,\mathcal{P},a} + \mu_{H,t}^i i_t \right].} \tag{119}$$

The entire Foreign contingent plan is held fixed when Eq. (119) is differentiated. The Foreign national Lagrangian is

$$\boxed{\mathfrak{L}_F^{N,R,\mathcal{P}} = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t^* - \frac{(N_t^*)^{1+\varphi}}{1+\varphi} + \sum_{a \in \mathcal{J}_F^{R,\mathcal{P}}} \lambda_{F,a,t} F_{F,t}^{R,\mathcal{P},a} + \mu_{F,t}^i i_t^* \right],} \tag{120}$$

with the complete Home contingent plan held fixed. These are two national Lagrangians, not a world-planner Lagrangian.

For any Home own-plan variable x_t, define

$$\mathcal{D}_{H,x_t}^{R,\mathcal{P}} \equiv \sum_{a \in \mathcal{J}_H^{R,\mathcal{P}}} \left[\lambda_{H,a,t} \frac{\partial F_{H,t}^{R,\mathcal{P},a}}{\partial x_t} + \beta^{-1} \lambda_{H,a,t-1} \frac{\partial F_{H,t-1}^{R,\mathcal{P},a}}{\partial x_t} + \beta \lambda_{H,a,t+1} \frac{\partial F_{H,t+1}^{R,\mathcal{P},a}}{\partial x_t} \right]. \tag{121}$$

The Home first-order conditions are therefore, for PCP,

$$0 = C_t^{-1} + \mathcal{D}^{PCP}_{H,C_t}, \qquad 0 = -N_t^{\varphi} + \mathcal{D}^{PCP}_{H,N_t}, \qquad 0 = \mathcal{D}^{PCP}_{H,S_t}, \tag{122}$$

$$\begin{aligned} &0 = \mathcal{D}^{PCP}_{H,Y_t}, \qquad 0 = \mathcal{D}^{PCP}_{H,\Pi_{H,t}}, \qquad 0 = \mathcal{D}^{PCP}_{H,\widetilde{p}_{H,t}}, \qquad 0 = \mathcal{D}^{PCP}_{H,\mathcal{K}_{H,t}}, \\ &0 = \mathcal{D}^{PCP}_{H,\mathcal{Z}_{H,t}}, \qquad 0 = \mathcal{D}^{PCP}_{H,Z_{H,t}}. \end{aligned} \tag{123}$$

For importer LCP, the PCP producer-block conditions are evaluated with $\mathcal{J}_H^{ILCP,\mathcal{P}}$ and Y_t remains an own choice, while the additional conditions are

$$\mathcal{D}^{ILCP,\mathcal{P}}_{H,\Psi_t} = \mathcal{D}^{ILCP,\mathcal{P}}_{H,\Pi^{\mathcal{P}}_{F,t}} = \mathcal{D}^{ILCP,\mathcal{P}}_{H,\widetilde{p}^{\mathcal{P}}_{F,t}} = \mathcal{D}^{ILCP,\mathcal{P}}_{H,\mathcal{K}^{\mathcal{P}}_{F,t}} = \mathcal{D}^{ILCP,\mathcal{P}}_{H,\mathcal{Z}^{\mathcal{P}}_{F,t}} = \mathcal{D}^{ILCP,\mathcal{P}}_{H,Z_{F,t}} = 0. \tag{124}$$

