Martingale measures for the geometric Lévy process models

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Abstract

The equivalent martingale measures for the geometric Lévy processes are investigated. They are separated to two groups. One is the group of martingale measures which are obtained by Esscher transform. The other one is such group that are obtained as the minimal distance martingale measures. We try to obtain the explicit forms of the martingale measures, and we compare the properties of the martingale measures to each other. Those discussions help for us to do the fitness analysis of the pricing models.

1 Introduction

The well-known Black-Scholes model

$$S_t = S_0 e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma W_t} \tag{1.1}$$

or equivalently

$$dS_t = S_t\left(\mu dt + \sigma dW_t\right) \tag{1.2}$$

is a very good model for the option pricing, even so this model has many week points, for example the gap between the historical volatility and the implied volatility, fat tail property and asymmetry property of the distribution of

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the log returns, etc. And so we need to introduce new models which may illustrate those properties.

The geometric Lévy process model is one of them. This model is an incomplete market model, so there are many equivalent martingale measures. As the first candidate for the equivalent martingale measure the minimal martingale measure was introduced in [13]. After that several candidates have been offered, for example the Esscher martingale measure([18]), the variance optimal martingale measure ([41]), the minimal entropy martingale measure ([30]) and etc.

In this paper we survey the properties of those martingale measures for the geometric Lévy processes. At first in §2 we explain the geometric Lévy process model. Next in §3 we review the martingale measures for the geometric Lévy processes, and in §4 we compare the martingale measures to each other. Finally in §5 we give the concluding remarks.

2 Geometric Lévy process

Suppose that a probability space $(\Omega, \mathcal{F}, P)$ and a filtration $\{\mathcal{F}_t, 0 \leq t \leq T\}$ are given. A geometric Lévy precess (GLP) is given by

$$S_t = S_0 e^{Z_t} \tag{2.1}$$

where Z_t is a Lévy process with the generating triplet $(\sigma^2, \nu(dx), b)$. The price process S_t has the following another expression

$$S_t = S_0 \mathcal{E}(\tilde{Z})_t, \tag{2.2}$$

where $\mathcal{E}(\tilde{Z})_t$ is the Doléans-Dade exponential of $\tilde{Z}_t$, and the generating triplet of $\tilde{Z}_t$, say $(\tilde{\sigma}^2, \tilde{\nu}(dx), \tilde{b})$, is

$$\tilde{\sigma}^2 = \sigma^2 \tag{2.3}$$

$$\tilde{\nu}(dx) = (\nu \circ J^{-1})(dx), \quad J(x) = e^x - 1, \tag{2.4}$$

$$\left(\quad i.e. \quad \tilde{\nu}(A) = \int 1_A(e^x - 1)\nu(dx) \quad \right)$$

$$\tilde{b} = b + \frac{1}{2}\sigma^2 + \int \left((e^x - 1)1_{\{|e^x - 1| \leq 1\}} - x1_{\{|x| \leq 1\}} \right) \nu(dx). \tag{2.5}$$

Remark 1 (i) S_t *satisfies*

$$dS_t = S_{t-} d\tilde{Z}_t. \tag{2.6}$$

(ii) *It holds that supp* $\tilde{\nu} \subset (-1, \infty)$.
(iii) *If* $\nu(dx)$ *has the density* $n(x)$*, then* $\tilde{\nu}(dx)$ *has the density* $\tilde{n}(x)$ *and* $\tilde{n}(x)$ *is given by*

$$\tilde{n}(x) = \frac{1}{1+x} n(\log(1+x)). \tag{2.7}$$

• Candidates for the suitable Lévy process.

As the candidate of the Lévy process for the underlying price process, what follow are proposed.
(1) Stable process (Mandelbrot, Fama(1963))
(2) Jump diffusion process (Merton(1973))
(3) Variance Gamma process (Madan(1990))
(4) Generalized Hyperbolic process (Eberlein(1995)
(5) CGMY process (Carr-Geman-Madam-Yor(2000))
(6) Normal inverse Gaussian process (Barndorff-Nielsen)
(7) finite moment log stable process(Carr-Wu(2003))

3 Equivalent martingale measures for GLP

3.1 Candidates for the suitable equivalent martingale measure

Since the geometric Lévy process model permits many equivalent martingale measures, we have to select one martingale measure for the option pricing. The candidates are as follows.
(1) Minimal Martingale Measure (MMM) (Föllmer-Schweizer(1991))
(2) Variance Optimal Martingale Measure (VOMM)(Schweizer(1995))
(3) Mean Correcting Martingale Measure (MCMM)
(4) Esscher Martingale Measure (ESMM) (Gerber-Shiu(1994), B-D-E-S(1996))
(5) Minimal Entropy Martingale Measure (MEMM) (Miyahara(1996), Frittelli(2000))
(6) Utility Based Martingale Measure (U-MM)

3.2 Classification of Martingale measures for geometric Lévy processes

The martingale measures listed in the above are separated to two groups. The one is the Esscher transformed martingale measures group, and the other is the minimal distance martingale measures group.

The martingale measures which belong to Esscher transformed martingale measures group are

(3) Mean Correcting Martingale Measure (MCMM),

(4) Esscher Martingale Measure,

(5) Minimal Entropy Martingale Measure (MEMM).

(See Esscher('32), Gerber-Shiu('94), B-D-E-S('96), Kallsen-Shiryaev('02), etc.)

The martingale measures which belong to the minimal distance martingale measures group are

(2) Variance Optimal Martingale Measure (VOMM),

(5) Minimal Entropy Martingale Measure (MEMM),

(6) Utility Based Martingale Measure (U-MM).

(See He-Pearson('91), Goll-Rüschendorf('01), Frittelli('02), Kallsen('02), etc.)

3.3 Esscher transformed martingale measures

3.3.1 Esscher transform and Esscher transformed martingale measure

Definition 1 *Let R be a risk variable and h be a constant. Then the probability measure $P_{R,h}^{(ESS)}$ defined by*

$$\frac{dP_{R,h}^{(ESS)}}{dP}|_{\mathcal{F}} = \frac{e^{hR}}{E[e^{hR}]} \tag{3.1}$$

is called the Esscher transformed measure of P by the random variable R and h, and this measure transformation is called the Esscher transform by the random variable R and h.

Definition 2 *Let $R_t, 0 \leq t \leq T$, be a risk process. Then the Esscher transformed measure of P by the process R_t and a constant h is the probability* *measure* $P^{(ESS)}_{R_{[0,T]},h}$*, which is defined by*

$$\frac{dP^{(ESS)}_{R_{[0,T]},h}}{dP}|_{\mathcal{F}} = \frac{e^{hR_T}}{E[e^{hR_T}]} \tag{3.2}$$

(*Remark that* $P^{(ESS)}_{R_{[0,T]},h} = P^{(ESS)}_{R_T,h}$.)

and this measure transformation is called the Esscher transform by the process R_t and a constant h.

Definition 3 *In the above definition, if the constant h is chosen so that the* $P^{(ESS)}_{R_{[0,T]},h}$ *is a martingale measure of* S_t*, then* $P^{(ESS)}_{R_{[0,T]},h}$ *is called the Esscher transformed martingale measure of* S_t *by the process* R_t*, and it is denoted by* $P^{(ESS)}_{R_{[0,T]}}$ *or* $P^{(ESS)}_{R_T}$.

3.3.2 Corresponding Risk Processes

When we give a certain risk process R_t, we obtain a corresponding Esscher transformed martingale measure if it exists. As we have seen in the previous section, the GLP has two kinds of representation such that

$$S_t = S_0 e^{Z_t} = S_0 \mathcal{E}(\tilde{Z})_t.$$

The processes Z_t and $\tilde{Z}_t$ are candidates for the risk process.

Definition 4 *The process* $\tilde{Z}_t$ *is called the simple return process of* S_t*, and the process* Z_t *is called the compound return process of* S_t.