For full LCP,

$$\begin{aligned} &0 = C_t^{-1} + \mathcal{D}^{FLCP,\mathcal{P}}_{H,C_t}, \qquad 0 = -N_t^{\varphi} + \mathcal{D}^{FLCP,\mathcal{P}}_{H,N_t}, \qquad 0 = \mathcal{D}^{FLCP,\mathcal{P}}_{H,S_t}, \\ &0 = \mathcal{D}^{FLCP,\mathcal{P}}_{H,C_{H,t}}, \qquad 0 = \mathcal{D}^{FLCP,\mathcal{P}}_{H,C^*_{H,t}}, \qquad 0 = \mathcal{D}^{FLCP,\mathcal{P}}_{H,S^{*,\mathcal{P}}_t}, \\ &0 = \mathcal{D}^{FLCP,\mathcal{P}}_{H,\Pi_{H,t}}, \qquad 0 = \mathcal{D}^{FLCP,\mathcal{P}}_{H,\Pi^{*,\mathcal{P}}_{H,t}}, \qquad 0 = \mathcal{D}^{FLCP,\mathcal{P}}_{H,\widetilde{p}_{H,t}}, \\ &0 = \mathcal{D}^{FLCP,\mathcal{P}}_{H,\widetilde{p}^{*,\mathcal{P}}_{H,t}}, \qquad 0 = \mathcal{D}^{FLCP,\mathcal{P}}_{H,\mathcal{K}_{H,t}}, \qquad 0 = \mathcal{D}^{FLCP,\mathcal{P}}_{H,\mathcal{Z}_{H,t}}, \\ &0 = \mathcal{D}^{FLCP,\mathcal{P}}_{H,\mathcal{K}^{*,\mathcal{P}}_{H,t}}, \qquad 0 = \mathcal{D}^{FLCP,\mathcal{P}}_{H,\mathcal{Z}^{*,\mathcal{P}}_{H,t}}, \qquad 0 = \mathcal{D}^{FLCP,\mathcal{P}}_{H,Z_{H,t}}, \qquad 0 = \mathcal{D}^{FLCP,\mathcal{P}}_{H,Z^*_{H,t}}. \end{aligned} \tag{125}$$

For numerical bookkeeping, the DCP implementation retains the same-dimensional starred Foreign-destination variables as full LCP. Under the Baseline common-PCP restriction, however, those starred Home-side variables are copies of the common Home producer-price block rather than an independent reset-price choice:

$$\begin{aligned} &0 = C_t^{-1} + \mathcal{D}^{DCP,\mathcal{P}}_{H,C_t}, \qquad 0 = -N_t^{\varphi} + \mathcal{D}^{DCP,\mathcal{P}}_{H,N_t}, \qquad 0 = \mathcal{D}^{DCP,\mathcal{P}}_{H,S_t}, \\ &0 = \mathcal{D}^{DCP,\mathcal{P}}_{H,C_{H,t}}, \qquad 0 = \mathcal{D}^{DCP,\mathcal{P}}_{H,C^*_{H,t}}, \qquad 0 = \mathcal{D}^{DCP,\mathcal{P}}_{H,S^{*,\mathcal{P}}_t}, \\ &0 = \mathcal{D}^{DCP,\mathcal{P}}_{H,\Pi_{H,t}}, \qquad 0 = \mathcal{D}^{DCP,\mathcal{P}}_{H,\Pi^{*,\mathcal{P}}_{H,t}}, \qquad 0 = \mathcal{D}^{DCP,\mathcal{P}}_{H,\widetilde{p}_{H,t}}, \\ &0 = \mathcal{D}^{DCP,\mathcal{P}}_{H,\widetilde{p}^{*,\mathcal{P}}_{H,t}}, \qquad 0 = \mathcal{D}^{DCP,\mathcal{P}}_{H,\mathcal{K}_{H,t}}, \qquad 0 = \mathcal{D}^{DCP,\mathcal{P}}_{H,\mathcal{Z}_{H,t}}, \\ &0 = \mathcal{D}^{DCP,\mathcal{P}}_{H,\mathcal{K}^{*,\mathcal{P}}_{H,t}}, \qquad 0 = \mathcal{D}^{DCP,\mathcal{P}}_{H,\mathcal{Z}^{*,\mathcal{P}}_{H,t}}, \qquad 0 = \mathcal{D}^{DCP,\mathcal{P}}_{H,Z_{H,t}}, \qquad 0 = \mathcal{D}^{DCP,\mathcal{P}}_{H,Z^*_{H,t}}. \end{aligned} \tag{126}$$