Remark 2 *The terms* 'simple return' *and* 'compound return' *were introduced in* B-D-E-S('96) p.294.

3.3.3 Examples of Esscher transformed martingale measure

(1) ESMM

Definition 5 *If the Esscher transformed martingale measure* $P^{(ESS)}_{Z_T}$ *is well-defined, then this measure is called the Compound Return Esscher transformed martingale measure or the Esscher Martingale Measure (ESMM), and is denoted by* $P^{(ESMM)}$.

(2) MEMM

Definition 6 *If the Esscher transformed martingale measure* $P^{(ESS)}_{\tilde{Z}_T}$ *is well-defined, then this measure is called the Simple Return Esscher transformed martingale measure, or based on the following Proposition, the Minimal Entropy Martingale Measure (MEMM), and is denoted by* $P^{(MEMM)}$ *or* P^*.

Definition 7 (MEMM) *If an equivalent martingale measure* P^* *satisfies*

$$H(P^*|P) \leq H(Q|P) \qquad \forall Q : EMM, \tag{3.3}$$

then P^* *is called the minimal entropy martingale measure* (MEMM) *of* S_t*. Where* $H(Q|P)$ *is the relative entropy of* Q *with respect to* P

$$H(Q|P) = \left\{ \begin{array}{ll} \int_\Omega \log[\frac{dQ}{dP}]dQ, & if \quad Q \ll P, \\ \infty, & otherwise, \end{array} \right\}. \tag{3.4}$$

From the proof of Fujiwara-Miyahara('03) Theorem 3.1, it follows that

Proposition 1 *The simple return Esscher transformed martingale measure* $P^{(ESS)}_{\tilde{Z}_{[0,T]}}$ *of* S_t *is the minimal entropy martingale measure* (MEMM) *of* S_t.

(3) MCMM

Definition 8 *Suppose* $\sigma > 0$*, and let* σW_t *be the diffusion part of* Z_t*. If the Esscher transformed martingale measure* $P^{(ESS)}_{W_T}$ *is well-defined, then this measure is called the Mean Correcting Martingale Measure (MCMM), and is denoted by* $P^{(MCMM)}$.

3.4 Minimal Distance Martingale Measures

3.4.1 Utility functions and Duality

The problem to obtain the minimax martingale measure for a given utility function $u(x)$ is equivalent to the problem to obtain the minimal distance martingale measure for the dual distance function $u^*(y)$ defined by

$$u^*(y) = \sup_x \left(u(x) - xy\right). \tag{3.5}$$

The minimal distance martingale for the distance function $F(x)$ means the following minimization problem.

$$E[F\left(\frac{dP^*}{dP}\right)] = \min_{Q:EMM}\left\{E[F\left(\frac{dQ}{dP}\right)]\right\}. \tag{3.6}$$

Here are examples of distance functions and the corresponding minimal distance martingale measures (See [1] or [19]):

1) $F(x) = x\log x$, minimal relative entropy martingale measure (MEMM)

2) $F(x) = -\log x$, minimal reverse relative entropy MM

3) $F(x) = |x-1|$, minimal total variation MM

4) $F(x) = -\sqrt{x}$, minimal Hellinger distance MM

5) $F(x) = |x-1|^p$, minimal p-moment MM

6) $F(x) = |x-1|^2$, minimal variance MM (variance optimal MM (VOMM))

3.4.2 Existence of Minimal Distance Martingale measures

The existence conditions in the general form have been studied in Frittelli('02) and Gundel('05).

3.4.3 Minimal Distance MM for Geometric Lévy Processes

We will see the explicit form of minimal distance MM for Geometric Lévy processes in the case of VOMM. (Jeanblanc-Miyahara [23])

Theorem 1 (i) For the existence of the VOMM, it is sufficient that the following equation for $(f, g(x), \mu)$ has a solution.

$$f = \mu\sigma \tag{3.7}$$

$$(e^{g(x)} - 1) = \mu(e^x - 1) \tag{3.8}$$

$$\mu\sigma^2 + \int(\mu(e^x-1)^2 + (e^x-1) - x1_{|x|\leq 1})\nu(dx) = \beta. \tag{3.9}$$

(ii) When the above equation has a solution $(f^*, g^*(x), \mu^*)$, then the martingale measure $Q^{L_T(f^*,g^*)}$ is the VOMM. ($\frac{dQ^{L_T(f^*,g^*)}}{dP} = L_T(f^*,g^*)$)

where

$$\mu^* = \frac{\beta - \int(e^x - 1 - x1_{|x|\leq 1})\nu(dx)}{\sigma^2 + \int(e^x-1)^2\nu(dx)}. \tag{3.10}$$

Remark 3 *(i) Explicit form of L_t^* is*

$$L_t^* = \exp\Big(f^* W_t - \Big(\frac{1}{2}(f^*)^2 + \int (e^{g^*(x)} - 1 - g^*(x) 1_{|x| \le 1}) \nu(dx) \Big) t + \int^t \int_{\{|x|>1\}} g^*(x) N(ds, dx) + \int^t \int_{\{|x \le 1\}} g^*(x) \widetilde{N}(ds, dx) \Big). \quad (3.11)$$

(ii) The Lévy measure of X_t under $P^{(VOMM)}$ is

$$\nu^{(VOMM)}(dx) = (1 + \mu^*(e^x - 1)) \, \nu(dx) \quad (3.12)$$

The $\sigma^{(VOMM)}$ is $\sigma^{(VOMM)} = \sigma$, and the $b^{(VOMM)}$ is determined from the martingale condition. [23]

The proof of this theorem is based on Kunita's representation theorem for positive martingale processes (See [26]). The idea of the proof of the above theorem can be applied to the power function distance, and we can obtain the similar explicit form for the power function distance. (See [23]).

3.5 Summary of the Explicit Forms of MM for Geometric Lévy Processes

Here is a list of the Lévy measures for Z_t (if exist) under the martingale measures.

(1) MEMM:

$$\nu^{(MEMM)}(dx) = e^{\theta^*(e^x - 1)} \nu(dx) \quad (3.13)$$

where θ^* is the solution of

$$b + (\frac{1}{2} + \theta)\sigma^2 + \int_{-\infty}^{\infty} \Big((e^x - 1) e^{\theta(e^x - 1)} - x 1_{\{|x| \le 1\}}(x) \Big) \, \nu(dx) = r. \quad (3.14)$$

Usually $\theta^* < 0$

(2) ESMM:

$$\nu^{(ESMM)}(dx) = e^{h^* x} \nu(dx) \quad (3.15)$$

where h^* is the solution of

$$b + (\frac{1}{2} + h)\sigma^2 + \int_{-\infty}^{\infty} \Big((e^x - 1) e^{hx} - x 1_{\{|x| \le 1\}} \Big) \, \nu(dx) = r. \quad (3.16)$$

(3) MCMM:

$$\nu^{(MCMM)}(dx) = \nu(dx) \quad (3.17)$$

(4) VOMM:

$$\nu^{(VOMM)}(dx) = (1 + \mu^*(e^x - 1))\,\nu(dx) \tag{3.18}$$

where

$$\mu^* = \frac{\beta - \int(e^x - 1 - x1_{|x|\le 1})\nu(dx)}{\sigma^2 + \int(e^x - 1)^2\nu(dx)}. \tag{3.19}$$

(5) Power-function MM:

Distance function: $F(x) = ax^b$, where $(a > 0, b > 1)$ or $(a < 0, b < 1)$.

$$\nu^{(PfMM)}(dx) = e^{g^*(x)}\nu(dx) \tag{3.20}$$

where g^* is given by

$$b(e^{(b-1)g^*(x)} - 1) = \mu^*(e^x - 1) \tag{3.21}$$

and $\mu*$ is the solution of

$$\frac{\mu\sigma^2}{b(b-1)} + \int\left((1 + \frac{\mu(e^x - 1)}{b})^{\frac{1}{b-1}}(e^x - 1) - x1_{|x|\le 1}\right)\nu(dx) = \beta \tag{3.22}$$

4 Comparison of MEMM and others

0) Law of Z_t under MMs given above

They are all Lévy processes.