For every regime, the Home policy-rate condition is

$$\frac{\lambda_{H,E,t}}{(1+i_t)^2} + \mu^i_{H,t} = 0, \qquad i_t \geq 0, \qquad \mu^i_{H,t} \geq 0, \qquad \mu^i_{H,t} i_t = 0. \tag{127}$$

The Foreign FOCs are the exact Home–Foreign mirrors, including

$$\frac{\lambda_{F,E,t}}{(1+i^*_t)^2} + \mu^i_{F,t} = 0, \qquad i^*_t \geq 0, \qquad \mu^i_{F,t} \geq 0, \qquad \mu^i_{F,t} i^*_t = 0. \tag{128}$$

The private-equilibrium restrictions, the Home FOCs, the Foreign FOCs, and both complementarity conditions are solved jointly. Their fixed point is the nonlinear national Nash–Ramsey allocation used in the quantitative sections.

E Price-setting details

This appendix records the common algebra behind the main-text reset-price equations. If a price P^s_t is Calvo sticky with reset price $\bar{P}^s_t$, elasticity ϵ_s, and non-reset probability θ_s, then

$$(P^s_t)^{1-\epsilon_s} = \theta_s (P^s_{t-1})^{1-\epsilon_s} + (1-\theta_s)(\bar{P}^s_t)^{1-\epsilon_s}. \tag{129}$$

For a price setter with nominal marginal cost $MC^{n,s}_{t+k}$, let χ^s_{t+k} convert the chosen sticky price into the seller's period-$t+k$ receipt in the currency in which $MC^{n,s}_{t+k}$ is measured. The reset-price condition can then be written directly with the chosen price on the left-hand side:

$$\bar{P}^s_t = \mathcal{M}_s \frac{\mathrm{E}_t \sum_{k=0}^{\infty} \theta_s^k \Lambda^s_{t,t+k} Q_{t+k|t} MC^{n,s}_{t+k}}{\mathrm{E}_t \sum_{k=0}^{\infty} \theta_s^k \Lambda^s_{t,t+k} Q_{t+k|t} \chi^s_{t+k}}. \tag{130}$$

For PCP and ILCP in the currency of the price setter, $\chi^s_{t+k} = 1$ after the relevant tariff-inclusive acquisition cost is included in $MC^{n,s}_{t+k}$. For exporter-set full LCP, and for the identical Foreign-to-Home exporter block under DCP, $\chi^s_{t+k} = 1/\mathcal{E}_{t+k}$ under PreTP and $\chi^s_{t+k} = 1/[(1+\tau_{M,t+k})\mathcal{E}_{t+k}]$ under PostTP. The Home side of Baseline DCP is instead the common PCP block, so no separate Home-to-Foreign DCP price-setting block is introduced. Thus PostTP uses the cum-tariff chosen price while the tariff path enters either tariff-inclusive acquisition cost for an importer or the conversion from purchaser price to net receipt for an exporter. Linearization of Eq. (130) and Eq. (129) gives the NKPCs stated in Sections 3 and 4.

F Welfare-center definitions

This appendix makes explicit how every hatted welfare variable used in the main text is constructed. After substituting the first-order equilibrium restrictions, write the real part of Home's second-order national loss as a quadratic polynomial in

$$v_t \equiv (y_t, s_t, \psi_t, \psi^*_t, \tau_{M,t}, \tau^*_{M,t})'.$$

Let $H^{R,(1)}$ denote the symmetric Hessian of that polynomial for pricing regime R. We complete squares sequentially in exactly the order used to generate the reported welfare criteria: (y, s) under PCP; (y, ψ, s, ψ^*) under ILCP; and (y, ψ^*, s, ψ) under full LCP and DCP. Suppose z_i is the variable completed at step k, $H^{R,(k)}$ is the Hessian remaining at that step, and $\mathcal{R}_k$ is the set of real variables completed later. Its center is

$$z^{w,R}_{i,t} \equiv -\frac{1}{H^{R,(k)}_{ii}} \left(\sum_{j \in \mathcal{R}_k} H^{R,(k)}_{ij} z_{j,t} + H^{R,(k)}_{i,\tau_M} \tau_{M,t} + H^{R,(k)}_{i,\tau^*_M} \tau^*_{M,t} \right), \qquad \hat{z}^{w,R}_{i,t} \equiv z_{i,t} - z^{w,R}_{i,t}.$$

The Hessian for the next step is the Schur complement of the completed variable,

$$H^{R,(k+1)}_{ab} \equiv H^{R,(k)}_{ab} - \frac{H^{R,(k)}_{ai} H^{R,(k)}_{ib}}{H^{R,(k)}_{ii}}.$$

These two expressions define the welfare centers used in the completed representation. Because the completion is sequential, a center generally depends on relative-price variables completed at later steps as well as on the tariff wedges. For the equal-size Baseline ($\nu = 0.50$) and the unilateral Home-tariff experiment ($\tau^*_{M,t} = 0$), the resulting centers are

$$\begin{aligned}
\text{PCP:}\quad & y^w_t = 0, \qquad s^w_t = -1.138493\, \tau_{M,t}; \\
\text{ILCP PreTP:}\quad & y^w_t = 0, \\
& \psi^w_t = 0.306359\, s_t - 0.317444\, \psi^*_t + 1.306359\, \tau_{M,t}, \\
& s^w_t = 0.871615\, \psi^*_t - 1.931221\, \tau_{M,t}, \qquad \psi^{*,w}_t = 0.262517\, \tau_{M,t}; \\
\text{ILCP PostTP:}\quad & y^w_t = 0, \\
& \psi^w_t = 0.306359\, s_t - 0.317444\, \psi^*_t + 0.401035\, \tau_{M,t}, \\
& s^w_t = 0.871615\, \psi^*_t - 1.259955\, \tau_{M,t}, \qquad \psi^{*,w}_t = 0.298246\, \tau_{M,t}; \\
\text{Full LCP/DCP:}\quad & y^w_t = 0.079380\, s_t + 0.096390\, \psi_t + 0.323190\, \psi^*_t - 0.017010\, \tau_{M,t},
\end{aligned}$$

$$\psi_t^{*,w} = -0.157454\, s_t - 0.216236\, \psi_t + 0.058782\, \tau_{M,t},$$
$$s_t^w = 0.346076\, \psi_t - 1.346076\, \tau_{M,t}, \qquad \psi_t^w = \tau_{M,t}.$$

The hatted real variables used in the main text are defined by subtracting the corresponding center: $\hat{y}_t^w \equiv y_t - y_t^w$, $\hat{s}_t^w \equiv s_t - s_t^w$, $\hat{\psi}_t^w \equiv \psi_t - \psi_t^w$, and $\hat{\psi}_t^{*,w} \equiv \psi_t^* - \psi_t^{*,w}$. The square-completion order is (y, s) under PCP, (y, ψ, s, ψ^*) under ILCP, and (y, ψ^*, s, ψ) under full LCP and DCP. The corresponding $\nu = 0.27$ centers are reported in Section 6.

The Home-PPI terms require a separate observation. Under PCP, ILCP, and full LCP the Home-PPI square is centered at zero, so the welfare criteria correctly use $\pi_{H,t}^2$ itself. Under DCP, Home domestic inflation and Home-currency Foreign-destination inflation are linked. Let

$$\zeta_t^{\text{PreTP}} \equiv \Delta\psi_t^*, \qquad \zeta_t^{\text{PostTP}} \equiv \Delta\psi_t^* - \Delta\tau_{M,t}^*.$$