1) Corresponding risk process

- MEMM: Simple return process $\tilde{Z}_t$
- ESMM: Compound return process Z_t
- MCMM: Wiener process W_t

2) Conditions for the Existence

- MEMM $P^{(MEMM)}$:

$$\int_{\{|x|>1\}} |(e^x - 1)| e^{\theta^*(e^x - 1)}\,\nu(dx) < \infty. \tag{4.1}$$

This condition is satisfied for wide class of Lévy measures, if $\theta^* < 0$.

- ESMM $P^{(ESMM)}$:

$$\int_{\{|x|>1\}} |e^x - 1|\, e^{h^* x}\nu(dx) < \infty \tag{4.2}$$

The MEMM may be applied to the wider class of models than the ESMM. The difference is clear in the stable process cases. In fact, the MEMM method can be applied to the geometric stable model, but the ESMM method can not be applied to this model.

- MCMM $P^{(MCMM)}$:

$$\sigma > 0, \quad \text{and} \quad \int_{\{|\mathrm{x}|>1\}} |\,(\mathrm{e}^{\mathrm{x}} - 1)\,|\nu(\mathrm{dx}) < \infty \tag{4.3}$$

- VOMM $P^{(VOMM)}$:

$$\int (e^x - 1)^2 \nu(dx) < \infty. \tag{4.4}$$

3) Corresponding utility functions

- The MEMM is corresponding to the exponential utility function. (Frittelli('00) or Goll-Rüschendorf('01)).
- The ESMM is corresponding to power utility function or logarithm utility function. (Gerber-Shiu('94) or Goll-Rüschendorf('01)). But the corresponding power parameter depends on the process.

Remark 4 *In the case of MEMM, the relation of the MEMM to the utility indifference price is known.* (See Fujiwara-Miyahara('03, §4), etc.). *This result is generalized by C. Stricker*('04).

4) Minimal distance to the original probability

- The relative entropy is called Kullback-Leibler Information Number(see Ihara('93, p.23) or Kullback-Leibler distance (see Cover-Thomas('91, p.18) in the field of information theory. We can state that the MEMM is the nearest equivalent martingale measure to the original probability P in the sense of Kullback-Leibler distance.
- The VOMM is the nearest equivalent martingale measure to the original probability P in the sense of variance.
- The ESMM is the nearest equivalent martingale measure to the original probability P in the sense of power function metric. But the minimal distance MM for the power function is not always ESMM.

5) Large deviation property of MEMM: The large deviation theory is closely related to the minimum relative entropy analysis, and the Sanov's theorem or Sanov property is well-known (see, e.g. Cover-Thomas('91 pp.291-304) or Ihara('93, pp.110-111)). This theorem results to saying that the MEMM is the most possible empirical probability measure of paths of price process in the class of the equivalent martingale measures.

5 Concluding Remarks

5.1 Volatility Smile/Smirk Property

It is shown that the geometric Lévy process models have the volatility smile/smirk property. (See [35]).

5.2 Methods for Option Pricing

(1) Risk Neutral Pricing

If the market is efficient enough, the risk neutral pricing (i.e. martingale measure pricing) is reasonable.

(2) Utility indifference Pricing

For the pricing of derivatives based on the non-tradable asset, the utility indifference pricing is effective. This pricing has strong relations with risk measures.

5.3 Fitness analysis of models

We ask a question, "Which model is most fitting to the empirical data?", and we have to do the fitness analysis of models.

In this paper we have obtained the explicit forms of equivalent martingale measures for the geometric Lévy process models. So we have now finished the preparation for the numerical analysis and the empirical analysis of the geometric Lévy process models.

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