Before completion, the relevant nominal block is

$$\Lambda_H \pi_{H,t}^2 + \Lambda_{H^*}(\pi_{H,t} - \zeta_t^{\mathcal{P}})^2,$$

where

$$\Lambda_H = \frac{\varepsilon}{\kappa}, \qquad \Lambda_{H^*} = -\frac{\varepsilon}{\kappa}(\eta - 1)\omega_F \Theta_n, \qquad \Theta_n \equiv \frac{1/M + \varphi}{1 + \varphi(\omega_H + \eta\omega_F)}, \qquad M \equiv (1 - \tau)\mathcal{M}_P.$$

Here M is the effective steady-state producer markup factor; $M = 1$ in the Baseline specification. The same signed destination-price coefficient Λ_{H^*} is the coefficient on Home export-price inflation under full LCP. Since $\eta > 1$, $\omega_F > 0$, and $\Theta_n > 0$ at the Benchmark Parameterization, $\Lambda_{H^*} < 0$: this is the strategic national-welfare component associated with the Foreign-market markup and terms of trade, not a negative price-dispersion cost. Completing this pair gives

$$(\Lambda_H + \Lambda_{H^*})(\pi_{H,t} - \pi_{H,t}^{w,DCP,\mathcal{P}})^2 + \frac{\Lambda_H \Lambda_{H^*}}{\Lambda_H + \Lambda_{H^*}}(\zeta_t^{\mathcal{P}})^2,$$

with

$$\pi_{H,t}^{w,DCP,\mathcal{P}} \equiv \frac{\Lambda_{H^*}}{\Lambda_H + \Lambda_{H^*}}\zeta_t^{\mathcal{P}} = -\frac{(\eta - 1)\omega_F \Theta_n}{1 - (\eta - 1)\omega_F \Theta_n}\zeta_t^{\mathcal{P}}.$$

Thus the hatted Home-PPI object is needed only under DCP. It is defined by

$$\hat{\pi}_{H,t}^{w,DCP,\mathcal{P}} \equiv \pi_{H,t} - \pi_{H,t}^{w,DCP,\mathcal{P}}.$$

At the equal-size Benchmark Parameterization the coefficient multiplying $\zeta_t^{\mathcal{P}}$ is -0.076800; at $\nu = 0.27$ it is -0.113380. In the unilateral Home-tariff experiments $\tau_{M,t}^* = 0$, so the DCP inflation center is the same under PreTP and PostTP even though the tariffed Foreign-to-Home private price-setting block differs.

References

[1] Amiti, Mary, Stephen J. Redding, and David E. Weinstein (2019), "The Impact of the 2018 Tariffs on Prices and Welfare," *Journal of Economic Perspectives*, 33(4), 187–210.

[2] Auray, Stéphane, Michael B. Devereux, and Aurélien Eyquem (2025), "Trade Wars and the Optimal Design of Monetary Rules," *Journal of Monetary Economics*, 151, 103726.

[3] Auray, Stéphane, Michael B. Devereux, and Aurélien Eyquem (2026), "The Pass-through of Tariffs and Exchange Rates," *NBER Working Paper* No. 35398.

[4] Barbiero, Omar, Emmanuel Farhi, Gita Gopinath, and Oleg Itskhoki (2019), "The Macroeconomics of Border Taxes," *NBER Macroeconomics Annual*, 33, 395–457.

[5] Benigno, Pierpaolo, and Michael Woodford (2004), "Inflation Stabilization and Welfare: The Case of a Distorted Steady State," manuscript, New York University and Princeton University.

[6] Bergin, Paul R., and Giancarlo Corsetti (2023), "The Macroeconomic Stabilization of Tariff Shocks: What Is the Optimal Monetary Response?" *Journal of International Economics*, 143, 103758.

[7] Bergin, Paul R., and Giancarlo Corsetti (2025), "Exchange Rates and Monetary Stabilization of Tariff Shocks," *NBER Working Paper* No. 33845, revised February 2026.

[8] Bianchi, Javier, and Louphou Coulibaly (2025), "The Optimal Monetary Policy Response to Tariffs," *NBER Working Paper* No. 33560.

[9] Buiter, Willem H. (2017), "Exchange Rate Implications of Border Tax Adjustment Neutrality," *Economics: The Open-Access, Open-Assessment E-Journal*, 11 (2017-12), 1–41.

[10] Cavallo, Alberto, Gita Gopinath, Brent Neiman, and Jenny Tang (2021), "Tariff Pass-Through at the Border and at the Store: Evidence from US Trade Policy," *American Economic Review: Insights*, 3(1), 19–34.

[11] Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc (2007), "Optimal Monetary Policy and the Sources of Local-Currency Price Stability," *NBER Working Paper* No. 13544.

[12] Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc (2010a), "Optimal Monetary Policy and the Sources of Local-Currency Price Stability," in Jordi Galí and Mark Gertler (eds.), *International Dimensions of Monetary Policy*, University of Chicago Press, 319–370.

[13] Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc (2010b), "Optimal Monetary Policy in Open Economies," *Handbook of Monetary Economics*, Vol. 3, 861–933.

[14] Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc (2023), "Exchange Rate Misalignment and External Imbalances: What Is the Optimal Monetary Policy Response?" *Journal of International Economics*, 144, 103771.

[15] Devereux, Michael B., and Charles Engel (2003), "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange-Rate Flexibility," *Review of Economic Studies*, 70, 765–783.

[16] Devereux, Michael B., Kang Shi, and Juanyi Xu (2007), "Global Monetary Policy under a Dollar Standard," *Journal of International Economics*, 71(1), 113–132.

[17] Faia, Ester, and Tommaso Monacelli (2004), "Ramsey Monetary Policy and International Relative Prices," *International Finance Discussion Papers* No. 798, Board of Governors of the Federal Reserve System.

[18] Fujiwara, Ippei, and Jiao Wang (2017), "Optimal Monetary Policy in Open Economies Revisited," *Journal of International Economics*, 108, 300–314.

[19] Galí, Jordi, and Tommaso Monacelli (2016), "Understanding the Gains from Wage Flexibility: The Exchange Rate Connection," *American Economic Review*, 106(12), 3829–3868.

[20] Gopinath, Gita, Oleg Itskhoki, and Roberto Rigobon (2010), "Currency Choice and Exchange Rate Pass-Through," *American Economic Review*, 100(1), 304–336.

[21] Gopinath, Gita, Emine Boz, Camila Casas, Federico J. Díez, Pierre-Olivier Gourinchas, and Mikkel Plagborg-Møller (2020), "Dominant Currency Paradigm," *American Economic Review*, 110(3), 677–719.

[22] Hwang, Yu-Ning, and Stephen J. Turnovsky (2013), "Exchange Rate Pass-through and the Effects of Tariffs on Economic Performance and Welfare," *Journal of International Money and Finance*, 33, 81–102.

[23] International Monetary Fund (2026), "World Economic Outlook Database," April 2026, GDP at current prices in U.S. dollars, World and United States.

[24] Monacelli, Tommaso (2005), "Monetary Policy in a Low Pass-Through Environment," *Journal of Money, Credit and Banking*, 37(6), 1047–1066.

[25] Monacelli, Tommaso (2025), "Tariffs and Monetary Policy," *CEPR Discussion Paper* No. 20142.

[26] Okano, Eiji (2007), "The Choice of the Inflation Rate as a Target in an Economy with Pricing-to-Market," *Japan and the World Economy*, 19(1), 48–67.

[27] U.S. Bureau of Economic Analysis (2026), "National Income and Product Accounts: Imports of Goods and Services and Personal Consumption Expenditures," annual series, retrieved through FRED, Federal Reserve Bank of St. Louis, July 28.

[28] Werning, Iván, Guido Lorenzoni, and Veronica Guerrieri (2025), "Tariffs as Cost-Push Shocks: Implications for Optimal Monetary Policy," *NBER Working Paper* No. 33772.